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The Efficiency of a Bremsstrahlung Interrogator

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Beam Research Program

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The Efficiency of a Bremsstrahlung Interrogator

Abstract

One can estimate the energetics of using gamma radiation produced by a high energy electron beam as a means of ascertaining the mass of distant objects. Several approaches to converting beam energy to radiation are compared, including a nonmaterial "bremsstrahlung" converter.

I. Scenario

This note provides an estimate of the efficiency of using a bremsstrahlung source driven by a high energy electron beam to ascertain the mass of distant objects. The geometry of the problem is illustrated in Fig. 1. Consider a primary electron beam of voltage $E[\text{MeV}]$ and total energy $\Sigma[J]$. This beam contains N_e electrons, where

$$N_e = (\Sigma / E) 1.6 \times 10^{13} \text{ electrons} . \quad (1)$$

Let the beam impinge upon a converter, which produces high energy ($> 2 \text{ MeV}$) gammas into an angle $\theta = (10\gamma)^{-1}$ radians with an efficiency $C(\theta)$. The gammas have a bremsstrahlung energy spectrum—equal energy per decade—with a mean value, $\epsilon_\gamma [\text{MeV}]$.

The high energy gammas travel without significant loss to illuminate a target of area $T[\text{m}^2]$ at a distance $R[\text{km}]$ from the source. In that case, the number of gammas striking the target is

$$\begin{aligned} N_\gamma &= (T / \pi(\theta R)^2) C(E / \epsilon_\gamma) (\Sigma / E) (1.6 \times 10^7) \\ &= (TE^2 / \pi R(\text{mc}^2)^2) C(\Sigma / \epsilon_\gamma) (1.6 \times 10^9) \\ &= (TE^2 / R^2) C(\Sigma / \epsilon_\gamma) (2.2 \times 10^9) . \end{aligned} \quad (2)$$

If the target is sufficiently massive or dense, the hard gammas will produce an electromagnetic shower yielding $[T B(\epsilon_\gamma / 0.511)]$ annihilation photons isotropically, where B is the bremsstrahlung production probability

Figure 1. Geometry for bremsstrahlung interrogator (where E = voltage, Σ = energy, T = target area, R = distance of target from source, r = distance at which annihilation photons are detected, and A = active area of the fast detector).

(proportional to target mass). The annihilation photons can be detected at a distance $r[\text{km}]$ with a fast detector of active area, $A[\text{m}^2]$, and efficiency, f . Assuming that the detector is matched to expected return flux so that dead-time losses are insignificant, one will record N_c counts, where

$$N_c = (A f 10^{-6} / 4\pi r^2) (1 \beta \epsilon_\gamma / 0.511) N_\gamma$$

$$= 350 (A \beta T E^2 \Sigma / r^2 R^2) (C_t f) . \quad (3)$$

For illustration, consider targets with $\beta T = 1$; hence,

$$N_c = (350 A E^2 \Sigma / r^2 R^2) \times (\text{efficiency product}) , \quad (4)$$

or

$$N_c < 350 (E / 1 \text{ GeV})^2 (1000 \text{ km} / r)^2 (\Sigma / 1 \text{ MJ}) (A / 1 \text{ m}^2) (1000 \text{ km} / R)^2 . \quad (5)$$

To determine whether this number of counts is usefully large, we must estimate typical values of the efficiency product, $C_t f$.

Computer simulations (Fig. 2) with radiation transport codes show that the hard gammas come from the first twentieth of a radiation length of converter with a distribution of energy roughly equally partitioned per decade ($> 10 \text{ keV}$). For the gammas to have an appreciable pair-production probability, their energy must exceed 1.2 MeV . Thus, the bremsstrahlung production efficiency in the converter is roughly

$$C \leq (1 - e^{-0.05}) (\log(E / 1.2 \text{ MeV}) / \log(E / 10^{-2} \text{ MeV}))$$

$$< 3 \% . \quad (6)$$

Typically, gamma detectors have efficiencies $f \geq 50\%$. This value could be appreciably lower if there are dead-time corrections.

