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PLANE AND TOROIDAL CONDUCTING
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**PLASMA PHYSICS
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TRANSIENT EDDY CURRENTS ON FINITE PLANE AND TOROIDAL CONDUCTING SURFACES

by

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ABSTRACT

This report applies a previously presented mesh analysis method to calculate transient eddy currents in conducting surfaces. Example calculations are presented for a planar conducting sheet of finite dimensions and also for a toroidal conducting surface which represents the vacuum vessel of the TFTR.

For the toroidal sheet, branch inductances are initially calculated by numerically integrating the vector potential function, then the branch matrix is transformed into a mesh matrix. For the flat sheet, an analytic expression is given which enables direct calculation of the mesh inductance matrix.

Streamline plots of the eddy current distributions are shown at successive time steps for each example.

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I. INTRODUCTION

Accurate prediction of transient eddy currents which arise from external magnetic field variations has become important in the design of magnetically confined fusion experiments. We shall confine our attention to eddy currents in conducting surfaces in this report. In particular, we present example calculations for a planar conducting sheet of finite dimensions and also for a toroidal conducting surface which represents the vacuum vessel of the TFTR.

This report is a companion to U. R. Christensen's theoretical presentation.[1] We calculate only transient eddy currents here although applications of the method which we use to a broader variety of problems are given in the companion paper.

Basically for both the flat sheet and the toroidal sheet we divide the surface into a network of rectangular or nearly rectangular meshes. Each mesh is composed of branches which have resistance and inductance, both self and mutual. The resulting branch resistance and branch inductance matrices are transformed into mesh matrices by a conventional lumped parameter network procedure.

The mesh matrices establish a set of simultaneous differential equations which, expressed in matrix form, is

$$\{\dot{I}\} = [L]^{-1} \left\{ \{V\} - [R]\{I\} \right\} . \quad (1)$$

where

- $I \equiv$ mesh current
- $\dot{I} \equiv$ derivative of I with respect to time
- $L \equiv$ mesh inductance
- $R \equiv$ mesh resistance
- $V \equiv$ mesh emf

We found the mesh currents, I , by solving Eq. (1) numerically using a Runge-Kutta technique. After calculating the mesh current matrix we can transform to the branch current matrix. The branch currents are needed for calculations such as parasitic magnetic fields and power dissipation.

Although rather large networks can be solved using this procedure, particularly if advantage is taken of symmetry, we have been limited by computer core size. Our present technique requires the entire inductance matrix in core simultaneously in order to invert the matrix. If this restriction could be removed, even larger systems could be solved.

The primary difficulty in actually applying this method is the calculation of L . Generally the branch inductances are calculated initially by numerically integrating the magnetic vector potential function, A , then the branch matrix is transformed into the mesh matrix. This method is used for the toroidal sheet. A considerable simplification is possible for a flat sheet. We can analytically calculate the magnetic flux resulting from a current in a rectangular mesh through coplanar rectangles oriented parallel to the original mesh as presented in the next section. This enables us to arrive at the mesh inductance matrix directly.

We originally found it convenient to display graphically the branch currents as arrows at the center of each mesh pointing in the direction of average branch current flow and with length proportional to the average branch current magnitude. Our original graphical display has been largely replaced by our streamline plotting program.

The mesh currents are proportional to the scalar electric potential as demonstrated in the companion report.[1] Thus by associating the center of each mesh with its respective mesh current magnitude we can create a contour plot of the mesh current streamlines. In addition we can indicate a direction of stream flow with each contour by arrowheads along the line. Appendix I gives a description of the procedure used to draw the stream plots included in this report.

II. FLAT SHEET

A. Rectangular Sheet Flux

We previously presented formulas for the magnetic field resulting from a rectangular sheet.[2] Consider a rectangular sheet centered at the origin and in the xz plane with side $2g$ parallel to the x axis and with side $2h$ parallel to the z axis. The sheet has a surface current density K in the x direction only uniformly distributed across its width. Then the y component of the magnetic flux density at a point (x, z) in the xz plane is

$$B_y = \frac{\mu_0 K}{4\pi} \sum_{i,k=1}^2 (-1)^{i+k} \ln(x_i + r_{ik}) \quad (2)$$

where

$$x_i \equiv x - (-1)^i g$$

$$z_k \equiv z - (-1)^k h$$

$$r_{ik} \equiv (x_i^2 + z_k^2)^{1/2}.$$

We can calculate the total magnetic flux from the active sheet through any other rectangle in the xz plane with sides parallel to the x and z axes by integrating Eq. (2) across the area of the passive rectangle. Consider a rectangle bounded by sides $z = a$ to $z = b$ and $x = c$ to $x = d$ then the total flux is

$$\Phi = \Phi_y = \int_{x=c}^{x=d} \int_{z=a}^{z=b} B_y \, dx \, dz \quad (3)$$

Using Eq. (2) we can write Eq. (3) as

$$\Phi = \frac{\mu_0 K}{4\pi} \sum_{i,k=1}^2 (-1)^{i+k} \int_{x=c}^{x=d} \int_{z=a}^{z=b} \ln(x_i + r_{ik}) dx dz \quad (4)$$

If we define

$$T_{ik} = \int_{x=c}^{x=d} \int_{z=a}^{z=b} \ln(x_i + r_{ik}) dx dz \quad (5)$$

then we can write

$$\Phi = \frac{\mu_0 K}{4\pi} \sum_{i,k=1}^2 (-1)^{i+k} T_{ik} \quad (6)$$

Equation (5) can be integrated to give

$$T_{ik} = \sum_{m,n=1}^2 (-1)^{m+n} \left\{ U_{m,i} V_{n,k} \ln \left[U_{m,i} + \left(U_{m,i}^2 + V_{n,k}^2 \right)^{1/2} \right] \right. \\ \left. - U_{m,i} V_{n,k} + \frac{1}{2} U_{m,i}^2 \ln \left[V_{n,k} + \left(U_{m,i}^2 + V_{n,k}^2 \right)^{1/2} \right] \right. \\ \left. - \frac{1}{2} V_{n,k} \left(U_{m,i}^2 + V_{n,k}^2 \right) \right\} \quad (7)$$

where

$$x'_k \equiv -(-1)^i \frac{(d-c)}{2} \quad z'_k \equiv -(-1)^k \frac{(b-a)}{2}$$

$$U_{1,i} \equiv x'_i + c \quad U_{2,i} \equiv x'_i + d$$

$$V_{1,k} \equiv z'_k + a \quad V_{2,k} \equiv z'_k + b$$

Therefore, if we consider a flat sheet divided into rectangular meshes each mesh will consist of four branches. We associate a rectangular plate with each branch as previously shown in the companion paper.[1] We can calculate the resulting flux from a particular mesh with unit current in each branch through every other mesh including itself. Thus we arrive at the flux linkage per unit current which is the inductance. This method was used to find the inductance matrices for the following flat sheet examples.

B. Uniform Field – Sudden Appearance

The first example we shall consider is a flat square sheet in the xy plane which experiences the sudden appearance of a uniform magnetic field. The uniform field is in the z direction as illustrated in Fig. 1. The magnitude of the field versus time is shown in Fig. 3(a).

We divided the sheet into a 9×9 mesh system. This means there are 10 sets of branches in the x direction and 10 sets of branches in the y direction. The x branches are numbered from 1 to 90 and the y branches are numbered from 91 to 180. The mesh incidence matrix is shown in Fig. 2. In this case each of the four sides is a streamline boundary.

Each mesh which results from this division is also square. Thus the branch resistances are all identical. The self-inductance values are also identical. In addition the mutual inductance values are repeated throughout the mesh. It is only necessary to make 81 inductance calculations and distribute the resulting values appropriately throughout the matrix. Actually for a square sheet even less calculations are required, but for a general rectangle one needs the values from a corner mesh to all the other meshes.

