Non-Critical Superstrings from Two Dimensional N=1 Supergravity

Zongan Qiu

Department of Physics
University of Florida
Gainesville, FL 32611

We describe a class of non-critical superstrings of $\hat{d} < 1$ arising from N=1 superconformal field theories coupled to 2-d supergravity. One loop partition functions are found and shown to be modular invariant. Furthermore the one loop partition functions vanish as consequences of identities of the character formula.

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1. Introduction

Recently considerable progress has been made in non-critical string theories of $d < 1$ arising from minimal models coupled to 2-d gravity[1]. These theories can be regarded as toy models of the critical bosonic string theory, and the non-perturbative understanding from matrix model[2] approach yields new insight into the structure of string theory. Nevertheless the more interesting region of $d > 1$ remains an unknown territory. The problem is believed to be closely related to the tachyon of the bosonic string. In critical superstrings the tachyon is projected out by a suitable GSO projection[3]. Therefore the understanding of non-critical superstrings might offer a resolution to the above problem.

Many results in 2-d gravity have been generalized to 2-d supergravity[4][5]. The connection of these theories to strings has not been addressed clearly[6]. Alternatively, the non-critical superstrings proposed by Kutasov and Seiberg[7] are based on introducing an auxiliary superfield in the Liouville sector and demanding global $N = 2$ supersymmetry. The origin of the auxiliary field is not clear. These theories are probably “compactifications” of some fundamental non-critical superstrings. Therefore the fundamental understanding of non-critical superstrings remains an outstanding problem.

In this letter we describe a class of non-critical superstrings arising from 2-d discrete superconformal field theories (SCFT) coupled to 2-d supergravity. These theories have a finite number of physical states and therefore possess the same simplicity as $d < 1$ non-critical bosonic string theories. The crucial observation leading to our construction is matching of dressing coefficients between Neveu-Schwarz (NS) and Ramond (R) fields. Furthermore we find a GSO projection which leads to non-critical superstrings. This projection is different from the GSO projection in minimal SCFT. It is analogous to the situation of the critical superstring where the the GSO projection corresponding $N=1$, $\hat{d} = 10$ SCFT will give a string theory of space-time bosons including tachyon. While the superstring is obtained by another GSO projection which projects out the tachyon and gives supersymmetric spectrum. The existence of the GSO projection crucially depends on the matching of NS and R fields in the theory. This is a highly non trivial condition and therefore it is remarkable that we are able to find matching fields and suitable GSO projection in $\hat{d} < 1$ SCFT coupled to 2-d supergravity. We will give a brief description of the cases where the matter theories are unitary. More detailed discussion and general cases will be given in a separate publication[8].

2. Spectrum and the Critical exponents

Let's first discuss the matter sector of the theories. The discrete N=1 SCFT of $\hat{d} < 1$ have central charges given by[9]

$$\hat{d} = 1 - \frac{8}{m(m+2)}, \quad m = 2, 3, \dots \quad (2.1)$$

and the SCFT for each m is constructed out of a set of primary fields of conformal weights

$$h = \frac{[p(m+2) - p'm]^2 - 4}{8m(m+2)} + \frac{1}{32} [1 - (-1)^{p-p'}] \quad 1 \leq p \leq m-1, 1 \leq p' \leq m+1. \quad (2.2)$$

SCFT has two sectors, NS and R sectors. $p + p'$ even and odd in (2.2) correspond to fields in these two sectors respectively. The fields in the NS sector are superfields and those in the R sector are ordinary conformal fields distinct from superfields and in fact non-local(double valued) with respect to fermionic part of superfields.