In the target, pair production competes with Compton scattering as the energy loss mechanism for the gammas. The respective cross sections for these processes are equal for most materials for gamma energies in the 4- to 12-MeV range. The Compton cross section decreases inversely with the gamma energy. The pair-production cross section should scale with energy like the bremsstrahlung cross section, $\sim \ln E$, by time-reversal invariance. Thus, we can estimate t as

$$t \approx (\ln \epsilon_\gamma / \ln 4) / [(\ln \epsilon_\gamma / \ln 4) + (4 / \epsilon_\gamma)] . \quad (7)$$

The mean energy, ϵ_γ , is related to the beam energy and minimum energy cutoff, $\epsilon_\gamma \approx E / \log E_{\min} \approx E / 8 \approx 120 \text{ MeV}$ for $E = 1 \text{ GeV}$. At 120 MeV , $t = 0.99$; even at 15 MeV , $t = 0.88$. Therefore, for the entire cascade process we can take $t = 0.9$. Thus, the overall efficiency product,

$$C_t f \leq 0.015 . \quad (8)$$

Multiplying eq. (5) by $C_t f$, one can estimate that the number of counts registered in the detector will be

$$N_c < 5 (E / 1 \text{ GeV})^2 (1000 \text{ km} / r)^2 (\Sigma / 1 \text{ MJ}) (A / 1 \text{ m}^2) (1000 \text{ km} / R)^2 . \quad (9)$$

II. Calculations of Bremsstrahlung Angular Distribution

The major limitation to the performance of this interrogation scheme is the efficiency of generating bremsstrahlung in such a narrow cone. Though Monte Carlo computations of electron-photon transport with codes such as SANDYL and TIGER have had widespread acceptance in the calculation of radiation production, the codes are seldom used for such forward narrow angles. Therefore, results such as displayed in Fig. 2 should not be accepted at

Figure 2. Monte Carlo calculations of bremsstrahlung in central zone due to 1-GeV electrons.

face value. In traversing a converter of thickness X , a parallel beam of electrons will undergo multiple Coulomb scattering to acquire a mean square angular spread given by the Rossi-Greisen formula,

$$\langle \theta^2 \rangle = (21/E)^2 (X/X_{\text{rad}}), \quad (10)$$

where X_{rad} is the radiation length. The spread of the bremsstrahlung cone will be dominated by the multiple scattering of the primary electrons in the converter unless the converter is extremely thin. In this case eq. (10) grossly overestimates the width of the angular distribution of the small-angle scattering. A better approximation,¹ based on a fit to Moliere theory, is

$$\langle \theta^2 \rangle^{1/2} = (14.1/E) (X/X_{\text{rad}})^{1/2} [1 + (1/9) \log (X/X_{\text{rad}})]. \quad (11)$$

For the scenario described in Sec. I, eq. (10) predicts that small-angle Coulomb scattering broadens the angular distribution of the electrons to

$$\langle \theta^2 \rangle^{1/2} = (10\gamma)^{-1} \text{ for } X/X_{\text{rad}} = 5 \times 10^{-6}. \quad (12)$$

In contrast, eq. (11) predicts that the converter be ≈ 15 times thicker to produce this degree of angular broadening. Unfortunately, this thickness ($\approx 10^{-4} X_{\text{rad}}$) is still woefully small for efficient bremsstrahlung production. These results are graphically displayed in Fig.3, which includes the effect of radiation attenuation in the converter. Clearly, the predictions of eq.(10) or (11) are inconsistent with the calculations of Fig. 2.

