ANALYSIS OF THE $^{235}$U NEUTRON CROSS SECTIONS IN THE RESOLVED RESONANCE RANGE

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ABSTRACT
Using recent high-resolution measurements of the neutron transmission of $^{235}$U and the spin-separated fission cross-section data of Moore et al., a multilevel analysis of the $^{235}$U neutron cross sections was performed up to 300 eV. The Dyson Metha $\Delta_3$ statistics were used to help locate small levels above 100 eV where resonances are not clearly resolved even in the best resolution measurements available. The statistical properties of the resonance parameters are discussed.

INTRODUCTION
An accurate resonance analysis of the $^{235}$U neutron cross sections was recently completed for ENDF/B-VI.1,2 In this analysis, the resolved resonance formalism is used up to 2250 eV. However, since the level spacing in $^{235}$U is approximately 0.5 eV the resonances cannot be truly resolved over that range, and at the higher energies the goal of the analysis was mainly to provide pseudo-parameters that represented well the observed structures of the cross sections. Such a representation is expected to yield a better estimate of resonance self-shielding than the often-used unresolved formalism.3,4

As part of this analysis, an attempt was made to determine as accurately as possible the resonance energies up to 300 eV. Even at this lower energy it is not possible to resolve unambiguously all the levels. However, the availability of spin-separated fission cross-section data5 and of high-resolution measurements done on samples cooled to the liquid nitrogen temperature6,7 to reduce Doppler broadening of the resonances, allows resolving levels up to 300 eV with reasonable confidence. The Dyson Metha $\Delta_3$ statistics8 introduce strong correlations in the expected level spacings and help determine the energies of small resonances that cannot be unambiguously resolved.

METHOD OF ANALYSIS
The resonance parameter analysis code SAMMY9 was used to perform a consistent R-matrix multilevel analysis of selected neutron cross section and transmission measurements. The computer code SAMMY uses the Reich-Moore formalism and a fitting procedure based on the Bayes' method which allows the successive incorporation of new data in a consistent manner. The code allows for searching not only resonance parameters but also experimental parameters such as sample thickness, residual backgrounds, local renormalization, effective temperature and the parameters of the resolution function, all consistent with predetermined uncertainty limits. A special feature of SAMMY, programmed for this analysis, allows the fitting of separated spin contributions to the fission cross section.

UTILIZATION OF THE $\Delta_3$ STATISTICS
The $\Delta_3$ statistics of Dyson and Metha8 are particularly sensitive to long-range correlations within a set of levels. The mean-square deviation $\Delta_3$ is defined by a sequence of $n$ consecutive resonance energies in the range $(L, +L)$ by:

$$\Delta_3 = \min_{A, B} \left\{ \frac{1}{2L} \int_{-L}^{+L} [N(E) - AE - B]^2 dE \right\},$$

where $N(E)$ is the cumulative number of levels versus neutron energy, and $AE + B$ is the line fitted to the histogram. The expected value of $\Delta_3$ is $\frac{1}{2} (\ln n - 0.0686)$ with a variance of 0.012 where $n$ is the total number of levels. Using the results of the high-resolution low Doppler-broadening transmission and fission measurements and the spin-separated fission cross-section data, essentially all the resonances can be resolved up to 60 eV. The $\Delta_3$ statistics can then be used to determine precise values of the level spacings for each spin levels.
Above 60 eV, the $\Delta_3$ statistics which constrain the number of levels per unit energy are used again to determine the probable spin of a level where the spin-separated fission cross-section data have large uncertainties or to decide where to place additional levels where the resolutions of the measurements do not permit an unambiguous interpretation of the resonance structure.

RESULTS OF THE ANALYSIS

The results of the $\Delta_3$ statistical test and the average values of the resonance parameters for each spin state are summarized in Table 1. Figure 1 shows histograms of the cumulative number of observed levels versus energy for each spin state as well as the lines fitted to these histograms. Figures 2 and 3 show the distributions of the reduced neutron widths for each spin state compared to an expected Porter-Thomas distribution. In Figs. 4 and 5 the nearest neighbor level spacing distributions observed for each spin state are compared to theoretical Wigner distributions. Finally, in Figs. 6 to 8 the transmission ratios and fission cross sections computed with our resonance parameters are compared to results of measurements. These figures illustrate how accurately the resonance parameters represent the experimental measurements. However, there are significant differences between observed and expected distributions of the spin 4 levels. These differences are under investigation. More extensive comparisons between cross sections computed with the resonance parameters and results of measurements are given in Refs. 1 and 2.

CONCLUSIONS

Recent high-resolution measurements and the spin-separated fission cross-section data permit resolving essentially all the levels in $^{235}$U up to 60 eV. The use of the $\Delta_3$ statistical test allows the extension of a multilevel analysis up to 300 eV. The resonance parameters obtained represent the available experimental data accurately. The observed distributions of the resonance parameters are consistent with the distribution predicted by R-matrix theory. Of course, the partial fission width is not unique since rotations and other transformations of the fission vectors leave the fission cross section invariant. A discussion of the fission vectors orientations has been presented elsewhere.

ACKNOWLEDGEMENTS

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REFERENCES


Table 1. Average value of observed $^{235}\text{U}$ resonance parameters up to 300 eV

<table>
<thead>
<tr>
<th>Parameter</th>
<th>$J = 3$</th>
<th>$J = 4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Level spacing (eV)</td>
<td>$1.39 \pm 0.09$</td>
<td>$0.90 \pm 0.04$</td>
</tr>
<tr>
<td>Reduced neutron width (meV$^{1/2}$)</td>
<td>$0.12 \pm 0.09$</td>
<td>$0.09 \pm 0.08$</td>
</tr>
<tr>
<td>$s$-wave strength function $\times 10^4$</td>
<td>$0.89 \pm 0.09$</td>
<td>$1.01 \pm 0.08$</td>
</tr>
<tr>
<td>Fission width (meV)</td>
<td>$239$</td>
<td>$189$</td>
</tr>
<tr>
<td>Capture width (meV)</td>
<td>$38 \pm 7$</td>
<td>$35 \pm 3$</td>
</tr>
<tr>
<td>Observed $\Delta_3$</td>
<td>$0.57$</td>
<td>$0.53$</td>
</tr>
<tr>
<td>Expected $\Delta_3$</td>
<td>$0.54 \pm 0.11$</td>
<td>$0.58 \pm 0.11$</td>
</tr>
</tbody>
</table>

FIG. 1. CUMULATIVE SUM OF OBSERVED LEVELS
FIG. 2. DISTRIBUTION OF THE REDUCED NEUTRON WIDTH (SPIN STATE 3)

FIG. 3. DISTRIBUTION OF THE REDUCED NEUTRON WIDTH (SPIN STATE 4)
FIG. 4. LEVEL SPACING DISTRIBUTION (SPIN STATE 3)

FIG. 5. LEVEL SPACING DISTRIBUTION (SPIN STATE 4)
FIG. 6. COMPARISON OF COMPUTED AND MEASURED TRANSMISSION RATIOS.
(0.00234 atm/b, 0.03269 atm/b)

FIG. 7. COMPARISON OF COMPUTED AND MEASURED FISSION CROSS SECTIONS
FIG. 8. TOTAL AND SPIN SEPARATED FISSION CROSS SECTION

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