The coupling of SCFT with 2-d supergravity has been studied in both "light-cone" gauge[4] and conformal gauge[5]. In the conformal gauge, supergravity can be represented by one superfield, the super-Liouville field[10]. Moreover on the quantum level, as in bosonic case[11], the super-Liouville field can be represented essentially by a free scalar superfield[5]. We will work in the conformal gauge and use the formalism developed by Distler, Hlousek and Kawai[5]. In conformal gauge the partition function of the theory is given by

$$\int [dY][DX][D\Phi][Db][Dc][D\beta][D\gamma] e^{-S_X - S_{gh} - S_l} \quad (2.3)$$

where X denotes the matter fields and the super-Liouville action S_l is given in terms of components of $\Phi = \phi + \bar{\theta}\bar{\psi} + \theta\psi$ as

$$S_l = \frac{1}{2\pi} \int d^2z \left(\partial\phi\bar{\partial}\phi - \frac{1}{4}Q\hat{R}\phi - \bar{\psi}\partial\bar{\psi} - \psi\bar{\partial}\psi \right) + S_c \quad (2.4)$$

Notice that we have performed integral over $\theta, \bar{\theta}$ and the S_c is a term used to set the scale of the theory. The choice of S_c depends on the field contents of the matter theory and the detail analysis of it for N=1 supergravity theories is given in [6]. We will give the result for the non-critical superstrings below.

The stress tensors of the super-Liouville theory are given by

$$\begin{aligned} T_\Phi(z) &= -\frac{1}{2}(\partial\phi\partial\phi + Q\partial^2\phi - \psi\partial\psi) \\ G_\Phi(z) &= -\frac{1}{2}(\psi\partial\phi + Q\partial\psi) \end{aligned} \quad (2.5)$$

which fix the constant Q in terms of the central charge $\hat{d}$ of the matter theory, $Q = \sqrt{\frac{9-\hat{d}}{2}}$.

The supergravity will dress up a matter superfield in the NS sector to form a vertex operator $e^{\beta\Phi}\Psi_{NS}$ of weight $(\frac{1}{2}, \frac{1}{2})$. Therefore the dress coefficient β is determined by the equation

$$h - \frac{1}{2}\beta(\beta + Q) = \frac{1}{2} \quad (2.6)$$

which has two solutions $\beta_{\pm}$ given by

$$\beta_{\pm} = -\frac{1}{2\sqrt{2}}(\sqrt{9-\hat{d}} \mp \sqrt{1-\hat{d}+16h}) \quad (2.7)$$

The $\beta_+ = \beta$ is the physical solution[5][12]. A vertex operator from the R sector of the theory is a dressed field of weight $(1, 1)$. It has the following form

$$e^{\beta\phi}\Psi_R^{gh}\Psi_R^l\Psi_R = e^{\beta\phi}\Psi_R^t \quad (2.8)$$

with Ψ_R^{gh} , Ψ_R^l are conformal fields of weights $\frac{3}{8}$, $\frac{1}{16}$ and denote the spin fields for the ghost and super-Liouville sectors respectively[13][5][14]. Therefore the dressing coefficient in the R sector is determined by the equation

$$h + \frac{3}{8} + \frac{1}{16} - \frac{1}{2}\beta(\beta + Q) = 1 \quad (2.9)$$

The physical solution of the above equation is given by

$$\beta = -\frac{1}{2\sqrt{2}}(\sqrt{9-\hat{d}} - \sqrt{1-\hat{d}+16(h-\frac{1}{16})}) \quad (2.10)$$

The critical exponents of 2-d supergravity are the characteristic properties of the theory under scaling. In the non-critical superstrings we will see below that the identity operator in the NS sector has the smallest dressing coefficient. The scale term in the Liouville action is therefore given by

$$S_c = -\frac{\mu}{4\pi} \int d^2z \psi \bar{\psi} e^{\alpha\phi} \quad (2.11)$$