Two simple calculations suggest the source of the discrepancy. First, the Rutherford scattering formula, upon which the fundamental scattering theory is based, does not hold for scattering angles less than the de Broglie wavelength of the electron divided by the radius of the converter atom. This limiting angular deflection is approximately

$$\Theta_{\min} = (Z^{1/3} / 137) (0.511 / E) \approx 1/30\gamma \quad (13)$$

for a high-Z converter. Scattering through $\Theta_{\min}$ produces an energy loss of

$$\Delta E = E (m / M) \Theta \approx 2.5 \gamma \Theta \quad (\text{eV}) \quad (14)$$

or

$$\Delta E_{\min} \approx 8 \times 10^{-3} \text{ eV} . \quad (15)$$

Figure 3. Bremsstrahlung intensity versus converter thickness with effects of multiple scattering and radiation attenuation included.

Moreover, scattering through $1/10\gamma$ leads to an energy loss of less than 0.1 eV. Consequently, we can see that the accurate computation of the radiation production from GeV electron beams at such small forward angles requires an energy resolution of 1 part in 10^{10} in the code. Such resolution is far from customary; typical energy resolution is 10 eV for high energy problems. Whether the requisite resolution is practical (or even possible) in standard codes is under investigation.

Using eq. (11) one can rewrite the scaling equations derived in Sec. I. The number of gammas produced in the forward cone is

$$\begin{aligned} N_\gamma &= (T/\pi(\theta R)^2) C (E/\varepsilon_\gamma) (\Sigma/E) (1.6 \times 10^7) \\ &= (T/\pi R^2) C E^2 (\Sigma/\varepsilon_\gamma) (8 \times 10^4) (X_{\text{rad}}/X) [1 + (1/9) \log(X/X_{\text{rad}})]^{-2} \\ &\approx (T/\pi R^2) C E^2 (\Sigma/\varepsilon_\gamma) (X_{\text{rad}}/X) (10^5). \end{aligned} \quad (16)$$

Therefore, the number of counts is

$$N_c < 6 \times 10^{-3} (E/1 \text{ GeV})^2 (1000 \text{ km}/r)^2 (\Sigma/1 \text{ MJ}) (A/1 \text{ m}^2) (1000 \text{ km}/R)^2 (X_{\text{rad}}/X) (Ct f) \quad (17)$$

Using the values for eq. (6) through (9) for $(X_{\text{rad}}/X) = 20$ to evaluate the efficiency product, $Ct f$, one obtains

$$N_c < 2 \times 10^{-3} (E/1 \text{ GeV})^2 (1000 \text{ km}/r)^2 (\Sigma/1 \text{ MJ}) (A/1 \text{ m}^2) (1000 \text{ km}/R)^2. \quad (18)$$

These arguments suggest that the predictions of eq. (9) may be overly optimistic by three orders of magnitude. If so, the remote bremsstrahlung interrogator will be energetically impractical unless one can either circumvent the problem of small-angle multiple scattering or find a target return which has restricted directionality.

III. Circumventing Small-Angle Scattering

A. Converters With Beam Cooling

Under certain circumstances² a focused electron beam that maintains its collective integrity in the converter can be made to produce radiation with an angular distribution roughly independent of beam depth within the converter. For this situation to obtain, the converter must be thin; that is, the beam should not lose a large fraction of its energy in the converter. Secondly, the rate of change of the mean angle of the electrons with depth due to scattering, eq (12) should be small in comparison with the betatron spatial frequency of the beam in the focusing field. In these circumstances, small-angle, multiple-Coulomb scattering still heats the beam slowly, but the beam expands adiabatically against the focusing field so as to keep the mean betatron angle constant. Thus, the actual angular distribution of the radiation, which is the convolution of the bremsstrahlung cone distribution with the betatron angular distribution, remains the same as the beam traverses the converter.

