The contract work was initiated by J. Epstein. The emphasis of the proposal was on a) the continuation of DWIA calculations of the $\gamma, \pi^\pm$ reaction in the $\Delta(1232)$ region with the extension to the $\gamma, \pi^0$ reaction and b) the application of the DWIA approach to the $\gamma, K^\pm$ reaction. The $\gamma, \pi^0$ work was intended to be in close collaboration with the experimental studies underway at Bates Linear Accelerator and the $\gamma, K^\pm$ calculations are relevant to proposed high duty factor electron accelerators in the 1-4GeV region.

DWIA calculations were carried out for coherent $\gamma, \pi^0$ reactions in $^{12}$C. Although the DWIA approach seems less fundamental that the current $\Delta$-hole model calculations, we had hopes that it could be more readily applicable to incoherent reactions leaving the target nucleus in the excited state. Epstein hoped to improve the reliability of the DWIA calculations for coherent $\gamma, \pi^0$ production by better treatment of the $\Delta$ propagating in the nuclear medium.

In parallel with the $\gamma, \pi^0$ work, Epstein carried on an active collaboration with William Donnelly on the $\gamma, K^\pm$ problem. They had succeeded in a relatively complete description of the reaction for $^{12}$C, and were beginning to extend the results to heavy nuclei, when Epstein abruptly resigned his academic post at Boston University (in August 1983) and took an industrial position.

The present principal investigator, E. Booth, asked to be appointed in the expectation that he could interest a theorist with expertise in the field to finish the $\gamma, K$ calculations in $^{12}$C, and extend the work to heavy nuclei. In spite of assurances from two individuals that they could carry on such a program, in both cases, after the fact, they were unable to do so as a major commitment. However, one of these, Justus Koch of NIKEF, was able to turn his attention to the project during the late spring of 1984. Working together with Donnelly, and assisted by modest per diem funds from this grant, the $\gamma, K^\pm$ calculations were carried forward. A copy of the talk by Donnelly entitled “Photo-and Electron-Production of Kaons and the Study of Hypernuclei” given October 29, 1984 at BadHonnef, Germany is enclosed. The talk covers the results of the work initiated by Epstein and Donnelly, and completed by Koch and Donnelly.

The residual funds are to be returned to the Department of Energy.

DISCLAIMER

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PHOTO- AND ELECTRO-PRODUCTION OF KAONS AND THE STUDY OF HYPERNUCLEI

T.W. Donnelly

Center for Theoretical Physics
Laboratory for Nuclear Science and Department of Physics
Massachusetts Institute of Technology
Cambridge, MA 02139

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INTRODUCTION

The present summary represents an updating of the theoretical study begun by G.N. Epstein and myself /1/; this work is still in progress and is now being undertaken in collaboration with J.H. Koch. Our primary focus is on electromagnetic interactions with nuclei (involving either real or virtual photons), where kaons are produced and the final state is a bound hypernucleus. That is, we are interested in $X(\gamma, K)^\Lambda Y$ and $X(e, e'K)^\Lambda Y$ reactions involving nuclear targets $X$ and final-state hypernuclei $^\Lambda Y$ (or possibly $^\Sigma Y$), as indicated in Fig. 1.

![Diagrams for photo- and electro-production of kaons from nuclei (X) leading to hypernuclei (Y). The electro-production reaction, $X(e, e'K^\pm)^\Lambda Y$, is treated in the one-photon-exchange approximation.](image)

The basic features of such studies are indicated schematically in Fig. 2.
The elementary process in this specific example involves a proton target with the $(\gamma, K)$ or $(e, e'K)$ reaction initiating the hadronic transition $p \rightarrow \Lambda^0$. For the many-body situation shown in the lower part of the figure additional complications arise: (1) the same elementary process occurs, now possibly modified by the presence of the other nucleons in the nucleus; (2) the outgoing kaon can be rescattered in the nuclear medium before exiting from the final-state hypernucleus; (3) the initial and final states are not just free-space $p$ and $\Lambda^0$ wave functions, but now involve many-body nuclear and hypernuclear wave functions. We take as given a simple model for the elementary process (see the next section) and focus mainly on these three basically nuclear physics problems.

Before proceeding to specific results let us place the discussion in context by listing various hadronic and electromagnetic processes involving nucleon-to-hyperon transitions (see Table I).
### Table I. Elementary Processes involving Kaons and Hyperons

<table>
<thead>
<tr>
<th>(\Delta\Omega) <em>Baryons</em></th>
<th>Reactions</th>
<th>Nucleon-Hyperon Transitions</th>
</tr>
</thead>
<tbody>
<tr>
<td>+1</td>
<td>((\bar{K}^0, \pi^-), (\pi^+, K^0))</td>
<td>(n \rightarrow \Sigma^+)</td>
</tr>
<tr>
<td>0</td>
<td>((K^-, \pi^-), (\bar{K}^0, \pi^0), (\pi^+, K^+), (\pi^0, K^0), (\gamma, K^0))</td>
<td>(n \rightarrow \Lambda^0) (\rightarrow \Sigma^0) (p \rightarrow \Sigma^+)</td>
</tr>
<tr>
<td>-1</td>
<td>((K^-, \pi^0), (\bar{K}^0, \pi^+), (\pi^0, K^+), (\pi^0, K^0), (\gamma, K^+))</td>
<td>(p \rightarrow \Lambda^0) (n \rightarrow \Sigma^- p \rightarrow \Sigma^0)</td>
</tr>
<tr>
<td>-2</td>
<td>((K^-, \pi^+), (\pi^-, K^+), (\pi^-, K^0))</td>
<td>(p \rightarrow \Sigma^-)</td>
</tr>
</tbody>
</table>

*In the electromagnetic reactions here, \((\gamma, K^0,^+\), the "\(\gamma\)" can be a real photon or can be the virtual photon in electro-production, \((e, e'K^0', +\). The reactions of principal interest are underlined.

