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*Observation of the Negative
Muonium Ion in Vacuum*

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Abstract

The negative muonium ion (M^-), which is the bound system of a positive muon and two electrons, has been produced and observed for the first time [Arn86,Har86,Kua87]. Its counterpart H^- is well known, and spectroscopy and collision studies with H^- have yielded many fruitful results. Noteworthy are recent investigations of the photoionization of a relativistic H^- beam [Bry81]. The negative positronium ion has also been formed and observed [Mil81b]. The discovery of M^- provides us with a new leptonic system for spectroscopy and collision studies, which may reveal interesting physics associated with mass effects. Since M^- is a charged particle, it can also be used to produce a beam of exotic atoms with a small phase space. This dissertation is a detailed account of the observation of M^- .

The experiment was conducted at LAMPF, utilizing a subsurface μ^+ beam [Bad85] and the beam-foil technique. When a μ^+ beam of about 10 MeV/c momentum passes through a thin foil, the outgoing species are μ^+ , M , and M^- due to charge exchange of μ^+ with the foil material. In the experiment, the M^- ions were accelerated electrostatically to about 20 keV. A magnetic spectrometer selected the negatively charged particles and bent the ion beam by 60° relative to the direction of the incoming beam. The ions were then focused by a solenoid onto a microchannel plate detector (MCP). Two pairs of scintillators were mounted around the MCP to detect the decay positrons for the measurement of the muon lifetime. The incident μ^+ beam was monitored by a thin plastic scintillation counter. The time-of-flight of the particles from the muon counter to the MCP detector was measured. The M^- was identified in the experiment as a particle of unit negative charge with a mass of about 107 MeV/c² and the lifetime of the muon. The fact that the system contains

a muon was further confirmed by the characteristic Michel energy spectrum of the positrons from muon decay measured with a NaI(Tl) detector. Three foil materials (Al, Au, Be) were used in the experiment. The M^- yield at the focal plane was about $1.4 \times 10^{-5}/\mu_{inc}^+$.

In addition, the low energy positive muons of up to 20 keV kinetic energies coming out of the foils were investigated. The results provide information on the energy distribution and the angular distribution of the emerging μ^+ beam.

A Monte Carlo scheme of simulating the energy loss, the multiple scattering, and the charge equilibrium states and of computing the time-of-flight has been developed. The results are compared with experimental data and good agreements are found. The Monte Carlo code can thus be used to predict results of future beam-foil experiments.

Chapter 1

Introduction

The negative hydrogen ion H^- has been known since the development of quantum mechanics [Bet29,Bet33] and over the decades many spectroscopy and collision studies of H^- have been made [Mas69,Bet77]. Recently, photoionization of relativistic H^- beams has been investigated [Bry81]. Moreover, H^- beams are playing an important role in several major accelerator laboratories.

H^- can be produced by the beam-foil method [Phi55]. When a proton beam passes through a thin foil, the protons undergo charge capture and loss processes. A fraction of the protons will capture one electron to form hydrogen atoms, a small fraction can capture two electrons to form H^- . For many reasons the charge changing processes have drawn great attention [All58,Taw73].

The leptonic atoms, positronium, Ps (e^+e^-), and muonium, M (μ^+e^-), are important simple systems for the study of fundamental interactions and atomic structure. Leptons are believed to be structureless particles. Because of the absence of the size effect of the nucleus, the structure of leptonic atoms can, in principle, be computed very precisely by Quantum Electrodynamics (QED). Experimental measurements of their structure, therefore, provide rigorous tests of the theory.

Muonium was first produced in vacuum by the beam-foil method [Bol81]. Since then, great effort has been devoted to measure the Lamb shift of the muonium atom in the $n = 2$ state [Ora84,Bad84], although improvement on the experimental

precision is necessary in order to compete with the precision of theoretical computations [Owe73]. Recently, the M system has also been used in order to probe the lepton number conservation law adopted by the standard model of the electroweak interaction [Gla61, Sal68, Wei67]. A search for spontaneous conversion of muonium to antimuonium was conducted, leading to an upper limit on the coupling constant of $G_{\overline{M}M} < 7.5 G_F$ (at 90% confidence level) [Ni87].

On the other hand, Ps was also found by the beam-foil method [Can74, Mil85]. A measurement of the decay rate of orthopositronium (1^3S_1) has shown a discrepancy with theory [Gid78, Wes87]. The present theoretical calculation of the decay rate has been carried out to $O(\alpha^2 \ln(\alpha^{-1}))$. Higher order corrections are therefore needed. The positronium negative ion, $\text{Ps}^- (e^+ e^- e^-)$, was also observed [Mil81b], which led to measurements of its annihilation lifetime [Mil83].

The negative muonium ion, $M^- (\mu^+ e^- e^-)$, can be produced in a similar fashion by the beam-foil method. Since a positive muon beam is easily obtainable in many laboratories, the charge capture process to form the negative muonium ion can be studied in great detail. Comparisons between the three different negative ions H^- , M^- , and Ps^- in spectroscopy and collision studies may reveal interesting physics associated with mass effects and lead to a better understanding of these three-body systems.

Recently, the thermal muonium atom has been produced with greater efficiency [Mil86, Bee86]. However, muonium atoms in the $n = 2$ metastable state are not expected to be formed with comparable efficiency in the thermal energy regime. Formation of muonium in the $2S$ state is essential for the Lamb shift measurement. Since M^- is charged, it is possible to form a beam of exotic atoms with a very small phase space by accelerating the ions to higher energy. This beam may be useful for further measurements of the muonium $n = 2$ Lamb shift.

The production and observation of the negative muonium ion in a beam-foil experiment is the main subject of this dissertation. The results of the studies on low energy μ^+ will also be given. They will be compared with Monte Carlo simulations. The principles and theories will be discussed in Chapter 2. In Chapter 3, the experimental techniques and the apparatus will be described. The observations and

data analysis will form the contents of Chapter 4 and 5. Discussion of the results will be given in Chapter 6. The Monte Carlo code, MUTRACE, is described in Appendix A.

Chapter 2

Physical Principles

The experiment utilized a μ^+ beam originating from positive pions stopped some distance from the surface inside the target for the primary proton beam. This μ^+ beam is called a “subsurface” μ^+ beam. When a μ^+ beam passes through a thin foil, some fraction of the μ^+ will capture one electron to form muonium atoms, and a smaller fraction should capture two electrons to form negative muonium ions. The physical principles underlying the production and the observation of the negative muonium ion will be presented in this chapter. Some properties of the ion will also be discussed.

2.1 Properties of the Negative Muonium Ion and Muon Decay

The negative muonium ion (M^-) is a bound system of a positive muon (μ^+) and two electrons (e^-). It is a light isotope of H^- , the negative hydrogen ion. Since the muon mass is about one ninth that of the proton and about 207 times that of the electron, M^- is expected to have similar properties to H^- . Because of the importance of H^- to astrophysics and atomic collisions and its simplicity as a negative ion, it has drawn attention of both theorists and experimentalists. An energy level diagram of H^- is shown in Fig. 2.1. The only bound state of the system is the ground state

$^1S_0$. All other states are autoionizing states except a metastable $2p^2\ ^3P^e$ state, as yet unobserved, 0.0095 eV below the energy level of a $n = 2$ hydrogen atom and a free electron [Dra70].

Figure 2.1: Lowest energy levels of H^- based on [Bur68].

The ground state of M^- has a binding energy of 0.525 a.u. (14.287 eV) based on theoretical calculations [Pet87]. The electron affinity, which is the binding energy against breaking up into a muonium atom and a free electron, is therefore about 0.0275 a.u. (0.747 eV). The binding energies of H^- , M^- , and Ps^-

are listed in Table 2.1 for comparison. The values of the fundamental constants,

Table 2.1: Binding energies of H^- , M^- , and Ps^- , and their electron affinities calculated based on [Pet87]. The reduced masses are also listed.

Ion	Binding Energy (eV)	Electron Affinity (eV)	Reduced Mass (in electron mass units)
H^-	14.361	0.763	0.999
M^-	14.287	0.747	0.995
Ps^-	7.130	0.327	0.500

$m_e = 0.51099906(15) \text{ MeV}/c^2$, $m_\mu = 105.658389(34) \text{ MeV}/c^2$, $m_p = 938.27231(28) \text{ MeV}/c^2$, and $R_\infty = 13.6056981(40) \text{ eV}$, are taken from [Coh36]. One can see that the binding energies are approximately proportional to the reduced masses. However, these are three-body systems, so that mass-polarization and other effects will certainly have influence on the energy levels. One of the interesting consequences is that the energy levels of the $2p^2 \ ^3\text{P}^e$ states of H^- and M^- are below $n = 2$ threshold, while that of the $2p^2 \ ^3\text{P}^e$ state of Ps^- is not [Mil81a,Bha83].

The photodetachment cross section of M^- has been computed recently. The maximum is on the order of 10^{-17} cm^2 [Bha87].

Since M^- contains a μ^+ , it will be advantageous to detect the μ^+ when confirming the observation of the ion. The positive muon decays via

$$\mu^+ \rightarrow e^+ \nu_e \bar{\nu}_\mu \quad (2.1)$$

with a lifetime of $2.19703(4) \text{ } \mu\text{sec}$ [Agu86]. It is a three-body decay. Thus, the decay positron has a continuous energy spectrum, the so-called Michel spectrum [Mic50,Com83]. Let us define the following parameters

$$\begin{aligned} W &= (m_e^2 + m_\mu^2)/2m_\mu, \\ x &= E_e/W, \\ x_0 &= m_e/W, \end{aligned} \quad (2.2)$$

where E_e is the energy of the decay positron, m_μ the muon mass, and m_e the electron or positron mass. The differential decay probability for a positron with

energy between x and $x + dx$, emitted at an angle between θ and $\theta + d\theta$ with respect to the muon spin direction is given by

$$\begin{aligned} \frac{d^2\Gamma^{(0)}(x, \theta)}{dx d(\cos\theta)} &= \frac{m_\mu W^4 G_F^2}{24\pi^3} \sqrt{x^2 - x_0^2} \{ (3x - 2x^2 - x_0^2) \\ &\quad + \sqrt{x^2 - x_0^2} (2x - 1 - \frac{m_e}{m_\mu} x_0) \cos\theta \} , \end{aligned} \quad (2.3)$$

where $G_F = 1.16637(2) \times 10^{-5} \text{GeV}^{-2}$ is the Fermi coupling constant in units where $\hbar = c = 1$. The radiative corrections are not included in the expression. Both the muon lifetime and the Michel spectrum can be measured in the experiment.

2.2 Charge Capture Processes

It has long been recognized that charge transfer processes play an important role in atomic collisions and have been studied by many experimentalists and theorists [All58, Taw73]. A great deal of work has been done on hydrogen passing through gaseous targets. For a proton in gas the processes can be described as follows. A particle experiences electron capture and loss processes during its traveling through the gas. After it travels a certain distance, equilibrium is reached. For a three-component system, such as H^- , H^0 , and H^+ , there are six charge-changing processes. The corresponding cross sections are σ_{+0} , σ_{+-} , σ_{0-} , σ_{0+} , σ_{-0} , and σ_{-+} . They represent single and double electron capture by H^+ , single electron capture and loss by H^0 , and single and double electron loss by H^- , respectively. The equilibrium condition for the charge states can be written as

$$\begin{aligned} (\sigma_{+0} + \sigma_{+-})n_+ - \sigma_{0+}n_0 - \sigma_{-+}n_- &= 0 , \\ \sigma_{+0}n_+ - (\sigma_{0+} + \sigma_{0-})n_0 + \sigma_{-0}n_- &= 0 , \\ n_+ + n_0 + n_- &= n , \end{aligned} \quad (2.4)$$

where n_+ , n_0 , and n_- are the numbers of H^- , H^0 , and H^+ , respectively. The fractions of the charge states are therefore

$$\Phi_+ = \frac{n_+}{n} = \frac{\sigma_{-+}\sigma_{0+} + \sigma_{-+}\sigma_{0-} + \sigma_{-0}\sigma_{0+}}{D} , \quad (2.5)$$

$$\Phi_0 = \frac{n_0}{n} = \frac{\sigma_{+0}\sigma_{-0} + \sigma_{+-}\sigma_{-0} + \sigma_{+0}\sigma_{-+}}{D}, \quad (2.6)$$

$$\Phi_- = \frac{n_-}{n} = \frac{\sigma_{0-}\sigma_{+-} + \sigma_{0+}\sigma_{+-} + \sigma_{+0}\sigma_{0-}}{D}, \quad (2.7)$$

where

$$D = \sigma_{-+}\sigma_{0+} + \sigma_{-+}\sigma_{0-} + \sigma_{-0}\sigma_{0+} + \sigma_{+0}\sigma_{-0} + \sigma_{+-}\sigma_{-0} + \sigma_{+0}\sigma_{-+} \\ + \sigma_{0-}\sigma_{+-} + \sigma_{0+}\sigma_{+-} + \sigma_{+0}\sigma_{0-}. \quad (2.8)$$

The electron capture processes by a proton in solids is much more complicated. The three charge states for hydrogen ions from solids were first intensively studied by Phillips [Phi55]. The experiment was repeated later with a deuterium beam [Ber72]. The general observations of the charge capture processes are that the fraction of a charge state is 1) strongly dependent on the velocity of the projectile, 2) weakly dependent on the target material, and 3) independent of the mass of the projectile. Therefore, a velocity scaling rule can be applied, i.e., the charge fraction for a positive muon is the same as that for a proton at the same velocity. This scaling rule was verified down to 4 keV/amu in both transmission experiments [Phi55,Ber72] and backscattering experiments [Bha80,Eck76] in which the charge fractions of reflected particles were measured.

Based on the proton data [Phi55], the fractions of the charge states for muonium ions emerging from a solid foil can be plotted as a function of the energy as shown in Fig. 2.2. Most of the neutral muonium atoms have less than 20 keV kinetic energy, while the fraction of the negative muonium ions only becomes significant for kinetic energies below 5 keV. The integrated rate of production of the negative muonium ions is about two orders of magnitude lower than that of the neutral muonium atoms.

It appears that the conventional description of the charge state equilibrium of particles at higher velocities ($> Z^{2/3}v_0$, where v_0 is the Bohr velocity of 2.2×10^8 cm/sec.) in gases can be adapted to solids. At these velocities, the capture and loss processes are due to the interaction with the inner shells of the target atoms. A particle experiences electron capture and loss processes as it travels through the solid, and the equilibrium of the charge states is obtained after some collisions.

Figure 2.2: Charge fractions of emerging muonium ions.

The charge-state fractions can then be obtained using cross sections computed for gaseous targets. This picture has been successfully used to describe the neutral fraction emerging from solid carbon with incident protons [Cro77]. Since the fraction of the negative ions is very small at these velocities, a two-component system, i.e. positive and neutral states, is assumed.

However, the concept of capture and loss inside a solid may be meaningless for velocities below the Fermi velocity of the electrons in the solid. (A typical metal Fermi velocity is about the same as the Bohr velocity v_0 .) In this case, because of the collective screening by the target electrons in a solid or collision broadening of the bound states, there is no bound state in the solid [Bra75b]. Therefore, the surface effects become important. At the surface, processes such as Auger-neutralization, resonance-neutralization, and resonance-ionization can take place [Hag54]. The recombination process [Kit76] can also occur. However, due to the complexity of

the processes, there is no single theory which can explain all experimental data.

At low velocities ($\leq v_0$) formation of the negative ions becomes significant. The fraction of the negative ions increases monotonically with decreasing energy down to 4 keV/amu [Phi55,Ber72]. In fact, it was found that the fractions of H^0 and H^- depend exponentially on energy in the region from 30 to 340 keV [Ros86]

$$\Phi = \exp(-(E + E_0)/E_1) , \quad (2.9)$$

where E is the kinetic energy of the ions, E_0 and E_1 are the fitting parameters. For H^- , E_0 is about 136 keV and E_1 is about 35.4 keV. The processes were also studied in a backscattering experiment for energies below 4 keV/amu [Ver80]. As the energy decreases, Φ_- reaches a maximum at about 3 keV/amu [Ver80] for most metallic surfaces. A strong correlation between the work function of the target material and the formation fraction of the negative ions was observed [Yu78]. As the work function decreases, the maximum of the charge fraction moves towards lower energy and the amplitude of the maximum increases.

The theories for the formation of the negative ions at a surface are developed for alkali covered surfaces. The general description is that an H^+ is first neutralized near the surface, then captures another electron at some distance from the surface. It may then lose an electron by resonance-neutralization [His79]. The production of the negative ion can be written as a product of the formation probability and the survival probability. Assuming these are adiabatic processes, that is the atom moves slowly with velocity $v_\perp$ normal to the surface, the formation probability can be qualitatively written as

$$1 - \exp(-\alpha/v_\perp) \quad (2.10)$$

and the survival probability is

$$\exp(-\beta/v_\perp) , \quad (2.11)$$

where α and β are constants, independent of the exit angle of the ion. Therefore, the fraction of the negative ions is

$$\Phi_- \sim (1 - \exp(-\alpha/v_\perp)) \exp(-\beta/v_\perp) . \quad (2.12)$$

Comparison between this model and the experimental data shows that Eq. (2.12) gives a reasonable trend for alkali metal surfaces such as Cs. However it can not be applied directly to other metal surfaces. In fact an angular dependence of α and β has been found [Ver80]. At present, there is no reliable theory for the formation of negative ions in beam-foil experiments.