The scaling of the partition function can be obtained by inserting a delta function

$$\delta(\int d^2z \psi \bar{\psi} e^{\alpha\phi} - L) \quad (2.12)$$

into the partition function (2.3). The dressing coefficient α is given by (2.7) with $h = 0$. The partition function on the surface with Euler number χ is then given by

$$F_\chi(L) = (L/L_0)^{\gamma_\chi - 3} F_\chi(L_0) \quad (2.13)$$

from a simple scaling argument. The exponent γ_χ is given by

$$\gamma_\chi = 2 + \frac{\chi Q}{2\alpha} \quad (2.14)$$

The string susceptibility γ is the exponent on the sphere

$$\gamma = \gamma_2 = \hat{d} - 1 - \sqrt{(9 - \hat{d})(1 - \hat{d})} \quad (2.15)$$

We are now ready to describe the spectrum of the non-critical superstrings. The bosonic and fermionic states are represented by vertex operators which are densities on the modular space of punctured surfaces. It is essential to find fermionic states in the spectrum. A fermion vertex operator is a dressed field (NS,R) or (R,NS) which requires the existence of NS and R fields in the matter theory to have the same dressing coefficient. This matching condition is a necessary condition for constructing non-critical superstrings[6]. Comparing (2.2)(2.7)(2.10), it is straightforward to check that the matching condition only exists for fields in the discrete SCFT $m = 2l \geq 4$ even[9]. Vertex operators of the non-critical superstring correspond to dressed fields with $1 \leq p \leq l-1, 1 \leq p' \leq l$. The bosonic vertex operators are dressed (NS, NS) and (R,R) fields

$$\begin{aligned} V_{p,p'}^B d^2 z d^2 \theta &= \Psi_{p,p'}(z, \theta) \Psi_{p,p'}(\bar{z}, \bar{\theta}) e^{\beta_{p,p'} \Phi(z, \bar{z}, \theta, \bar{\theta})} d^2 z d^2 \theta, \quad p + p' \text{ even} \\ V_{p,p'}^B d^2 z &= \Psi_{p,p'}^t(z) \Psi_{p,p'}^t(\bar{z}) e^{\beta_{p,p'} \phi(z, \bar{z})} d^2 z, \quad p + p' \text{ odd} \end{aligned} \quad (2.16)$$

and the fermionic vertex operators are dressed (NS,R) and (R, NS) fields and given by

$$\begin{aligned} V_{p,p'}^F d^2 z d\theta &= \\ \Psi_{p,p'}(z, \theta) \Psi_{l-p, l+1-p'}^t(\bar{z}) e^{\beta_{p,p'} (\phi(z, \bar{z}) + \theta \psi(z))} d^2 z d\theta, \quad p + p' \text{ even} \\ V_{p,p'}^F d^2 z d\bar{\theta} &= \\ \Psi_{p,p'}^t(z) \Psi_{l-p, l+1-p'}(\bar{z}, \bar{\theta}) e^{\beta_{p,p'} (\phi(z, \bar{z}) + \bar{\theta} \bar{\psi}(\bar{z}))} d^2 z d\bar{\theta}, \quad p + p' \text{ odd} \end{aligned} \quad (2.17)$$

where we keep the explicit dependence on the world-sheet coordinates for clarity. Of course we have done the proper chiral projection in the NS and R sectors such that the resulting

vertex operators are local with respect to each other and the degeneracy in the R sector is removed. In the next section we will see that not all the vertex operators in (2.16)(2.17) appear in the final theory.

The important reason we are able to construct fermionic vertex operators (2.17) is that the corresponding NS and R matter fields share the same dressing coefficient and the super-Liouville zero mode can be included in the dressing. The matching of the dressing coefficient is a necessary condition for constructing a fermionic vertex operator in 2-d supergravity[6]. Furthermore the role of world sheet supersymmetry in the R sector of the matter theory is unclear because in the final theory the supersymmetric ground state will be projected out.