In each case the overall strangeness is assumed to be conserved and, since the relevant hyperons (the \(\Lambda\) and \(\Sigma\)) have \(S = -1\), the \(K + \pi, \pi + K\) or \(\gamma + K\) transitions have \(\Delta S = +1\). In particular, this means that the electromagnetic processes involve \(K^0\) or \(K^+\) mesons and not \(\bar{K}^0\) or \(K^-\) mesons. For convenience some of the properties of the particles involved in this study are collected in Table II.

### Table II. Particle Properties (from Ref. /2/)

<table>
<thead>
<tr>
<th>Particle</th>
<th>Mass (MeV)</th>
<th>(J^\pi)</th>
<th>(T)</th>
<th>(S)</th>
<th>Mean Life (sec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(p)</td>
<td>938.3</td>
<td>1/2(^+)</td>
<td>1/2</td>
<td>0</td>
<td>stable</td>
</tr>
<tr>
<td>(n)</td>
<td>939.6</td>
<td>1/2(^+)</td>
<td>1/2</td>
<td>0</td>
<td>898</td>
</tr>
<tr>
<td>(\Lambda^0)</td>
<td>1115.6</td>
<td>1/2(^+)</td>
<td>0</td>
<td>-1</td>
<td>2.63 \times 10^{-10}</td>
</tr>
<tr>
<td>(\Sigma^+)</td>
<td>1189.4</td>
<td>1/2(^+)</td>
<td>1</td>
<td>-1</td>
<td>0.80 \times 10^{-10}</td>
</tr>
<tr>
<td>(\Sigma^0)</td>
<td>1192.5</td>
<td>1/2(^+)</td>
<td>1</td>
<td>-1</td>
<td>5.8 \times 10^{-10}</td>
</tr>
<tr>
<td>(\Sigma^-)</td>
<td>1197.3</td>
<td>1/2(^+)</td>
<td>1</td>
<td>-1</td>
<td>1.48 \times 10^{-10}</td>
</tr>
<tr>
<td>(\pi^\pm)</td>
<td>139.6</td>
<td>0(^-)</td>
<td>1</td>
<td>0</td>
<td>2.60 \times 10^{-8}</td>
</tr>
<tr>
<td>(\pi^0)</td>
<td>135.0</td>
<td>0(^-)</td>
<td>1</td>
<td>0</td>
<td>0.83 \times 10^{-8}</td>
</tr>
<tr>
<td>(K^+)</td>
<td>493.7</td>
<td>0(^-)</td>
<td>1/2</td>
<td>+1</td>
<td>1.24 \times 10^{-8}</td>
</tr>
<tr>
<td>(K^-)</td>
<td>493.7</td>
<td>0(^-)</td>
<td>1/2</td>
<td>-1</td>
<td>1.24 \times 10^{-8}</td>
</tr>
<tr>
<td>(K^0)</td>
<td>497.7</td>
<td>0(^-)</td>
<td>1/2</td>
<td>+1</td>
<td>1.24 \times 10^{-8}</td>
</tr>
<tr>
<td>(K^{*0})</td>
<td>497.7</td>
<td>0(^-)</td>
<td>1/2</td>
<td>-1</td>
<td>1.24 \times 10^{-8}</td>
</tr>
<tr>
<td>(K^{*+})</td>
<td>892.1</td>
<td>1(^-)</td>
<td>1/2</td>
<td>+1</td>
<td>width = 51 MeV</td>
</tr>
<tr>
<td>(K^{*0})</td>
<td>896.5</td>
<td>1(^-)</td>
<td>1/2</td>
<td>+1</td>
<td>width = 50 MeV</td>
</tr>
</tbody>
</table>

\(K^{*0}\) width = 51 MeV

\(K^{*+}\) width = 50 MeV
Next consider how mesons interact with nucleons (and hence with nuclear matter) at the energies of interest here. In Fig. 3 several cases are shown schematically.

![Diagram](https://via.placeholder.com/150)

**Figure 3.** Schematic representations of $\pi^+ p$, $K^- p$ and $K^+ p$ interactions in terms of some of the quark configurations entering in microscopic descriptions of the mesons and baryons. Similar figures can be drawn for $\pi^+ N$, $\pi^- N$ and $\pi^0 N$ interactions (analogous to class a), for $K^- N$ and $K^0 N$ interactions (analogous to class b), and for $K^+ N$ and $K^0 N$ interactions (analogous to class c), where $N = p$ or $n$.