2.3 Subsurface Positive Muon Beam

To achieve a high formation fraction of negative muonium ions, one has to obtain large number of low energy ($v \leq v_0$) positive muons at the exit surface of the foil target. Therefore, a positive muon beam of high stopping density is required. The positive muon beam can be produced via the reactions

where p is a proton and N is a target nucleus. The π^+ production is achieved by bombarding a carbon target with a proton beam of energy above the threshold T_i , which is given by (see for example [Bet55])

$$T_i \simeq 2m_\pi + \frac{m_\pi^2}{2m_N} \sim 290 \text{ MeV}, \quad (2.13)$$

where $m_\pi = 139.57 \text{ MeV}/c^2$ is the pion mass and $m_N \simeq 940 \text{ MeV}/c^2$ is the nucleon mass. The μ^+ originating from π^+ -decay can then be collected. An effective way of producing a high stopping density μ^+ beam was developed by Pifer, Bowen, and Kendall [Pif76] in which μ^+ from π^+ -decay at rest are directly collected off the primary beam target surface. The μ^+ from π^+ decay ($\tau = 26.03 \text{ nsec.}$) at rest has an isotropic angular distribution. Its energy and momentum are

$$E_\mu = \frac{m_\pi^2 + m_\mu^2}{2m_\pi} = 109.78 \text{ MeV}, \quad (2.14)$$

$$p_\mu = \frac{m_\pi^2 - m_\mu^2}{2m_\pi} = 29.79 \text{ MeV}/c, \quad (2.15)$$

where $m_\mu = 105.66 \text{ MeV}/c^2$ and the neutrino mass is assumed to be zero. Because of nonconservation of parity in the weak interaction the neutrino is assumed to have always left handed helicity. The μ^+ from π^+ -decay are therefore fully polarized with negative helicity in order to conserve momentum and spin.

The range of the μ^+ in this momentum region is approximately given by [Tro66]

$$R = kp^\alpha \quad (2.16)$$

as a function of momentum p , where k is a constant that depends on material and $\alpha \sim 3.5$. The range spread is therefore given by

$$\Delta R = \alpha R \Delta p / p . \quad (2.17)$$

Taking into account a range straggling of 10% of the range [Ste60], the total range spread can be written as

$$\Delta R = kp^\alpha \sqrt{(0.1)^2 + (\alpha \Delta p / p)^2} . \quad (2.18)$$

In order to obtain a high formation fraction of M and M^- , a high stopping density μ^+ beam is desirable. As one can see, a low-momentum μ^+ beam should be used. The μ^+ beam utilized in this experiment had a momentum between 8 MeV/c and 14 MeV/c. It was obtained by collecting μ^+ originating from positive pions stopped some distance inside the surface of the primary proton target. This beam is referred to as a subsurface μ^+ beam [Bad85].

Chapter 3

Experimental Technique and Apparatus

The experiment employed a beam-foil method and utilized a subsurface beam of positive muons. The stopped muon channel (SMC) at the Los Alamos Meson Physics Facility (LAMPF) was tuned to a low-momentum subsurface μ^+ beam [Bad85], such that the peak of the stopping profile was centered near the downstream surface of the production foil. The positron contamination of the μ^+ beam was greatly reduced by using a Wien filter. The experimental apparatus is shown in Fig. 3.1. The purified positive muon beam had a momentum p_μ of about 10 MeV/c and $\Delta p_\mu/p_\mu$ of 10% (FWHM). The average intensity was $140 \times 10^3 \text{ sec}^{-1}$ through a 7-cm thick lead collimator with a 6% duty cycle. The beam passed through a thin 2 mg/cm² scintillator which served both as a degrader and as a muon counter, and a 0.2 mg/cm² aluminum protection foil which prevented sparking at high voltage. The production foil was mounted on the high-voltage end of an electrostatic accelerator column capable of operation up to $\pm 25 \text{ kV}$, while a highly transparent copper mesh held at ground potential maintained a uniform accelerating field. Three different foils were used as charge exchange media. The particles were selected for charge state and momentum by a wedge-shaped 60° bending magnet with double-focusing properties symmetric about the central trajectory of the particles. A solenoid of 130 cm length and 712.5 G central field transported the ions away from the direct beam

Figure 3.1: Schematic diagram of the experimental apparatus for observation of the charge states of the outgoing beams when positive muons pass through a thin foil. The insert in the lower left corner is an enlarged view of the accelerator.

into a low background region. As a result of its fringe field, this solenoid focused the beam onto a microchannel plate (MCP) detector. The decay positrons from positive muons were observed in either of two pairs of scintillator telescopes installed above and below the MCP (Fig. 3.2). The characteristic Michel energy spectrum of the

Figure 3.2: The horizontal view of the detector arrangement at the end of the 60° channel.

decay positrons was taken with a NaI(Tl) detector. Neutral muonium atoms were stopped directly downstream in the straight channel on a Teflon plate subtending a solid angle of about 8 msr at the production foil. Decay positrons from this beam stop were also observed with another NaI(Tl) detector. Time-of-flight spectra were

taken between the muon counter (plastic scintillator), microchannel plate, and scintillator telescopes. The data were read into a PDP-11/34 computer via a CAMAC interface. The technical details will be described in the following sections of this chapter.

3.1 Separated Subsurface Beam of Positive Muons

The linear accelerator at LAMPF provided an 800 MeV proton beam with an average current of 635 μA and a duty cycle of 6.0% [LAM80]. The proton beam had a microstructure of nanosecond-wide bursts separated by 5 nsec and a macrostructure consisting of 500- μsec bursts at a rate of 120 sec^{-1} . The proton beam went through the A-2 production target which was nominally carbon, producing pions in the target region. The subsurface positive muon beam, originating from positive pions stopped in the A-2 production target, was transported through the stopped muon channel (SMC) [Tho79]. Figure 3.3 shows the layout of the channel, which consists of 21 quadrupole magnets and 4 bending dipole magnets. The overall length of the channel is about 30 m. The channel was originally designed for a muon beam from pions decaying in flight as well as for a pion beam. It can be viewed as having three separate sections: (1) a pion collection and analyzing portion, (2) a π -decay and muon collection portion, and (3) a pion rejection and muon momentum analysis portion. However, it was used to transport a subsurface positive muon beam of 8 MeV/c to 14 MeV/c momentum in this experiment by simply tuning the channel to low momentum. The typical phase space of the subsurface μ^+ beam in the west cave (CAVE A) is [Bad85]

$$\begin{aligned}\sigma_x \cdot \sigma_{x'} &= 95 \text{ cm} \cdot \text{mrad} , \\ \sigma_y \cdot \sigma_{y'} &= 46 \text{ cm} \cdot \text{mrad} , \\ \frac{\Delta p}{p} &= \pm 5\% .\end{aligned}\tag{3.1}$$

The channel was extended with six quadrupole magnets and a Wien filter, ($\mathbf{E} \times \mathbf{B}$) separator, to reduce e^+ contamination in the μ^+ beam. The extension is shown in Fig. 3.4. The quadrupole magnets were arranged to be FDDFFD in the

Figure 3.3: Layout of the stopped muon channel (SMC). The present experiment was carried out in the west cave (CAVE A).

Figure 3.4: Layout of the channel extension which connects the stopped muon channel (SMC) to the experimental apparatus.

horizontal plane, where F and D stand for focusing and defocusing, respectively. It was attached directly to the valve SV-A of the SMC. The separator has two high voltage plates of 100 cm long and 15 cm wide, and the gap between the plates is 10 cm. The effective length of the magnetic field region is 38 cm. The magnetic field was set at 300.55 G. The corresponding electric field for transmitting particles with velocity βc is given approximately by E (kV/cm) = $112\beta B$ (kG). In the experiment, the electric field was tuned to maximize the positive muon rate. The optimum was found at 2.95 kV/cm for 9.75 MeV/c positive muons. A Monte Carlo calculation of the transmission efficiency was performed using the computer code MUTRACE, which is described in Appendix A. The transmission efficiency was found to be 74% for 9.75 MeV/c positive muons, while the transmission efficiency for positrons at 9.75 MeV/c was less than 0.01%. The e^+ contamination was reduced to $e^+/\mu^+ < 4$. Most of the positrons which went through the channel were from positive muons decaying in flight. The e^+ contamination is higher than that reported before [Bad85] due to the unequal lengths of **E** and **B** field regions in the separator which prevented us from increasing the fields.

Radioactive gas in the channel was another major source of background [Bad85]. Because of the high temperature of the A-2 production target (400 K at an average proton current of 650 μ A), the gaseous, radioactive spallation products with a mean lifetime of the order of 1 sec diffuse out of the target before they decay. It is necessary to reduce the radioactive gas in our experimental apparatus, since the thin scintillation counter and the microchannel plate detector are very sensitive to them. The main components of the radioactive gas are ^6He ($t_{1/2} = 0.807$ sec.) and ^{12}N ($t_{1/2} = 0.011$ sec.). A gas barrier of 1.5 μm mylar was inserted in the beam line between the separator and the downstream apparatus. The purpose of the gas barrier was the following. It left the vacuum upstream at about 5×10^{-3} Torr, thus retarding the diffusion of the radioactive gas such that most of it decayed before reaching the gas barrier. The downstream vacuum was about 3×10^{-6} Torr. The 1.5 μm gas barrier itself can effectively prevent nitrogen from diffusing through.

The positive muon rate was monitored by a thin plastic scintillator. The beam spot size at the scintillator was about 5 cm (FWHM), as was calculated using the

computer code TRANSPORT [Bro67,Bro77]. It was collimated with 7 cm of lead before it hit the scintillator. An average flux of $140 \times 10^3 \mu^+ \text{ sec}^{-1}$ was obtained. The subsurface positive muon beam has the macroscopic time structure of the primary proton beam. However, because of the π -decay time of 26 nsec, the microstructure is washed out. The μ^+ beam is nearly 100% polarized.

3.2 Production Foil

Muonium atoms and negative muonium ions can be produced with a positive muon beam passing through a thin foil in analogy to proton beam-foil experiments [Phi55]. The formation of muonium atoms and negative muonium ions is believed to take place in the last several atomic layers of the foil. It is indicated in the proton experiment that the fractions of the neutrals and negatives are almost independent of the foil material [Phi55]. Since the atomic electrons in the foil have a typical velocity of αc , it is most probable for positive muons of a few keV kinetic energy to capture electrons in the foil. Therefore, the more positive muons that have an energy on this order at the exit surface of the foil, the more muonium atoms and negative muonium ions will be produced. In order to achieve high yields of muonium atoms and negative muonium ions, it is necessary to have as little material as possible in the beam, thus, allowing the use of a low momentum beam of small range straggling and high stopping density.

Since energy loss [Lin61,Fan63,Var70] and multiple scattering [Mol47,Mol48,Bet53] are strongly Z -dependent, it is interesting to investigate these effects with foils of different atomic numbers. Three foils were used in the experiment: beryllium ($Z=4$), aluminum ($Z=13$), and gold ($Z=79$). Their properties and thicknesses are listed in Table 3.1. The momentum of the beam was tuned to accommodate the thicknesses of the different foils. For the Al foil the beam momentum was tuned in steps of 0.25 MeV/ c and the muonium rate was optimized in the straight channel. For the Be and Au foils, the beam momenta were chosen based on range and energy loss calculations such that the centroids of the energy distributions of the outgoing positive muons were close to that for the Al foil.

Table 3.1: Properties and thicknesses of foils used as charge capture media in the experiment.

Foil	Z	A	Density (g/cm ³)	Thickness (μ m)
Be	4	9.01	1.848	24.9
Al	13	26.98	2.70	0.8
Au	79	196.97	19.32	2.0

3.3 Accelerator

The accelerator used in the experiment to accelerate the negative muonium ions and low energy positive muons consists of an electrostatic column. It is shown in the insert of Fig. 3.1. The production foil described in section 3.2 was mounted on the upstream ring. High voltage, variable in both magnitude and polarity, was applied to the foil. A highly transparent copper mesh (30 lines per inch, 90% transmission) was mounted on the downstream ring which was at ground potential. The distance between the foil and the mesh was about 6 cm. The voltages on the two center rings were distributed through three resistors of 500 M Ω . The electric field can be computed with POISSON group programs, which solve Poisson differential equations with cylindrical symmetry or in two dimensions (2-D) [Men87,War87]. The geometry and the equipotential lines are shown in Fig. 3.5. The electric field is uniform to within 1% in the region of 4 cm radius along the beam axis in the accelerator. The rings were polished to eliminate rough edges. A protection foil of 0.8 μ m Al at ground potential was mounted between the thin scintillation counter and the accelerator. It reduced sparking and prevented fringe fields from influencing the incident positive muon beam. The accelerator was tested with high voltage in a vacuum of 3×10^{-6} Torr. It can be run at -25 kV to $+25$ kV without sparking. The current was monitored to be less than 20 μ A.

Figure 3.5: Electrostatic accelerator and the equipotential lines computed using POISSON group programs.

3.4 Magnetic Spectrometer

The bending magnet shown in Fig. 3.1 served as both a charge and a momentum selector. Particles with the appropriate sign of charge and momentum were deflected by 60° . Since the intensity of the M^- beam is very small, it is desirable to use a wedge magnet with a double focusing property [Cam51,Cro51]. TRANSPORT calculations show that a wedge magnet of “effective” 28 degrees will give the best focusing effect in both the horizontal and the vertical planes. However, the question arises as to how to produce an effective field region of 28-degree wedge. This is obviously a three-dimensional (3-D) problem. In order to answer the question, we again used the POISSON group programs, which are 2-D programs. The 3-D effect is simulated with a virtual return path of the field. The studies indicate that the pole pieces and the coils should also be 28-degree wedges. The pole pieces of an existing H-frame dipole magnet were modified. New coils were made to accommodate the new pole pieces. The field of the magnet was mapped. The data show that the wedge angle of the effective field region is 27.9° , which is in good agreement with the design. Monte Carlo calculation shows that the magnet has a momentum acceptance of about 30% (FWHM). The relation between the current (I) and the magnetic field (B) in the center was measured, and is given by

$$B \text{ (G)} = 3.14I \text{ (A)} , \quad (3.2)$$

which is a linear least-squares fit to the data. The magnet was operated at 85 A providing a central field of 267 G. The bending radius is 22.7 cm.

3.5 Solenoid

The fringe field of a solenoid can provide a focusing effect on a charged particle beam passing through it. Unlike quadrupole magnets, a solenoid has no defocusing effect on particle beams. It is preferable to use a solenoid as a focusing element for a low-energy beam.

The effect of a solenoid on a charged particle can be described as a transformation matrix [Ban66],

$$T_{\text{solenoid}} = \begin{pmatrix} \cos^2 \frac{\theta}{2} & \frac{2l}{\theta} \sin \frac{\theta}{2} \cos \frac{\theta}{2} & \sin \frac{\theta}{2} \cos \frac{\theta}{2} & \frac{2l}{\theta} \sin^2 \frac{\theta}{2} \\ -\frac{\theta}{2l} \sin \frac{\theta}{2} \cos \frac{\theta}{2} & \cos^2 \frac{\theta}{2} & -\frac{\theta}{2l} \sin^2 \frac{\theta}{2} & \sin \frac{\theta}{2} \cos \frac{\theta}{2} \\ -\sin \frac{\theta}{2} \cos \frac{\theta}{2} & \frac{2l}{\theta} \sin^2 \frac{\theta}{2} & \cos^2 \frac{\theta}{2} & \frac{2l}{\theta} \sin \frac{\theta}{2} \cos \frac{\theta}{2} \\ \frac{\theta}{2l} \sin^2 \frac{\theta}{2} & -\sin \frac{\theta}{2} \cos \frac{\theta}{2} & -\frac{\theta}{2l} \sin \frac{\theta}{2} \cos \frac{\theta}{2} & \cos^2 \frac{\theta}{2} \end{pmatrix}, \quad (3.3)$$

where l is the effective length of the solenoid with magnetic field B along its axis, $\theta = qBl/cp$, c is the speed of light, and q and p are the charge and the momentum of the particle. The transformation matrix acts on a vector

$$X = \begin{pmatrix} x \\ x' \\ y \\ y' \end{pmatrix}, \quad (3.4)$$

specifying the coordinates of the particle, where x and y are the horizontal and vertical displacements, respectively. x' and y' are the angles of the momentum of the particle with the beam axis in the local rectangular system. The transformation matrix can also be written as a product of two matrices

$$T_{\text{solenoid}} = \begin{pmatrix} \cos \frac{\theta}{2} & \frac{2l}{\theta} \sin \frac{\theta}{2} & 0 & 0 \\ -\frac{\theta}{2l} \sin \frac{\theta}{2} & \cos \frac{\theta}{2} & 0 & 0 \\ 0 & 0 & \cos \frac{\theta}{2} & \frac{2l}{\theta} \sin \frac{\theta}{2} \\ 0 & 0 & -\frac{\theta}{2l} \sin \frac{\theta}{2} & \cos \frac{\theta}{2} \end{pmatrix} \cdot \begin{pmatrix} \cos \frac{\theta}{2} & 0 & \sin \frac{\theta}{2} & 0 \\ 0 & \cos \frac{\theta}{2} & 0 & \sin \frac{\theta}{2} \\ -\sin \frac{\theta}{2} & 0 & \cos \frac{\theta}{2} & 0 \\ 0 & -\sin \frac{\theta}{2} & 0 & \cos \frac{\theta}{2} \end{pmatrix}. \quad (3.5)$$

The second matrix in Eq. (3.5) represents a simple rotation of coordinates and is

insignificant. The first matrix represents focusing in both “horizontal” and “vertical” planes. Since the solenoid is cylindrically symmetric, we consider only the motion in the “horizontal” plane. Suppose a particle starts from a point located at a distance l_o from the upstream end of the solenoid. The initial coordinates of the particle are

$$X_o = \begin{pmatrix} x_o \\ x'_o \end{pmatrix}. \quad (3.6)$$

The “image” of the particle is located at a distance l_i from the downstream end of the solenoid. The final coordinates are

$$X_i = \begin{pmatrix} x_i \\ x'_i \end{pmatrix}. \quad (3.7)$$

They are given by

$$\begin{pmatrix} x_i \\ x'_i \end{pmatrix} = \begin{pmatrix} 1 & l_i \\ 0 & 1 \end{pmatrix} \begin{pmatrix} \cos \frac{\theta}{2} & \frac{2l}{\theta} \sin \frac{\theta}{2} \\ -\frac{\theta}{2l} \sin \frac{\theta}{2} & \cos \frac{\theta}{2} \end{pmatrix} \begin{pmatrix} 1 & l_o \\ 0 & 1 \end{pmatrix} \begin{pmatrix} x_o \\ x'_o \end{pmatrix}. \quad (3.8)$$