The other critical exponents of the theory are defined from scaling behavior of the two point function of vertex operators with respect to L by

$$\langle V(z)V(w) \rangle_L = (L/L_0)^{2(1-\Delta)} \langle V(z)V(w) \rangle_{L_0} \quad (2.18)$$

One can easily check that the exponent Δ is given by

$$\Delta = 1 - \frac{\beta}{\alpha} \quad (2.19)$$

We therefore have the following result for critical exponents

$$\begin{aligned} \gamma &= -\frac{1}{l}, \\ \Delta_{p,p'} &= \frac{|(l+1)p - lp'| - 1}{2l} \end{aligned} \quad (2.20)$$

It is straightforward to check that $V_{p,p'}^B$ and $V_{p,p'}^F$ have the same critical exponent Δ . This is expected in a supersymmetric theory because supersymmetry commutes with scaling. One should also notice that the set of exponents in (2.20) are identical to those of l th discrete series coupled with 2-d quantum gravity.

3. One Loop Modular Invariance

The remaining question is the consistence of the theories. For this purpose we will study the modular invariance in one loop, i.e. on the torus. Partition function of the theory has the following form in the path integral

$$Z_1 \sim \int_{\mathcal{F}} \int [d\Upsilon][\mathcal{D}X][\mathcal{D}\Phi][\mathcal{D}b][\mathcal{D}c][\mathcal{D}\beta][\mathcal{D}\gamma] e^{-S_X - S_{gh} - S_l} \quad (3.1)$$

where the z -integration runs over a torus with modular parameter $\tau = \tau_1 + i\tau_2$ and $\mathcal{F}$ denotes the fundamental region of the modular group: $\tau_2 > 0, |\tau| > 1, -\frac{1}{2} \leq \tau_1 < \frac{1}{2}$. The summation over spin structures is included in $d\Upsilon$.

The contribution of the matter sector are given in terms of the characters of the superconformal algebra. We list results for the discrete SCFT here. The character of a representation of superconformal algebra defined by

$$\chi_{p,q} = \text{tr} e^{2\pi i \tau (L_0 - \frac{d}{16})} = \text{tr} q^{L_0 - \frac{d}{16}} \quad (3.2)$$

will give us the counting of states on the torus. It is well-known for representations given in (2.1)(2.2) that the characters are given by [15]

$$\chi_{p,p'}^{NS} = f^{NS}(q) \Gamma_{p,p'}^{2l}, \quad f^{NS}(q) = q^{\frac{-1}{48}} \eta^{-1}(q) \prod_{n=1}^{\infty} (1 + q^{n-\frac{1}{2}}) \quad (3.3)$$

for the NS sector and

$$\chi_{p,p'}^R = f^R(q) \Gamma_{p,p'}^{2l}, \quad f^R(q) = q^{\frac{1}{24}} \eta^{-1}(q) \prod_{n=1}^{\infty} (1 + q^n) \quad (3.4)$$

for the R sector where $\eta(q)$ is the Dedekind function, $1 \leq p \leq m-1, 1 \leq p' \leq m+1$ and $p-p'$ is even in the NS sector and odd in the R sector. The functions $\Gamma_{p,p'}^{2l}$ are defined by

$$\Gamma_{p,p'}^{2l}(q) = \sum_{n \in \mathbb{Z}} \left(q^{a_{p,p'}^{2l}(n)} - q^{b_{p,p'}^{2l}(n)} \right) \quad (3.5)$$

where

$$\begin{aligned} a_{p,p'}^{2l}(n) &= \frac{[4l(l+1)n + p(l+1) - p'l]^2}{8l(l+1)} \\ b_{p,p'}^{2l}(n) &= \frac{[4l(l+1)n + p(l+1) + p'l]^2}{8l(l+1)} \end{aligned} \quad (3.6)$$

We also have the projected characters in the NS sector as

$$\tilde{\chi}_{p,p'}^{NS} = \text{tr} (-1)^F q^{L_0 - \frac{d}{16}} = \tilde{f}^{NS}(q) \tilde{\Gamma}_{p,p'}^{2l}, \quad \tilde{f}^{NS}(q) = q^{\frac{-1}{48}} \prod_{n=1}^{\infty} (1 - q^{n-\frac{1}{2}}) \eta^{-1}(q) \quad (3.7)$$

where F is the fermion number operator and the functions $\tilde{\Gamma}$ are given by

$$\tilde{\Gamma}_{p,p'}^{2l}(q) = \sum_{n \in \mathbb{Z}} \left(q^{a_{p,p'}^{2l}(n)} - (-1)^{pp'} q^{b_{p,p'}^{2l}(n)} \right) \quad (3.8)$$