As an example, consider a beam that is self-focused in a gas converter. The beam radius, a , is related to the beam current, I , and energy by

$$a = (\varepsilon_n / \gamma) (17 \gamma / I [\text{kA}])^{1/2}, \quad (19)$$

where ε_n is the normalized beam emittance. If the current is partially neutralized or if focusing is provided by an external field, then I is replaced by an effective current. The quantity

$$\theta_\beta = (17 \gamma / I_{\text{eff}} [\text{kA}])^{-1/2} \quad (20)$$

is the rms betatron angle. For $E = 3$ GeV, the effective current must be ≈ 3 A for $\theta_\beta = 1/\gamma$. The betatron wavelength,

$$\lambda_\beta = a (2\pi / \theta_\beta) = 2\pi a \gamma, \quad (21)$$

will set the length of the converter. A gas converter of practical length should have a betatron wavelength less than 3 m; therefore, $a \approx 80 \mu\text{m}$. From eq.(19) the corresponding normalized emittance of the beam must be $\approx 1 \times 10^{-2}$ cm-rad, a small value, but one that is achievable with state-of-the-art rf guns and one that is common in storage rings. The corresponding focusing field in the converter is ≈ 80 G.

In this example, the converter thickness, X , must be such that

$$\begin{aligned} \langle \theta_{av}^2 \rangle^{1/2} / X &= (14.1 / E) (X X_{rad})^{-1/2} \{ 1 + (1/9) \log (X / X_{rad}) \} \\ < \theta_\beta / \lambda_\beta &\approx 6 \times 10^{-7} \text{ rad/cm}. \end{aligned} \quad (22)$$

As a high-Z converter, gaseous Xe at STP has a density of $5.8 \times 10^{-3} \text{ g-cm}^{-3}$ and a corresponding radiation length of 14.6 m. Passage of the beam through a 40-cm-wide jet of Xe at 0.2 Torr ($10^{-4} X_{rad}$) will increase the mean electron angle by $\approx 6 \times 10^{-7} \text{ rad/cm}$, satisfying eq.(22). If the gas jet is located in a storage ring of 25-m circumference, the beam will traverse $0.01 X_{rad}$ in $\approx 8 \mu\text{s}$. The beam will e-fold in size in a Nordseick length,

$$L_N = 0.135 (E / 1 \text{ GeV}) (I / 1 \text{ kA}) X_R (760 \text{ Torr/P}). \quad (23)$$

One finds, therefore, that the beam will expand to ≈ 10 times its original radius.

B. Converters With Reacceleration

A related way to maintain the narrowness of the radiation cone during the conversion process is to follow the gas-jet converter described in the previous section by acceleration cells as schematically depicted in Fig. 4. During its

Figure 4. Schematic of recirculating beam with bremsstrahlung converter and reaccelerating cell.

passage through the gas jet, the beam will suffer an increase of emittance due to gas scattering. The mean betatron angle will increase as

$$\theta \rightarrow \theta_1 = \theta + \Delta\theta = [\theta^2 + (14/E)^2(X/X_{\text{rad}})\{1 + (1/9)\log(X/X_{\text{rad}})\}^2]^{1/2}, \quad (24)$$

where the angle, beam radius, and beam emittance are related by

$$\varepsilon = (\varepsilon_n / \gamma) \theta a. \quad (25)$$

The consequent increase in normalized emittance is

$$\varepsilon \rightarrow \varepsilon' = \varepsilon_n (1 - \Delta\theta / \theta). \quad (26)$$

In contrast the bremsstrahlung emission during the passage of the beam through the converter will serve to cool the beam as the radiation is emitted tangent to the electrons' trajectory. In other words, bremsstrahlung leaves the un-normalized emittance invariant. Hence, if

$$\gamma \rightarrow \gamma (1 - X/X_{\text{rad}}), \quad (27)$$

then

$$\varepsilon \rightarrow \varepsilon_n (1 - X/X_{\text{rad}}). \quad (28)$$

The combined radiation and scattering process has

$$\varepsilon_n / \gamma \rightarrow \varepsilon_n (1 - X/X_{\text{rad}}) (1 - \Delta\theta / \theta) / (\gamma (1 - X/X_{\text{rad}})) \equiv \varepsilon_1' / (\gamma (1 - X/X_{\text{rad}})). \quad (29)$$