We obtain

$$X_i = \begin{pmatrix} \cos \frac{\theta}{2} - \frac{\theta l_i}{2l} \sin \frac{\theta}{2} & (l_o + l_i) \cos \frac{\theta}{2} + \frac{2l}{\theta} \sin \frac{\theta}{2} - \frac{\theta l_o l_i}{2l} \sin \frac{\theta}{2} \\ -\frac{\theta}{2l} \sin \frac{\theta}{2} & \cos \frac{\theta}{2} - \frac{\theta l_i}{2l} \sin \frac{\theta}{2} \end{pmatrix} X_o. \quad (3.9)$$

Therefore, the condition for point to point focusing is

$$(l_o + l_i) \cos \frac{\theta}{2} + \frac{2l}{\theta} \sin \frac{\theta}{2} - \frac{\theta l_o l_i}{2l} \sin \frac{\theta}{2} = 0. \quad (3.10)$$

It can be assumed that the spatial distribution of the outgoing beam intensity is Gaussian on the focal plane

$$P(\rho, p) = \frac{1}{2\pi\sigma^2(p)} \exp\left(-\frac{1}{2} \frac{\rho^2}{\sigma^2(p)}\right), \quad (3.11)$$

where ρ is the radius vector in the polar coordinate on the focal plane and $\sigma(p)$ is approximately proportional to x_i , which is a function of the momentum, p . Therefore, the probability for a particle of momentum p to hit a detector of radius r on

the focal plane is given by

$$\begin{aligned}
 P(\rho \leq r, p) &= \int_0^{2\pi} \int_0^r \frac{1}{2\pi\sigma^2(p)} \exp\left(-\frac{1}{2}\frac{\rho^2}{\sigma^2(p)}\right) \rho d\rho d\phi \\
 &= 1 - \exp\left(-\frac{1}{2}\frac{r^2}{\sigma^2(p)}\right).
 \end{aligned} \tag{3.12}$$

The width of the distribution depends also on the size of the detector, r . A computer simulation of the momentum acceptance using MUTRACE is shown in Fig. 3.6. The

Figure 3.6: Momentum acceptance of the solenoid based on MUTRACE calculation. $l_o = 40$ cm, $l_i = 25$ cm, $B = 713$ G, $l = 135$ cm.

momentum acceptance at $p = 2$ MeV/c is about 8.5% (FWHM) for a detector of 3.8 cm radius, which is much smaller than the acceptance of the bending magnet. The peaks at 2 MeV/c and 3.2 MeV/c are relevant to the experiment. In the later

stages of the experiment we observed the peak at 2 MeV/c and part of the peak at 3.2 MeV/c in the time-of-flight measurements. It is also useful to study the line shape of the solenoid, since it determines the acceptance of our apparatus. One can expand $\sigma^2(p)$ as a Taylor series in $(p - p_0)$ around p_0 , where $\sigma^2(p)$ has its minimum. Since the first derivative of $\sigma^2(p)$ vanishes at $p = p_0$ and the second derivative is positive at $p = p_0$, we have

$$\sigma^2(p) = \sigma^2(p_0) + k(p - p_0)^2 + O((p - p_0)^3), \quad (3.13)$$

where $k > 0$.

In general, the phase space of the incident beam to the solenoid is not an erect ellipsoid. Therefore we can write the expression for $\sigma^2(p)$ as

$$\begin{aligned} \sigma^2(p) = & \sigma_{11} \left(\cos \frac{\theta}{2} - \frac{\theta l_i}{2l} \sin \frac{\theta}{2} \right)^2 \\ & + 2\sigma_{12} \left(\cos \frac{\theta}{2} - \frac{\theta l_i}{2l} \sin \frac{\theta}{2} \right) \left(l_i \cos \frac{\theta}{2} + \frac{2l}{\theta} \sin \frac{\theta}{2} \right) \\ & + \sigma_{22} \left(l_i \cos \frac{\theta}{2} + \frac{2l}{\theta} \sin \frac{\theta}{2} \right)^2, \end{aligned} \quad (3.14)$$

where σ_{ij} are the matrix elements of the phase space. Equations (3.12) and (3.14) will be used later in the data analysis.

The coils of the solenoid are made of 2.54 cm wide and 50.8 μm thick Al strips. They have an inner diameter of 28.7 cm, an outer diameter of 39.1 cm, and are 128.9 cm long. Cooling water flows through two sets of copper tubing between the vacuum pipe and the coils. The whole solenoid is covered with a cylindrical tube of 0.95 cm thick iron and of 45.8 cm o.d. and the two ends are covered with 0.95 cm thick plates of iron, providing a return path for the magnetic field. The field along the axis was mapped and found to be in good agreement with the POISSON calculation shown in Fig. 3.7 by a smooth curve. The current density used to produce 380 G in the measurement was 62.0 A/cm², while the current density is 59.7 A/cm² in the POISSON calculation. The difference is due to the spacing between windings. The effective length was measured to be 135 cm. The solenoid was operated at 15 A, which gave 712.5 G along the axis. The magnet got hot because of the

inefficient cooling system. However, the temperature became stable after two hours of operation.

Figure 3.7: Magnetic field along the axis of the solenoid. The solid points with error bars represent the measurement, while the curve is based on POISSON calculation.

3.6 Transport System

The design of the transport system for low energy positive muons or negative muonium ions is based on calculations using the computer code TRANSPORT. The beam envelope is computed, assuming a phase space of $\sigma_x \cdot \sigma_{x'} = \sigma_y \cdot \sigma_{y'} = 4 \text{ cm} \cdot 200 \text{ mrad}$ and a momentum spread of $\pm 5\%$ at 2 MeV/c central momentum after the accelerator stage. It is shown in Fig. 3.8. Because of the limitation in space, the

Figure 3.8: Beam envelope of the transport system.

optimum double-focusing condition was not applied. However, the system resulted in a symmetric beam envelope in horizontal and vertical planes. The beam ellipses at different locations are shown in Fig. 3.9. The size of the beam spot on the microchannel plate is about 4 cm in radius. The transport efficiency of the system is about 36% as calculated using TURTLE [Bro74]. In the experiment, the solenoid was set at 712.5 G, which is the highest field one can obtain without over-heating of the magnet. The field of the bending magnet was first tuned to have maximum transmission of positive muons. The accelerating voltage was varied to accelerate

low energy positive muons to the momentum acceptance of the spectrometer. Negative muonium ions were studied, by reversing the polarities of the accelerating voltage and the field in the bending magnet.

Figure 3.9: Beam ellipses at different locations of the transport system.

3.7 Detectors and Logic

The detectors used in the experiment are a thin plastic scintillation counter, a microchannel plate, several thick plastic scintillators and a NaI crystal scintillator.

In order to have a minimum amount of material in the beam, a thin plastic scintillator (NE102A) of 20 μm thickness was used as a μ^+ -beam monitor. This detector

is essentially identical to the one used in the experiment on formation of muonium in the 2S state and observation of the Lamb shift transition [Bad84,Woo85]. It is expected that 670 scintillation photons can be produced by a positive muon of 9.75 MeV/c passing through the scintillator, while only 30 photons will be produced by a 1 to 40 MeV positron. The scintillation light was transported through four Lucite light guides to four photomultiplier tubes. These tubes are RCA 8850 type photomultipliers which are designed for low light level measurement applications.

Since the efficiency of the scintillator for detecting a positron was less than 18%, while the efficiency for a positive muon was about 92% in each phototube, a three-out-of-four logic, which required that more than two of the four tubes fired in coincidence, was set up to increase the detection efficiency for positive muons and, at the same time, to reduce the efficiency for positrons. The logic diagram is shown in Fig. 3.10. E1, E2, E3, and E4 are the signals from the four tubes. Each signal was amplified and then triggered a leading-edge timing discriminator, providing pulses of 20 nsec width. The discriminator thresholds were set just below the single photoelectron peak, in other words, in the noise band. The $\mu_{3/4}$ signal had an efficiency of 97% for positive muons and of 2% for positrons and it was used as the signature of a positive muon entering the apparatus. This signal was used to measure the time-of-flight, which will be discussed later in this section, and, together with the other signals from coincidence levels 1, 2, and 4, was read into several CAMAC [Clo82] scalars for an efficiency check of the scintillator. It should be noted that the analog sum of E1, E2, E3, and E4 can also be used as a signature for a positive muon with a properly set discriminator threshold, since a positive muon gives a much larger pulse height than a positron does.

The Michel energy spectrum of positrons from positive muon decay can be measured with a NaI(Tl) crystal scintillator. The positrons from positive muon decay have energies up to 53 MeV with a maximum range of about 5 cm in NaI. The NaI(Tl) crystal is a cylinder of 25 cm diameter and 25 cm height. It has a photon yield of about one photon per 25 eV of deposited energy. The scintillation light was viewed by seven phototubes with the gains adjusted to be equal. Hence, the pulse height is approximately proportional to the deposited energy [Bir64,Yua61]. The

Figure 3.10: Logic diagram for muon counter.

energy resolution was found to be $22\%/E^{1/4}$ (FWHM) [Hug72], where E is in MeV. The analysis will be presented in Chapter 5.

A scintillator telescope consisting of two circular plastic scintillators, T1 and T2, 0.5 cm thickness and 6 cm radius, was placed between the beam stop and the NaI crystal. The telescope subtended a fractional solid angle of 2.9% at the center of the beam stop in the straight direction. An anti-coincidence counter, A, located above the beam stop was used to veto signals from cosmic rays. It was a 0.8 cm thick plastic scintillator and had an area of 30 cm×30 cm. The NaI was also surrounded by four plastic scintillators, A1, A2, A3, and A4, each of them bent to form a quarter of a

circle. Lead bricks surrounded this anti-coincidence shield. EMI 9813B phototubes were used for these detectors. These scintillation counters had an efficiency of about 95%. A block diagram of the logic for measuring the Michel spectrum is shown in Fig. 3.11. The analog signal was amplified by a Tennelec TC203BLR amplifier with

Figure 3.11: Logic diagram for measuring the Michel spectrum.

a gain of 14.5 and 0.25 μsec shaping. The delayed unipolar output was taken as the input to a LRS 2259 amplitude-to-digital converter (ADC). The prompt bipolar output of the amplifier was given to a Canberra single channel analyzer (SCA) operated in dual discriminator mode, which set a lower level threshold of 0.25 V

and an upper level threshold of 10 V. This signal was gated by the scintillator logic signal together with the beam gate, which represents the macro time structure of the primary proton beam, to form an interrupt to the CAMAC and a gate of 1 μ sec for the ADC.

At the end of the 60°-channel, a microchannel plate (MCP) was mounted in the center of the vacuum chamber. It served as both a beam stop and a detector for positive muons and negative muonium ions transported through the 60°-channel. The MCP is a pair of microchannel plates in which the channel axes of the two plates are tilted by 16° with respect to one another in order to eliminate spurious noise caused by positive ion feedback [Wiz79]. It is a Chevron CEMA (Model No. 3075) manufactured by Galileo Electro-Optics Corp. shown in Fig. 3.12. It has an active diameter of 75 mm. Each channel plate has a bias angle of 8° with respect to the front surface. The channels have a length to diameter ratio of 90, with a diameter of 25 μ m and a center to center spacing of 32 μ m, which gives a 55% open area ratio. A stainless steel disk is mounted at the back of the microchannel plates as an anode for electrical signal readout. A 95% transparent grounded Cu mesh (20 lines per inch) is mounted in front of the MCP.

The MCP was operated at 815 V per plate with no interplate bias voltage. This mode of operation helped to discriminate heavy particles, which produce considerably larger pulses, from positrons. The pulse amplitude spectra were taken during the experiment with a LRS 2259A ADC. The detection efficiency of a MCP has been studied with H^+ , He^+ , and O^+ in the energy region of several keV [Gao84] and found to be $(65 \pm 10)\%$ [Ort85]. The same efficiency is expected for muons, although no direct measurement is available.

Two pairs of scintillators C1, C2 and C3, C4 were mounted, respectively above and below the MCP to detect the decay positrons from positive muons or negative muonium ions stopped in the MCP. These scintillators were essentially identical to the scintillator A described above. They subtended a fractional solid angle of 16.2% at the MCP. The logic diagram for the scintillator telescopes is shown in Fig. 3.13. The signature of a positive muon or of a negative muonium ion incident on the MCP was defined as a delayed coincidence of the signal from the MCP and 4.2 μ sec

Figure 3.12: Schematic diagram of the wiring for the MCP detector.

Figure 3.13: Logic diagram of the scintillator telescopes for detecting decay positrons from MCP.

gate, $(C12+C34) \cdot \text{BEAM}$, shown in Fig. 3.13. This coincidence signal is denoted as the START to the time-to-digital converters (TDC), shown in Fig. 3.14. One of the TDC's measured the muon lifetime (LIFE) and the other the time-of-flight (TOF) of positive muons or negative muonium ions from the muon counter to the MCP. The delayed $(C12+C34) \cdot \text{BEAM}$ signal was used as a stop in the measurement of muon lifetime, in which a LRS 2228A TDC, modified to have about a $5\text{-}\mu\text{sec}$ range, was used.

An EG&G TD104 TDC was used for the time-of-flight measurements. The $\mu_{3/4}$ signal was delayed by about $1\text{ }\mu\text{sec}$. The time-of-flight was measured in reversed timing, since the instantaneous counting rate of the MCP is less than 200 sec^{-1} , which is significantly lower than the instantaneous rate of the $\mu_{3/4}$ of $2.3 \times 10^6\text{ sec}^{-1}$.

Figure 3.14: Logic diagram for muon lifetime and time-of-flight measurements.

The logic shown in Fig. 3.14 was set up in a cascading fashion to allow three stop signals, STOP1, STOP2, and STOP3, corresponding to three hits in the muon counter for each MCP signal. This TDC was also modified to have about a 1- μ sec full range.

The TDC gains were calibrated with the setup shown in Fig. 3.15. The frequency

Figure 3.15: Circuit diagram for calibrating TDC's.

synthesizer, a FLUKE 6160A, generates a sine wave of very stable frequency. The frequency was monitored using a FLUKE 1953A frequency counter/timer. The width, Δt , of the gate from the gate generator was adjusted to be close to the range of the TDC. This gate generator acts as a pileup gate. The random pulser was a scintillation counter. The START randomly picks up some signals from the

frequency synthesizer. The circuit generates a set of peaks in the TDC spectrum with known distance apart, given by the period of the pulses from the synthesizer. The centroid of each peak was recorded and fitted to a straight line. The gains of the LRS 2228A and the EG&G TD104 were found to be 2.5267 ± 0.0008 nsec/channel and 0.999 ± 0.001 nsec/channel, respectively. The relation between the time-of-flight and the channel number in the EG&G TD104, obtained using a pulser, is

$$t_{TOF}(\text{nsec}) = 916 - 0.999n_{TDC} \pm 20, \quad (3.15)$$

where t_{TOF} is the time-of-flight of a particle from the muon counter to the MCP and n_{TDC} is the channel number in the TDC. The 20-nsec uncertainty is due to the absolute time calibration.

The CAMAC modules, such as scalars, ADC's and TDC's, were all set up in a single CAMAC crate as shown in Fig. 3.16. The scalars in station 1 were not gated by any signal to record the total number of counts of the detectors, while the scalars in station 2 and 3 were gated by the primary proton beam. Therefore, average rates and instantaneous rates of the detectors could be monitored and recorded. A pulser running at 10^4 sec^{-1} was fed into an ungated scaler channel and a gated scaler channel for time measurements. A 5- μsec TDC in station 4 was connected to measure the muon lifetime. The peak-sensing ADCs in stations 6 and 7 were used for measurements of pulse amplitude of MCP and NaI, respectively. A 1- μsec TDC in station 16 was for the time-of-flight measurement. Three of the four channels were used by STOP1, STOP2, and STOP3. They shared a common start from the MCP. The interrupt register generated CAMAC interrupts from an external signal, such as the gated NaI logic signal or the delayed coincidence signal of MCP shown in Fig. 3.14 as PI. The LAM (look-at-me) grader built at Yale buffers the interrupts generated by each module, allowing the computer to determine which module needs service. The crate controller was a Standard Engineering CC-A1. The data were sent to a PDP-11/34 computer via a CAMAC interface, the branch driver, and the PDP-11 Unibus. The data acquisition PDP-11/34 computer, manufactured by Digital Equipment Corporation, is a 16-bit computer and has 124k 16-bit words of core memory, which is the maximum extensible memory space on this type of machine. It

CAMAC CONTROLLER	25
	24
YALE LAM GRADER	23
	22
	21
	20
VISUAL BRANCH TERMINATOR	19
	18
DATAWAY DISPLAY	17
EG&G IR026 INTERRUPT REGISTER	16
	15
TDC	14
TDC	13
EG&G TD104 TDC, 4 CH, 10 BITS	12
ADC	11
ADC	10
ADC	9
	8
LRS 2259 ADC, 12 CH, 10 BITS	7
LRS 2259A ADC, 12 CH, 11 BITS	6
	5
LRS 2228A TDC, 8 CH, 11 BITS	4
LRS 2551 SCALER, 12 CH, 24 BITS	3
LRS 2551 SCALER, 12 CH, 24 BITS	2
LRS 2551 SCALER, 12 CH, 24 BITS	1

Figure 3.16: CAMAC module arrangement in the crate.

ran on the RT-11 operating system. The PDP-11 Unibus interconnects the central processing unit (CPU), the core memory and all the peripherals. There was a DEC VT100 terminal with Retro-Graphics support for normal operations, controlling the experiment and displaying histograms, a DECwriter teletype terminal for logging messages and printing the on-line analysis data, and a printer for program listings. Two RK05 disk drives were connected to the computer. One of them served as the system disk which keeps the RT-11 operating system, the utilities and the data acquisition programs, the other as the data disk to store the on-line analysis data and histograms. Two magnetic tape drives were used for recording the raw data during normal data taking.