The sudden appearance of the magnetic field results in eddy currents which completely cancel the net field across the sheet. Thus the initial eddy current distribution is controlled by the inductance alone. The initial eddy currents must

produce a flux through each mesh which is equal in magnitude but opposite in direction to the imposed flux. Thus the initial eddy currents must satisfy the relationship

$$[L] \{I\} = - \{ \Phi_m \} \quad (8)$$

where $\Phi_m \equiv$ mesh flux. Since the initiating field is uniform across the sheet, the mesh fluxes are identical and readily calculated by multiplying the mesh area by B.

We solved Eq. (8) for $\{I\}$ using a linear equation computer subroutine which was in IMSL, many other similar routines could be used. The initial current distribution is shown in Fig. 4 at $t = 0$. The ten streamline contours were chosen by finding the maximum and minimum mesh current values including the value zero for the sides. The difference between the maximum and minimum was then divided by eleven and this gives ten equally spaced contour values excluding the minimum and maximum.

The decay of the eddy currents in time is illustrated by the remaining plots in Fig. 4, the plots represent equal time increments after time zero. The time increment was chosen such that at ten time increments even the smallest contour value would not be present, but at nine time increments at least the smallest contour value would be present.

The direction of the streamlines at $t = 0$ is clockwise which would result in a magnetic field in the negative z direction as expected. The current is initially concentrated around the edges of the sheet. As the current pattern becomes more influenced by the resistance, the current is more uniformly distributed around the center. The current dies away progressively slower as time progresses, and the net field asymptotically approaches the initiating uniform B.

In addition to this sudden appearance example, calculations were also made for a linearly ramped uniform field. This puts a constant $\dot{\Phi}$ through each mesh and hence a constant V around each mesh. The initial eddy currents are zero and after

sufficient time the eddy currents stabilize at constant values. This means that the $\dot{I}$ are zero and the distribution is therefore resistively controlled.

The resistive distribution could also be found by setting $\left\{ \dot{I} \right\}$ equal to zero in Eq.

(1). We thus obtain

$$\left\{ V \right\} = [R] \left\{ I \right\}$$

which can be solved readily using many standard methods. The current distributions found in this manner were identical to those found using the time series method.

An infinite series formula exists for the stabilized eddy current distribution in a rectangular sheet. The formula is presented in Appendix II. We compared the total current flowing as calculated by the network program with the current calculated by the formula. Appendix II summarizes these calculations for several test cases. The methods were found to be in very good agreement.

C. Straight Wire Field - Sudden Disappearance

Our second example is a flat rectangular sheet located near an infinitely long conductor as shown in Fig. 5. The sheet and the wire are in xy plane. The wire is parallel to the x axis and initially carries a current in the positive x direction which suddenly disappears as illustrated in Fig. 3(b). The magnetic field varies inversely as the distance from the wire as suggested in Fig. 5.

As before we divided the sheet into a 9 by 9 mesh system. The mesh incidence matrix is the same as shown in Fig. 2 for the first example. This time, however, each mesh is a rectangle. Thus the branch resistances have one value for the x direction branches and a different value for the y direction branches. The same procedure as in the first example was used to calculate the inductance matrix.

The sudden disappearance of the magnetic field results in eddy currents which completely maintain the net field across the sheet. This is the same effect as previously described, but in the opposite direction. In example I we had total flux exclusion, here we have total flux inclusion. Thus Eq. (8) is modified by a negative sign

$$[L] \{I\} = \{\Phi_m\} \quad (9)$$

The mesh flux was found with sufficient accuracy for this example by multiplying the B at the center of a mesh by the mesh area. We solved Eq. (9) for $\{I\}$ again using the ISML subroutine.

The initial current distribution is shown in Fig. 6 at $t = 0$. The ten streamline contours were chosen using the same method as in example 1. As before the time increment between the plots in Fig. 6 was chosen for no contours at $t = 10$.

In this example the direction of the streamlines at $t = 0$ is counterclockwise which would result in a magnetic field in the positive z direction as expected. The current is initially concentrated in the side of the sheet nearest the wire. As the current pattern becomes more influenced by the resistance the streamline center moves to the center of the sheet.

We also investigated the eddy currents resulting from an exponential decay of the current in the straight wire. Figure 7 illustrates the total eddy current flowing in the sheet versus time for three different time constants compared with the flux inclusion example. The exponential decay curves each rise rapidly to a maximum value which is below the initial flux inclusion value. The curves then overshoot the flux inclusion curve slightly and eventually become coincident with it. As the time constant becomes shorter the exponential curves more closely approach the flux inclusion curve. Flux inclusion represents a limit curve for an infinitely short time constant.

D. Straight Wire Field - Linear Disappearance

The third example is the same rectangular sheet as in example 2 experiencing a straight wire field as shown in Fig. 5. However, in this case, the field decays linearly to zero as shown in Fig. 3(c). The mesh resistance matrix and the mesh inductance matrix are the same as in example 2. The same contour values were used as in Fig. 6, but the time increment was modified so that at $t = 10$ no contours were present as in the previous examples.

Since there is no sudden field change, in this case, the initial currents are all zero. Figure 8 illustrates the resulting current distribution in time. The eddy currents start at zero at $t = 0$, build rapidly to a maximum when the current in the wire drops to zero, between $t = 2$ and $t = 3$, then slowly decay to zero. The maximum number of contour lines is about six compared to ten in the flux inclusion example. As in Fig. 6, the pattern is initially concentrated toward the straight wire side of the sheet and becomes more uniform about the center of the sheet as the resistive influence gains predominance.

E. Dipole Field - Sudden Appearance

The final flat sheet example is a square sheet which experiences the sudden appearance of a dipole field. The dipole is located slightly above the center of the sheet as shown in Fig. 9 and oriented in the positive z direction. The magnitude of the field versus time is shown in Fig. 3(d).

We take advantage of the symmetry in this system and use a quarter-sheet matrix for actual computation. Consider the quarter-sheet in the positive x , positive y quadrant. The upper side and the right side are stream boundaries, while the lower side and the left side are equipotential boundaries.

We divided the quarter-sheet into a 15 by 15 mesh system. This means there are 15 sets of branches in the x direction and 15 sets of branches in the y direction. This is one less set in each direction than in the previous examples, because of the equipotential boundaries. The mesh incidence matrix is shown in Fig. 10.

As in example 1 the resulting meshes are square and thus the branch resistances are all identical. As in the previous examples the inductance values are repeated throughout the mesh. For the full sheet it is only necessary to calculate 900 inductances.

The original 900 full-sheet inductance values are simultaneously contracted into 225 quarter-sheet values and distributed appropriately throughout the inductance matrix. The contraction is accomplished by summing the flux passing through a

passive mesh in the first quadrant which results from an active mesh in the first quadrant with the flux passing through the same passive mesh which results from the images of the active mesh in the other three quadrants.

As in example 1 the sudden appearance of the magnetic field results in eddy currents which initially completely cancel the net field across the sheet. Thus to find the initial mesh currents we must solve

$$[L] \{I\} = - \{ \Phi_m \} \quad (8)$$

The mesh flux was calculated by numerically integrating the B from a perfect dipole across a mesh area using double Gaussian quadrature.

The ten streamline contours were chosen as in examples 1 and 2. The time increment between plots was chosen as before so that no contours were present at $t = 10$.

The initial eddy currents are shown in Fig. 11 at $t = 0$. The full sheet is plotted by distributing the information in the quarter-sheet appropriately throughout the full-sheet. The increased mesh divisions result in much smoother streamlines than in the previous examples. In contrast to the uniform field the eddy currents here are concentrated near the center initially, but not immediately at the center, and decrease to the sides. As expected, the direction of the streamlines is in the clockwise direction.

Figure 11 illustrates that as the eddy currents decay they become more uniform about the center. Note that at the center of the plate the streamlines are nearly perfect circles, as would be expected in an infinite plane. The streamlines become more squarish at the edges of the sheet. We would expect that as the dipole moves close to the sheet, compared to the dimensions of the sides, the sheet would approximate an infinite plane in the neighborhood of the dipole.

