ENHANCED RESISTIVE WALL INSTABILITY FOR OFF-CENTERED BEAMS

E.D. Courant, M. Monch, C. Pellegrini and J.M. Wang

Summary

Beam occupation of a large fraction of the available vacuum chamber, typical of high energy proton storage ring designs, results in an enhancement of the resistive wall instability. The effect is considered for ISABELLE during the current stacking procedure. Results for the coasting stack in its initial phase as well as for the injected bunches are presented.

1. INTRODUCTION

High energy proton storage rings are designed to make maximal use of the available vacuum chamber aperture. This is dictated primarily by economic considerations. The accumulation of current in a typical high energy ring creates a rather unusual beam configuration. In particular, we can have a ribbon beam in a circular chamber set off the central axis toward one side of the chamber in the median plane. It might be anticipated that such a condition could produce an enhanced resistive wall instability. Since the threshold is a strong function of the chamber radius, we could even guess, a priori, that for an off-centered beam, the threshold would significantly decrease and be roughly related to an "effective radius" which is simply the distance of closest approach to the chamber.

To use the chamber aperture optimally for high current accumulation, we would like to be able to position the beam as close as possible to the chamber on one side and to complete the ribbon with continued addition of injected pulses. The total beam within the chamber, therefore, has a variety of phases: 1) Injected bunch, 2) "Small" stack-close to wall, 3) Wide ribbon-off center, and 4) Very wide centered ribbon for bunched stack. In the latter two phases, the standard treatment of the resistive wall instability, leading to a dispersion relation for the coherent frequency shift, is not adequate. Since the induced fields are sensitive to beam position within the vacuum chamber, the resulting coherent modes have variations across the beam width. In other words, there is a coupling of one part of the beam to another through the image fields. This problem which involves the solution of an integral equation for the coherent oscillation mode has been treated by E.D. Courant and M. Monch. They show that even for a wide ribbon there is a significant effect, but that the enhancement tends to be most severe for a narrow off-centered beam. We will, therefore, restrict ourselves to the case of a small stack.

We treat, therefore, the two phases, that of a "small" stack close to one wall, corresponding to the initial period of the stacking process, and that of the injected bunches close to the other wall. The transverse aperture is shown in Figure 1. The case of the small coasting stack is presented in section 2, while that of the injected bunches is detailed in section 3. ISABELLE parameters are used throughout. Some implications for the ISABELLE design resulting from the resistive wall enhancement are discussed in section 4.

2. NARROW COASTING STACK

The development of coherence in the beam is inhibited by a frequency spread. We take this spread to arise from the spread in linear betatron tunes. In terms of a normalized tune variable, \( u \), we can show by standard methods that the dispersion relation for the coherent frequency, \( \omega_c \), can be written

\[
1 = \int g(u) \, du
\]

where \( g(u) \) is the beam position in the vacuum chamber as a fraction of the chamber radius, and \( \delta \) is a strong function of the chamber radius, we could even guess, a priori, that for an off-centered beam, the threshold would significantly decrease and be roughly related to an "effective radius" which is simply the distance of closest approach to the chamber.

To use the chamber aperture optimally for high current accumulation, we would like to be able to position the beam as close as possible to the chamber on one side and to complete the ribbon with continued addition of injected pulses. The total beam within the chamber, therefore, has a variety of phases: 1) Injected bunch, 2) "Small" stack-close to wall, 3) Wide ribbon-off center, and 4) Very wide centered ribbon for bunched stack. In the latter two phases, the standard treatment of the resistive wall instability, leading to a dispersion relation for the coherent frequency shift, is not adequate. Since the induced fields are sensitive to beam position within the vacuum chamber, the resulting coherent modes have variations across the beam width. In other words, there is a coupling of one part of the beam to another through the image fields. This problem which involves the solution of an integral equation for the coherent oscillation mode has been treated by E.D. Courant and M. Monch. They show that even for a wide ribbon there is a significant effect, but that the enhancement tends to be most severe for a narrow off-centered beam. We will, therefore, restrict ourselves to the case of a small stack.

We treat, therefore, the two phases, that of a "small" stack close to one wall, corresponding to the initial period of the stacking process, and that of the injected bunches close to the other wall. The transverse aperture is shown in Figure 1. The case of the small coasting stack is presented in section 2, while that of the injected bunches is detailed in section 3. ISABELLE parameters are used throughout. Some implications for the ISABELLE design resulting from the resistive wall enhancement are discussed in section 4.