The data acquisition program called PION was developed at the University of Heidelberg. It was written in assembly and FORTRAN languages. The program could handle up to 6 LRS scaler modules and 20 programmed-data-transfer (PDT) parameters when this experiment was conducted. The capability was later expanded. The scaler modules have to be installed one next to another starting from station 1, while modules other than scalers can be set up in any order after the scaler modules. The program read the scalers every second. The readout of the data from modules other than scalers were requested through interrupt. The data were then buffered and written to magnetic tape after the buffer was full. The program also processed the data for on-line histograms. It has some limitation on on-line processing, though it is adequate for monitoring the experiment. The data on tape contains the information on when a run was started, the structure of the event data buffer, the number of parameters, the name of each parameter defined, the data buffer, the time when the run was stopped, and the sum of the scaler data. A replay program running on a VAX computer of Digital Equipment Corporation with a VMS operating system was then developed. It can do essentially all the data processing tasks that PION can. Q routines developed by the MP-1 group at LAMPF can also be incorporated in the program which allow much more flexibility in data manipulation. The data can be transferred directly to the VAX by copying one record at a time or using the FLX utility on a PDP-11/RSX-11M system to transfer a RT-11 volume on RK05 disk into a Files-11 volume used on RSX-11M

and VMS systems, since there was no RK05 disk drive available on the VAX. All the data replay was done on the VAX computer.

Chapter 4

Experimental Observations

The primary design consideration of the apparatus was optimization of the signal rate. The formation of neutral muonium was observed with the NaI detector. As pointed out in Chapter 2, the electron capture probability is very sensitive to the velocity of the incident positive muon. In order to maximize the formation fraction of muonium atoms, the momentum of the incident positive muon beam was tuned by varying the setting of the SMC. At the optimum momentum of the incident beam, the formation of the negative muonium ions was observed. The times-of-flight of the positive muons and the negative muonium ions through the 60° channel were measured and from these the masses of the particles can be computed. The fact that the negatively charged particles contain muons was further verified by measuring both the lifetime of the particles and the characteristic Michel spectrum of the decay positrons from positive muons. In this chapter, the experimental procedure and observations will be described.

4.1 Observation of Muonium Formation

The initial running stage of the experiment was a search for neutral muonium atoms with the Al production foil in place. In order to ensure that the NaI detector located downstream in the straight channel was working properly, the Michel spectrum of decay positrons from positive muons was measured by bringing positive muons to

the straight channel by turning off the magnetic field in the bending magnet. The spectrum shown in Fig. 4.1 is later used for energy calibration for the detector. The beam related background dominates the region of the spectrum below channel 70.

Figure 4.1: ADC spectrum of the NaI detector taken when the field in the bending magnet was switched off.

This was verified by turning off both the electric and magnetic fields in the beam separator, allowing both positive muons and positrons to go into the apparatus. A NaI spectrum taken in this mode is shown in Fig. 4.2. With the bending magnet

Figure 4.2: ADC spectrum of the NaI detector taken when the fields in the bending magnet and the separator were turned off.

fully up (1.1 kG), positive muons were swept out, leaving only neutral muonium atoms traversing downstream of the bending magnet. The radius of curvature of a charged particle of 10 MeV/c momentum is about 30 cm in this field. Figure 4.3 is an ADC spectrum of the NaI detector. The signal rate is much reduced due to the

Figure 4.3: ADC spectrum of the NaI detector taken when the field in the bending magnet was on (1.1 kG).

low formation fraction of muonium atoms. The number of decay positrons detected in the NaI is determined by setting a window in the ADC spectra from channel 160 to channel 360, which corresponds to 66% of the full Michel spectrum. A flat background rate based on the data from channel 500 to channel 800 in the ADC spectrum was subtracted from the windowed data. As is described in Chapter 3, the acceptance of the NaI is about 2.9% . Therefore, the rate of the muonium atoms

at the beam stop was calculated from the number of decay positrons observed. The momentum setting of the SMC was then tuned in order to optimize this rate. The rates in NaI and projected rates of muonium atoms at the beam stop are listed in Table 4.1. The number of incident positive muons μ_{inc}^+ (also in Table 4.1) is given by

$$\mu_{inc}^+ = \frac{(\mu_{3/4})_{gated} t_{ungated} - (\mu_{3/4})_{ungated} t_{gated}}{t_{ungated} - t_{gated}}, \quad (4.1)$$

where t is the experimental running time and the subscripts *gated* and *ungated* denote beam gated and not-beam gated counts respectively. The observed muonium rates at the beam stop are plotted as solid dots with error bars, in Fig. 4.4, against the momentum of the incident positive muon beam. The maximum rate is at

Figure 4.4: Muonium rate at the beam stop measured at various momenta of the incident positive muon beam.

about 9.75 MeV/c. The Monte Carlo simulations using MUTRACE show the same behavior and are also presented in Fig. 4.4 as a histogram. A similar measurement was also done with a Au foil. The incident positive muon beam was tuned to 10.67 MeV/c momentum at which the average rate of muons was $140 \times 10^3 \text{ sec}^{-1}$. The rate in the NaI was found to be $3.18(18) \times 10^{-7} / \mu_{inc}^+$. The projected rate of muonium atoms at the beam stop is therefore $1.66(9) \times 10^{-5} / \mu_{inc}^+$.

Table 4.1: Average rates of μ_{inc}^+ , rates in NaI and projected rates of muonium atoms at the beam stop.

p_μ (MeV/c)	μ_{inc}^+ Rate (10^3 sec^{-1})	Rate in NaI ($10^{-7} / \mu_{inc}^+$)	Projected Rate at Beam Stop ($10^{-5} / \mu_{inc}^+$)
9.02	80	2.75(28)	1.44(15)
9.49	106	8.88(37)	4.64(19)
9.75	122	10.24(41)	5.35(21)
10.00	99	9.92(38)	5.18(20)
10.25	117	8.20(40)	4.29(21)

4.2 Studies of Low Energy Positive Muons

Once the optimum condition for muonium formation was found, the low energy positive muons could be studied in the 60°-channel. The field of the solenoid was set to 712.5 G. At this field, the solenoid has a high transmission efficiency for 2 MeV/c particles, as calculated using MUTRACE shown in Fig. 3.6. The positive muons were used to tune the bending magnet so that the acceptance of the bending magnet was centered at about 2 MeV/c. A bump was found on a nearly linearly increasing distribution of the rates in the MCP at 267 G, shown in Fig. 4.5. Fine tuning was then done around 267 G. The event rates were calculated using the counts in the regions from channel 288 to 488 in the TDC spectra of the time-of-flight of the positive muons. The data are shown in Fig. 4.6. The magnetic field of the bending magnet was set to 267 G for the rest of the measurements based on these tuning curves.

Figure 4.5: Tuning curve of the bending magnet to maximize the transport efficiency of the spectrometer.

Figure 4.6: Fine tuning curve of the bending magnet to maximize the transport efficiency of the spectrometer. The event rates were calculated using the counts in the regions from channel 288 to 488 in the TDC spectra of the time-of-flight of the positive muons.

The time-of-flight of positive muons was measured. Figure 4.7 shows the TDC spectra taken with three stop signals, STOP1, STOP2, and STOP3, described in the previous chapter for the time-of-flight measurement from the muon counter to the MCP. Because of the high counting rate in the muon counter, the uncorrelated events are more probable to be taken as a stop in the lower channels of the first TDC spectrum. Since STOP2 has to be enabled by STOP1, the probability to have an uncorrelated event from muon counter in the lower channels of the second TDC spectrum is reduced. This reduction is even greater in the third TDC spectrum using STOP3. This rate effect has been carefully studied by several authors [Mat71,Low72,Hof82].

As shown in Fig. 3.6, the acceptance of the solenoid reaches a maximum at many different regions of the particle momentum. Since the momentum acceptance of the bending magnet is very large (30%), fractions of the positive muons with high enough momentum in neighboring focusing regions of the solenoid can also be detected by the MCP. This lower order focusing effect on higher momentum particles was also observed, as shown in Fig. 4.7 at about channel 620.

The center of the main peak is at channel 444, which corresponds to a time-of-flight of 472 nsec. The width of about 45 nsec (FWHM) is partly due to the finite momentum acceptance of the apparatus and partly due to the different path lengths of the particles in the bending magnet. By scaling down the settings of the magnetic fields of the bending magnet and the solenoid by a factor (1.415), a TDC spectrum was taken with zero accelerating voltage. With an extra delay of 128 nsec for the stop signals, a peak was found at channel 382, corresponding to 662 nsec with a width of 50 nsec (FWHM). This confirmed that the time-of-flight spectrometer was working properly.

Different accelerating voltages on the accelerator were then used to collect positive muons with different initial kinetic energies. As the accelerating voltage was increased to 19 kV, the width of the peak became narrower, due to the cutoff of the energy distribution at zero energy. The TDC spectra taken at 19 kV are shown in Fig. 4.8.

In the on-line analysis, the counts from channel 388 to 488 in the first TDC

Figure 4.7: TDC spectra for the time-of-flight measurements of positive muons with Al foil and zero accelerating voltage on the accelerator. The timing is reversed.

Figure 4.8: TDC spectra for the time-of-flight measurements of positive muons with Al foil and accelerating voltage of 19 kV on the accelerator. The timing is again reversed.

spectrum were taken as candidates for positive muons incident on the MCP. These rates are designated as the raw rates. The results are listed in Table 4.2, together with the incident positive muon rates and the interrupt rates. The number of incident positive muons was calculated according to Eq. (4.1). It should be noted

Table 4.2: Average rates of μ_{inc}^+ , interrupt rates and raw rates of positive muons taken with Al foil at various accelerating voltages. The momentum of the incident positive muon beam was 9.75 MeV/c.

Accelerating Voltage (kV)	μ_{inc}^+ Rate (10^3 sec^{-1})	Interrupt Rate ($10^{-7}/\mu_{inc}^+$)	Raw Rate ($10^{-7}/\mu_{inc}^+$)
21.0	132	43(2)	6(1)
20.0	104	69(2)	18(1)
19.0	132	99(2)	32(1)
18.0	133	142(2)	49(1)
15.0	129	188(3)	62(2)
12.0	132	192(4)	66(2)
6.0	121	206(5)	75(3)
0.0	121	203(2)	68(1)
-24.2	114	205(4)	76(3)

that the raw rates contain part of the signals and also some background counts. But it does give us some idea on how the rates vary at different accelerating voltages. Because of unequal acceptance of the channel for different accelerating voltages, this rate dependence on accelerating voltages can not be interpreted directly as an energy distribution of the positive muons.

As mentioned before, it is interesting to study the behavior of the secondary beams with foils of different atomic numbers Z . Two other foils were chosen for these studies, a 25 μm Be foil of $Z=4$ and 2 μm Au foil of $Z=79$. The momentum of the incident positive muon beam was chosen such that the expected average final energy of the positive muons coming out the foils is about the same as that for the Al foil.

For the Be foil, a 12.63 MeV/c incident positive muon beam was used. The data shown in Table 4.3 have similar behavior as those of the Al foil. The rate for the Be foil increases more slowly than that for the Al foil as the accelerating voltage

Table 4.3: Average rates of μ_{inc}^+ , interrupt and raw rates of positive muons taken with Be foil at various accelerating voltages. The momentum of the incident positive muon beam was 12.63 MeV/c.

Accelerating Voltage (kV)	μ_{inc}^+ Rate (10^3 sec^{-1})	Interrupt Rate ($10^{-7}/\mu_{inc}^+$)	Raw Rate ($10^{-7}/\mu_{inc}^+$)
21.0	125	33(1)	5(1)
20.0	127	46(1)	9(1)
19.0	125	61(2)	13(1)
17.0	110	104(2)	28(1)
15.0	126	142(3)	40(1)
10.0	130	163(3)	43(2)
6.0	128	171(4)	44(2)
0.0	130	191(3)	48(2)

decreases. This indicates that the energy distribution of the positive muons from Be foil is broader or flatter than that of the Al foil.

The Au data listed in Table 4.4 were taken with a 10.67 MeV/c incident positive muon beam. We clearly see a maximum rate in the neighborhood of 15 kV to 17.5

Table 4.4: Average rates of μ_{inc}^+ , interrupt and raw rates of positive muons taken with Au foil at various accelerating voltages. The momentum of the incident positive muon beam was 10.67 MeV/c.

Accelerating Voltage (kV)	μ_{inc}^+ Rate (10^3 sec^{-1})	Interrupt Rate ($10^{-7}/\mu_{inc}^+$)	Raw Rate ($10^{-7}/\mu_{inc}^+$)
22.5	147	15(1)	2(1)
20.0	145	30(1)	7(1)
19.0	147	47(1)	14(1)
17.5	146	67(2)	22(1)
15.0	145	68(2)	21(1)
10.0	141	50(1)	12(1)
0.0	145	43(1)	11(1)

kV in accelerating voltage. Therefore, a narrower energy distribution with the centroid closer to zero is expected. A more detailed analysis will be presented in the next chapter.

It should be mentioned that the time spectrum of decay positrons from positive muons and the pulse height spectrum of the MCP were also taken. The time spectrum shown in Fig. 4.9 has three distinct regions. I) The region below channel 294 corresponds to uncorrelated events. II) The spike at channels 294 to 300 is partly due to the decay positrons giving signals in both the MCP and the scintillator telescope and partly due to cosmic rays. III) The region above channel 300 below channel 1610 gives the normal time spectrum of muon decay. The cutoff at channel 1610 was caused by hardware gate. Therefore the effective window in which the decay positrons were looked for was $3.3 \mu\text{sec}$ which accepts 77.9% of all decay positrons hitting the scintillator telescope.

Figure 4.9: TDC spectrum for the lifetime measurements of positive muons with Al foil.

By setting the cuts looking at these three regions separately three pulse height spectra of the MCP were obtained. The ADC spectra are shown in Fig. 4.10. The

Figure 4.10: ADC spectra of the MCP pulse height. Cuts in different regions in Fig. 4.9 are used.

low energy positive muons give much large pulses than positrons do. This is because a low energy positive muon generates more secondary electrons than a positron does when it hits the surface of the microchannel plate.

The events corresponding to the signals below channel 301 in the lifetime spectrum give an uncorrelated time-of-flight spectrum. One of such spectrum from STOP1 is shown in Fig. 4.11. The spectrum has an exponential decay structure with a time constant of about $0.37^{+0.13}_{-0.02} \mu\text{sec}$, which is in good agreement with the instantaneous counting rate of $2.3 \times 10^6 \text{ sec}^{-1}$ in the muon counter.

Figure 4.11: Uncorrelated time-of-flight spectrum. It is taken from STOP1 in Fig. 4.7 with cuts requiring a count falling below channel 294 in the spectrum shown in Fig. 4.9.

4.3 Observation of the Formation of the Negative Muonium Ion

By reversing the polarity of the bending magnet negative muonium ions were observed. The SMC was again tuned to 9.75 MeV/c subsurface positive muon beam for the Al production foil. The time-of-flight spectra taken with -17 kV accelerating voltage are shown in Fig. 4.12. The center of the peak is at channel 419 corresponding to a time-of-flight of about 497 ± 20 nsec. Taking into account 4-nsec flight time from the muon counter to the production foil and neglecting the initial energy of the particle, the mass of the particle is estimated to be 104 ± 8 MeV/c². This mass value is consistent with the expected mass of 106.7 MeV/c² for negative muonium ion. On the other hand if we take the mass of the ion to be 106.7 MeV/c², the initial kinetic energy of the ion can be computed. The length of the accelerator is 6 cm. The central path length from the downstream end of the accelerator to the MCP detector is 256 cm. The initial kinetic energy of the ion is estimated to be 0.3 ± 1.3 keV. The relatively large error is mainly due to the 20-nsec uncertainty in the time calibration [see Eq. (3.15)].

The accelerating voltage was then varied. the event rates at different accelerating voltages are listed in Table 4.5. A maximum was found at about -17.5 kV. The

Table 4.5: Average rates of μ_{inc}^+ , interrupt rates, and raw rates of negative muonium ions taken with Al foil at various accelerating voltages. The momentum of the incident positive muon beam was 9.75 MeV/c.

Accelerating Voltage (kV)	μ_{inc}^+ Rate (10^3 sec^{-1})	Interrupt Rate ($10^{-7}/\mu_{inc}^+$)	Raw Rate ($10^{-7}/\mu_{inc}^+$)
-14.00	137	11(1)	1.6(0.2)
-15.50	126	16(1)	4.2(0.3)
-17.00	129	17(1)	6.4(0.2)
-17.50	114	24(2)	7.2(0.5)
-17.75	105	18(1)	5.5(0.3)
-18.50	136	19(1)	5.7(0.4)
-20.00	133	11(1)	2.8(0.3)

Figure 4.12: Time-of-flight spectra of negative muonium ions taken with Al foil. The momentum of the incident positive muon beam was 9.75 MeV/c. The timing is reversed.

width is about 4.5 kV (FWHM) which is close to the acceptance of the apparatus of 8.5% in momentum. This indicates that the energy distribution of the negative muonium ion is on the order of a few keV. This narrow distribution can also be seen in the time-of-flight spectra shown in Fig. 4.12. Another difference between the energy distributions of the positive muons and of the negative muonium ions is that there is no high energy negative muonium ions. Therefore the second peak shown up in the positive muon time-of-flight spectra does not exist in the time-of-flight spectra of the negative muonium ions.