The holomorphic contributions of the ghost fields to the partition function for different spin structures are well-known except for a phase factor which is absent in the final theories. The holomorphic contributions from the oscillators part of the super-Liouville theory to the partition function for different spin structures are the same as those of one free scalar superfield. They are given in the following table

	<i>ghost</i>	<i>Liouville</i>
$(-, -):$	$(f^{NS}(q))^{-2}$	$f^{NS}(q)$
$(-, +):$	$(\tilde{f}^{NS}(q))^{-2}$	$\tilde{f}^{NS}(q)$
$(+, -):$	$(f^R(q))^{-2}$	$f^R(q)$

(3.9)

The contribution of the zero mode of the super-Liouville field is the same for all the spin structures and denoted as $\frac{Z_1^0(\tau)}{\sqrt{\tau_2}}$.

Collecting contributions from matter, Liouville and ghost sectors with the same spin structure and summing over spin structures, the torus partition function is given by

$$Z_1 = N \int_{\mathcal{F}} d^2\tau \frac{Z_1^0}{\sqrt{\tau_2^3}} \sum M_{i,j} \bar{\Gamma}^j(\tau) \Gamma^i(\tau) \quad (3.10)$$

where the i, j refer to both spin structures and (p, p') . The exact form of Z_1^0 is not important for our purpose and has been computed by Bershadsky and Klebanov[16] to be independent of τ . The results for general 2-d supergravity can be found in [8][6]. The terms in the integrand are clearly modular forms.

All string theories arising from $\hat{d} < 1$ SCFT coupled to 2-d supergravity are modular invariant solutions $M_{i,j}$ of (3.10). The classification of these solutions will be addressed in a separate publication[8]. Here we will give a series of generic solutions. Let's define functions Θ and θ as

$$\begin{aligned} \Theta_{p,p'}^{2l-} &= \begin{cases} \frac{1}{2} \left[\Gamma_{p,p'}^{2l-} + \tilde{\Gamma}_{p,p'}^{2l-} \right] - \Gamma_{l-p,l+1-p'}^{2l-} & p, p' \text{ odd} \\ \frac{1}{2} \left[\Gamma_{p,p'}^{2l-} + \tilde{\Gamma}_{p,p'}^{2l-} \right] - \Gamma_{l-p,l+1-p'}^{2l-} & p, p' \text{ even} \end{cases} \\ \theta_{p,p'}^{2l-} &= \begin{cases} \frac{1}{2} \left[\Gamma_{p,p'}^{2l-} - \tilde{\Gamma}_{p,p'}^{2l-} \right] + \Gamma_{l-p,l+1+p'}^{2l-} & p, p' \text{ odd} \\ \frac{1}{2} \left[\Gamma_{p,p'}^{2l-} - \tilde{\Gamma}_{p,p'}^{2l-} \right] & p, p' \text{ even} \end{cases} \\ &1 \leq p \leq l-1, 1 \leq p' \leq l, \end{aligned} \quad (3.11)$$

and

$$\theta_{p,q}^{2l+} = \frac{1}{2} \left[\Gamma_{p,p'}^{2l+} - \tilde{\Gamma}_{p,p'}^{2l+} \right] \quad p, p' \text{ even}$$

where $\Gamma^\pm$ are defined by

$$\Gamma_{p,p'}^{2l\pm} = \Gamma_{p,p'}^{2l} \pm \Gamma_{p,2(l+1)-p'}^{2l} \quad (3.12)$$