When the beam passes through the accelerating cell, it will receive an increment of energy,

$$\Delta E = \Delta \gamma m_e c^2. \quad (30)$$

As the acceleration process leaves the normalized emittance invariant,

$$\varepsilon_1' / (1 - X/X_{\text{rad}}) \rightarrow \varepsilon_1' / \gamma_2 \equiv \varepsilon_1' / \gamma (1 - X/X_{\text{rad}}) + \Delta\gamma / \gamma. \quad (31)$$

The incremental energy can be chosen so that the angle of the radiation is invariant or actually shrinks. For example, one can choose $\Delta\gamma$ so that

$$\varepsilon_1' / \gamma_2 = \varepsilon_n / \gamma, \quad (32)$$

in which case

$$\Delta\gamma / \gamma = (1 - X/X_{\text{rad}}) \{ [1 + (a\gamma/\varepsilon_n)^2 (28/\gamma) (1 + (1/9)\log(X/X_{\text{rad}}))^2]^{1/2} - 1 \}. \quad (33)$$

If X/X_{rad} is sufficiently small that $\theta_0 \gg \langle \theta_s^2 \rangle$, then one can rewrite eq. (31) as

$$\Delta\gamma / \gamma = (1 - X/X_{\text{rad}}) \{ (1/2)(a\gamma/\varepsilon_n)^2 (28/\gamma)^2 (1 + (1/9)\log(X/X_{\text{rad}}))^2 \}. \quad (34)$$

As in the previous section, let $\theta_0 = 1/\gamma$; then

$$\{ (1/2)(28)^2 (1 + (1/9)\log(X/X_{\text{rad}}))^2 \} < 1, \quad (35)$$

which in turn implies that $X / X_{\text{rad}} \leq 10^{-3}$. In this case, eq.(34) simplifies to

$$\Delta\gamma / \gamma \approx 390 (X / X_{\text{rad}}) (1 + (1/9)\log(X / X_{\text{rad}}))^2. \quad (36)$$

The incremental energy increase must be much larger than the incremental energy loss due to bremsstrahlung. If $X / X_{\text{rad}} = 10^{-4}$, for instance, $\Delta\gamma / \gamma \approx 1.2 \times 10^{-2}$. For the beam to pass through 0.05 radiation lengths of converter, the energy of a 1-GeV beam must be increased by ≈ 6 GeV to keep the emittance constant. This value is much larger than the original energy. A more reasonable approach is to begin with a higher energy beam and dispense with re-acceleration.

One might object that since beam radius will vary with beam energy, the acceleration should be chosen to keep the mean electron angle constant rather than the emittance. In that case, one sets

$$\theta_1 = \varepsilon_1 / (\gamma(1 - X / X_{\text{rad}}) + \Delta\gamma) \quad a_1 = \varepsilon_0 / \gamma a = \theta_0, \quad (37)$$

where a_1 is the new equilibrium radius. Combining eq. (36) and (24), one obtains

$$\Delta\gamma / \gamma = (a / a_1)(1 - X / X_{\text{rad}})[1 + (a\gamma / \varepsilon_n)^2(28 / \gamma)(1 + (1/9)\log(X / X_{\text{rad}}))^2]^{1/2} - (1 - X / X_{\text{rad}}). \quad (38)$$

Again, choose $\theta_0 = 1 / \gamma$; then for $X / X_{\text{rad}} \ll 1$, eq. (38) reduces to eq.(36) plus terms of order $(\Delta\gamma/\gamma)^2$.