Clearly pions interact strongly with the nuclear medium and important states involving the $\Delta$ are to be found at intermediate energies. In an analogous fashion $K^-$ or $K^0 (S = -1)$ mesons interact strongly and form $\Lambda^*$ and $\Sigma^*$ intermediate states. On the other hand, $K^+$ and $K^0 (S = +1)$ mesons behave rather differently (compare diagrams b and c in Fig. 3) and so feel less strong interactions. This is reflected for example in the large mean free path of the $K^+$ in nuclei (= 3.5 fm, based on a $K^+ p$ total cross section of 18 mb at 1.5 GeV/c kaon momentum /2/). Hypernuclei are generally studied using the hadronic reaction $X (K^-, \pi^-)_\Lambda Y$ (also $\Sigma Y$) (see for example Refs. /3/), in which the initial $K^-$-nucleus and final $\pi^-$-hypernucleus interactions are both strong. This leads to a predominantly surface-peaked reaction mechanism in which substitutional transitions occur. Hadronic reactions such as $X(\pi^+, K^+)_\Lambda Y$ involve $K^+$-hypernucleus interactions in the final state and so need not be as surface dominated; however, they
still have the strong $\pi^+ - \text{nucleus}$ interaction in the initial state. Electromagnetic reactions such as $X(\gamma, K^+)_A Y$ have the advantage of the least distorting effects in both incident and exiting channels, but, of course, have the disadvantage of being intrinsically weaker in strength. Our primary goal in studying the electromagnetic processes as complements to hadronic production of hypernuclei is to exploit the lack of surface peaking to probe deep-lying (non-substitutional) states in a one-step, relatively well-understood reaction mechanism.

Let us conclude this introductory discussion by bringing out a few more differences between hadronic and electromagnetic production reactions. Due to the pseudoscalar nature of pions and kaons, forward-angle ($K^-, \pi^-$) and ($\pi^+, K^+$) reactions prefer to excite natural parity (non-spin-flip) states; ($\gamma, K^+$) reactions, on the other hand, excite non-natural parity (spin-flip) states as well. For lambda hypernuclei the ($K^-, \pi^-$) and ($\pi^+, K^+$) reactions involve the elementary transition $n + A^0$, whereas the ($\gamma, K^+$) reaction involves a transition $p + A^0$ (see Table I). This means that in typical nuclei with $N > Z$ the former excites both $T_>$ and $T_<$ states, whereas the latter excites only $T_>$ states. Finally the kinematics involved in ($K^-, \pi^-$), with an incident heavy particle and an exiting light particle, are different from ($\pi^+, K^+$) or ($\gamma, K^+$) where the situation is reversed. In the former it is possible the reach the zero momentum transfer (recoilless) situation, whereas in the latter two this is not the case. For example, in ($\gamma, K^+$) the minimum momentum exchange (i.e., the nuclear+ hypernuclear momentum difference) is attained when the angle between the photon and the kaon is zero. In a heavy nucleus we can neglect the recoil energy in the kinematics; even so the lowest exchange momentum possible is $M_A - M_p \approx 177$ MeV/c = 0.90 fm$^{-1}$ (neglecting binding energies) and this is approached only at high energies where the kaon rest mass can be neglected with respect to its momentum. As we shall see later, typical exchange momenta lie in the vicinity of 250 - 350 MeV/c and so permit the excitation of high-spin states in hypernuclei.
The general kinematical situation for kaon photoproduction is shown in Fig. 4.

Figure 4. Kinematics for the photo-production reaction $X(\gamma, K^+) \rightarrow Y$, where $X$ is the target nucleus and $Y$ is the produced hypernucleus. We take $Q$ to be the 3-momentum of the photon and $K$ to be the 3-momentum of the kaon; the 3-momentum exchange in the process (i.e., the difference between the 3-momenta of the hypernucleus and the nucleus) is then given by $T = Q - K$. In the laboratory system $T$ is just the 3-momentum of the hypernucleus. When used in the CM system all quantities will be indicated with asterisks: $Q^*$, $\theta^*_Y$, etc.

A few comments on conventions are in order here: 4-vectors are indicated by capital letters, $Q$; 3-vectors have arrows, $\vec{Q}$, and their magnitudes are indicated by lower-case letters, $q = |\vec{Q}|$; the metric and spinor conventions of Bjorken and Drell /4/ are employed and so $Q^2 = (Q^\mu Q_\mu) - q^2$; we take $\hbar = c = 1$ throughout.

Let us now focus on the elementary process $\gamma p \rightarrow \Lambda^0 K^+$ with the same kinematics as shown in Fig. 4 and now with $X = p$ and $Y = \Lambda^0$. The threshold for this process occurs at a laboratory photon energy of 911 MeV (although we shall not consider $\Sigma$ hypernuclei in the present overview, we note that the corresponding thresholds for $\gamma p \rightarrow \Sigma^0 K^+$ and $\gamma n \rightarrow \Sigma^- K^+$ occur at 1046 and 1052 MeV respectively). Following Thom /5/ we shall describe this process in terms of the Born diagrams shown in Fig. 5.