The torus partition function of the non-critical superstrings can be written in terms of those functions as

$$Z_1 \sim \int_{\mathcal{F}} d^2\tau \frac{Z_1^0}{\sqrt{\tau_2^3}} \left\{ \sum_{\substack{1 \leq p \leq l-1, 1 \leq p' \leq l \\ p, p' \text{ odd}}} \left(\bar{\Theta}_{p,p'}^{2l-} \Theta_{p,p'}^{2l-} + \bar{\theta}_{p,p'}^{2l-} \theta_{p,p'}^{2l-} \right) + \sum_{\substack{1 \leq p \leq 2l-1, 1 \leq p' \leq 2l+1 \\ p, p' \text{ even}}} \frac{\bar{\theta}_{p,p'}^{2l+} \theta_{p,p'}^{2l+}}{2} \right\} \quad (3.13)$$

The modular invariance is easily verified. Furthermore we have identities $\Theta_{p,p'}^{2l-} = 0$ and $\theta_{p,p'}^{2l\pm} = 0$, therefore the torus partition function vanishes identically. The theories are therefore “space-time” supersymmetric, i.e. superstrings. This concludes our discussion of the consistency of the theories.

Several comments are in order. The fermions(2.17) enter partition function with a minus sign and are consistent with statistics. Only a subset of the physical states of (2.16)(2.17) is contained in the theory, namely $V_{p,p'}^B$, $V_{l-p,l+1-p'}^B$, $V_{p,p'}^F$ and $V_{l-p,l+1-p'}^F$ with p, p' odd. The fusion algebra of SCFT is closed within these fields. Since the vanishing of partition function is the consequence of holomorphic identities one could construct non-critical heterotic superstrings[17] from the level matching consideration[8].

There are other solutions to the condition of modular invariance of (3.10). The modular invariant spin models are solutions. They have only bosonic states but nevertheless are interesting string theories. In particular the R fields in those theories have smaller dressing coefficients and therefore set the scale of the theories. The critical exponents(2.20) for the m th theory are therefore modified to be[6]

$$\gamma = \begin{cases} -\frac{2}{2m+1}, & m \text{ odd} \\ 0, & m \text{ even} \end{cases}, \quad \Delta_{p,p'} = \begin{cases} \frac{|p(m+2)-p'm|-1}{2m+1}, & m \text{ odd} \\ \frac{|p(m+2)-qm|}{2(m+1)}, & m \text{ even} \end{cases} \quad (3.14)$$

Notice that in the non-critical superstrings discussed here, the scale can also be set by the dressed R field $\Psi_{l-1,l}^t e^{\beta_{l-1,l}\phi}$. The scaling behavior is unchanged. The fact that the R field sets the scale of the theory seems to be the generic property of 2-d supergravity theories. With suitable GSO projection one can successfully cross the $\hat{d} = 1$ barrier.

4. Conclusions

We have constructed a class of non-critical superstrings from 2-d discrete SCFT coupled to 2-d supergravity. It is straightforward to generalize the results to non-unitary minimal SCFT. Our construction will only apply to class II minimal SCFT, (p, q) theories with both p, q even, where there exist matching between NS and R fields[6].

The non-critical superstrings have only a finite number of physical degrees and the same simplicity as $d < 1$ bosonic strings. It is therefore conceivable to have a similar non-perturbative description. The close analogy with critical superstrings make them an excellent toy models for study supersymmetry breaking and the off-shell structure of superstrings.

The matching condition is more general than the requirement of "space-time" supersymmetry. It can also be used to construct non-critical fermionic strings analogy to the $SO(16) \otimes SO(16)$ heterotic string[18].

Generalization of our results to rational SCFT with $\hat{d} > 1$ seems straightforward. The dressing coefficients will remain to be real for $\hat{d} \leq 9$ from the requirement of matching between NS and R fields[6]. But the question of how to incorporate SCFT of flat space(-time) is still unclear because there is, in general, no matching between NS and R fields [6], e.g. in three dimensions where the superstring is suggested to be relevant to the phase transition of the 3-d Ising model[10][19].

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