The field produced by eddy currents in an infinite plane sheet induced by the instantaneous appearance of a dipole source can be calculated using the image

method. Maxwell [3] originally noted that the field due to the eddy currents appears, from either side of the sheet, to come from an image of the inducing source on the opposite side receding with a uniform velocity. Smythe [4] gives the uniform velocity as

$$v = \frac{2s}{\mu_0} \quad (10)$$

where s = surface resistivity.

We calculated the field due to eddy currents at selected points on both sides of the sheet along the z axis for each time step. We compared these values with analytically computed values using image dipoles. For points on the positive z side of the sheet, we summed the inducing dipole, at $z = z_{\text{dip}}$, with a negative image dipole initially at $z = -z_{\text{dip}}$ and moving in the negative z direction at velocity v . For points on the negative z side of the sheet, we summed the inducing dipole with a negative image dipole initially at $z = z_{\text{dip}}$ and moving in the positive z direction at velocity v . We obtained very good agreement between both sets of values at all time steps for points located between $-z_{\text{dip}}$ to z_{dip} . The results are summarized in Appendix III.

III. TOROIDAL SHEET

A. Uniform Resistivity

A circular cross-section toroidal sheet of uniform resistivity was used for a benchmark comparison to determine the accuracy of the inductance calculations. Since all the magnetic flux configurations with which we were concerned possessed azimuthal symmetry, it was possible to model the torus as a branch network of concentric rings as shown in Fig. 12. Additionally, since there is symmetry about the midplane, it was possible to contract the network matrices and use only the top half of the torus for actual computation.

The incident matrix for the system is as follows:

MESH

1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18

BRANCH

0 -1 -2 -3 -4 -5 -6 -7 -8 -9 -10 -11 -12 -13 -14 -15 -16 -17

1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18

where each branch consists of two rings: top (+) and bottom (-).

Note that Fig. 12 depicts rings and not branches. The branch current direction in the incident matrix applies to both + and - rings. Thus, for example, mesh #8 consists of a current which flows negatively in branch 7, rings +7 and -7, and which flows positively in branch 8, rings +8 and -8. Mesh #1 consists of only branch 1 which encompasses all the flux passing through the inner major circumference of the torus. Each of the other meshes encompasses the flux between two adjacent branches.

As shown in Fig. 12, the concentric rings are spaced at equal angles around the minor circumference of the torus and all rings have the same cross-section. Thus the resistance of each ring is directly proportional to its radius. Since this system exhibits the s-mirror symmetry described in Section 6 of the companion report, [1] the value used for the branch resistance is the top ring resistance only.

The concentric rings were assumed to be ideal solenoids in the calculation of the inductance matrix. This means they have axial length, but no radial thickness. An active solenoid of radius r_0 centered at z_0 produces a flux through a concentric circle of radius r centered at z . This flux was calculated using the method presented by Garrett.[5]

The mesh inductance is the flux through a passive mesh which results from a unit current flowing in the branches of an active mesh. As an example, consider the flux from mesh #2 to mesh #4. A unit current flows in the positive direction in branch 2, rings +2 and -2, and a unit current flows in the negative direction in branch 1, rings +1 and -1. Here again the s-mirror symmetry considerations of

Section 6 of the companion report [1] apply. Thus the mesh inductance is the net flux passing between rings +3 and +4 only.

In general the flux from an ideal solenoid (radius = r_0 , center = z_0 , length = ℓ , $I = 1$) through a concentric circle (radius = r , center = z) is identical to the flux from a complementary ideal solenoid (radius = r , center = z , length = ℓ , $I = 1$) through a complementary circle (radius = r_0 , center = z_0). Using this fact it is readily seen that the flux from a first mesh, top and bottom rings active, through a second mesh, top rings only, is identical to the flux from the second mesh, top and bottom rings active, through the first mesh, top rings only. Thus the mesh inductance matrix is symmetric.

B. Non-Uniform Resistivity - TFTR Vacuum Vessel

The TFTR vacuum vessel is depicted in Fig. 13. The vessel consists of 20 segments each subtending 18° of the major circumference. Each segment consists of a stainless steel port section and a high resistance bellows flanked on each side by a supporting ring, which we refer to respectively as the port, bellows, and rib sections.

In the actual TFTR, six of the segments have the bellows sections replaced by neutral beam injector nozzels, which are resistively equivalent to the stainless steel, but inductively identical to the other segments. Therefore, we modeled the vessel as 20 identical segments to compute the inductance matrix. The resistance matrix is also initially based on a model of 20 segments which is adjusted to compensate for the six missing bellows sections. The adjustment algorithm is presented in the resistance matrix section.

In our model we use an average radius of the port, rib, and bellows to form a sheet surface with sections having coincident edges. This approximation introduces negligible error into the inductance calculations. The correct thickness of each section is used for resistance calculations.

There are two types of symmetry in this model, s-mirror and e-mirror which are described in Section 6 of the companion report. [1] The s-mirror is about the

midplane of the machine and the e-mirrors are about the 40 equipotential planes spaced azimuthally every 9° around the vessel. The equipotential planes bisect each bellows section and each port section. Thus we can model the entire vessel as the top half of a 9° segment as shown in Fig. 14.

The TFTR vacuum vessel segments are actually composed of circular cylinders. The bellows section is a right circular cylinder, the rib section is a right circular cylinder, and the port section is a circular cylinder truncated by $\pm 9^\circ$ planes. Our model assumed the port section to be a true toroidal segment. This surface simplified the programming and is negligibly different from the true surface. It is important to realize that the axis of the port toroidal segment is not the same as the axis of the vacuum vessel.

As shown in Fig. 14, the plane which bisects the port section intersects the port section as a semicircle. This semicircle was divided into 17 meshes, each subtending 10° of the minor circumference, starting with 5° . This defines the mesh corners shown on the 0° semicircle of Fig. 14. The additional mesh corners are found from the intersection of the vacuum vessel with planes parallel to the midplane which pass through the 0° mesh corners.

The 9° segment is divided into six meshes azimuthally. The mesh sizes are determined from the overlapping branch plates which are represented in Fig. 15. Since the port section model is toroidal, actually unrolling the 9° segment would produce distortions which are not shown.

The solid dots in Fig. 15 correspond to the mesh corners in Fig. 14. The azimuthal plates are illustrated as lying below the radial plates. The only plates which are entirely rib is one strip of radial plates. For any azimuthal line the azimuthal dimensions of the port plates are equal. The azimuthal dimensions of the bellows plates are equal and constant. These constraints completely determine the branch plate parameters and hence the mesh parameters.

The mesh incidence matrix is illustrated in Fig. 16. Recall that the azimuthal branches are bounded by streamlines and the radial branches are bounded by equipotentials. The azimuthal branches are numbered from 1 to 108 (from 6×18). The radial branches are numbered from 109 to 193 (from $5 \times 17 + 108$). Note that mesh #103 consists of the six innermost azimuthal branches. This corresponds to mesh #1 in the uniform resistivity benchmark and encompasses all the flux passing through the inner major circumference of the torus.

C. Inductance Matrix

The mesh inductances are not calculated directly for the vacuum vessel as they were for the flat sheet. First, the branch inductances are calculated by numerically integrating the vector potential A , then the branch matrix is transformed into the mesh matrix. The details of this operation are described in the companion paper. [1]

The symmetry of the vacuum vessel allows the upper half of a 9° segment to be used for actual calculation. Thus, the inductance matrix can be contracted as illustrated in Fig. 17. The mesh relationships are shown for clarity.

A typical active mesh is shown in the 9° segment which has 19 images carrying identical current in the same sense of rotation on the top half of the vessel. In addition, there are 20 images on the bottom half, which when viewed from the top, carry identical current in the same sense, but when viewed from the bottom have the opposite sense. All these meshes produce a flux through a typical passive mesh in the 9° segment.

The exceptional case is mesh #103. When mesh #103 is active, it forms with its images a complete loop around the inner major circumference of the vessel. When mesh #103 is passive, the net flux received is $1/40$ of the total flux transmitted through the inner major circle of the vessel by a particular active mesh and its associated images.