This should be viewed as an initial representation of the elementary process for use in the nuclear environment; in future extensions of this work it will be straightforward to include contributions which go beyond this simple Born description since the amplitudes used in our analysis have been cast in the general CGLN form /6/.
Figure 5. Born terms considered in describing the $\gamma p + \Lambda^0 K^+$ reaction. As exchanged particles we consider the following: $p(938)$, $\Lambda(1116)$ $\Sigma^0(1193)$, $K^+(494)$ and $K^{*+}(892)$ with properties given in Table II.

Let us outline the basic structure of the problem. The $T$-matrix for the process may be written in terms of invariant amplitudes $A_i$ and spinor matrix elements:

$$T_{\Lambda p} = \sum_{i=1}^4 A_i \left( \bar{u}_{\Lambda}(P') M_i u_{p}(P) \right),$$

where the initial proton has 4-momentum $P$ and the final lambda has 4-momentum $P'$, so that the exchange momentum is $T = P' - P = \Omega - K$ (see also Fig. 4). The Dirac $\gamma$-matrix structures which occur in the spinor matrix elements have the form

$$\begin{align*}
M_1 &= -\gamma_5 \left( \gamma \cdot \varepsilon \right) \left( \gamma \cdot \Omega \right) \\
M_2 &= 2\gamma_5 \left\{ \left( P \cdot \varepsilon \right) \left( P' \cdot \Omega \right) - \left( P' \cdot \varepsilon \right) \left( P \cdot \Omega \right) \right\} \\
M_3 &= \gamma_5 \left\{ \left( \gamma \cdot \varepsilon \right) \left( P \cdot \Omega \right) - \left( \gamma \cdot \Omega \right) \left( P \cdot \varepsilon \right) \right\} \\
M_4 &= \gamma_5 \left\{ \left( \gamma \cdot \varepsilon \right) \left( P' \cdot \Omega \right) - \left( \gamma \cdot \Omega \right) \left( P' \cdot \varepsilon \right) \right\},
\end{align*}$$

where $\varepsilon$ is the photon polarization vector. The photo-production cross section may then be written

$$\left( \frac{d\sigma}{d\Omega} \right)^*_{\gamma p + \Lambda^0 K^+} = \left\{ \frac{k}{q} \left| \langle \chi_f | F | \chi_i \rangle \right|^2 \right\}^*,$$

where $\chi_i$ and $\chi_f$ are initial and final Pauli spinors respectively and where the asterisk indicates that all quantities are evaluated in the CM system. The spin-space operator $F$ is given by
\[ F^* = \left\{ F_1 (\hat{\sigma} \cdot \hat{\epsilon}) + F_2 (i \hat{\sigma} \cdot \hat{u} \hat{\sigma} \cdot \hat{\epsilon} \times \hat{u}) \right\} + F_3 (\hat{\sigma} \cdot \hat{u} \hat{\sigma} \cdot \hat{\epsilon} \hat{u}) + F_4 (\hat{\sigma} \cdot \hat{u} \hat{\sigma} \cdot \hat{\epsilon} \hat{u}) \}^*, \quad (4) \]

where \( \hat{\sigma} \) is the Pauli spin operator. The CM amplitudes \( F^*_i \) are related to the invariant amplitudes by

\[
\begin{align*}
F^*_1 &= \frac{g^*}{4\pi} \left( \frac{E^*_\Lambda + M_\Lambda}{2W^*} \right)^{1/2} \left\{ A_1 - \frac{W^* + M_p}{2} A_3 - \frac{Q \cdot P'}{W^* - M_p} A_4 \right\} \\
F^*_2 &= \frac{g^*}{4\pi} \left( \frac{E^*_\Lambda - M_\Lambda}{2W^*} \right)^{1/2} \left\{ -A_1 - \frac{W^* - M_p}{2} A_3 - \frac{Q \cdot P'}{W^* + M_p} A_4 \right\} \\
F^*_3 &= \frac{k^*g^*}{4\pi} \left( \frac{E^*_\Lambda + M_\Lambda}{2W^*} \right)^{1/2} \left\{ -(W^* - M_p) A_2 + A_4 \right\} \\
F^*_4 &= \frac{k^*g^*}{4\pi} \left( \frac{E^*_\Lambda - M_\Lambda}{2W^*} \right)^{1/2} \left\{ (W^* + M_p) A_2 + A_4 \right\}, \quad (5) \\
\end{align*}
\]

where \( E^*_\Lambda \) is the lambda CM energy and where

\[ W^* = \left\{ M_p (2E_\gamma + M_p) \right\}^{1/2} \quad (6) \]

is the total CM energy given in terms of the laboratory photon energy. In turn the invariant amplitudes may be evaluated given a specific model for the process. Here with the Born diagrams shown in Fig. 5 we have a set of pole terms involving the s, t and u channels. For example, amplitude \( A_1 \) has a proton pole in the s-channel, \( K^+ \) and \( K^{*+} \) poles in the t-channel, and \( \Lambda^0 \) and \( \Sigma^0 \) poles in the u-channel with similar forms for amplitudes \( A_2 \), \( A_3 \) and \( A_4 \). In Thom's analysis /5/ the coupling strengths were adjusted to obtain a best fit to the existing photoproduction data. He concluded that the Born terms alone (with, in fact, couplings not too far from the SU(3) predictions) were already quite successful in reproducing the photo-production cross section for the first few hundred MeV above threshold, with effects of assumed resonance states occuring at only the 20\% level. We have repeated the calculations using only the Born terms with Thom's couplings and obtain the results shown in Figs. 6 and 7.