To ensure the peaks are real the spectrometer settings were again scaled down by a factor (1.415). The accelerating voltage was tuned. In order to take the time-of-flight spectra, an extra 128 nsec delay was added to the stop signals. A maximum rate of events at about 8.7 kV confirms that the spectrometer was working. The raw rates are calculated based on counts in channel 281 to 440 of the first time-of-flight spectra. They are listed in Table 4.6. These measurements at low channel settings

Table 4.6: Average rates of μ_{inc}^+ , interrupt rates, and raw rates of negative muonium ions taken with Al foil at various accelerating voltages. The momentum of the incident positive muon beam was 9.75 MeV/c. The spectrometer settings were scaled down by a factor (1.415).

Accelerating Voltage (kV)	μ_{inc}^+ Rate (10^3 sec^{-1})	Interrupt Rate ($10^{-7}/\mu_{inc}^+$)	Raw Rate ($10^{-7}/\mu_{inc}^+$)
-8.00	111	9.1(0.3)	3.6(0.2)
-8.50	131	10.0(0.4)	4.4(0.3)
-8.75	121	9.8(0.2)	4.1(0.1)
-9.00	125	9.7(0.3)	3.9(0.2)
-9.50	128	8.6(0.3)	3.4(0.2)

together with the measurements at normal channel settings also enable us to make better estimate of the initial energy of M^- . Since the TDC gain was calibrated very precisely, from the differences of the times-of-flight at low channel settings and at normal channel settings the ambiguity in the time calibration can be eliminated from the estimate of the centroid. A more precise value of 0.16(9) keV is obtained, which is consistent with the observation of the negative hydrogen ions.

The time spectra of the decay positrons were taken at the same time. A typical muon lifetime was observed. The spectrum is shown in Fig. 4.13. To further verify

Figure 4.13: TDC spectrum for the lifetime measurements of negative muonium ions with Al foil.

that the particle contains a muon the NaI detector was moved to the end of the 60°-channel. The characteristic Michel spectrum was observed. The MCP signal was included in the trigger to further reduce the background. The spectrum is shown in Fig. 4.14. The ADC gain was recalibrated by bringing the positive muons into the 60°-channel.

Figure 4.14: Michel spectrum of decay positrons from negative muonium ions.

The formation of the negative muonium ions with the Be and Au foils was also observed. The SMC was tuned to a 12.63 MeV/c positive muon beam to accommodate the thicker Be foil. The rates are listed in Table 4.7. Again a maximum was found at about -17.5 kV accelerating voltage. The fraction of formation, however, is reduced by almost a factor of two as compared with that of the Al foil.

Table 4.7: Average rates of μ_{inc}^+ , interrupt rates, and raw rates of negative muonium ions taken with Be foil at various accelerating voltages. The momentum of the incident positive muon beam was 12.63 MeV/c.

Accelerating Voltage (kV)	μ_{inc}^+ Rate (10^3 sec^{-1})	Interrupt Rate ($10^{-7}/\mu_{inc}^+$)	Raw Rate ($10^{-7}/\mu_{inc}^+$)
-16.00	136	15(1)	3.1(0.3)
-17.50	145	18(1)	3.9(0.3)
-18.50	128	15(1)	3.8(0.3)
-20.00	133	14(1)	3.5(0.3)

Data with the Au foil were taken only in one condition, i.e. at -17.5 kV accelerating voltage. The incident positive muon beam was at 10.67 MeV/c with an average rate of $146 \times 10^3 \text{ sec}^{-1}$. The rates are also listed in Table 4.8 for easy comparison with the other foils. The formation fraction of the negative muonium ions is comparable with that of the Al foil.

Table 4.8: Average rate of μ_{inc}^+ , interrupt rate, and raw rate of negative muonium ions taken with Au foil. The momentum of the incident positive muon beam was 10.67 MeV/c.

Accelerating Voltage (kV)	μ_{inc}^+ Rate (10^3 sec^{-1})	Interrupt Rate ($10^{-7}/\mu_{inc}^+$)	Raw Rate ($10^{-7}/\mu_{inc}^+$)
-17.50	146	21(1)	6.5(0.4)

Chapter 5

Data Analysis

There are basically three types of spectra in the experiment, the pulse height spectrum of the NaI crystal detector, the time spectrum of the decay positrons, and the time-of-flight spectrum of the particles from the muon counter to the MCP detector. The energy distribution of the decay positrons from the positive muons is the characteristic Michel spectrum and was observed using the NaI crystal detector. The time spectra of the decay positrons give the lifetime of positive muons. A lifetime of $2.197 \mu\text{sec}$ will indicate the existence of the muons. The time-of-flight spectra will give us some insight into the energy distribution of the particles. In order to estimate the formation fraction of the negative muonium ions the integrated rates of the ions in the time-of-flight spectra were computed. The rates of the low energy positive muons were also obtained. These rates are compared with those of the Monte Carlo simulations. The computer code for function minimization, MINUIT [Jam75], was used for most of the least-squares fits. An iteration method was employed to obtain the undistorted time-of-flight spectra based on the TDC spectra. The methods of the data analysis will be described in this chapter.

5.1 The Michel Spectrum

The pulse height spectra of the NaI detector were calibrated using a positive muon beam. It was done by turning off the magnetic field of the bending magnet for the

NaI detector located at the end of the straight channel. By integrating over the direction of the positive muon spin θ , the Michel spectrum, Eq. (2.3), becomes

$$M(x) = 2\sqrt{x^2 - x_0^2} (3x - 2x^2 - x_0^2), \quad (5.1)$$

up to a proportional constant, where $x = E_e/W$, $W = 52.831$ MeV is the maximum energy of a positron from positive muon decay at rest, and $x_0 = 9.67 \times 10^{-3}$ is the normalized electron mass. This expression for the energy distribution of decay positrons detected by the NaI detector is valid for unpolarized muons. However, it is also a good approximation for polarized muons, since the NaI detector is at 90° to the beam direction.

In order to calibrate the spectrum, a linear relation between the energy of the positron E_e and the pulse height n_{ADC} is assumed, i.e.

$$E_e = E_0 + kn_{ADC}, \quad (5.2)$$

where E_0 is the offset to the energy scale in MeV and k is the energy scale of the ADC, which has units of MeV/channel. Both E_0 and k are fitting parameters to be computed. Because of the resolution and the finite size of the NaI detector two corrections are necessary. The effect of the detector resolution can be expressed as a Gaussian function

$$g(x, z) = \frac{1}{\sqrt{2\pi}\sigma^2} \exp\left(-\frac{(x - z)^2}{2\sigma^2}\right), \quad (5.3)$$

where $\sigma = a/x^{1/4}$ is assumed [Hug72] and a is a fitting parameter. Taking into account the effect that some of the high energy decay positrons passed through a corner of the crystal and deposited a fraction of their energy in the crystal, a function,

$$h(x) = 1 - sx, \quad (5.4)$$

is introduced, where s again is a fitting parameter. The parameters a and s specify the properties of the NaI detector. The fitting function is therefore

$$N(z) = N_0 \int_{x_0}^1 M(x)g(x, z)h(x)dx + b, \quad (5.5)$$

where N_0 and b are the only parameters which depend on the running time and the beam condition of the measurements. By fitting the function (5.5) to the data in channels 65 to 560, the parameters are found to be

$$\begin{aligned}
 N_0 &= 15.2(5) \times 10^2, \\
 b &= 0.9(4), \\
 E_0 &= 3.7(6) \text{ MeV}, \\
 k &= 0.142(2) \text{ MeV/channel}, \\
 a &= 0.035(4), \\
 s &= 0.91(2).
 \end{aligned} \tag{5.6}$$

The result is shown in Fig. 5.1. The curve is the best fit to the experimental

Figure 5.1: NaI spectrum taken with positive muons at the end of the straight channel for the energy calibration of the detector.

data. The first four data points were not used in the fit, since they are background dominated. Therefore the counts in channel 160 to channel 360 of the ADC spectra used to estimate the rates of the muonium production constitute 65.9% of the full spectra. These cuts correspond to an energy window of from 26.4 MeV to 54.8 MeV. By folding in this 65.9% energy cut and the solid angle of 365 msr for the detector, the muonium rates at the beam stop can be computed and they are shown in Fig. 4.4. The parameter a gives an expression for the resolution of the detector $22\%/E^{1/4}$ (FWHM), where E is in MeV.

It should be noted that the decay positrons were viewed by the NaI detector through a 0.13 mm Al window at the end of the straight channel, while there was no special window for decay positrons at the end of the 60°-channel. The decay positrons had to go through the 1.6 mm Al wall of the vacuum can. Therefore the NaI detector has to be recalibrated for data taken in the 60°-channel. The same fitting function as Eq. (5.5) was used. The parameters a and s are fixed to be 0.035 and 0.91 respectively. The fit is shown in Fig. 5.2. The cutoff at 20 MeV is caused by the threshold of the discriminator. The data point at 20 MeV was not used for the fit. The parameters are found to be

$$\begin{aligned}
 N_0 &= 3.03(7) \times 10^2, \\
 b &= 0.5(3), \\
 E_0 &= 13.5(1.4) \text{ MeV}, \\
 k &= 0.121(4) \text{ MeV/channel}.
 \end{aligned}
 \tag{5.7}$$

The larger offset E_0 to the energy scale than that in the straight channel is due to bigger energy loss of the positron when it passed through the wall of the vacuum can.

For the data taken with the negative ions the parameters E_0 and k are fixed to be 13.5 and 0.12 respectively given in Eq. (5.7) as well as a and s determined by the positive muon data taken in the straight channel [Eq. (5.6)]. The fit is shown in Fig. 5.3. The parameters are found to be

$$\begin{aligned}
 N_0 &= 0.68(4) \times 10^2, \\
 b &= 1.8(5).
 \end{aligned}
 \tag{5.8}$$

Figure 5.2: NaI spectrum taken with positive muons at the end of the 60°-channel for the energy calibration of the detector.

Figure 5.3: NaI spectrum taken with negative muonium ions at the end of the 60°-channel.

The χ^2 is 8 for 13 degrees of freedom. The first data point was not used in the fit. This good fit indicates that the energy spectrum of the secondary particles from the negative ions stopped in the MCP is a Michel spectrum. Hence the negative ion contains a muon.

5.2 Muon Lifetime

The time spectra of the decay positrons from positive muons should have a characteristic of exponential decay. The behavior can be expressed analytically as

$$N(n_{TDC}) = N_0 \int_{n_{TDC} - \frac{w}{2}}^{n_{TDC} + \frac{w}{2}} \frac{1}{\tau} \exp\left(-\frac{t}{\tau}\right) dt, \quad (5.9)$$

where n_{TDC} is the channel number of the center of each bin, w is the number of channels in each bin, and N_0 and τ are fitting parameters. A least-squares fit has been performed in which w was taken to be 32. All data taken with low energy positive muons transported to the end of the 60°-channel are added together. The data are shown in Fig. 5.4, together with the fitting function (5.9), which is a least-squares fit to the data from channel 321 to channel 1600. A computed flat background based on the counts in channels 1 to 160 has been subtracted from the data prior to the fitting. The parameters are found to be

$$\begin{aligned} N_0 &= 876(5) \times 10^2, \\ \tau_\mu &= 870(11) \text{ channels}, \\ &= 2.20(3) \mu\text{sec}, \end{aligned} \quad (5.10)$$

where the TDC calibration constant of $2.5267 \pm 0.0008 \mu\text{sec/channel}$ was used.

The data taken with the negative muonium ions shown in Fig. 5.5 are treated in the same way. The best fit gives the following values for the parameters

$$\begin{aligned} N_0 &= 180(2) \times 10^2, \\ \tau_{M^-} &= 856(26) \text{ channels}, \\ &= 2.16(7) \mu\text{sec}. \end{aligned} \quad (5.11)$$

Figure 5.4: Time spectrum of the decay positrons from positive muons at the end of the 60°-channel.

Figure 5.5: Time spectrum of the decay positrons from the negative muonium ions at the end of the 60°-channel.

The measured lifetimes of both the low energy positive muons and the negative muonium ions are in good agreement with the present best value of muon lifetime of $2.19703(4) \mu\text{sec}$ [Agu86].

5.3 The Time-of-Flight Spectrum

Because of the high counting rate in the muon counter the TDC spectra for the time-of-flight measurements are distorted. An earlier stop signal uncorrelated to the start signal has a higher probability to be recorded in the TDC than a stop signal coming later. This effect can be seen in Figs. 4.7, 4.8, and 4.12. It is necessary to correct this rate effect in order to obtain the undistorted time-of-flight distribution, i.e. the distribution for correlated events.

One way to correct this effect is to accept only those events for which there is no more than one stop signal in some optimized time window. Since there will be no second stop signal in this window, the rate effect will be greatly reduced. However there will be a compromise between the cleanness of the resultant spectrum and the overall efficiency of the time-of-flight spectrometer, since a wider window will result in a cleaner spectrum and fewer correlated counts in the spectrum. A time window, channel 140 to channel 650, is set up to obtain clean time-of-flight spectra. It requires having one and only one stop signal in this window. The spectra for low energy positive muons taken at $HV=0.0$ kV and $HV=19.0$ kV, and negative muonium ions at $HV=-17.0$ kV with an Al foil are shown in Figs. 5.6, 5.7, and 5.8 for comparison with the raw spectra shown in Figs. 4.7, 4.8, and 4.12 respectively.

As pointed out in Chapter 4, the width (45 nsec) of the main peak in the spectrum shown in Fig. 5.6 taken at $HV = 0.0$ kV with the Al foil is due to the momentum acceptance of the spectrometer and the different path lengths of the particles in the bending magnet. At high accelerating voltage (19.0 kV), very low energy positive muons were collected and hence the peak in Fig. 5.7 is narrower. There is a small peak at about channel 620 in both Figs. 5.6 and 5.7, due to the the solenoid focusing effect at 3.2 MeV/c. Since there are few negative ions with initial kinetic energies above 5 keV, there is only one peak in Fig. 5.8.

Figure 5.6: Time-of-flight spectrum of positive muons obtained by requiring no more than one signal falling into a window from channel 140 to channel 650. The accelerating voltage was 0.0 kV.

Figure 5.7: Time-of-flight spectrum of positive muons obtained by requiring no more than one signal falling into a window from channel 140 to channel 650. The accelerating voltage was 19.0 kV.

Figure 5.8: Time-of-flight spectrum of negative muonium ions obtained by requiring no more than one signal falling into a window from channel 140 to channel 650. The accelerating voltage was -17.0 kV.

Another way to analyze the data is to study the relation between a TDC spectrum and a distribution undistorted by uncorrelated events. An undistorted distribution can be obtained, if one can derive an analytical expression for the TDC spectrum based on the undistorted distribution. In order to do so we consider the counting rate of a stop signal $R(t)$ as a function of time. We suppose that the counting process is purely Poisson. The probability that k stop pulses occur in a time interval from 0 to t is given by

$$P(k) = \frac{\left(\int_0^t R(\tau) d\tau\right)^k}{k!} \exp\left(-\int_0^t R(\tau) d\tau\right). \quad (5.12)$$

Therefore the probability that the first stop pulse occurs at time t is the same as the probability that none occurs in the time interval from 0 to t ; i.e.

$$P(0) = \exp\left(-\int_0^t R(\tau) d\tau\right). \quad (5.13)$$

Hence the TDC spectrum would be

$$M(t) = NR(t) \exp \left(- \int_0^t R(\tau) d\tau \right) , \quad (5.14)$$

where N is the number of measurements, i.e. the number of start pulses in the TDC. Let us first consider the spectrum of STOP1 in which the first pulse of the stop counter is recorded. If a start signal is given by a pulse which is uncorrelated to the stop pulses (call it type I start), the probability density that a stop pulse occurs at time t is

$$P_1^U(t) = Ue^{-Ut} , \quad (5.15)$$

where U is the instantaneous counting rate of the detector used for the stop signal.

If a start signal is given by a pulse which is correlated with one of the pulses in the stop counter (call it type II start), there are two types of possible stop pulses in the STOP1 spectrum. Let us assume that $C(t)$ is the probability distribution of the correlated events, which is what we are mostly interested in. The probability density of having an uncorrelated stop in STOP1 spectrum with such a start is

$$P_1^{C1}(t) = Ue^{-Ut} \left(1 - \int_0^t C(\tau) d\tau \right) , \quad (5.16)$$

while the probability density of having a correlated stop with such a start is

$$P_1^{C2}(t) = C(t)e^{-Ut} . \quad (5.17)$$

If we assume N such measurements and that there is one correlated pulse from the stop counter for each of the N_C measurements (type II start), the TDC spectrum would be given by

$$\begin{aligned} M_1(t) = & (N - N_C) Ue^{-Ut} + N_C Ue^{-Ut} \left(1 - \int_0^t C(\tau) d\tau \right) \\ & + N_C C(t) e^{-Ut} . \end{aligned} \quad (5.18)$$

This can also be written as

$$M_1(t) = N \left(Ue^{-Ut} \left(1 - \int_0^t C'(\tau) d\tau \right) + C'(t) e^{-Ut} \right) , \quad (5.19)$$

where

$$C'(t) = \frac{N_C}{N} C(t). \quad (5.20)$$

The equation for $M_1(t)$ satisfies the normalization condition

$$\int_0^\infty M_1(t) dt = N . \quad (5.21)$$

$M_1(t)$ gives the STOP1 TDC spectra. Eq. (5.19) will be used to derive the undistorted distribution of the time-of-flight spectrum.