2. NARROW COASTING STACK

The development of coherence in the beam is inhibited by a frequency spread. We take this spread to

\[
1 = \int g(u) \, du
\]

where \( g(u) \) is the beam position in the vacuum chamber as a fraction of the chamber radius, and \( \delta \) is a strong function of the chamber radius, we could even guess, a priori, that for an off-centered beam, the threshold would significantly decrease and be roughly related to an "effective radius" which is simply the distance of closest approach to the chamber.

To use the chamber aperture optimally for high current accumulation, we would like to be able to position the beam as close as possible to the chamber on one side and to complete the ribbon with continued addition of injected pulses. The total beam within the chamber, therefore, has a variety of phases: 1) Injected bunch, 2) "Small" stack-close to wall, 3) Wide ribbon-off center, and 4) Very wide centered ribbon for bunched stack. In the latter two phases, the standard treatment of the resistive wall instability, leading to a dispersion relation for the coherent frequency shift, is not adequate. Since the induced fields are sensitive to beam position within the vacuum chamber, the resulting coherent modes have variations across the beam width. In other words, there is a coupling of one part of the beam to another through the image fields. This problem which involves the solution of an integral equation for the coherent oscillation mode has been treated by E.D. Courant and M. Monch. They show that even for a wide ribbon there is a significant effect, but that the enhancement tends to be most severe for a narrow off-centered beam. We will, therefore, restrict ourselves to the case of a small stack.

We treat, therefore, the two phases, that of a "small" stack close to one wall, corresponding to the initial period of the stacking process, and that of the injected bunches close to the other wall. The transverse aperture is shown in Figure 1. The case of the small coasting stack is presented in section 2, while that of the injected bunches is detailed in section 3. ISABELLE parameters are used throughout. Some implications for the ISABELLE design resulting from the resistive wall enhancement are discussed in section 4.

2. NARROW COASTING STACK

The development of coherence in the beam is inhibited by a frequency spread. We take this spread to

\[
1 = \int g(u) \, du
\]

where \( g(u) \) is the beam position in the vacuum chamber as a fraction of the chamber radius, and \( \delta \) is a strong function of the chamber radius, we could even guess, a priori, that for an off-centered beam, the threshold would significantly decrease and be roughly related to an "effective radius" which is simply the distance of closest approach to the chamber.

To use the chamber aperture optimally for high current accumulation, we would like to be able to position the beam as close as possible to the chamber on one side and to complete the ribbon with continued addition of injected pulses. The total beam within the chamber, therefore, has a variety of phases: 1) Injected bunch, 2) "Small" stack-close to wall, 3) Wide ribbon-off center, and 4) Very wide centered ribbon for bunched stack. In the latter two phases, the standard treatment of the resistive wall instability, leading to a dispersion relation for the coherent frequency shift, is not adequate. Since the induced fields are sensitive to beam position within the vacuum chamber, the resulting coherent modes have variations across the beam width. In other words, there is a coupling of one part of the beam to another through the image fields. This problem which involves the solution of an integral equation for the coherent oscillation mode has been treated by E.D. Courant and M. Monch. They show that even for a wide ribbon there is a significant effect, but that the enhancement tends to be most severe for a narrow off-centered beam. We will, therefore, restrict ourselves to the case of a small stack.

We treat, therefore, the two phases, that of a "small" stack close to one wall, corresponding to the initial period of the stacking process, and that of the injected bunches close to the other wall. The transverse aperture is shown in Figure 1. The case of the small coasting stack is presented in section 2, while that of the injected bunches is detailed in section 3. ISABELLE parameters are used throughout. Some implications for the ISABELLE design resulting from the resistive wall enhancement are discussed in section 4.

2. NARROW COASTING STACK

The development of coherence in the beam is inhibited by a frequency spread. We take this spread to
rate, \( \text{Im}(w) = 1/T \geq 0 \). The threshold is the limit of zero growth rate, i.e., \( \text{Im}(w) = 0 \). Since \( u \) and \( w \) differ in sign, a threshold solution to the dispersion relation (1) is found by taking the limit \( \text{Im}(u^c) \to 0 \).