During its trip around the ring, the beam will also cool due to the emission of synchrotron radiation. The fractional change in energy is given by

$$\partial\gamma / \gamma = 1.18 \times 10^{-14} (\gamma^3 / \rho), \quad (39)$$

where ρ [meters] is the radius of curvature of the ring. For the case discussed above, one can take a radius of 2 m, which leads to $\partial\gamma / \gamma = 5 \times 10^{-5} \ll \Delta\gamma / \gamma = 1.2 \times 10^{-2}$. As eq.(39) is a rapidly increasing function of energy, the beam energy will increase until

$$\partial\gamma / \gamma = \Delta\gamma / \gamma, \quad (40)$$

which occurs at $E \approx 6$ GeV. Thus, one concludes that re-acceleration of a beam passing through a tenuous converter is not a viable alternative to adiabatic cooling for producing radiation in a narrow cone.

C. Magnetic Bremsstrahlung Converters

A novel way to eliminate the effects of multiple scattering is to replace the ordinary material converter with a region of megagauss fields in which the beam rapidly decelerates by emitting incoherent synchrotron radiation. The large magnetic field need not be applied externally; it can be generated by the beam itself. This process, magnetic bremsstrahlung (beamsstrahlung), has the added advantage of radically transforming the energy distribution of the emitted radiation to one peaked at very high energy ($\gg 10$ MeV). Therefore, nearly all the gammas produced are capable of generating return signals from the target.

Compact accelerators (gradients exceeding 300 MeV/m) now under development for the high energy physics community will be capable of producing 10-GeV beams with a peak current of 500 A and with a self-focused beam radius of 1 μm . The confining pinch field of such a beam is 1 MG. The electrons in the confined beam will undergo betatron motion with a mean angle and wavelength

$$\begin{aligned} \theta_\beta &= 1.2 \times 10^{-3} \text{ rad}, \\ \lambda_\beta &= 0.52 \text{ cm}. \end{aligned} \quad (41)$$

The temporal frequency of the electron oscillations is

$$\Omega = c / \lambda_B = 5.7 \times 10^{10} \text{ Hz} . \quad (42)$$

Electrons experiencing such rapid oscillations will emit copious synchrotron radiation, thereby losing energy and shrinking in radius with an e-folding length,³

$$L_B = 8 \times 10^3 (a / 8.7 \mu\text{m})^2 (2 \times 10^4 / \gamma) (250 \text{ A} / I_{\text{eff}})^2 \text{ m} \\ = 28 \text{ m} . \quad (43)$$

The angular width of the synchrotron radiation is somewhat narrower than (approximately half) the betatron angle, which will expand by $\sqrt{e}$ in the process. For the electron beam to create the strong pinch field, the beam's space charge must be neutralized. The neutralization can be accomplished by injecting the beam into a tube of length, L_B , filled with low pressure gas with a large radiation length. Helium at ≈ 20 Torr will provide sufficient background ions to focus the beam, while constituting only 6×10^{-4} radiation lengths of scatterer. From eq. (11), one finds that the multiple scattering in the beamsstrahlung "converter" leads to a scattering angle of $15 \mu\text{rad}$, a value much smaller than the betatron angle.

The most startling feature of the beamsstrahlung converter is the hardness of the radiation which possesses a synchrotron spectrum⁴ (Fig. 5). The spectrum is characterized by a critical frequency above which little radiation is generated. The critical frequency, ω_c , is given by

$$\omega_c = 3 (\omega_0 \gamma^3) , \quad (44)$$

where ω_0 is the frequency of the electrons' motion in the laboratory frame. For the example, the betatron wavelength is 0.5 cm, corresponding to a frequency of 6×10^{10} Hz. From eq.(44) one obtains a critical frequency of

Figure 5. Synchrotron radiation spectrum (energy radiation per unit frequency interval) as a function of frequency.