For the sake of completeness we now study the distributions of STOP2 and STOP3 spectra. STOP2 is not enabled immediately by STOP1, since there is a delay time τ_{d2} . It causes an effective dead time of τ_{d2} . For type I start the probability density of having a stop signal at time t is

$$P_2^U(t) = \begin{cases} U^2 (t - \tau_{d2}) e^{-U(t-\tau_{d2})} & \text{if } t \geq \tau_{d2} , \\ 0 & \text{otherwise .} \end{cases} \quad (5.22)$$

For type II start, one of the following three cases can happen.

1. If the pulses taken in STOP1 and STOP2 are both uncorrelated with the start pulse, the probability density of having an uncorrelated stop signal in STOP2 spectrum at time t is

$$P_2^{C1}(t) = \begin{cases} U^2 e^{-U(t-\tau_{d2})} \int_0^{t-\tau_{d2}} (1 - \int_0^\tau C(\tau_1) d\tau_1) \\ \times (1 - \int_{\tau+\tau_{d2}}^t C(\tau_2) d\tau_2) d\tau & \text{if } t \geq \tau_{d2} , \\ 0 & \text{otherwise .} \end{cases} \quad (5.23)$$

2. If the pulse taken in STOP1 is correlated with the start pulse, the probability density of having an uncorrelated stop pulse in STOP2 spectrum at time t is

$$P_2^{C2}(t) = \begin{cases} U e^{-U(t-\tau_{d2})} \int_0^{t-\tau_{d2}} C(\tau) d\tau & \text{if } t \geq \tau_{d2} , \\ 0 & \text{otherwise .} \end{cases} \quad (5.24)$$

3. If the pulse taken in STOP1 is uncorrelated with the start pulse, the probability density of having a correlated stop pulse in STOP2 spectrum at time t is

$$P_2^{C3}(t) = \begin{cases} U C(t) e^{-U(t-\tau_{d2})} \int_0^{t-\tau_{d2}} (1 - \int_0^\tau C(\tau_1) d\tau_1) d\tau & \text{if } t \geq \tau_{d2} , \\ 0 & \text{otherwise .} \end{cases} \quad (5.25)$$

Therefore the STOP2 spectrum can be expressed as

$$M_2(t) = (N - N_C) P_2^U(t) + N_C (P_2^{C1}(t) + P_2^{C2}(t) + P_2^{C3}(t)) . \quad (5.26)$$

Again one has the normalization condition

$$\int_0^\infty M_2(t) dt = N . \quad (5.27)$$

In the same fashion one can derive the expression for STOP3 spectrum. We assume a dead time τ_{d3} due to the fact that STOP3 was not enabled until a time interval τ_{d3} after STOP2 was recorded. For type I start one can write down the probability density of having a stop pulse in STOP3 spectrum at time t as

$$P_3^U(t) = \begin{cases} \frac{1}{2} U^3 (t - \tau_{d2} - \tau_{d3})^2 e^{-U(t - \tau_{d2} - \tau_{d3})} & \text{if } t \geq \tau_{d2} + \tau_{d3} , \\ 0 & \text{otherwise .} \end{cases} \quad (5.28)$$

For type II start one of the following four cases can happen.

1. If the pulses taken in STOP1, STOP2, and STOP3 spectra are all uncorrelated with the start pulse, the probability density of having an uncorrelated stop pulse in STOP3 spectrum at time t is

$$P_3^{C1}(t) = \begin{cases} U^3 e^{-U(t - \tau_{d2} - \tau_{d3})} \\ \times \int_{\tau_{d2}}^{t - \tau_{d3}} \left(\int_0^{\tau - \tau_{d2}} (1 - \int_0^{\tau_3} C(\tau_1) d\tau_1) \right. \\ \times \left(1 - \int_{\tau_3 + \tau_{d2}}^{\tau} C(\tau_2) d\tau_2 \right) d\tau_3 \\ \times \left(1 - \int_{\tau + \tau_{d3}}^t C(\tau_4) d\tau_4 \right) d\tau & \text{if } t \geq \tau_{d2} + \tau_{d3} , \\ 0 & \text{otherwise .} \end{cases} \quad (5.29)$$

2. If the pulse taken in STOP1 spectrum is uncorrelated with the start pulse, while the pulse taken in STOP2 spectrum is correlated with the start pulse, the probability density of having an uncorrelated stop pulse in STOP3 spectrum at time t is

$$P_3^{C2}(t) = \begin{cases} U^2 e^{-U(t - \tau_{d2} - \tau_{d3})} \int_{\tau_{d2}}^{t - \tau_{d3}} C(\tau) \\ \times \left(\int_0^{\tau - \tau_{d2}} (1 - \int_0^{\tau_2} C(\tau_1) d\tau_1) d\tau_2 \right) d\tau & \text{if } t \geq \tau_{d2} + \tau_{d3} , \\ 0 & \text{otherwise .} \end{cases} \quad (5.30)$$

3. If the pulse taken in STOP1 spectrum is correlated with the start pulse, while the pulse taken in STOP2 spectrum is uncorrelated with the start pulse, the probability density of having an uncorrelated stop pulse in STOP3 spectrum at time t is

$$P_3^{C3}(t) = \begin{cases} U^2 e^{-U(t-\tau_{d2}-\tau_{d3})} \int_{\tau_{d2}}^{t-\tau_{d3}} \left(\int_0^{\tau-\tau_{d2}} C(\tau_1) d\tau_1 \right) d\tau & \text{if } t \geq \tau_{d2} + \tau_{d3} , \\ 0 & \text{otherwise .} \end{cases} \quad (5.31)$$

4. If the pulses taken in STOP1 and STOP2 spectra are both uncorrelated with the start pulse, the probability density of having a correlated stop pulse in STOP3 spectrum at time t is

$$P_3^{C4}(t) = \begin{cases} UC(t) e^{-U(t-\tau_{d2}-\tau_{d3})} \\ \times \int_{\tau_{d2}}^{t-\tau_{d3}} \left(\int_0^{\tau-\tau_{d2}} (1 - \int_0^{\tau_3} C(\tau_1) d\tau_1) \right. \\ \times \left(1 - \int_{\tau_3+\tau_{d2}}^{\tau} C(\tau_2) d\tau_2 \right) d\tau_3 \Big) d\tau & \text{if } t \geq \tau_{d2} + \tau_{d3} , \\ 0 & \text{otherwise .} \end{cases} \quad (5.32)$$

Hence the distribution of STOP3 can be written as

$$M_3(t) = (N - N_C) P_3^U(t) + N_C (P_3^{C1}(t) + P_3^{C2}(t) + P_3^{C3}(t) + P_3^{C4}(t)) . \quad (5.33)$$

One can verify that it satisfies the normalization condition

$$\int_0^\infty M_3(t) dt = N . \quad (5.34)$$

In principle any one of the three spectra can be used to extract the undistorted time-of-flight spectrum. However, in practice, because of the difficulty in estimating the dead time τ_{d2} and τ_{d3} of the electronics circuitry and the complexity of the expressions, we will therefore concentrate on the spectrum of STOP1.

The undistorted time-of-flight distribution is given by $N_C C(t)$. Equation (5.19) can be rearranged as

$$\begin{aligned} N_C C(t) &= NC'(t) \\ &= M_1(t) e^{Ut} - NU + U \int_0^t NC'(\tau) d\tau . \end{aligned} \quad (5.35)$$

This equation can be regarded as an integral equation of $NC'(t)$. One method of solving the equation is using an iteration method. We replace the integration in the equation by a summation, i.e.

$$NC'(t_i) = M_1(t_i) e^{Ut_i} - NU + U \sum_{j=0}^i NC'(t_j) \Delta t, \quad (5.36)$$

where

$$t_j = j\Delta t, \quad j = 0, 1, 2, \dots \quad (5.37)$$

Equation (5.36) is a good approximation of Eq. (5.35) for very small Δt compared to the range of the TDC. Therefore we have

$$NC'(t_i) = \frac{1}{1 - U\Delta t} \left(M_1(t_i) e^{Ut_i} - NU + U \sum_{j=0}^{i-1} NC'(t_j) \Delta t \right), \quad (5.38)$$

where $M_1(t)$ is the TDC spectrum of STOP1, N is the total number of measurements (i.e. the number of start pulses), U is the instantaneous rate in the stop counter which is on the order of $2.3 \times 10^6 \text{ sec}^{-1}$, and Δt is taken to be the time interval of one channel of the TDC, which is about 1 nsec.

For each spectrum N is based on all the counts in the spectrum including overflow counts and U is based on the counts in the muon counter and on the beam-gated running time recorded in the scalers for each run. The origin of the time is taken to be at channel 130 of the spectrum. A first check of Eq. (5.38) is to apply the equation to the spectrum for uncorrelated events shown in Fig. 4.11. The result shown in Fig. 5.9 is satisfactory. The average number of counts is slightly above zero due to the background. The error bars are computed with error propagation.

Applying Eq. (5.38) to the STOP1 spectra shown in Figs. 4.7, 4.8, and 4.12, the undistorted spectra of the time-of-flight are obtained and shown in Figs. 5.10, 5.11, and 5.12 respectively as solid dots with error bars. To compare these results with those obtained earlier with the requirement of having no more than one stop pulse in a time window, the spectra shown in Figs. 5.6, 5.7, and 5.8 are normalized to the same peak heights as those obtained using Eq. (5.38) and plotted in Figs. 5.10, 5.11, and 5.12 as histograms. The agreements are excellent.

Figure 5.9: Spectrum for uncorrelated events obtained by applying Eq. (5.38) to Fig. 4.11.

Figure 5.10: Time-of-flight spectrum of positive muons obtained by applying Eq. (5.38) to the STOP1 spectrum shown in Fig. 4.7. The accelerating voltage was 0.0 kV. The distribution is plotted as solid points with error bars.

Figure 5.11: Time-of-flight spectrum of positive muons obtained by applying Eq. (5.38) to the STOP1 spectrum shown in Fig. 4.8. The accelerating voltage was 19.0 kV. The distribution is plotted as solid points with error bars.

Figure 5.12: Time-of-flight spectrum of negative muonium ions obtained by applying Eq. (5.38) to the STOP1 spectrum shown in Fig. 4.12. The accelerating voltage was -17.0 kV. The distribution is plotted as solid points with error bars.

Finally, to demonstrate that Eqs. (3.12) and (3.14) can be used to describe the time-of-flight spectra, an undistorted spectrum taken at $HV = -24.2$ kV with the Al foil was obtained. It is shown in Fig. 5.13 as solid points with error bars. The

Figure 5.13: Time-of-flight spectrum of positive muons decelerated by -24.2 kV. The smooth curve is a fit to the data using function (5.39).

reason that this spectrum taken with decelerated positive muons is chosen is that it yields flatter energy distribution than those taken at other accelerating voltages due to the difference in the distribution in different energy regions. The function used for the least-squares fit is

$$N(n_{TDC}) = A \int_{l_c - l_t}^{l_c + l_r} P(\rho \leq r, p) ds, \quad (5.39)$$

where the integrand $P(\rho \leq r, p)$ is given by Eqs. (3.12) and (3.14). $l_c = 254$ cm is the central path length measured from the downstream mesh of the accelerator

to the MCP detector, $l_l = 6.4$ cm and $l_r = 10.4$ cm are introduced to account for the difference in path length due to the 60° bend. Assuming a uniform distribution within a small angle $\sqrt{\sigma_{22}}$, the average momentum p can be computed by

$$p_z = \frac{m_\mu s}{t_0 - 0.999n_{TDC}}, \quad (5.40)$$

and

$$p = \frac{2p_z}{1 + \cos\sqrt{\sigma_{22}}}. \quad (5.41)$$

The parameters l and l_i in Eq. (3.14) are taken to be 135 cm and 25 cm respectively. The parameter r in Eq. (3.12) is 3.8 cm, the radius of the MCP detector. The continuous curve shown in Fig. 5.13 is the least-squares fit to the data. The parameters in the fitting are found to be

$$\begin{aligned} A &= 720(65), \\ t_0 &= 892(1), \\ \sqrt{\sigma_{11}} &= 9.85(4) \text{ cm}, \\ \sigma_{12} &= -7.99(4) \text{ cm}, \\ \sqrt{\sigma_{22}} &= 0.854(4). \end{aligned} \quad (5.42)$$

The numbers are in reasonable agreement with the estimation made independently. However these results are only approximate, since they depend on the energy and angular distributions of the positive muons.

5.4 Rates of the Low Energy Positive Muons and the Negative Muonium Ions

From the undistorted time-of-flight spectra obtained by Eq. (5.38), the number of positive muons and negative muonium ions stopped at the MCP detector can be computed. The spectra can be divided into two regions. The number of counts was computed by integrating each spectrum over time from channel 339 to channel 538. A flat background computed by integrating the spectrum over time from channel 141 to channel 338 was subtracted from the spectrum.

The counting rates of the scintillator telescopes at various accelerating voltages with the Al foil are listed in Table 5.1 together with the projected rates at the focal plane where the MCP detector was located. The projected rates are computed by

Table 5.1: Average rates of μ_{inc}^+ , telescope rates, and projected rates of low energy positive muons at focal plane taken with Al foil at various accelerating voltages. The momentum of the incident positive muon beam was 9.75 MeV/c.

Accelerating Voltage (kV)	μ_{inc}^+ Rate (10^3 sec^{-1})	Telescope Rate ($10^{-6}/\mu_{inc}^+$)	Rate at Focal Plane ($10^{-6}/\mu_{inc}^+$)
21.0	132	0.8(3)	10.8(4.0)
20.0	104	3.2(3)	43.5(3.9)
19.0	132	4.7(3)	63.5(3.6)
18.0	133	7.9(3)	106.9(3.7)
15.0	129	10.0(4)	134.7(5.6)
12.0	132	11.0(5)	148.5(6.2)
6.0	121	12.8(9)	164.1(12.3)
0.0	121	11.2(2)	151.6(2.9)
-24.2	114	13.0(6)	175.4(7.4)

taking into account the detection efficiency of the scintillator telescopes (90%), the time window of the telescopes ($3.3 \mu\text{sec}$, 77.9%), the fractional solid angle subtended by the telescopes (16.2%), and the detection efficiency of the MCP detector (65%). The central momentum of the incident positive muon beam was 9.75 MeV/c.

The results of the measurements on the low energy positive muons performed with the Be foil and the Au foil are listed in Tables 5.2 and 5.3 respectively. The Be data were taken with a 12.63 MeV/c incident positive muon beam, while a 10.67 MeV/c positive muon beam was used for the Au foil.

The data taken with the negative muonium ions are analyzed using the same method as for positive muons. Table 5.4 lists the rates of the negative muonium ions at various accelerating voltages with the Al foil. The results for data taken at low spectrometer settings are also given here in Table 5.5. Because of different time-of-flight and of an extra 128-nsec delay for the stop signal the time window used here is from channel 231 to channel 431. The background levels are estimated based on the counts in the region from channel 141 to channel 230. These background

Table 5.2: Average rates of μ_{inc}^+ , telescope rates, and projected rates of low energy positive muons at focal plane taken with Be foil at various accelerating voltages. The momentum of the incident positive muon beam was 12.63 MeV/c.

Accelerating Voltage (kV)	μ_{inc}^+ Rate (10^3 sec^{-1})	Telescope Rate ($10^{-6}/\mu_{inc}^+$)	Rate at Focal Plane ($10^{-6}/\mu_{inc}^+$)
21.0	125	0.6(1)	7.4(1.8)
20.0	127	1.2(1)	15.8(1.9)
19.0	125	1.6(2)	21.3(2.4)
17.0	110	4.0(2)	53.9(2.9)
15.0	125	5.7(3)	77.2(3.7)
10.0	130	6.2(3)	83.4(4.5)
6.0	128	6.4(4)	86.6(5.5)
0.0	130	6.6(3)	89.1(4.7)

Table 5.3: Average rates of μ_{inc}^+ , telescope rates, and projected rates of low energy positive muons at focal plane taken with Au foil at various accelerating voltages. The momentum of the incident positive muon beam was 10.67 MeV/c.

Accelerating Voltage (kV)	μ_{inc}^+ Rate (10^3 sec^{-1})	Telescope Rate ($10^{-6}/\mu_{inc}^+$)	Rate at Focal Plane ($10^{-6}/\mu_{inc}^+$)
22.5	147	0.2(1)	1.9(0.9)
20.0	145	0.9(1)	12.7(1.8)
19.0	147	2.0(2)	27.5(2.7)
17.5	146	3.5(2)	47.4(3.4)
15.0	145	3.2(2)	43.4(2.8)
10.0	141	1.7(2)	23.6(2.3)
0.0	145	1.4(2)	19.5(2.0)

Table 5.4: Average rates of μ_{inc}^+ , telescope rates, and projected rates of negative muonium ions at focal plane taken with Al foil at various accelerating voltages. The momentum of the incident positive muon beam was 9.75 MeV/c.

Accelerating Voltage (kV)	μ_{inc}^+ Rate (10^3 sec^{-1})	Telescope Rate ($10^{-7}/\mu_{inc}^+$)	Rate at Focal Plane ($10^{-6}/\mu_{inc}^+$)
-14.00	137	3.24(0.78)	4.38(1.06)
-15.50	126	6.42(0.99)	8.67(1.34)
-17.00	129	10.40(0.41)	14.04(0.55)
-17.50	114	9.74(0.97)	13.14(1.30)
-17.75	105	8.90(0.65)	12.02(0.88)
-18.50	136	9.48(1.02)	12.80(1.38)
-20.00	133	4.13(0.89)	5.58(1.21)

Table 5.5: Average rates of μ_{inc}^+ , telescope rates, and projected rates of negative muonium ions at focal plane taken with Al foil at various accelerating voltages. The momentum of the incident positive muon beam was 9.75 MeV/c. The spectrometer fields were scaled down by a factor (1.415).