The reasonable success seen in this energy region provides us with a starting point for discussion of \( (\gamma, K) \) reactions in nuclei; at a later stage it will be straightforward to refine our analysis to have greater confidence in the predictions further away from threshold. In Fig. 8 the experimental total cross section is shown.
Figure 6. Differential cross sections for the process $\gamma p \rightarrow \Lambda^0 K^+$ at three angles shown as functions of the incident laboratory photon energy $E_\gamma = |\vec{\gamma}|$. The quantities with asterisks are given in the CM system. The solid curves result from using the Born terms (see Fig. 4) as described in the text; the data are taken from Ref. /7/.

Figure 7. As for Fig. 6, but now given for two photon energies as functions of CM angle $\theta_K^*$. 
Clearly our main focus for studies of hypernuclei will be at energies below $E_\gamma \sim 2$ GeV, where the cross section is maximum.

![Figure 8](image)

**Figure 8.** Total $\gamma p \rightarrow \Lambda^0 K^+$ cross section as a function of laboratory photon energy $E_\gamma$. The data are taken from Ref. /7/.

Finally it should be remarked that a similar analysis can easily be done for the $\gamma p \rightarrow \Sigma^0 K^+$ reaction; the experimental cross sections /7/ for that case are known to be comparable to the $\gamma p \rightarrow \Lambda^0 K^+$ results shown above.

**PHOTO-PRODUCTION OF HYPERNUCLEI**

Let us now consider photo-production of kaons from nuclei leading to hypernuclear final states (see Fig. 4). As we saw above from examining the $\gamma p \rightarrow \Lambda^0 K^+$ cross sections we expect to be dealing with laboratory photon energies of about $E_\gamma = 2$ GeV or lower. The next question in defining the relevant kinematical region of greatest interest is: what is the range of momentum transfer that is important? Recall that the exchange momentum is given in terms of the photon and kaon momenta by

$$\hat{T} = \hat{Q} - \hat{K}.$$ (7)
In calculating nucleus-to-hypernucleus matrix elements we have expressions of the following kind,

\[ M_{fi} \sim \langle \psi^i_{\text{hypernucleus}} | \phi^\dagger_{K^+} e^{i\hat{Q} \cdot \hat{x}} O_j | \psi^i_{\text{nucleus}} \rangle, \quad (8) \]

where \( O_j \) is a transition operator, possibly containing Pauli spin or gradient operators (see below). The plane-wave factor represents the incident photon and \( \phi^\dagger \) represents the out-going kaon. If the latter can also be taken to be a plane wave (i.e., neglecting distortion effects for the kaon), then these two factors combine to give

\[ \phi^\dagger_{K^+} e^{i\hat{Q} \cdot \hat{x}} = e^{-i\hat{K} \cdot \hat{x}} e^{i\hat{Q} \cdot \hat{x}} = e^{i\hat{T} \cdot \hat{x}}, \quad (9) \]

a single plane-wave factor containing the exchange momentum \( T \). Upon making the familiar multipole analyses as used in studies of the electroweak interaction in nuclei, we obtain form factors (now nucleus-to-hypernucleus transition form factors) which are functions of

\[ t = |\hat{T}| = |\hat{Q} - \hat{K}| = \left( q^2 + k^2 - 2ak \cos \theta_{K^+} \right)^{1/2}. \quad (10) \]

Just as with the more familiar nuclear electromagnetic form factors, typically the transition form factors which occur here may be large at intermediate values of \( t \), but beyond 350 - 400 MeV/c fall-off very rapidly with increasing exchange momentum. For the specific case of \( ^{12}\text{C}(\gamma, K^+)_\Lambda^{12}\text{B} \) (g.s.) we have the situation shown in Fig. 9. Clearly we begin to enter the interesting region for \( E_\gamma > 1.2 \text{ GeV} \) and, given the fall-off of the elementary cross section for \( E_\gamma > 2 \text{ GeV} \) (see Fig. 8), it would appear that our interest should be focussed primarily in the energy range defined by these two numbers. Note that the threshold on a heavy nucleus, where the recoil energy can be neglected, is much lower: \( E_\gamma \) (threshold) = 671 MeV - \( \Delta B_{\Lambda^0_p} \), where \( \Delta B_{\Lambda^0_p} \) is the difference in binding energy (final-state hyperon - initial-state nucleon). Analogous thresholds for \( X(\gamma, K^+)_\Sigma^0 Y \) and \( X(\gamma, K^+)_\Sigma^- Y \) are \( 746 \text{ MeV} - \Delta B_{\Sigma^0_p} \) and \( 751 \text{ MeV} - \Delta B_{\Sigma^-_n} \) respectively. The problem is clearly illustrated by Fig. 9, however - the exchange momentum near threshold is quite large and the cross section correspondingly small.
Figure 9. Exchanged momentum $t$ (defined in Eq. 10) versus angle $\theta_K$ for several photon energies for the ($\gamma, K^+$) reaction in the $A=12$ system. All quantities are in the laboratory system.