1.4×10^{24} Hz. Recalling that $1 \text{ eV} = 2 \times 10^{14}$ Hz, one finds that the critical energy is $\approx 7 \text{ GeV}$. Although this value is nearly equal to the incident beam energy, the quantum mechanical correction to the distribution of Fig. 5 is not substantial as nearly all frequencies are kinematically allowed. The peak of the frequency distribution is at $\approx 1.5 \text{ GeV}$ with substantial radiation down to 50 MeV .

Thus, we see that the beamsstrahlung converter is very efficient at producing radiation, nearly all of which will produce many annihilation gammas. Moreover, the Compton-scattered radiation will have an energy distribution unlike artificial backgrounds that the interrogator may encounter.

For the beamsstrahlung source, the number of gammas at the target will be

$$N_\gamma = (TE / I_{\text{eff}} R^2) C (\Sigma / \epsilon_\gamma) (7 \times 10^8), \quad (45)$$

which will produce

$$N_C = (370 A E \Sigma / I_{\text{eff}} r^2 R^2) \times (\text{efficiency product}) \quad (46)$$

counts from the target. For this case, however, $C = 0.70$ and $t = 0.99$. Therefore, one can expect

$$N_C = 2.6(E / 10 \text{ GeV})(0.5 \text{ kA} / I_{\text{eff}})(\Sigma / 1 \text{ MJ})(A / 1 \text{ m}^2)(10^3 \text{ km} / r)^2 (10^3 \text{ km} / R)^2, \quad (47)$$

which on an energetics basis is only slightly inferior to eq. (9).

IV. Alternate Interaction Schemes

An evaluation of the energetic efficiency of radiation interrogation should include some comparison of alternate interaction schemes. One may increase the target return per unit by measuring an interaction with a larger cross section or with smaller background or by choosing an interaction for which the interaction products are dynamically or kinematically constrained to lie in a restricted solid angle. For example, the beamsstrahlung converter described above will produce copious gammas with energy above the pion production threshold. The decay of the neutral pions will produce a distinctive gamma ray signal at $\approx 70 \text{ MeV}$ where backgrounds are expected to be small. Alternatively, one could choose to measure the high energy Compton-scattered photons, which will be kinematically restricted in angular distribution.

To make such estimates, it is convenient to characterize the target by a path length, $L[\text{atoms-cm}^{-2}]$, which is the product of the mean number density of the target and its thickness. Note that L implicitly contains a measure of the density and mean atomic number of the target material. For illustrative purposes, consider $L \approx 10^{24} \text{ atoms-cm}^{-2}$. The beam-target interaction is characterized by a cross section, $\sigma[\text{barns}]$.

Pions are readily produced by the formation of the spin 3/2 nucleon resonance, Δ , at 1232 MeV ;

$$\gamma + N \rightarrow \Delta(1232) \rightarrow N + \pi. \quad (48)$$

In almost all cases in which a π^0 is produced, the pion will subsequently decay into two gammas. As can be seen from Fig. 6, the cross section for this process in the formation channel is $\approx 550 \mu\text{b}$. For this estimate, it is sufficient to scale the cross section to the case of a target atom with atomic number, A , by

$$\sigma_{\gamma A} = \sigma_{\gamma p} A^{2/3}. \quad (49)$$

Thus, the cross section for 300-MeV gammas to produce pions by striking a lead target is $\approx 20 \text{ mb}$.

The number of Δ 's formed is, therefore,

$$N_\Delta = N_\gamma \sigma L. \quad (50)$$

Figure 6. Gamma-proton total cross section versus energy.

Assuming that all the gammas have an energy of 300 MeV and evaluating N_γ from eq. (45) with the scale values shown in eq. (47), one can estimate the maximum number of pions produced in the target from eq. (50):

$$N_\pi = N_\Delta \approx 6 \times 10^8, \quad (51)$$

of which $\approx 30\%$ are π^0 .