Each branch inductance of the vacuum vessel is calculated using basically the same method as was used for the flat sheet. The branch inductance is the integrated

vector potential along a passive branch due to a unit current in an active branch plus a unit current in all the images of the active branch. Section 3 of the companion paper [1] describes the transformation of the branch inductance matrix to the mesh inductance matrix.

The vector potential method requires the specification of the centerline, normal, and width of the active branch and the centerline of the passive branch. No branch plate of the vacuum vessel is actually a true, flat rectangle. However, consideration of Figs. 14 and 15 shows that the mesh corners of the bellows and rib meshes form rectangles which inscribe the cylindrical surfaces. In the bellows and rib sections we can approximate the true surface using this set of inscribed rectangles. Thus we can specify for each branch a centerline, normal, and width for inductance calculations.

The port section branch plates are slightly more complicated. The port section mesh corners nearly form rectangles, but not exactly. However, the centerline and normal of each branch is completely specified. We can approximate the true surface using rectangles having these branch centerlines and normals together with their respective average widths. Thus the necessary specifications are made for inductance calculations in the port section too.

D. Resistance Matrix

Essentially the same branch plates as in the inductance matrix calculation are used for the resistance matrix. In this case, however, the plate thickness and resistivity are needed to calculate the resistance. As in the inductance calculations, for the non-rectangular plates, an average plate width is used to calculate the resistance.

Most of the plates have a single thickness and resistivity. The exception is the central pair of azimuthal plates. As shown in Fig. 15, these plates consist of either a half-bellows-plate and a half-rib-plate or a half-rib-plate and a half-port-plate.

As mentioned previously in the actual TFTR, six of the 18° segments, i.e., twelve 9° segments, have their bellows sections replaced by stainless steel. This replacement does not affect the inductance model, but does require modification of the resistance model. To exactly model this situation would require using a 180° model which is prohibitively large. Thus a synthetic device was developed to correct the 9° model.

At the inductive extreme, when $\{I\} = 0$ and $\{\dot{I}\} \neq 0$, the voltage distribution is inductively controlled. In this case the emf across each 9° segment is $1/40$ of the total emf around the entire vacuum vessel.

At the resistive extreme, when $\{\dot{I}\} = 0$ and $\{I\} \neq 0$, the voltage distribution is resistively controlled. The azimuthal resistance of the vacuum vessel is composed almost entirely of the bellows contribution. Thus, the resistance of the vacuum vessel is effectively reduced by $28/40$ when twelve of the 9° bellows sections are removed. It follows that, in the resistive case, the emf across each 9° segment is $1/28$ of the total emf around the entire vacuum vessel.

Between these two extremes the emf across each 9° segment must vary between $1/40$ to $1/28$ of the total emf around the entire vacuum vessel. Thus the emf must be partially inductively controlled and partially resistively controlled. There must be a relationship for the ratio of the inductive part to the resistive part which yields the correct emf for any combination of $\{I\}$ and $\{\dot{I}\}$.

Since the radial boundaries of a 9° segment are equipotential surfaces, the emf between any point on one boundary and any point on the other boundary will be constant. Examination of Fig. 16 reveals that the emf through mesh #103 is equal to the emf across the 9° segment.

We can rewrite Eq. (1) for mesh #103 as follows:

$$V_{103} - \sum_{i=1}^{102} \dot{I}_i L_{i,103} = I_{103} \sum_{i=1}^{103} R_{i,103} + \dot{I}_{103} L_{103,103} \quad (11)$$

Let us define E as the net emf imposed on mesh #103, which is the same as the net emf across the 9^0 segment. Thus:

$$E = V_{103} - \sum_{i=1}^{102} \dot{I}_i L_{i,103} \quad (12)$$

It can be seen from Eq. (12) that E is composed of an externally imposed emf, V , and an internally imposed emf due to the mutual inductances from mesh #103 to all the other meshes.

Let us define the ratio of the inductive contribution to the net emf as

$$p \equiv \frac{\dot{I}_{103} L_{103,103}}{E} \quad (13)$$

Similarly let us define the ratio of the resistive contribution to the net emf as

$$q \equiv \frac{I_{103} \sum_{i=1}^{103} R_{i,103}}{E} \quad (14)$$

We know that in the inductive extreme $p = 1$ and $q = 0$ and in the resistive extreme $p = 0$ and $q = 1$. Thus both p and q vary between 0 and 1 and using Eqs. (11) and (12)

$$p + q = 1 \quad (15)$$

Let us define E_T as the total emf around the vacuum vessel. Then using our assumed relation of the 9^0 segment voltage to the total voltage we can write:

$$E = \frac{E_T}{(28 + 12p)} \quad (16)$$

or

$$E = \frac{E_T}{(40 - 12q)} \quad (17)$$

Using Eqs. (11) and (12) with (13) and (16) or (14) and (17) we find the relation:

$$\frac{28}{40} I_{103} \sum_{i=1}^{103} R_{i,103} + \dot{I}_{103} L_{103,103} = \frac{E_T}{40} \quad (18)$$

It can be seen from consideration of Eq. (18) that the desired correction to the 9° model can be made by simply multiplying the new vector $R_{i,103}$ by the factor $28/40$. This device forces the correct driving emf to appear across the 9° segment at all times. Since we have assumed any inductive differences between the actual vessel and the repeated 9° segments to be negligible, we therefore arrive at an eddy current distribution which represents an average for the segments with bellows.

E. OH External Source

This example compares the eddy currents flowing in the TFTR vacuum vessel model with the uniform resistivity model for complete flux exclusion. An OH type field was used as the external emf source which means there is essentially no flux which penetrates the vessel itself. All the flux passes down inside the inner major circumference of the torus and returns up outside the outer major circumference of the torus.

In the uniform resistivity example this flux configuration means that all the mesh emf's, except mesh #1, are equal to zero. Similarly, in the vacuum vessel model all the mesh emf's, except mesh #103, are equal to zero.

Since the resistance does not enter into the flux exclusion calculation, we expect the eddy currents to be identical, if the vacuum vessel inductances have been correctly calculated. Because of the symmetry of the problem, the same current must flow azimuthally at a given radius. Similarly, there can be no radial current flow anywhere. This allows the contraction of the 103 by 103 vacuum vessel mesh inductance matrix into an 18 by 18 matrix which is directly comparable to the uniform resistivity model.

The contraction is accomplished as follows. Meshes #1 to #102 consist of 17 sets of 6 adjacent meshes each, with each set having a common major radius. The inductance between any two sets is found by summing the 36 mesh inductances which correspond to the 6 meshes of each set. For example, if set #2 consists of meshes #1

to #6, and set #5 consists of meshes #19 to #24, then the contracted inductance, L^* , between set #2 and set #5 is

$$L_{2,5}^* = \sum_{i=1}^6 \sum_{j=19}^{24} L_{ij} \quad (19)$$

Since mesh #103 of the vacuum vessel model consists of six adjacent branches already, only one sum is required. For example, if set #1 consists of only mesh #103, then L^* between set #1 and set #5 is

$$L_{1,5}^* = \sum_{j=19}^{24} L_{103,j} \quad (20)$$

A unique case is the contracted self inductance of set #1 which is just

$$L_{1,1}^* = L_{103,103} \quad (21)$$

The vacuum vessel model is based on nine degrees, but the uniform resistivity model is based on complete circles. Thus each of the contracted inductances must be multiplied by 40 to directly compare the calculations.

As in the flat sheet examples we solved

$$[L^*] \{I\} = - \{\Phi_m\} \quad (22)$$

to find the eddy currents which initially completely cancel the net field through the torus. A unit flux of one weber was used and the resulting azimuthal currents normalized to the value at the inner major radius of the uniform resistivity model.

Figure 18 shows the extremely close correspondence between the vacuum vessel model and the uniform resistivity model. The major part of the current flow is seen to be initially on the inner portion of the torus.

Further calculations were made using the full inductance matrix and an actual OH configuration for the TFTR. The flux through the center of the torus was calculated using the thick solenoid method described by Garrett.[5] This method numerically integrates the ideal solenoid fluxes previously mentioned.