In ISABELLE, the frequency for the lowest unstable mode \( (\alpha = 23) \) is \( f_c = 79.64 \text{ kHz} \) at 30.26 kHz. At this frequency the skin depth for stainless steel, \( \delta_s = 2.89 \text{ mm} \) and for copper, \( \delta_c = 0.38 \text{ mm} \). These interesting in that the ISABELLE vacuum chamber is a double walled structure, with the inner stainless steel thickness, \( 1 \text{ mm} \). Thus, the fields at this low frequency penetrate through the stainless steel into the copper where they are attenuated. In this case, the skin depth is a complex number. Satisfying the appropriate boundary conditions at the stainless steel surface as well as at the stainless steel-copper interface, we can find the effective skin depth in general:

\[
\delta_{\text{eff}} = \delta_s \left[ (1+\delta_c/\delta_s) e^{-2\delta_s q} \right]^{1/2}
\]

where \( q \) is the stainless steel thickness, and \( u_c = (1-1)/\delta_s \). For the ISABELLE parameters listed in Table I, we find for the lowest \( n = 23 \) mode, \( \delta_{\text{eff}} = (1.414 - 0.8161) \text{ mm} \).

The ISABELLE beam parameters are given in Table II while the "small" beam is defined in Table III.

The threshold depends on the "current-tune" density, \( I = I/2d \). This quantity has a design value essentially independent of the amount of current stacked. We can therefore use this value even in the case of a 20% stack. To be definite we define a normalized current, \( \eta = I/2d = I/I_0 \), where the value \( \eta = 1 \) corresponds to the threshold for a small centered beam with a uniform tune distribution. Introducing \( \eta \) explicitly, we can write for the dispersion relation (1),

\[
1 = \frac{\eta}{\omega} \int \frac{d(u)}{1 - \eta u} du,
\]

where \( \overline{G} \) is evaluated with the nominal current and tune spread given in Table II. Thus, we can see how the threshold changes for a different distribution or as a function of where the beam is within the chamber.

### Table II ISABELLE Beam Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average vertical ( \beta_v )</td>
<td>28.3 m</td>
</tr>
<tr>
<td>Average horizontal ( \beta_h )</td>
<td>46.7 m</td>
</tr>
<tr>
<td>Vertical dispersion, ( \chi_v )</td>
<td>2.27 m</td>
</tr>
<tr>
<td>Horizontal dispersion, ( \chi_h )</td>
<td>4.73 m</td>
</tr>
<tr>
<td>Average current, ( I )</td>
<td>1.6 A</td>
</tr>
<tr>
<td>Nominal full current, ( I_0 )</td>
<td>433.5 A/unit</td>
</tr>
</tbody>
</table>

### Table III Beam Parameters with Point Beam Model

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Beam current, ( I )</td>
<td>1.6 A</td>
</tr>
<tr>
<td>Tune spread, ( \pm d )</td>
<td>1.84 X 10^-3</td>
</tr>
<tr>
<td>Momentum spread, ( \pm \delta )</td>
<td>0.12</td>
</tr>
<tr>
<td>Momentum vertical size, ( \delta )</td>
<td>2.27 mm</td>
</tr>
<tr>
<td>Horizontal size, ( \Delta )</td>
<td>7.0 mm</td>
</tr>
</tbody>
</table>

In Figure 2, we plot the threshold current, \( \eta(t_o) \), as a function of position in the chamber.

### Figure 2. Current threshold function for "small beam" case. \( \eta \) is normalized to 1 at chamber-center with uniform distribution. \( t_o = \text{distance off center to chamber center} \). I. Uniform density distribution. II. Equal \( \sin^2 \) flanks: 80% flat.

The curves in this figure are thresholds in the sense that \( \text{Im}(u^c) = 0 \). Each solution corresponds to a frequency shift function, \( u^c(t_o) = \Re(u_c) \), when \( \text{Im}(u_c) = 0 \). This frequency shift function is plotted in Figure 3.

Thus, we see that for the "small" coasting stack, the threshold at the chamber center is determined by the beam self-field term (the so-called capacitive term). As we move off center, the image fields tend...
This quantity gives us a feeling for the demands on a feedback system to control growing oscillations if Landau damping from tune spread is insufficient.