Let us continue with our specific example, $^{12}\text{C}(\gamma, K^+)_{\Lambda}^{12}\text{B}$. In Fig. 10 a schematic representation of a possible nucleus-to-hypernucleus transition in this system is shown. Here a proton in the target nucleus ($^{12}\text{C}$) is replaced by a lambda which may occupy any of the single-particle levels in its effective potential well. The contrast with charged-pion photo- and electro-production should be stressed: in those cases the baryonic transitions are $p+n$ or $n+p$ and the final-state nucleon must obey the Pauli Exclusion Principle when it tries to occupy single-particle levels in the final-state nucleus. This generally means that deep-lying levels are inaccessible for nucleons, whereas the hyperon in the present situation can go to the lowest level in its well.
Figure 10. Schematic representation of possible states involved in the reaction $^{12}\text{C}(\gamma, K^+)^{12}\Lambda^0\text{B}$. The initial-state nucleus $^{12}\text{C}$ is considered to have six protons and six neutrons occupying the ls and lp shells, and of course no lambda present. The final-state hypernucleus $^{12}\Lambda^0\text{B}$ has one less proton than $^{12}\text{C}$ and now has a lambda present. The specific configuration shown is $(ls)_A^1(lp)_B^0$; clearly other particle-hole configurations are formed in a similar way.

A summary of the procedures followed in going from the elementary amplitudes discussed in the previous section to the formalism needed in discussing the nuclear/hypernuclear problem is the following:

1. The amplitudes obtained in treating the reaction $\gamma p + \Lambda^0 K^+$ (see above) undergo a non-relativistic reduction where only the leading terms of order $(p/m)_{\text{barvon}}$ are retained. This leads to specific expressions for the quantities $O_j$ in Eq. 8. For example the form $\sigma \cdot \mathbf{E}$, familiar from studies of charged-pion photo-production, is one of those found for $(\gamma, K)$ reactions as well.

2. Multipole projections are made in the standard fashion (i.e., as in electroweak studies in nuclei), resulting in nine classes of multipole operators for use with nuclear and hypernuclear states which have specific angular momentum and parity quantum numbers.

3. Distorted waves are used for the outgoing kaon. We use solutions in the $K^+$ - nucleus optical potential previously also used by Dover and Walker /8/ in analysing $K^+$ scattering from nuclei.
4. The resulting complete operator (as in Eq. 8) is used in DWIA with model nuclear and hypernuclear many-body wave functions. In the present work we use simple particle-hole configurations (see below) and harmonic oscillator single-particle radial wave functions with a common effective oscillator parameter.

Let us begin a discussion of specific results obtained using these procedures by examining the $1s+1s$ proton-to-lambda transition in the $A=12$ system. In Fig. 11 the CM differential cross section is shown for several laboratory photon energies $E_\gamma$. Clearly something like 1.2 GeV or greater is needed before significant cross sections are reached. In Fig. 12 results are given for this same transition at $E_\gamma = 2$ GeV now with and without final-state kaon distortion effects (i.e., from the presence of the $K^+$-nucleus optical potential); for such light systems under these conditions the effects are seen to be small, typically at or below the 10% level. This should be contrasted with hadronic production of hypernuclei via the $(K^-, \pi^-)$ reaction where initial - and final-state meson-nucleus interaction effects typically attenuate the cross section (i.e., for other than valence substitutional transitions) by about an order of magnitude.

![Figure 11](image_url)
Figure 12. Photo-kaon differential cross sections at 2 GeV photon energy for a given transition in the A=12 system. Results from the calculation with distorted-wave kaons (see text) are shown as a solid line; results obtained using plane-wave kaons are shown as a dashed line.

Proceeding now to other transitions in the A=12 system, in Fig. 13 results are given for the ground-state 1^-/2^- multiplet in \^{12}_\Lambda B. As the upper part of the figure indicates, the ratio of transition strength to these two hypernuclear levels varies by about 50% in going from \( \theta_K = 0^\circ \) to 8°. Next in Fig. 14 are shown results for the lp\{lp valence substitutional transitions. Clearly there is considerable richness in the angle-dependence of the various partial cross sections. Two things should be noted here: (1) High-spin states, such as the 3^+ in the present case, have large cross sections, since the exchange momentum reached is optimum for exciting them; (2) the filling-in of diffraction zeros (the 1^+: (lp_{3/2})\_\Lambda (lp_{3/2})\_P case would have had a zero near 2° if plane-wave kaons had been employed) reflects the presence of the K^+-nucleus optical potential. Finally in Fig.
15 are shown transitions to more highly excited (unbound) configurations. All of this information is presented as a rough excitation spectrum in Fig. 16.

![Graph](image)

Figure 13. Photo-kaon differential cross sections at 2 GeV photon energy for configurations in the A = 12 system which likely constitute the major part of the ground-state multiplet [(ls\(_{1/2}\))(lp\(_{3/2}\))]\(_{\Delta}^{-1}\). The upper part of the figure contains the ratio of strength in the two transitions.