The current through the OH was linearly ramped at a rate of $\dot{I} = -0.996 \times 10^6$ amp/sec. Figure 19 illustrates the resulting eddy current distribution in time. As in the flat sheet examples, ten equally spaced contours were chosen between extremes of current including zero. Each time step is normalized to itself so that the distributions can be compared, but the magnitudes cannot.

In this case the initial currents are all zero. If a plot were made at a very short time after $t = 0$ the streamlines would all be circles around the vacuum vessel. As shown in Fig. 19 the current at $t = 80 \mu\text{sec}$ is still almost entirely azimuthal. The initial distribution of current is as before inductively controlled. As time progresses the resistance becomes more of an influence, until eventually the current distribution is completely resistively controlled.

The resistance differences between the bellows and port sections are seen readily at $t = 480 \mu\text{sec}$. A distinct jog is seen at the rib. After $t = 480 \mu\text{sec}$, the distribution remains essentially constant. Independent calculations based on resistance only show that this is indeed the resistive distribution.

The resistive distribution is observed in Fig. 19 to be more uniform across the torus than the inductive distribution. The distribution is controlled by the radial differences in azimuthal resistance. The resistance varies approximately as the radius, thus we would expect the current to vary inversely with the radius as observed.

F. EF External Source

The last example to be considered is the eddy currents resulting from a linearly ramped EF system. An actual TFTR EF configuration was used to make these calculations. The EF current was ramped at a rate of $\dot{I} = -0.17 \times 10^6$ amp/sec.

In contrast to the OH system, flux penetrates both the vessel itself as well as the inner major circumference. Flux values were calculated by multiplying the field values at the center of each mesh by the respective mesh area. The flux through

mesh #103, however, was calculated using the method just described in the OH section.

As in the OH example ten equally spaced contours were chosen between extremes of current including zero. Each time step is normalized to itself so that the distributions can be compared, but the magnitudes cannot.

The initial currents are all zero. If the distribution were shown a very short time after $t = 0$, the current would be completely azimuthal, corresponding to complete inductive control.

Figure 20 illustrates the eddy current distribution in time. At $t = 40 \mu\text{sec}$ it is observed that the distribution is almost completely azimuthal, but that current flows in both directions. This means that a major part of the EF penetrates the vessel itself. When the resistance influence gains predominance at $t = 280 \mu\text{sec}$ half a turnaround loop is observed. With a little imagination one can envision the complete system of turnaround loops.

After $280 \mu\text{sec}$ the current distribution does not change significantly. The distribution is the same as that calculated by resistance alone. The bellows and port sections are distinct and the rib shows up as an abrupt transition in the streamlines.

IV. CONCLUSIONS

We have presented several examples of transient eddy currents in both flat sheets and toroidal sheets in this report. The expected qualitative behavior of the calculated eddy current distributions was observed. Other checks were made where possible to insure the accuracy of the results. In particular the distributions agreed in the limits with independently derived purely inductive and purely resistive distributions. In addition, when the flat sheet approached an infinite plane, the resulting net field which we calculated agreed at all times with the applicable analytic expression.

The effort required to set up a particular calculation is not trivial, but it must be recalled that we are dealing with a complicated physical phenomena. We are confident this method can be used successfully in a broad variety of applications.

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FIGURE CAPTIONS

Figure 1. Uniform field through a flat square sheet.

Figure 2. Mesh incidence matrix for rectangle divided 9 by 9 with stream boundaries on all four sides.

Figure 3. Magnitudes of field with time for flat sheet examples.

- (a) Example 1: sudden appearance of a uniform field.
- (b) Example 2: sudden disappearance of a straight wire field.
- (c) Example 3: linear disappearance of a straight wire field.
- (d) Example 4: sudden appearance of a dipole field.

Figure 4. Eddy current distributions in time for the sudden appearance of a uniform field through a flat square sheet.

Figure 5. Field from an infinite straight wire through a flat rectangular sheet.

Figure 6. Eddy current distributions in time for the sudden disappearance of the field from an infinite straight wire through a flat rectangular sheet.

Figure 7. Total eddy current flowing versus time for three different time constants of exponential decay compared to the flux inclusion case. All curves for field from an infinite straight wire through a flat rectangular sheet.

Figure 8. Eddy current distributions in time for the linear disappearance of the field from an infinite straight wire through a flat rectangular sheet.

Figure 9. Field from an ideal dipole located above center of flat square sheet.

Figure 10. Mesh incidence matrix for rectangle divided 15 by 15 with stream boundaries top and right sides and equipotential boundaries bottom and left sides.

Figure 11. Eddy current distributions in time for the sudden appearance of an ideal dipole located above the center of a flat square sheet.

Figure 12. Cross section of circular torus showing concentric ring model. Each branch has a top (+) ring and a bottom (-) ring.

Figure 13. TFTR vacuum vessel.

Figure 14. Top half of nine degree segment of TFTR vacuum vessel.

Figure 15. Detail of overlapping plates used for network model of TFTR vacuum vessel. Azimuthal plates shown below radial plates.

Figure 16. Mesh incidence matrix for top half of nine degree segment of TFTR vacuum vessel. Stream boundaries at outer and inner circumference. Equipotential boundaries at right and left sides. Six major circumference divisions and seventeen minor circumference divisions.

Figure 17. Typical vacuum vessel mesh inductance.

Figure 18. Azimuthal eddy current distribution on toroidal sheet for OH flux exclusion example. Uniform resistivity model compared to vacuum vessel model.

Figure 19. Eddy current distributions in time on TFTR vacuum vessel due to linearly ramped OH source.

Figure 20. Eddy current distributions in time on TFTR vacuum vessel due to linearly ramped EF source.

Figure 21. Streamline plotting technique applied to example rectangle.

Figure 22. Location of mesh current values which were used to create streamline plot of vacuum vessel eddy currents.

APPENDIX I

STREAMLINE PLOTTING

All the plots in this report were generated using a grid of rectangles. Fig. 21 illustrates an example rectangle. The flat plate examples naturally possess this configuration. The toroidal sheet was projected onto a rectangular grid as shown in Fig. 22. The toroidal sheet was plotted in this manner to present the current patterns clearly as well as to simplify the programming.

The network mesh current values are used as the corners of the rectangles. As shown in Fig. 21, each rectangle has four corner values, numbered from 1 to 4, counterclockwise from the lower left-hand corner. Each side of the rectangle is similarly numbered from 1 to 4 counterclockwise from the bottom side.

In Fig. 21 corner #1 has a value 0.1, #2 has 2.0, #3 has 0.1, and #4 has 3.9. Contours are desired for the values 1.0, 2.0, and 3.0. Each side is proportioned linearly to find intermediate values between beginning and end points. The direction of each line is always counterclockwise around the rectangle.

Side 1 begins with the value 0.1 and ends with the value 2.0. Two possible contour values, 1.0 and 2.0, exist on this side. Since the values increase in the direction of side 1, the intermediate values are indicated with a superscript + in Fig. 21.

Side 2 begins with the value 2.0 and ends with the value 0.1. Since the values decrease in the direction of side 2, the intermediate values are indicated with a superscript - in Fig. 21. Similarly sides 3 and 4 are proportioned in the same manner as 1 and 2.

The direction of a streamline contour is chosen from - to +. Thus a streamline is indicated by an arrow from side 4, value 3^- , to side 3, value 3^+ . This means that a value elevation will be encircled by counterclockwise streamlines while a value depression will be encircled by clockwise streamlines.

The streamline for the value 3 is unambiguous in the example. However, the values 1 and 2 each present two possibilities. Between two - and + pairs the pair with the shortest distance was chosen as the streamline. For example a streamline could be drawn from side 1, value 1^+ to either side 4, value 1^- , or side 2, value 1^- . The distance between the side 1-side 4 pair is shorter than the distance between the side 1-side 2 pair, thus the streamline is drawn from side 4, to side 1. This forces the remaining value 1 streamline to be drawn from side 2 to side 3.

An unusual possibility occurs in the example for the value 2. In the example the side 4-side 3 distance is shorter than the side 4-side 1 distance. This leaves side 1-side 2, which is the same point and thus only one arrow is actually drawn.