3. INJECTED BUNCHED BEAM

The ISABELLE current accumulation process involves the injection of a bunched beam with ~ 4.5 MHz bunch frequency from the AGS. Using five pulses from the AGS, the ISABELLE injection orbit is filled with 57 equally spaced bunches. Assuming these bunches to have equal intensity, there are 57 normal modes of transverse coherent oscillations. The normal mode frequencies are given by \( \omega_n = \omega_0 (tM + s - v_0) \) with \( s = 1, \ldots, M \). Here, \( M \) is the number of bunches (57 in the ISABELLE case treated) and \( s \) is the mode number; \( v_0 \) is the central betatron frequency for the bunch, \( \omega_0 \) is the revolution frequency in radians/sec, and \( \ell \) is any integer.

For each normal mode, we have a dispersion relation similar to (1),

\[
\omega \approx \omega_0 (nM + s - v_0) \quad \text{with} \quad s = 1, \ldots, M.
\]

Here, \( s = (\omega_0 - v)/\omega_0 \) with \( \omega_0 \) the half width of the tune distribution. This distribution is taken to be parabolic: \( \omega(u) = \frac{3}{4} (1 - u^2) \). The normalized coherent frequency \( \omega_0 \) is given by \( \omega_0 = (e^2 + s - v_0)/\omega_0 \). The quantity \( G_B \) takes into account the bunch structure of the beam: \( G_B = N_B (\omega_0 + v_0)/\omega_0 \), where \( N_B \) is the number of particles per bunch and

\[
\omega_0 + \omega = \frac{\omega_0}{N_B} (1 - \frac{h^2}{b^2 (1-\ell_0^2)^2} + \frac{\omega_0 + v_0}{b^2 (1-\ell_0^2)^2} + (1+\ell) A - \frac{s}{b^2 (1-\ell_0^2)^2} ).
\]

The direct beam term is written in terms of the beam space charge tune shift, \( \delta v \). For an elliptical beam of semiaxes \( A \), horizontal, and \( h \), vertical and for a parabolic density distribution, the vertical tune shift is given by

\[
\delta v = \frac{2 B \delta v}{\pi \gamma v_0^3 h (4h + B)}
\]

where \( B \) is the bunching factor, bunch length/bunch separation, \( B = B/L \), with \( L \) the bunch length. Notice that the bunch is taken to be parabolic in density, while the space density is almost uniform (we have taken 20% \( \sin^2 \) tails). The quantity \( A \) is given by

\[
A = \frac{\delta v}{2 B \gamma v_0^3}
\]

while \( \delta v \) is the effective complex skin depth defined by (3) for the double walled chamber. It is evaluated at the unstable frequency, \( \omega = (e^2 + s - v_0)/\omega_0 \).

In obtaining the dispersion relation (5), we have taken the betatron frequency spread to be associated with an "external" variable, the momentum spread. This provides Landau damping. In the bunched beam case this is only valid if we are dealing with a growth condition occurring on a time scale sufficiently shorter than the synchrotron oscillation period. A quantitative criterion for this is that the zero frequency spread growth period, \( T_0 \), be larger than the synchrotron period, \( T_s \); that is, \( T_0 < T_s \). We will see that this condition is satisfied in our case.
In evaluating the image contribution to $G$, the second term on the rhs of (6), we have taken the "point" bunch. While the imaginary part of $\xi^{\text{eff}}(\omega)$ is convergent, the real part is divergent. For a bunch of nonzero length, the skin depth $\delta^{\text{eff}}$ must be multiplied by a phase factor which ensures convergence. We can take account of the nonzero bunch length in a simple manner by introducing a cutoff in the sum. We terminate the series at $\xi_0$, corresponding to the inverse of the bunching factor: $\xi_0 = 1/\beta$. This is equivalent to limiting the contribution to only those frequencies with wavelengths longer than the bunch length, a valid approximation for studying rigid bunch motion.

The rise time $t_R$ of an unstable oscillation is determined by the term $\nu_0$: $1/\gamma = \nu_0 \Im(G_0) = \nu_0 \nu_0^2 \nu_0^2$. For the most unstable mode for which $s = \nu_0$, we have $\gamma = 50$ msec (on axis, $t_0 = 0$). This is enhanced by a factor of 35 at $t_0 = 0.8$, giving $\gamma = 1.4$ m sec. Even the growth period at the chamber center is about four times shorter than the synchrotron period for ISABELLE at injection which is about 180 m sec. See Table IV.