Accelerating Voltage (kV)	μ_{inc}^+ Rate (10^3 sec^{-1})	Telescope Rate ($10^{-7}/\mu_{inc}^+$)	Rate at Focal Plane ($10^{-6}/\mu_{inc}^+$)
-8.00	94	3.05(0.46)	4.11(0.62)
-8.00	129	4.15(0.41)	5.61(0.55)
-8.50	131	4.49(0.50)	6.06(0.67)
-8.75	121	4.16(0.24)	5.62(0.32)
-9.00	125	4.01(0.40)	5.42(0.55)
-9.50	128	3.73(0.40)	5.45(0.55)

levels were subtracted from the final results. An incident positive muon beam of 9.75 MeV/c momentum was used for these measurements.

The results on the negative muonium ions measured with the Be and Au foils are listed in Tables 5.6 and 5.7 respectively. The momenta of the incident positive muon beam were 12.63 MeV/c for the Be foil and 10.67 MeV/c for the Au foil.

Table 5.6: Average rates of μ_{inc}^+ , telescope rates, and projected rates of negative muonium ions at focal plane taken with Be foil at various accelerating voltages. The momentum of the incident positive muon beam was 12.63 MeV/c.

Accelerating Voltage (kV)	μ_{inc}^+ Rate (10^3 sec^{-1})	Telescope Rate ($10^{-7}/\mu_{inc}^+$)	Rate at Focal Plane ($10^{-6}/\mu_{inc}^+$)
-16.00	136	4.12(0.77)	5.57(1.04)
-17.50	145	4.21(0.80)	5.69(1.08)
-18.50	128	5.49(0.86)	7.41(1.16)
-19.00	139	4.84(0.80)	6.54(1.08)

Table 5.7: Average rate of μ_{inc}^+ , telescope rate, and projected rate of negative muonium ions at focal plane taken with Au foil. The accelerating voltage on the accelerator was -17.5 kV. The momentum of the incident positive muon beam was 10.67 MeV/c.

Accelerating Voltage (kV)	μ_{inc}^+ Rate (10^3 sec^{-1})	Telescope Rate ($10^{-7}/\mu_{inc}^+$)	Rate at Focal Plane ($10^{-6}/\mu_{inc}^+$)
-17.50	146	10.83(0.94)	14.6(1.3)

5.5 Monte Carlo Simulations

Because of the complicated nature of the transport system employed in the experiment it is desirable to have a Monte Carlo simulation for the system. A Monte Carlo computer code MUTRACE was, therefore, developed for this purpose. Its physical principles are described in Appendix A. The program simulates the processes of energy loss and straggling of positive muons and charge capture by positive muons

based on the data from hydrogen and of multiple scattering. It can also be used as a ray-tracing program for transport system. The main objective is to reproduce the experimental data using the simulation code. By comparison between the results from simulations and the results from experiment one can gain some understanding of the processes.

The phase space of the incident positive muon beam is described by

$$\begin{aligned}
 -3.0 \text{ cm} &\leq x \leq 3.0 \text{ cm}, \\
 -0.029 &\leq x' \leq 0.029, \\
 -2.3 \text{ cm} &\leq y \leq 2.3 \text{ cm}, \\
 -0.095 &\leq y' \leq 0.095, \\
 -5\% &\leq \delta \leq 5\%,
 \end{aligned} \tag{5.43}$$

where x and y are the horizontal and vertical displacements respectively, x' and y' are the tangents of the angles of the momentum of a particle with the beam axis in the local rectangular system, and $\delta = (p - p_0)/p_0$ represents the fractional deviation of the momentum p of the particle from the central momentum p_0 of the beam.

The experimental data for muonium production are easier to interpret than those of the low energy positive muons and the negative muonium ions, since the later were taken with more complicated channel. Therefore it is natural to study the muonium production first using the MUTRACE code. The results have been shown in Fig. 4.4 as a histogram, where a $0.8 \mu\text{m}$ Al foil was used as the production foil. The agreement is very good. The Monte Carlo simulations also give predicted muonium rates at the Al foil, which are shown in Fig. 5.14. Fig. 5.14 shows that the muonium formation probability is slightly less than 1%. As mentioned in Chapter 4 a measurement to observe the muonium formation with the Au foil gives a rate of $1.66(9) \times 10^{-5} / \mu_{inc}^+$ at the beam stop. The Monte Carlo simulation predicts a rate of $1.33 \times 10^{-5} / \mu_{inc}^+$ at the beam stop and the muonium formation probability of 1.8% at the foil. The fact that the rate at the beam stop for the Au foil is lower than that for the Al foil, while the formation fraction for the Au foil is higher than that for the Al foil is due to a wider angular distribution of the muonium atoms from the Au foil. We will discuss this effect in more detail later.

Figure 5.14: Muonium rates at the production foil predicted by Monte Carlo simulation with various momenta of the incident positive muon beam. The production foil was $0.8 \mu\text{m}$ Al.

The results of Monte Carlo calculations of the low energy positive muons corresponding to the data listed in Tables 5.1, 5.2, and 5.3 taken at the end of the 60° -channel can be generated. These are shown in Fig. 5.15. The solid points with error bars represent the experimental data listed in Table 5.1, 5.2, and 5.3. The simulations were performed in 1 kV steps and plotted as histograms. They agree with the experimental data better than a factor of two.

The energy distributions of the positive muons exiting the production foils are obtained by Monte Carlo simulations. The simulations are based on the experimental running conditions in which the central momenta of the incident positive muon beam are 9.75 MeV/c for the Al foil, 12.63 MeV/c for the Be foil, and 10.67 MeV/c for the Au foil, respectively. They are shown in Fig. 5.16. The distribution

Figure 5.15: Low energy positive muon rates at the MCP detector with various accelerating voltages.

Figure 5.16: Energy distributions of outgoing positive muons from (a) Al foil, (b) Be foil, and (c) Au foil calculated with the Monte Carlo code.

for the Be foil is peaked at higher energy than those for the Al and Au foils and is also wider. The distribution for the Au foil however is peaked at lower energy than those others. These characteristics confirm the observation made in Chapter 4.

The process of producing the negative muonium ions can also be simulated using the MUTRACE code. Again an incident positive muon beam of 9.75 MeV/c momentum was used for the Al foil. The results of the simulations are shown in Fig. 5.17 as a histogram. The solid points with error bars in the figure are taken from Table 5.4. The fact that the width of the distribution (4.5 kV FWHM) is comparable to the acceptance of the channel suggests that the energy distribution of the negative muonium ions is narrow (≤ 4.5 keV FWHM). The results of the

Figure 5.17: Rates of negative muonium ions at focal plane with Al foil. The histogram represents the Monte Carlo simulation.

Figure 5.18: Rate of negative muonium ions at focal plane with Be foil. The histogram represents the Monte Carlo simulation.

simulations for the Be foil are shown in Fig. 5.18 as a histogram. The momentum of the incident positive muon beam was 12.63 MeV/c for the Be foil. The experimental data listed in Table 5.6 are also shown in the figure as solid points with error bars.

Monte Carlo simulation also gives the rate at the focal plane for the Au foil with a 10.67 MeV/c incident positive muon momentum. The rate is $18.8 \times 10^{-6} / \mu_{inc}^+$ which is also in good agreement with the measured rate of $14.6(1.3) \times 10^{-6} / \mu_{inc}^+$ listed in Table 5.7.

Based on these simulations the projected formation fractions of negative muonium ions at the production foils can be obtained. They are listed in Table 5.8 together with some of the relevant experimental data.

The results for the negative muonium ions can be explained qualitatively as

Table 5.8: Projected formation fractions of the negative muonium ion at production foils.

	Foil thick. (mg/cm ²)	diam. (cm)	μ_{inc}^+		M ⁻ Rate at Focal Plane ($10^{-5}/\mu_{inc}^+$)		M ⁻ Formation Probability ($10^{-4}/\mu_{inc}^+$)
			$p_{\mu_{inc}^+}$ (MeV/c)	rate (10^3s^{-1})	Exp.	Monte Carlo	Monte Carlo
Be	4.6	7.6	12.63	145	0.57(11)	0.87	0.51
Al	0.2	7.6	9.75	129	1.40(6)	2.13	1.80
Au	3.8	6.4	10.67	146	1.46(13)	1.88	3.50

follows. The energy distribution of the outgoing particles from the Be foil is peaked at higher energy than for the Al and Au foils. Therefore the stopping density of positive muons at the surface of the foil is relatively low and hence the formation fraction of the negative muonium ions is lower. The energy distribution of the outgoing particles from the Au foil is peaked at lower energy and hence the formation fraction of the negative muonium ions is higher. Another interesting feature of the results in Table 5.8 is that the collection efficiency of the negative muonium ions, defined as the ratio of the rate at focal plane to the formation fraction, is not the same for these different foils. This is due to the fact that the angular distribution of outgoing particles from a foil with larger atomic number Z is expected to be wider than that from a foil with smaller atomic number due to the multiple scattering, as indicated in the Monte Carlo simulations. The angular distributions of the outgoing particles from Monte Carlo simulations are shown in Figs. 5.19, 5.20, and 5.21, respectively.

Finally, it should be mentioned that the Monte Carlo code, MUTRACE, is also capable of computing the time-of-flight of the particles. The result of one such simulation is shown in Fig. 5.22(a). The simulation is performed under the condition in which the spectrum shown in Fig. 5.10 was taken, i.e. the production foil is an Al foil, the momentum of the incident positive muon beam is 9.75 MeV/c, the accelerating voltage is 0.0 kV, and the bending magnet is set to accept positively charged particles. The data shown in Fig. 5.10 is plotted again in Fig. 5.22(b). The time axis has been reversed so that the physical time is from left to right. In both of these plots the main peak at about 470 nsec corresponds to about 2 MeV/c in

Figure 5.19: Angular distributions of (a) μ^+ , (b) M , and (c) M^- coming out of Al foil. The momentum of the incident positive muon beam was centered at 9.75 MeV/c.

Figure 5.20: Angular distributions of (a) μ^+ , (b) M , and (c) M^- coming out of Be foil. The momentum of the incident positive muon beam was centered at 12.63 MeV/c.

Figure 5.21: Angular distributions of (a) μ^+ , (b) M , and (c) M^- coming out of Au foil. The momentum of the incident positive muon beam was centered at 10.67 MeV/c.

momentum and the small peak at about 300 nsec, which is also due to the focusing effect of the solenoid, corresponds to 3.2 MeV/c (see Fig. 3.6). The momentum of the particle is first selected by the bending magnet and thus only part of the 3.2 MeV/c peak can be seen in the spectra.

Figure 5.22: Time-of-flight spectra of positive muons, (a) Monte Carlo, (b) measurement.

Chapter 6

Summary and Discussion

The negative muonium ion $M^- (\mu^+ e^- e^-)$ has been produced and observed for the first time. The ion was produced in a beam-foil experiment in which a subsurface positive muon beam of about 10 MeV/c momentum passed through a thin metal foil. The mass or the initial energy of the negative muonium ion was obtained by a time-of-flight technique. The mass of the ion is found to be 104 ± 8 MeV/c² in good agreement with the expected value of 106.7 MeV/c² for M^- . The initial kinetic energy of the ion is in the vicinity of 0.16(9) keV. This is consistent with the proton data and the velocity scaling rule. The lifetime of the ion is determined by the lifetime of the muon. The decay process was also observed in the experiment by measuring the energy and time spectra of the decay positrons.

The observed rate of the negative muonium ion at the spectrometer focal plane is about $10^{-5}/\mu_{inc}^+$ or 1.4 sec^{-1} (average). The production foils were placed in moderately good vacuum (3×10^{-6} Torr) and the last several atomic layers of the foils are believed to play an important role in charge capture processes. The formation fraction of the negative ions is expected to be the same for all foils. The slight difference in rates for Al, Be, and Au foils is likely caused by the different incident beam conditions and the range of the positive muons in the different foils.

The positive muons of a few keV to 20 keV coming out of the foils were studied. A rate of $1.5 \times 10^{-4}/\mu_{inc}^+$ or 20 sec^{-1} was observed with an acceptance of 4.5 keV (FWHM).

A Monte Carlo computer code, MUTRACE, was developed for this experiment (see Appendix A). It can reproduce essentially all our experimental data within a factor of two. The simulations were of substantial help in understanding the experimental data. The conversion probabilities for the M^- at the foils are about $10^{-4}/\mu_{inc}^+$. The differences in M^- rates for different foils are also demonstrated in the Monte Carlo simulation. From a careful study of the energy and angular distributions of the particles leaving the foils, these differences can be understood in terms of the different stopping distributions of the positive muons for different foils. Studies of the low energy positive muons leaving the foils indicate that the dE/dx is larger for high- Z material and the angular distribution is also wider. These are in agreement with general expectations.

The negative muonium ion itself is a very interesting system. A beam of such ions is now available for spectroscopy and collision studies. An interesting question to answer experimentally is whether the ions are polarized. In the $M^- \ ^1S_0$ ground state no depolarization will occur, but if muonium is formed in a first single e^- capture process, the μ^+ can be partially depolarized before the second e^- capture occurs.

Since the M^- is a charged particle, a beam of neutral muonium atoms can be generated from an M^- beam. The muonium beam would be useful for measurements of the Lamb shift of the muonium in the $n = 2$ state [Ora84,Bad84] and an experimental search for spontaneous conversion of muonium to antimuonium [Ni87]. Recently, advances have been made on a more intense muonium source in the thermal energy region [Mil86,Bee86]. This thermal muonium source would be very useful for the future experimental search for muonium to antimuonium conversion. However at present the fast muonium remains the practical source of muonium for a Lamb shift measurement. Therefore more careful studies of a fast muonium source would be beneficial for future Lamb shift experiments.

In the past it was often assumed that the angular distribution of the muonium atoms leaving a foil is isotropic because of a lack of experimental information. The Monte Carlo simulations developed for this experiment show that the angular distribution of the muonium atoms is considerably wider than that of positive

muons, since the muonium atoms have lower energy than do the positive muons. One might be able to take advantage of this characteristics of the angular distributions for Lamb shift experiments by using muonium atoms that are produced at an angle to the beam axis. An improvement on signal to background ratio of a factor of 30 to 95 can be achieved without much loss of signal rate (a factor of two at 30°), assuming that the 2S muonium atoms have the same distribution as the muonium atoms in all states. An improvement on Lamb shift measurement is therefore feasible with the apparatus used in the previous Lamb shift experiment. A detailed analysis will be given in Appendix A.

Although the present beam intensity for M^- is small ($\sim 1.4 \text{ sec}^{-1}$ average), increases of up to a factor of 10 could be achieved by optimizing the incident μ^+ channel and the accelerator-spectrometer arrangement. It is well known that the charge capture cross sections are larger for alkali metal vapors and alkali metal coated targets. One would expect a factor of 10 or more increase in formation of the negative muonium ions using such targets [Ehl80].

Positive muons of a few keV kinetic energy can also be utilized for making $M(2S)$. One of the effective ways to produce $H(2S)$ is the following process:

The cross section of this process is measured to be $10^{-15} \sim 10^{-14} \text{ cm}^2$ in the range from 5 to 10 keV kinetic energies of the incident H^+ [Sel67]. With the current setup a $M(2S)$ beam may be produced using the low energy positive muons. The intensity of this $M(2S)$ beam would be about 0.02 s^{-1} , using a Cs vapor target of $10^{11} \text{ atoms/cm}^2$, which is about an order of magnitude lower than that in the previous measurements of the muonium Lamb shift [Bad84]. Positive muons of even lower energies ($< 10 \text{ eV}$) emitted from LiF surface have been observed [Har86]. Unfortunately the intensity was also low ($\sim 10^{-7}/\mu_{inc}^+$). A more intense source of low energy positive muons with small phase space is needed for a dramatic increase in intensity of $M(2S)$.

Appendix A

MUTRACE

A.1 Introduction

Fast muonium (M or μ^+e^-) has been utilized in several experiments recently. Because of absence of the size effect of the nucleus, muonium has become an ideal system for testing Quantum Electrodynamics. The Lamb shift of muonium atoms in the $n = 2$ state has been measured [Ora84,Bad84], although improvements on the experimental precision are necessary for a sensitive test of the theory. A triple coincidence technique was used in these measurements. It first required a hit in the muon counter, subsequently a count in the Lyman- α detector, and then a hit in the microchannel plate (MCP). The timing of the signals depends on the time-of-flight of the muonium atoms in the 2S state. Thus some knowledge of the energy distribution of the muonium atoms is needed. A recent experimental search for muonium to anti-muonium ($\bar{M}$ or μ^-e^+) conversion gives a new upper limit on the branching ratio of the process [Ni87], which provides another way of testing the standard theory of the electroweak interaction [Gla61,Sal68,Wei67]. The signature of an anti-muonium atom is the detection of K_α and L_α x-rays from μ^- being captured by a high Z atom and a hit in a microchannel plate from secondary electrons created by the anti-muonium atom hitting the stopping target. Therefore the detection probability of such an event is dependent on the time-of-flight of muonium and anti-muonium atoms in the region of very low magnetic field. The ability to

compute the time-of-flight of muonium or anti-muonium atom was found to be very helpful in the data analysis. The muonium atoms with kinetic energy of several keV can be produced by a beam-foil method with a positive muon beam in vacuum environment [Boi81]. The processes involved are the energy loss of positive muons, multiple scattering of positive muons by target atoms and charge exchange between projectile and the target atoms. The mechanism of the processes are not fully understood especially in the intermediate energy region. However experimental data from hydrogen, with reasonable accuracy, is available. Computer simulation of these processes on phenomenological basis is therefore feasible. The formation of negative muonium ion (M^- or $\mu^+e^-e^-$, sometimes denoted as Mu^-) by double charge capture was also observed [Kua87]. It is very desirable to have a Monte Carlo type of computer simulation for helping us to analyze the data and to design new experiments. The physical principles of the simulation will be described in sections A.2, A.3, A.4, and A.5. In section A.6 some results will be presented and will be compared with experimental data. Finally, the application of the results will be discussed.