It could happen, however, that for the value 2, the side 4-side 1 distance is shorter than the side 4-side 3. Thus two arrows would be drawn, side 4-side 1, and side 2-side 3. Note that this possibility only depends on the order in which the points are computed in the program. It could still happen that only one arrow would be drawn. Either possibility could be more representative of the actual pattern.

With a little thought one can find other ambiguous possibilities particularly with identical corner values or corner values identical to contour values. This streamline method is admittedly rough and does not work well with a coarse grid. In practice, however, when mesh values are calculated to six or seven places internally in a computer, identical values are found only in cases of high symmetry. We have encountered spurious streamlines less than once in every thousand plots. Plots which change drastically with small time or parameter changes are indicative of too coarse a grid. The example shown in Fig. 21 is an extreme which really requires a finer grid to plot accurately.

Finally Fig. 22 illustrates how streamline and equipotential boundaries may be used to extend the area of the vacuum vessel eddy current plots. The actual calculated mesh center values are indicated by the solid dots 1 to 102. In addition

the value for mesh #103 is used at six locations along the inside major circumference, which is a stream boundary.

The outer major circumference is also a stream boundary and from symmetry we know that its value must be 0. These values are indicated by the solid squares in Fig. 22. The left and right boundaries of the vacuum vessel segment are equipotentials. Thus opposing values on each side of a boundary must be identical. These values are indicated by the open circles and squares in Fig. 22. Thus the values for the entire surface of the vacuum vessel are known to whatever accuracy the grid spacing allows.

APPENDIX II

EDDY CURRENTS IN A RECTANGULAR SHEET PRODUCED BY A LINEARLY RAMPED UNIFORM FIELD

Hague [6] presents the formulas applicable to the magnetostatic flux distribution inside a very highly permeable rectangular bar of infinite length carrying a uniform current. This problem is identical in form to the stabilized current distribution inside a very highly conducting rectangular bar of infinite length subjected to a uniform axial magnetic field increasing linearly in time. The following equivalences apply

$$\begin{aligned} \mu &\rightarrow \sigma \\ \vec{B} = \nabla \times \vec{A} &\rightarrow \vec{J} = \nabla \times \vec{T} \\ \Phi = \int \vec{B} \cdot d\vec{s} = \int \vec{A} \cdot d\vec{\ell} &\rightarrow I = \int \vec{J} \cdot d\vec{s} = \int \vec{T} \cdot d\vec{\ell} \\ \nabla^2 \vec{A} = -\mu \vec{J} &\rightarrow \nabla^2 \vec{T} = \sigma \vec{B} \end{aligned}$$

Consider a rectangular bar centered on the z axis with its sides parallel to the x and y axes. The expression for the vector current potential is

$$\begin{aligned} \vec{T}(x,y) = -\frac{1}{2} \dot{B} \sigma \left[x^2 + \frac{32a^2}{\pi^2} \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)^3 \cosh(2n+1) \frac{\pi}{2} \frac{b}{a}} \right. \\ \left. \cdot \cosh(2n+1) \frac{\pi y}{2a} \cos(2n+1) \frac{\pi}{2} \frac{x}{a} \right] \hat{z} \end{aligned} \quad (\text{II-1})$$

where

2a $\equiv$ x dimension of bar

2b $\equiv$ y dimension of bar

$\dot{B}$ $\equiv$ rate of change of B

σ $\equiv$ conductivity of bar

Using the relation

$$I = \int \vec{T} \cdot d\vec{l} \quad (\text{II-2})$$

it is readily seen that the total current flowing per unit length of the bar between any line (x,y) and the z axis is

$$I(x,y) = T(x,y) - T(0,0) \quad (\text{II-3})$$

Since we are not really interested in an infinite rectangular bar, consider now a rectangular sheet lying in the xy plane, centered at the origin. The same symmetry conditions apply and the equations still have the same form. The differences are that the conductivity is replaced by the surface conductivity and the current density is replaced by the surface current density.

As shown in Section 5 of the companion paper, [1] the vector current potential in a two-dimensional sheet effectively becomes a scalar current potential. The total current flowing between the sheet center and any point (x,y) is identical in form to Eq. (II-3), except that T is now a true scalar and not a vector magnitude.

Thus we can find the total current flowing between the sheet center and its edge by choosing the edge point $x = a$, $y = 0$, and substituting into Eq. (II-3), the result is

$$I_T = -\frac{1}{2} \frac{Ba^2t}{\rho} \left[1 - \frac{32}{\pi^3} \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)^3 \cosh(2n+1) \frac{\pi}{2} \left(\frac{b}{a}\right)} \right] \quad (\text{II-4})$$

where, in this case,

$t \equiv$ thickness of sheet

$\rho \equiv$ resistivity of sheet

This series converges very rapidly.

We made several test comparisons between this formula and the network program for a 9 x 9 mesh division. We used the following parameters for each of five cases.

$$\begin{aligned}
 a &= 1 \\
 t &= 10^{-3} \\
 \dot{B} &= -1 \\
 \rho &= 10^{-6} .
 \end{aligned}$$

We varied b/a for each run. The following table shows the results.

<u>b/a</u>	Total Current	
	<u>Program</u>	<u>Formula</u>
1.0	292.4	294.7
2.0	452.7	455.5
3.0	489.0	490.7
4.0	497.2	498.1
5.0	499.2	499.6

APPENDIX III

COMPARISON OF NETWORK PROGRAM WITH IMAGE FORMULA
FOR DIPOLE SOURCE OVER LARGE SHEET

The following calculations were all based on an ideal dipole with dipole moment, p , equal to 10^9 amp - m^2 located at $z_d = 0.1$ m above the center of a large rectangular sheet. On the axis of the sheet the field from the source dipole is given as

$$B_d = \frac{\mu_0}{2\pi} \frac{p}{|z - z_d|^3}.$$

For a suddenly appearing dipole image theory predicts the field due to the eddy currents on the axis to be given by

$$B_e^+ = -\frac{\mu_0}{2\pi} \frac{p}{\left| \left(z + z_d + \frac{2st}{\mu_0} \right) \right|^3}$$

$$B_e^- = -\frac{\mu_0}{2\pi} \frac{p}{\left| \left(z - z_d - \frac{2st}{\mu_0} \right) \right|^3}$$

on the nonsource side of the sheet where

s = surface resistivity

t = time from appearance.

Thus the total field on the source side is $B^+ = B_d + B_e^+$ and the total field on the nonsource side is $B^- = B_d + B_e^-$.

We used a square sheet, 0.5 m by 0.5 m, to simulate an infinite plane. The sheet had a surface resistivity, s , equal to 0.033. The eddy currents were calculated using a quarter-sheet divided into a 15 by 15 mesh. The results of the comparison between the total field calculated by both methods is shown in the following table. The time increment, Δt , used was 3.8×10^{-8} sec. Results are shown for selected times from zero to 500 Δt .