Angular momentum and parity considerations lead to an angular correlation of decay products that is described by a third-order Legendre polynomial. Unfortunately, as the interrogation scheme relies on a single detector and as no preferential dynamical axis is established by, for instance, the polarization of either incident gamma or target nucleon, the distribution of the 70-MeV gammas will be isotropic in the center-of-momentum frame. In this frame, the momentum of the nucleon is only $\approx (1/3)m_N c$. Consequently, kinematic restriction of the angular distribution due to relativistic effects will be small, yielding at most a factor of two enhancement of the signal. One concludes that the signal from pion production, though distinctive, is too weak for efficient mass interrogation.

Examining Fig. 7, one sees that Compton scattering from high-Z atoms has a cross section appreciably larger than that for pion production. For 300-MeV gammas, the Compton cross section in lead is ≈ 300 mb whereas the pair-production cross section is ≈ 30 b. Therefore, one expects $\approx 10^{12}$ pair productions and $\approx 10^{10}$ Compton scatterings.

Although there are fewer Compton scatters than pair productions, the angular distribution of the scattered photons is constrained by energy conservation via the well known Compton formula,

$$E_0 = E_i / [1 + (m c^2)^{-1} (1 - \cos \theta)], \quad (52)$$

Figure 7. Contributions to photon cross sections in (a) carbon and (b) lead where: τ = atomic photo-effect, σ^{COH} = Coherent scattering, σ^{INCOH} = Incoherent scattering, κ_n = Pair production, nuclear field, κ_e = Pair production, electron field, and $\sigma^{\text{PH.N.}}$ = Photonuclear absorption.

where E_0 and E_i are the outgoing and incoming photon energies, respectively, and where $\varnothing$ is the scattering angle. The Klein-Nishina cross section for Compton scattering is

$$\Phi(E_i, E_0) dE_0 \approx (C mc^2 / E_i) (dE_0 / E_0) [1 + (E_0 / E_i)^2]. \quad (53)$$

Integrating eq. (53) from $mc^2 / 2$ to E_i , one can estimate the fraction of scatters with $E_0 > 20$ MeV as a function of E_i . For $E_i \sim 300$ MeV, this fraction is $\approx 40\%$. From eq.(52) one determines that for these events $\varnothing \approx 1$. The net result is an enhancement by a factor of five in the hard gamma signal; for $E_i = 1$ GeV, the enhancement is nearly two orders of magnitude. In that case, the signal from the Compton photons will equal that from annihilation photons. The potential backgrounds for the hard Compton photons, however, will be far smaller.

V. Conclusions

Analytic estimates indicate that Monte Carlo calculations of the efficiency of converting electron-beam energy in gamma radiation may grossly underestimate the effect of multiple small-angle scattering in enlarging the width of the radiation cone. To remedy this deficiency, one could introduce beam cooling during the conversion process or employ novel "beamsstrahlung" converters in which multiple scattering is virtually eliminated. The latter technique has the added advantage of hardening the gamma spectrum so that most of the energy is above 20 MeV. With such hard spectra available from the probe, the detection of hard gammas Compton scattered into restricted angular regions becomes a far more intriguing possibility than detection of annihilation events.

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VII. References

1. V. L. Highland, Nucl. Inst. and Meth. **129**, 497 (1975) and **161**, 171 (1979). The literature on the subject of bremsstrahlung production and small-angle scattering is extensive. See also Motz, Olson and Koch, Rev. Mod. Phys., **36**, 881 (1964) and J. Jacob, Phys. Rev. A, **8**, 226 (1973).
2. A similar analysis has been given by R. F. Hubbard and D. A. Tidman, Bremsstrahlung Radiation from High Current Electron Beams and Radiation Converter Devices, JAYCOR Tech Note 350-79-007, Sept., 1979.
3. E. P. Lee, Radiation Damping of Betatron Oscillations, Lawrence Livermore National Laboratory, Livermore, Calif., UCID - 19381 (1982).
4. J. D. Jackson, Classical Electrodynamics, 2nd edition, p.678