A.2 Energy Loss and Range of Positive Muons

The stopping power of positive muons depends significantly on their velocity. The theory of energy loss at high velocity, i.e. $v \gg Z^{2/3}\alpha c$ with v being the velocity of the projectile, Z the atomic number of the target material, and α the fine structure constant, is well established. The inelastic collisions of positive muons with atoms can be described by Bethe's formula [Fan63]

$$-\left(\frac{dE}{dx}\right)_H = \frac{4\pi e^4 N Z}{m_e v^2} \left(\ln \left(\frac{2m_e v^2}{I} \right) + \ln \left(\frac{1}{1-\beta^2} \right) - \beta^2 - \frac{C}{Z} - \frac{1}{2}\delta^2 \right), \quad (A.1)$$

where $\beta = v/c$, m_e is the electron mass, v is the velocity of the projectile, N is the density of atoms, I is the mean excitation energy which is defined as a logarithmic mean over the excitation energies E_n weighted according to the corresponding oscillator strengths f_n :

$$\ln I = \sum_n f_n \ln E_n, \quad (A.2)$$

and C/Z and δ^2 represent the so called “inner shell corrections” and “density effect correction”, respectively.

The energy loss in the low energy region ($v < \alpha c$) is evaluated using the Thomas-Fermi model of atom [Lin61],

$$-\left(\frac{dE}{dx}\right)_L = \frac{8\pi e^2 a_0 N Z v}{(1 + Z^{2/3})^{3/2} \alpha c}, \quad (\text{A.3})$$

where $a_0 = \hbar^2/m_e e^2$ is the Bohr radius of hydrogen atom.

The mechanism of energy loss in the intermediate region ($v \sim \alpha c$), where the charge exchange processes are favorable, is not well understood. The energy loss in this region is bridged by [Var70]

$$\left(\frac{dE}{dx}\right)_I^{-1} = \left(\frac{dE}{dx}\right)_H^{-1} + \left(\frac{dE}{dx}\right)_L^{-1}. \quad (\text{A.4})$$

Equations (A.1) and (A.3) tell us that the energy loss in both high and low energy regions depends mainly on the velocity of the projectile. We thus make a further assumption that dE/dx is the same for protons as for positive muons at equal velocity. The same methods as used previously [Bol82] are adopted in energy loss calculation. The data of the energy loss and range of hydrogen [And77] are used in the simulations. Bragg’s additivity rule is employed in computing the range for compounds, although the rule may not hold exactly at 10% level [Par63,Sau65].

The projected path length is calculated based upon the formulae given by Schiøtt [Sch66]. The ratio of the projected path length to the total path length is independent of material. A Gaussian straggling of 10% of the total path length is assumed in the range calculation.

A.3 Multiple Scattering

The multiple scattering of a heavy particle in matter is well described by Molière’s theory [Mol47,Mol48]. The distribution is characterized by two parameters, the unit probability angle χ_c and the screening angle χ_a . The detailed derivation is given by Bethe [Bet53].

The number of positive muons in the angular interval $d\theta$ after traversing a thickness l is

$$f(\theta, l) \theta d\theta = \frac{\theta d\theta}{\chi_c^2} \int_0^\infty y dy J_0\left(\frac{\theta y}{\chi_c}\right) \exp\left(\frac{1}{4}y^2\left(-b + \ln \frac{1}{4}y^2\right)\right), \quad (\text{A.5})$$

$$\chi_c^2 = \sum_i \chi_{ci}^2, \quad (\text{A.6})$$

$$\chi_{ci}^2 = \frac{4\pi N_i l e^4 Z_i^2}{p^2 v^2}, \quad (\text{A.7})$$

where N_i is the number of scattering atoms of the i th species per cm^3 , Z_i is the atomic number of the i th species in the material, p and v are the momentum and velocity of the positive muon, $J_0(x)$ is Bessel function of first kind, and the parameter b is defined as

$$b = \ln \frac{\chi_c^2}{\chi_a^2} + 1 - 2C, \quad (\text{A.8})$$

where $C = 0.577 \dots$ is Euler's constant.

By averaging over different constituencies, the parameter χ_a can be written as

$$\chi_a = \exp\left(\frac{1}{\chi_c^2} \sum_i \chi_{ci}^2 \left(\ln \chi_{ai} - \frac{1}{2} \frac{D_i}{Z_i}\right)\right). \quad (\text{A.9})$$

Based on the Thomas-Fermi potential, Molière gave a formula for χ_{ai}

$$\chi_{ai}^2 = \chi_{0i}^2 \left(1.13 + 3.76 \alpha_i'^2\right), \quad (\text{A.10})$$

$$\chi_{0i} = \frac{1.13}{137} Z_i^{1/3} (m_e c / p), \quad (\text{A.11})$$

$$\alpha_i' = Z_i e^2 / \hbar v. \quad (\text{A.12})$$

The Thomas-Fermi model is not suitable for low- Z substances [Han51]. A more precise expression for χ_{ai} may be obtained using atomic screening functions [Sal87]. However the approximation is sufficient for our purposes.

The second term in Eq. (A.9) is the correction of the scattering of the atomic electrons. The explicit expression for the parameter D_i is given by Fano [Fan54]

$$D_i = \ln \left(1130 Z_i^{-4/3} / \left(\frac{c^2}{v^2} - 1 \right) \right) - u_{ini} - \frac{1}{2} \frac{v^2}{c^2}, \quad (\text{A.13})$$

where $-u_{ini}$ can be calculated using the incoherent scattering functions. The typical value for $-u_{ini}$ is about 5.

By defining a new parameter B as

$$B - \ln B = b, \quad (\text{A.14})$$

the projected distribution (A.5) can be expanded as

$$f(\theta, l) \theta d\theta = \vartheta d\vartheta \left(f^{(0)}(\vartheta) + B^{-1} f^{(1)}(\vartheta) + B^{-2} f^{(2)}(\vartheta) + \dots \right), \quad (\text{A.15})$$

where

$$f^{(n)}(\vartheta) = \frac{1}{n!} \int_0^\infty u du J_0(\vartheta u) \exp\left(-\frac{1}{4}u^2\right) \left(\frac{1}{4}u^2 \ln\left(\frac{1}{4}u^2\right)\right)^n, n = 0, 1, 2, \dots \quad (\text{A.16})$$

and the reduced angle ϑ is given by

$$\vartheta = \frac{\theta}{\chi_c B^{\frac{1}{2}}}. \quad (\text{A.17})$$

Selected values of functions $f^{(0)}(\vartheta)$, $f^{(1)}(\vartheta)$, and $f^{(2)}(\vartheta)$ are tabulated in Bethe's paper [Bet53].

The theory is considered to be good for $B > 4.5$ and $\chi_c^2/\chi_a^2 > 20$ [Mol47, Mol48]. The scatterers considered in our simulations are mylar ($\text{C}_5\text{H}_4\text{O}_2$), scintillator (CH), beryllium (Be), aluminum (Al), and gold (Au) foils. The typical average momentum of the positive muon beam is about 6.5 MeV/c. The values of B and χ_c^2/χ_a^2 for 6.5 MeV/c positive muon in these substances are listed in Table A.1.

Table A.1: Typical B and χ_c^2/χ_a^2 values for the substances used in the simulations.

Material	Thickness (μm)	B	χ_c^2/χ_a^2
Mylar	1.5	7.3	231
Scintillator	19.1	10.1	2839
Beryllium	24.9	11.6	10755
Aluminum	0.8	5.9	71
Gold	2.0	5.4	47

A.4 Charge Exchange

The secondary beam from a thin metal foil consists mainly of positive muons (μ^+), muonium atoms (M or μ^+e^-) and negative muonium ions (M^- or $\mu^+e^-e^-$). Much experimental and theoretical work has been done on hydrogen [All58,Taw73]. The formation of neutrals and negatives is believed to occur in the last several atomic layers of the exiting surface of the foil. The statistical approach presented by Brandt and Sizmann [Bra75a] gives a fairly good approximation for the processes at high velocity where the formation of the negative ion is negligible. With the assumption of two components of the beam, the fraction of neutrals can be written as [Cha76]

$$\Phi_0 = \frac{1}{1 + \sigma_l/\sigma_c} , \quad (\text{A.18})$$

where σ_c and σ_l are the electron capture and loss cross sections, respectively, given by

$$\sigma_c = \pi a_0^2 \frac{2^{18}}{5} \frac{Z^5}{v_1^6 \left(v_1^2 + \frac{2^6}{\sqrt[3]{40}} Z^{14/9} \right)^3} , \quad (\text{A.19})$$

and

$$\sigma_l = \pi a_0^2 \frac{Z^{2/3}}{Z^{2/3} + v_1} \frac{4Z^{1/3}(Z+1)}{4Z^{1/3}(Z+1) + v_1} , \quad (\text{A.20})$$

where

$$v_1 = v/\alpha c . \quad (\text{A.21})$$

At low energy ($E_\mu \leq 5$ keV) all three components have to be considered, since the formation of the negative ions becomes appreciable. With the velocity scaling rule the hydrogen data of Phillips' [Phi55] are used for positive muons with energy below 21 keV. For positive muons with energy above 21 keV the formulae (A.18), (A.19), and (A.20) give a fairly good estimate on the fraction of neutral muonium atoms.

A.5 Transport of Charged Particles

The motions of positive muons and negative muonium ions, unlike neutral muonium atoms, are affected by electric and magnetic fields. This property provides us a

means of transporting and focusing the particles. Therefore it is useful to include the simulations of the effects of beam handling devices often used in the code. The formalism of TURTLE [Bro74] is adopted. The coordinates of each particle is described by a matrix

$$X = \begin{pmatrix} x \\ x' \\ y \\ y' \\ t \\ \delta \end{pmatrix}, \quad (\text{A.22})$$

where x and y are the horizontal and vertical displacements respectively, x' and y' are the tangents of the angles of the momentum of the particle with the beam axis in the local rectangular system, t is the time of flight measured from some reference point, and $\delta = (p - p_0)/p_0$ represents the fractional deviation of the momentum p of the particle from the central momentum p_0 of the beam. Each element in the beam is represented by a transformation T . It gives the output coordinates of the particle directly from the input, i.e.

$$X_f = TX_i. \quad (\text{A.23})$$

Some special beam handling devices, such as electrostatic accelerator and electromagnetic separator, often used in our experiments are implemented in the ray tracing as well as the effect of the finite lifetime of the particles, although the decay products are not traced. Ease of changing and adding new elements has been the first consideration of programming. Thus the code becomes very general. Transport of positive muons through thin foil is treated in three steps, energy losses, multiple scattering and charge exchange. The time-of-flight of the projectiles is computed for easy comparison with experimental results, since it is the quantity that we measure frequently in the experiments.

A.6 Application to the Experimental Data

The secondary beams from positive muons passing through a foil have not been carefully studied. However, the experiment described in Ref. [Kua87] provides useful information on the rates of the three species. Therefore we will limit ourselves only on the data from this experiment. The simulations were performed according to the experimental setup shown in Fig. 1 of Ref. [Kua87]. Figure A.1 is a tuning curve of the muonium rates at the Teflon beam stop for different momenta of incident positive muon beams. The solid points with error bars represent the data taking in the experiment in the straight channel of the apparatus [Kua87].

In the simulation the positive muon beams go through a gas barrier of $1.5 \mu\text{m}$ thick mylar, a $19 \mu\text{m}$ plastic scintillator and two $0.8 \mu\text{m}$ thick aluminum foils. The teflon beam stop is located 1.4 m downstream of the Al production foil, which

Figure A.1: Muonium rates at beam stop for different momenta of incident beam.

subtends a solid angle of 8 msr. The simulation code MUTRACE takes an initial phase space of

$$\begin{aligned}
 -3.0 \text{ cm} &\leq x \leq 3.0 \text{ cm}, \\
 -0.029 &\leq x' \leq 0.029, \\
 -2.3 \text{ cm} &\leq y \leq 2.3 \text{ cm}, \\
 -0.095 &\leq y' \leq 0.095, \\
 -5\% &\leq \delta \leq 5\%,
 \end{aligned}
 \tag{A.24}$$

which is a typical phase space of the subsurface positive muon beam in the stopped muon channel at LAMPF. The central momentum p_0 of the beam was varied in steps of 0.25 MeV/c around 10 MeV/c. The histogram plot represents the result of our simulation, which agrees very well with the experimental measurements. The simulations also predict the fractions of muonium at the production foil. The results are shown in Fig. A.2. An earlier report on the conversion factor was based

Figure A.2: Muonium rates at production foil for different momenta of incident beam.

on the assumption of isotropic distribution in angle which may not be well found [Bad84]. The maximum yield of muonium atoms occurs at momentum of incident positive muon beam of 9.75 MeV/c. The simulation shows that half of the positive muons are stopped in the material at this momentum, indicating that the stopping distribution is centered at the exit surface of the production foil.

We have also tried to reproduce the experimental data shown in Table II of Ref. [Kua87] with MUTRACE, which was taken with the channel of 60° bend. The experimental data are corrected for the efficiency (90%) of the scintillator telescopes which was ignored in Ref. [Kua87]. The data of both experimental results and our simulation are shown in Fig. A.3. The simulation is considered to be satisfactory.

The counting rates of negative muonium ions at the MCP located at end of the 60° channel are also generated with MUTRACE. They are listed in Table A.2 together with the experimental data from Table I of Ref. [Kua87] and the Monte Carlo prediction on the conversion probabilities at the production foils. They agree with each other very well.

Table A.2: Rates of negative muonium ions at MCP with different production foils. The experimental data are taken from [Kua87].

Foil	Rate at MCP ($10^{-5}/\mu_{inc}^+$)		Conversion Probability (Monte Carlo) ($10^{-4}/\mu_{inc}^+$)
	(Experimental)	(Monte Carlo)	
Beryllium	0.84(9)	0.87	0.51
Aluminum	1.47(7)	2.13	1.80
Gold	1.66(10)	1.88	3.50

The direct outcome of the simulations are the energy and angular distributions of the three species. Figure A.4 shows the energy distribution of the outgoing positive muons with an incident positive muon beam of 9.75 MeV/c and an Al foil. The distribution is peaked at 130 keV with a FWHM of 200 keV. The graph has been expanded in Fig. A.5 for energies below 40 keV. The distributions of muonium atoms and negative muonium ions are also shown in Fig. A.5. Most of the muonium atoms have kinetic energies less than 20 keV while there are few negative muonium ions above 5 keV, which is consistent with the experimental findings.

Figure A.3: Positive muon rates at the MCP beam stop for different accelerating high voltages with (a) Al, (b) Be, and (c) Au foils. The solid points with error bars are the experimental data from Ref. [Kua87].

Figure A.4: Energy distribution of positive muons from Al foil. The momentum of the incident positive muon beam was 9.75 MeV/c.

Figure A.5: Energy distributions of positive muons, muonium atoms, and negative muonium ions below 40 keV from Al foil.

The angular distributions reveal some interesting differences among the species. Figure A.6 shows the distributions of positive muons (a), muonium atoms (b), and negative muonium ions (c). The fluctuations of the distributions of the muonium atoms and the negative muonium ions at small angle are a statistical effect. One can clearly see that the outgoing positive muons are much more forwardly directed than the muonium atoms, since most of positive muons have higher energy than the muonium atoms. The previous experimental apparatus for the Lamb shift measurements was set up along the beam axis. The detectors such as the microchannel plate saw directly not only the positrons but also the positive muons. The Bremsstrahlung radiations from positive muons and positrons hitting the material around also lead to high counting rates in the microchannel plate and the Lyman- α detector. The random coincidence of the counters constitutes the major source of the background. One might be able to take advantage of this characteristics of the angular distributions for Lamb shift experiments by looking for muonium atoms off the beam axis. An improvement on signal to background ratio of a factor of 30 to 95 can be achieved without much loss of signal rate (a factor of two at 30°), assuming that the 2S muonium atoms have the same distribution as muonium atoms in all states. A better measurement on Lamb shift is therefore feasible. An experiment is being undertaken at LAMPF to study the energy and angular distributions of the positive muons and of the muonium atoms. This experiment will provide detailed information concerning the distributions.

It should be noted that even though the charge exchange processes are insensitive to the material used as charge exchange media, the processes are highly velocity-dependent. A high- Z material will result in high stopping density according to Eq. (A.1). Hence a high fraction of muonium atom and negative ion formation is expected. On the other hand the multiple scattering is clearly Z -dependent, since the unit probability angle is proportional to the atomic number Z . In fact a preliminary study indicates an isotropic angular distribution of muonium atoms from a Au foil. Therefore it is also interesting to investigate the formation fraction and, especially, the angular distribution of the charge states with a high- Z material such as a Au foil. Future experiments on Lamb shift measurements may benefit

Figure A.6: Angular distributions of positive muons (a), muonium atoms (b), and negative muonium ions (c).

from these studies.

A.7 Conclusion

A Monte Carlo code, MUTRACE, has been developed. It allows us to simulate the experimental conditions in a beam-foil experiment with an incident positive muons. The transport of the secondary beam is incorporated in the code. It has been used for several experiments utilizing the secondary beams [Kua87,Ni87]. Good agreement with experimental observation has been achieved. More careful experimental studies on these complex processes are desirable to further check the theories and improve the accuracy of the simulations.

A.8 Acknowledgement

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Appendix B

First observation of the negative muonium ion produced by electron capture in a beam-foil experiment

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(In Table I, the thickness of the Be foil has been misprinted: 7.6 mg/cm² should read 4.6 mg/cm².)