TABLE

TOTAL FIELD ON AXIS OF SHEET IN TESLA

P = Network program
 F = Image formula
 $\Delta t = 3.8 \times 10^{-8}$

t	z = -0.08		z = -0.06		z = -0.04	
	P	F	P	F	P	F
0	0.02	0.00	0.02	0.00	0.02	0.00
10	0.97	0.93	1.49	1.45	2.45	2.40
20	1.61	1.55	2.43	2.38	3.91	3.85
30	2.05	1.98	3.07	3.00	4.85	4.78
40	2.37	2.29	3.51	3.43	5.48	5.41
50	2.60	2.52	3.83	3.74	5.92	5.84
60	2.78	2.69	4.06	3.97	6.23	6.15
70	2.91	2.82	4.24	4.14	6.47	6.37
80	3.02	2.92	4.37	4.27	6.64	6.55
90	3.10	3.00	4.47	4.37	6.78	6.68
100	3.16	3.06	4.56	4.45	6.88	6.78
200	3.40	3.33	4.84	4.77	7.24	7.16
300	3.43	3.39	4.88	4.84	7.28	7.24
400	3.43	3.41	4.88	4.86	7.29	7.26
500	3.43	3.42	4.88	4.87	7.29	7.28
∞	3.43	3.43	4.88	4.88	7.29	7.29
	$\times 10^4$		$\times 10^4$		$\times 10^4$	

t	z = -0.02		z = -0.00		z = 0.02	
	P	F	P	F	P	F
0	0.00	0.00	0.22	0.00	2.75	2.75
10	0.43	0.43	0.96	0.84	3.18	3.18
20	0.67	0.67	1.33	1.27	3.42	3.42
30	0.82	0.81	1.55	1.51	3.57	3.56
40	0.91	0.91	1.68	1.66	3.66	3.66
50	0.98	0.97	1.77	1.75	3.73	3.72
60	1.02	1.01	1.83	1.81	3.77	3.76
70	1.05	1.04	1.87	1.85	3.80	3.79
80	1.08	1.07	1.90	1.89	3.82	3.81
90	1.09	1.08	1.92	1.91	3.84	3.83
100	1.11	1.10	1.94	1.93	3.86	3.84
200	1.15	1.14	1.99	1.98	3.90	3.89
300	1.16	1.15	2.00	1.99	3.91	3.90
400	1.16	1.15	2.00	2.00	3.91	3.90
500	1.16	1.16	2.00	2.00	3.91	3.90
∞	1.16	1.16	2.00	2.00	3.91	3.91
	$\times 10^5$		$\times 10^5$		$\times 10^5$	

TABLE (Cont'd)

t	z = 0.04		z = 0.06		z = 0.08	
	P	F	P	F	P	F
0	8.53	8.53	3.08	3.08	2.50	2.50
10	8.78	8.77	3.09	3.09	2.50	2.50
20	8.92	8.92	3.10	3.10	2.50	2.50
30	9.02	9.01	3.11	3.11	2.50	2.50
40	9.08	9.07	3.11	3.11	2.50	2.50
50	9.12	9.11	3.11	3.11	2.50	2.50
60	9.15	9.15	3.12	3.12	2.50	2.50
70	9.18	9.17	3.12	3.12	2.50	2.50
80	9.19	9.18	3.12	3.12	2.50	2.50
90	9.21	9.20	3.12	3.12	2.50	2.50
100	9.22	9.21	3.12	3.12	2.50	2.50
200	9.25	9.25	3.12	3.12	2.50	2.50
300	9.26	9.25	3.12	3.12	2.50	2.50
400	9.26	9.26	3.12	3.12	2.50	2.50
500	9.26	9.26	3.12	3.12	2.50	2.50
∞	9.26	9.26	3.12	3.12	2.50	2.50
	$\times 10^5$		$\times 10^6$		$\times 10^7$

UNIFORM FIELD

Fig. 1. 784466

MESH INCIDENCE MATRIX

1	-10	-91	92	51	-60	-146	147
2	-11	-92	93	52	-61	-147	148
3	-12	-93	94	53	-62	-148	149
4	-13	-94	95	54	-63	-149	150
5	-14	-95	96	55	-64	-151	152
6	-15	-96	97	56	-65	-152	153
7	-16	-97	98	57	-66	-153	154
8	-17	-98	99	58	-67	-154	155
9	-18	-99	100	59	-68	-155	156
10	-19	-101	102	60	-69	-156	157
11	-20	-102	103	61	-70	-157	158
12	-21	-103	104	62	-71	-158	159
13	-22	-104	105	63	-72	-159	160
14	-23	-105	106	64	-73	-161	162
15	-24	-106	107	65	-74	-162	163
16	-25	-107	108	66	-75	-163	164
17	-26	-108	109	67	-76	-164	165
18	-27	-109	110	68	-77	-165	166
19	-28	-111	112	69	-78	-166	167
20	-29	-112	113	70	-79	-167	168
21	-30	-113	114	71	-80	-168	169
22	-31	-114	115	72	-81	-169	170
23	-32	-115	116	73	-82	-171	172
24	-33	-116	117	74	-83	-172	173
25	-34	-117	118	75	-84	-173	174
26	-35	-118	119	76	-85	-174	175
27	-36	-119	120	77	-86	-175	176
28	-37	-121	122	78	-87	-176	177
29	-38	-122	123	79	-88	-177	178
30	-39	-123	124	80	-89	-178	179
31	-40	-124	125	81	-90	-179	180
32	-41	-125	126				
33	-42	-126	127				
34	-43	-127	128				
35	-44	-128	129				
36	-45	-129	130				
37	-46	-131	132				
38	-47	-132	133				
39	-48	-133	134				
40	-49	-134	135				
41	-50	-135	136				
42	-51	-136	137				
43	-52	-137	138				
44	-53	-138	139				
45	-54	-139	140				
46	-55	-141	142				
47	-56	-142	143				
48	-57	-143	144				
49	-58	-144	145				
50	-59	-145	146				