For orientation it should be noted that a cross section of 0.1 \(\mu b\) sr\(^{-1}\) leads to a counting rate under optimum conditions on the order of 100 counts / hr for 1 MeV resolution in the hypernucleus /1/ with
correspondingly lower rates for better resolution. To resolve the interesting structural differences seen here probably entails resolutions of about a few x 100 keV.

ELECTRO-PRODUCTION OF HYPERNUCLEI

Let us now turn to a brief discussion of the coincidence reaction \((e, e' K^+)\) as a means of producing hypernuclei (see Fig. 17). The cross section may be written quite generally in the form /9/
Figure 15. As for Fig. 13, but now displaying higher-lying (likely broad, unbound) strength involving \((1p)_L(1s)^+_p\) configurations. 

\[
\left(\frac{d^3\sigma}{d\Omega_e d\Omega_K dE_K}\right)(e, e'K^+);
\]

\[
= \frac{1}{M_i} \sigma_M \left\{ v_L W_L + v_T W_T + v_{TT} W_{TT} \cos 2\phi_K + v_{TL} W_{TL} \cos \phi_K \right\},
\]  

(11)

where the Mott cross section is given by 

\[
\sigma_M = \left(\frac{\alpha \cos \theta e/2}{2\varepsilon \sin^2 \theta e/2}\right)^2,
\]

(12)

with \(\varepsilon\) the initial and \(\varepsilon'\) the final electron energies,
\begin{align*}
\nu_L &= (Q^2/q^2)^2 \\
\nu_T &= -\frac{1}{2} (Q^2/q^2) + \tan^2 \theta_e / 2 \\
\nu_{TT} &= \frac{1}{2} (Q^2/q^2) \\
\nu_{TL} &= \frac{1}{\sqrt{2}} (Q^2/q^2) \sqrt{-(Q^2/q^2) + \tan^2 \theta_e / 2}
\end{align*}

By varying these kinematical factors and using the explicit dependence on the kaon azimuthal angle \( \phi_K \), it is possible to isolate experimentally the four hadronic response functions \( W_L, W_T, W_{TT} \) and \( W_{TL} \) where \( L \leftrightarrow \) longitudinal and \( T \leftrightarrow \) transverse (referred to the direction \( \vec{Q} \) in Fig. 17). In the most ambitious experiment we will wish to use these kinematical "knobs" to map out the four response functions; for the present, however, we will specialize the discussion somewhat. We know from our previous treatment of photo-production that the exchange momentum \( t \) in Eq. 10 must be reasonably small for the cross section to be large. The off-shell nature of the virtual photon in electroproduction is unfavorable in keeping \( t \) small and so we are led to require almost real photons to obtain significant \( (e, e'K^+) \) cross sections. That is, we want \( q = \omega \) or equivalently \(-Q^2 = q^2 - \omega^2\) to be small; cast in terms of a dimensionless parameter, we want

\[ n \equiv \sqrt{-Q^2/q^2} = \sqrt{1 - (\omega/q)^2} \tag{14} \]

to be small. With this in mind we can rewrite Eq. (11) in the form

\[ \left( \frac{d^3 \sigma}{d\Omega_d d\Omega_K dE_K} \right)_{(e, e'K^+)} = \frac{1}{M_i} \left( \frac{\sigma'}{\sigma} \right) \left( \frac{\alpha}{q \tan \theta_e / 2} \right)^2 \\
\times \left\{ n^2 W_L - \frac{1}{\sqrt{2}} \left( 1 + \frac{q^2}{4 \epsilon \epsilon' \cos^2 \theta_e / 2} \right)^{1/2} n W_{TL} \cos \phi_K \\
+ \frac{1}{2} \left[ \left( 1 + \frac{q^2}{2 \epsilon \epsilon' \cos^2 \theta_e / 2} \right) W_T - W_{TT} \cos 2\phi_K \right] \right\}. \tag{15} \]

For \( n \ll 1 \) we can safely neglect the first two terms involving \( W_L \) and \( W_{TL} \), leaving only purely transverse response functions. Furthermore, the \( \theta_K \) - dependences which survive quite generally have the following forms when \( \theta_K \) is small:

\[ W_T \sim a + b \sin^2 \theta_K + ... \]
\[ W_{TT} \sim c \sin^2 \theta_K + ... \]  \( \tag{16} \)
and so $W_{TT}$ can be neglected as well. This allows us to write a simple relationship between the almost-real-photon electro-production cross section and the photo-production cross section discussed above:

$$
\left( \frac{d^3\sigma}{d\Omega_e d\Omega_K dE_K} \right)_{(e, e'K^+)} \begin{cases} 
|Q^2/q^2| \ll 1, 
\sin^2 \theta_K \ll 1 
\end{cases}
$$

$$
= \frac{\alpha}{8\pi^2} \left\{ 1 + \left( \frac{e'}{\epsilon} \right)^2 \right\} \frac{1}{E_{\gamma} \sin^2 \theta_{K}/2} \cdot \left( \frac{d\sigma}{d\Omega} \right)_{(\gamma, K^+)} ,
$$

(17)

where $E_{\gamma} \rightarrow q = \omega = \epsilon - \epsilon'$. We shall use this relationship throughout.