Table IV Injected Bunch Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Energy (GeV)</td>
<td>29.4</td>
</tr>
<tr>
<td>Number of protons/bunch</td>
<td>$N_b = 2.5 \times 10^{11}$</td>
</tr>
<tr>
<td>Horizontal emittance (rad-m)</td>
<td>$E_h = 15^\circ \times 10^{-6}$</td>
</tr>
<tr>
<td>Longitudinal emittance (e-sec)</td>
<td>$E_l = 15^\circ \times 10^{-6}$</td>
</tr>
<tr>
<td>Beam height (cm)</td>
<td>$2h = 0.58$</td>
</tr>
<tr>
<td>Beam width (cm)</td>
<td>$2a = 0.78$</td>
</tr>
<tr>
<td>Momentum spread</td>
<td>$\Delta p/p = 10^{-3}$</td>
</tr>
<tr>
<td>Bunch length (m)</td>
<td>$L = 10$</td>
</tr>
<tr>
<td>Synchrotron frequency $\nu_0$</td>
<td>$\nu_0 = 6.9 \times 10^{-3}$</td>
</tr>
<tr>
<td>Synchrotron period (sec)</td>
<td>$\tau = 0.18$</td>
</tr>
</tbody>
</table>

The dependence of $\tau_R$ on $t_0$ is, in fact, the same as for the coasting stack, Figure 3. However, the magnitude of $\tau_R$ should be increased by a factor of 5.

Thus, we have that Landau damping from linear tune spread will be effective, as we have assumed in writing the dispersion relation (5). This tune spread arises from bunch momentum spread and the ISABELLE chromaticity. With an overall momentum spread in the bunch of 0.1% and a chromaticity of $\chi = 2$, we have a frequency spread $\Delta \omega = 1 \times 10^{-3}$. Taking a parabolic distribution, we can calculate the threshold for transverse coherent rigid bunch oscillations. The result is shown in Figure 3 where the threshold, in terms of the maximum number of particles per bunch, $N_p$, is given as a function of the ratio of $x_b$, the distance of the bunch center from the chamber center to $b$, the chamber radius: $t_0 = x_b/b$. The peak in $N_p$ at $t_0 = 0.76$ corresponds to $x_b$ passing through the value zero, meaning a cancellation of the direct space charge with the reactive part of the image contribution. Once this cancellation occurs, the image term dominates and the enhancement is rapid and evident in the figure. The bunch parameters used in the calculation are given in Table IV.

4. Conclusions

We have reviewed the impact of the resistive wall instability as it relates to the use of phase displacement stacking. Specific computations for ISABELLE are presented. The essential features which cause an enhancement of the instability are (1) the use of a large part of the chamber aperture to accumulate large currents and (2) the injection of bunches with high transverse density for the purpose of optimizing the luminosity of the colliding beams.

The initial coasting stack is set off the chamber close to the wall. Also, the nominal allowed tune spread, a large reduction in the threshold current results with the details depending sensitively on how close to the wall the first pulses are stacked. A factor of 10 to 100 decrease in threshold occurs if 80%-90% of the aperture is to be used. The growth rates tend to be high under unstable conditions ($T > 0.1$ msec). Because of the high transverse density of the injected bunches, the threshold intensity, $N_p$, the maximum number of particles per bunch, is low. Even without wall enhancement, the threshold is $N_p = 4 \times 10^{10}$ protons/injected bunch. This is well below the design value of $2.5 \times 10^{11}$ protons per bunch. At the injection orbit, about 80% to the wall, the growth periods are short, $\tau_R = 1$ msec.

One way to increase the threshold is to increase the betatron frequency spread. This may be accomplished by increasing the chromaticity from its design value with a multipole arrangement or by adding an octupole term, which introduces a tune spread with higher amplitude. This means modifying the ISABELLE design so as to extend the available betatron tune working line. We might mention that an octupole distribution could produce "curvature in the working line" and thereby cause localized beam instabilities (brick-wall effect).

Another possibility to cope with the unstable conditions is to employ a direct feedback system. Such a system should provide coherence damping on a time scale less than 1 msec and should also be capable of acting on each injected bunch independently. Furthermore, the injected bunches and coasting stack must exist simultaneously, thus necessitating a combined feedback process. Finally, the injected bunches will have finite initial coherent amplitudes due to injection errors, therefore, placing an increased demand on the feedback system in both speed and strength.

References

3. ISABELLE - Proton-Proton Colliding Beam Facility, BNL 50718 (1978).