Fig. 2. 78817

(a) UNIFORM FIELD
SUDDEN APPEARANCE

(c) STRAIGHT WIRE
LINEAR DISAPPEARANCE

(b) STRAIGHT WIRE
SUDDEN DISAPPEARANCE

(d) DIPOLE FIELD
SUDDEN APPEARANCE

Fig. 3. [(a), (b), (c), (d)] 788115

$T=0$ $T=4$ $T=1$ $T=5$ $T=8$ $T=2$ $T=6$ $T=9$ $T=3$ $T=7$ $T=10$

Fig. 4. 784395

STRAIGHT WIRE FIELD

Fig. 5. 784467

$T=0$ $T=1$ $T=2$ $T=3$ $T=4$ $T=5$ $T=6$ $T=7$ $T=8$ $T=9$ $T=10$

Fig. 6. 784358

Fig. 7. 784351

$T=0$ $T=1$ $T=2$ $T=3$ $T=4$ $T=5$ $T=6$ $T=7$ $T=8$ $T=9$ $T=10$

Fig. 8. 784438

DIPOLE FIELD

FRESH INCIDENCE MATRIX

0	-1	C	226	36	-51	-275	276	86	-101	-325	326	136	-151	0	376	186	-201	-425	426
0	-2	-226	227	37	-52	-276	277	87	-102	-326	327	137	-152	-376	377	187	-202	-426	427
0	-3	-227	228	38	-53	-277	278	88	-103	-327	328	138	-153	-377	378	188	-203	-427	428
0	-4	-228	229	39	-54	-278	279	89	-104	-328	329	139	-154	-378	379	189	-204	-428	429
0	-5	-229	230	40	-55	-279	280	90	-105	-329	330	140	-155	-379	380	190	-205	-429	430
0	-6	-230	231	41	-56	-280	281	91	-106	0	331	141	-156	-380	381	191	-206	-430	431
0	-7	-231	232	42	-57	-281	282	92	-107	-331	332	142	-157	-381	382	192	-207	-431	432
0	-8	-232	233	43	-58	-282	283	93	-108	-332	333	143	-158	-382	383	193	-208	-432	433
0	-9	-233	234	44	-59	-283	284	94	-109	-333	334	144	-159	-383	384	194	-209	-433	434
0	-10	-234	235	45	-60	-284	285	95	-110	-334	335	145	-160	-384	385	195	-210	-434	435
0	-11	-235	236	46	-61	0	286	96	-111	-335	336	146	-161	-385	386	196	-211	0	436
0	-12	-236	237	47	-62	-286	287	97	-112	-336	337	147	-162	-386	387	197	-212	-436	437
0	-13	-237	238	48	-63	-287	288	98	-113	-337	338	148	-163	-387	388	198	-213	-437	438
0	-14	-238	239	49	-64	-288	289	99	-114	-338	339	149	-164	-388	389	199	-214	-438	439
0	-15	-239	240	50	-65	-289	290	100	-115	-339	340	150	-165	-389	390	200	-215	-439	440
1	-16	0	241	51	-66	-290	291	101	-116	-340	341	151	-166	0	391	201	-216	-440	441
2	-17	-241	242	52	-67	-291	292	102	-117	-341	342	152	-167	-391	392	202	-217	-441	442
3	-18	-242	243	53	-68	-292	293	103	-118	-342	343	153	-168	-392	393	203	-218	-442	443
4	-19	-243	244	54	-69	-293	294	104	-119	-343	344	154	-169	-393	394	204	-219	-443	444
5	-20	-244	245	55	-70	-294	295	105	-120	-344	345	155	-170	-394	395	205	-220	-444	445
6	-21	-245	246	56	-71	-295	296	106	-121	0	346	156	-171	-395	396	206	-221	-445	446
7	-22	-246	247	57	-72	-296	297	107	-122	-346	347	157	-172	-396	397	207	-222	-446	447
8	-23	-247	248	58	-73	-297	298	108	-123	-347	348	158	-173	-397	398	208	-223	-447	448
9	-24	-248	249	59	-74	-298	299	109	-124	-348	349	159	-174	-398	399	209	-224	-448	449
10	-25	-249	250	60	-75	-299	300	110	-125	-349	350	160	-175	-399	400	210	-225	-449	450
11	-26	-250	251	61	-76	0	301	111	-126	-350	351	161	-176	-400	401				
12	-27	-251	252	62	-77	-301	302	112	-127	-351	352	162	-177	-401	402				
13	-28	-252	253	63	-78	-302	303	113	-128	-352	353	163	-178	-402	403				
14	-29	-253	254	64	-79	-303	304	114	-129	-353	354	164	-179	-403	404				
15	-30	-254	255	65	-80	-304	305	115	-130	-354	355	165	-180	-404	405				
16	-31	0	256	66	-81	-305	306	116	-131	-355	356	166	-181	0	406				
17	-32	-256	257	67	-82	-306	307	117	-132	-356	357	167	-182	-406	407				
18	-33	-257	258	68	-83	-307	308	118	-133	-357	358	168	-183	-407	408				
19	-34	-258	259	69	-84	-308	309	119	-134	-358	359	169	-184	-408	409				
20	-35	-259	260	70	-85	-309	310	120	-135	-359	360	170	-185	-409	410				
21	-36	-260	261	71	-86	-310	311	121	-136	0	361	171	-186	-410	411				
22	-37	-261	262	72	-87	-311	312	122	-137	-361	362	172	-187	-411	412				
23	-38	-262	263	73	-88	-312	313	123	-138	-362	363	173	-188	-412	413				
24	-39	-263	264	74	-89	-313	314	124	-139	-363	364	174	-189	-413	414				
25	-40	-264	265	75	-90	-314	315	125	-140	-364	365	175	-190	-414	415				
26	-41	-265	266	76	-91	0	316	126	-141	-365	366	176	-191	-415	416				
27	-42	-266	267	77	-92	-316	317	127	-142	-366	367	177	-192	-416	417				
28	-43	-267	268	78	-93	-317	318	128	-143	-367	368	178	-193	-417	418				
29	-44	-268	269	79	-94	-318	319	129	-144	-368	369	179	-194	-418	419				
30	-45	-269	270	80	-95	-319	320	130	-145	-369	370	180	-195	-419	420				
31	-46	0	271	81	-96	-320	321	131	-146	-370	371	181	-196	0	421				
32	-47	-271	272	82	-97	-321	322	132	-147	-371	372	182	-197	-421	422				
33	-48	-272	273	83	-98	-322	323	133	-148	-372	373	183	-198	-422	423				
34	-49	-273	274	84	-99	-323	324	134	-149	-373	374	184	-199	-423	424				
35	-50	-274	275	85	-100	-324	325	135	-150	-374	375	185	-200	-424	425				

Fig. 10. 788118

T=0 **T=4** **T=1** **T=5** **T=8** **T=2** **T=6** **T=9** **T=3** **T=7** **T=10** **Fig. 11. 784439**

CONCENTRIC RING BRANCH NETWORK

Fig. 12. 788184

TFTR VACUUM VESSEL

Fig. 13. 784355

Fig. 14. 784429

Fig. 15. 788185

MESH INCIDENCE MATRIX

-1	7	0	-109	0	0	-61	67	0	-159	0	0
-2	8	109	-110	0	0	-62	68	159	-160	0	0
-3	9	110	-111	0	0	-63	69	160	-161	0	0
-4	10	111	-112	0	0	-64	70	161	-162	0	0
-5	11	112	-113	0	0	-65	71	162	-163	0	0
-6	12	113	0	0	0	-66	72	163	0	0	0
-7	13	0	-114	0	0	-67	73	0	-164	0	0
-8	14	114	-115	0	0	-68	74	164	-165	0	0
-9	15	115	-116	0	0	-69	75	165	-166	0	0
-10	16	116	-117	0	0	-70	76	166	-167	0	0
-11	17	117	-118	0	0	-71	77	167	-168	0	0
-12	18	118	0	0	0	-72	78	168	0	0	0
-13	19	0	-119	0	0	-73	79	0	-169	0	0
-14	20	119	-120	0	0	-74	80	169	-170	0	0
-15	21	120	-121	0	0	-75	81	170	-171	0	0
-16	22	121	-122	0	0	-76	82	171	-172	0	0
-17	23	122	-123	0	0	-77	83	172	-173	0	0
-18	24	123	0	0	0	-78	84	173	0	0	0
-19	25	0	-124	0	0	-79	85	0	-174	0	0
-20	26	124	-125	0	0	-80	86	174	-175	0	0
-21	27	125	-126	0	0	-81	87	175	-176	0	0
-22	28	126	-127	0	0	-82	88	176	-177	0	0
-23	29	127	-128	0	0	-83	89	177	-178	0	0
-24	30	128	0	0	0	-84	90	178	0	0	0
-25	31	0	-129	0	0	-85	91	0	-179	0	0
-26	32	129	-130	0	0	-86	92	179	-180	0	0
-27	33	130	-131	0	0	-87	93	180	-181	0	0
-28	34	131	-132	0	0	-88	94	181	-182	0	0
-29	35	132	-133	0	0	-89	95	182	-183	0	0
-30	36	133	0	0	0	-90	96	183	0	0	0
-31	37	0	-134	0	0	-91	97	0	-184	0	0
-32	38	134	-135	0	0	-92	98	184	-185	0	0
-33	39	135	-136	0	0	-93	99	185	-186	0	0
-34	40	136	-137	0	0	-94	100	186	-187	0	0
-35	41	137	-138	0	0	-95	101	187	-188	0	0
-36	42	138	0	0	0	-96	102	188	0	0	0
-37	43	0	-139	0	0	-97	103	0	-189	0	0
-38	44	139	-140	0	0	-98	104	189	-190	0	0
-39	45	140	-141	0	0	-99	105	190	-191	0	0
-40	46	141	-142	0	0	-100	106	191	-192	0	0
-41	47	142	-143	0	0	-101	107	192	-193	0	0
-42	48	143	0	0	0	-102	108	193	0	0	0
-43	49	0	-144	0	0	1	2	3	4	5	6
-44	50	144	-145	0	0						
-45	51	145	-146	0	0						
-46	52	146	-147	0	0						
-47	53	147	-148	0	0						
-48	54	148	0	0	0						
-49	55	0	-149	0	0						
-50	56	149	-150	0	0						
-51	57	150	-151	0	0						
-52	58	151	-152	0	0						
-53	59	152	-153	0	0						
-54	60	153	0	0	0						
-55	61	0	-154	0	0						
-56	62	154	-155	0	0						
-57	63	155	-156	0	0						
-58	64	156	-157	0	0						
-59	65	157	-158	0	0						
-60	66	158	0	0	0						

Fig. 16. 788116

TYPICAL VACUUM VESSEL MESH INDUCTANCE

Fig. 17. 788224

Fig. 18. 788223

$t = 80 \mu\text{sec}$

$t = 160 \mu\text{sec}$

$t = 320 \mu\text{sec}$

$t = 480 \mu\text{sec}$

Fig. 19. 794003

Fig. 20. 788299

EXAMPLE OF STREAMLINE PLOTTING TECHNIQUE

Fig. 21. 788311

Location of mesh current values
which were used to create streamline plot
of vacuum vessel eddy currents

Fig. 22. 788300

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