---

**Figure 17.** Kinematics for the reaction $X(e, e'K^+)\gamma$ to be compared with Fig. 5. The z-axis is chosen to be along $\vec{Q}$, the virtual-photon direction; the y-axis is chosen to lie along $\vec{K} \times \vec{K}$, the normal to the electron scattering plane; the x-axis is then given by $\vec{u} = \vec{u}_x \times \vec{u}_z$ and lies in the electron scattering plane as shown. The kaon is then presumed to be detected in a direction specified by the angles $\theta_K$ and $\phi_K$ as indicated.

For simplicity (and practicality) the coplanar geometry shown in Fig. 18 will be adopted. Let us now examine what range of exchange momentum $t$ is found under typical conditions (see Fig. 19). Several criteria must be met in practical experiments: (1) the angles $\theta_e$ and $\theta_K$ must be kept reasonably small or else $t$ becomes large and the cross sections correspondingly small; (2) the angles cannot be too small, on the other hand, due to practical physical limitations on the electron and kaon detectors; (3) the final electron energy $\epsilon'$ should not be too small compared to the incident beam energy $\epsilon$ or
Figure 18. Specialization of the general kinematics in Fig. 17 to coplanar geometry (compare also with Fig. 5). The angle $\theta_K$ specifies the kaon direction with respect to the virtual-photon direction; the angle $\theta_K^*$ specifies the kaon direction with respect to the incident electron direction, as indicated. ($\theta_K^* < 0$ corresponds to the kaon being detected on the opposite side of the beam direction from the scattered electron).

else the singles rates involved (which appear in the denominator of the true/accidental ratio) will grow uncomfortably (in the figure the reference point $e' = e/2$ is indicated); the kaon should have enough momentum or else it will be difficult to detect as it decays (in the figure the kaon momenta for case b with $\theta_e = 10^\circ$ are given). Putting all of this together we are led to electron energies of $e \sim 3-4$ GeV as representative of where the above criteria can be met. Going against this is the stringent requirement on resolution which becomes more and more of a problem as the energy is increased: the interesting hypernuclear physics is accessed when the energy balance in the reaction can be done to a few $\times$ 100 kev.

In Figs. 20 and 21 the $(e, e'K^+)$ cross sections are shown for fixed kaon kinematics varying $\theta_K^*$ and fixed kaon kinematics varying $\theta_e$, respectively.
Figure 19. Exchanged momentum $t$ (defined in Eq. 10) versus kaon angle $\bar{\theta}_K$ (see Fig. 18) for a variety of incident electron energies $\epsilon$ and electron scattering angles $\theta_e$. The solid lines indicate that the scattered electron energy $\epsilon'$ satisfies $\epsilon' > \epsilon/2$, whereas the dashed lines correspond to the regime where $\epsilon' < \epsilon/2$. For $\theta_e = 10^\circ$ the kaon 3-momentum $k = |\vec{k}|$ is also indicated.
Figure 20. Multiple-differential cross sections for the reaction $^{12}\text{C} (e, e' K^+) ^{12}\text{B} (1^- \text{g.s.})$ for different fixed electron kinematical conditions as functions of the kaon angle $\theta_k$. The special relationship to the real-photon cross section (Eq. 17) has been employed in obtaining these results.

From considerations of possible future tagged $\gamma$ or coincident $(e, e' K^+)$ measurements /10/, it would appear that event rates of a few counts/hr with true-to-accidental ratios of 1:1 or better may be expected.

SUMMARY

From our theoretical discussions up to the present it appears that electromagnetic reactions involving the production of kaons and leading to bound hypernuclear final states hold promise as a developing area of high-energy nuclear physics. Experimental studies will not be easy, however; they will require high-intensity photon and
electron beams in the several GeV region and will only develop into interesting programs (as against a few special experiments) if resolutions of a few x 100 keV in the hypernucleus can be achieved. Much remains to be done theoretically. For example, while distortion effects on the out-going kaon were seen not to be especially important for light nuclei such as the $A=12$ system, they must be for heavy systems. The situation indicated in Fig. 22 is potentially a very interesting case of this type.
Figure 22. Schematic representation of extreme nucleus-to-hypernucleus transitions proceeding directly from a high principal quantum number configuration (here $N=5$) to deep-lying final states ($N=0,1,\ldots$).

Here a direct transition is made from a valence proton level to very deep-lying hyperon levels; the hyperon in turn now probes the interior of the nucleus in an uncommon way - as a baryon which unlike the nucleon has a special tag, namely, non-zero strangeness. Studies of such heavy systems with kaon distortions included are now in progress.

Other aspects of the general problem remain to be explored more fully. For instance, electromagnetic production of $\Sigma$ hypernuclei should have cross sections similar to the $\Lambda$ cases discussed here. Also $K^0$ production may be of interest (see Table I); in this case the final mesonic state has both $K^0_L$ and $K^0_S$, where the latter decay very rapidly, while the former are relatively long-lived. If a practical detector could be devised which exploits the neutral-kaon regeneration idea, then $(\gamma, K^0)$ reactions would be of interest as well as the $(\gamma, K^+)$ processes that we have focussed on in the present work.

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