2
JAN 10 1991

SLAC-CN--384

DE91 005774

Stanford Linear Accelerator Center, Stanford University, Stanford, Ca. 94309

SINGLE PASS COLLIDER MEMO CN-384

AUTHOR: V. Ziemann

DATE: December 7, 1990

AFFILIATION: Stanford Linear Accelerator Center

TITLE: Qualitative Results from a Beamstrahlung Flight Simulator. ¹

Abstract

A simulation code for beam-beam deflections and beamstrahlung fluxes for tilted and elliptic beams is presented. Some qualitative features of beamstrahlung scans are discussed.

1 Introduction

In a previous collider note [1] the response of the beamstrahlung Cerenkov monitor was calculated and scaling laws for the beamstrahlung flux in the bunch length, energy and particle numbers were deduced. In this note the analysis will be extended and a full simulation code for the beamstrahlung emitted by elliptic and tilted beams is described. Using this code general features such as dips and asymmetric shapes in the observed beamstrahlung scans are interpreted qualitatively.

The simulation of the beamstrahlung fluxes assumes electron and positron bunches with longitudinal and transverse gaussian profiles. However, the transverse profile is allowed to be elliptic and tilted. The response of the Cerenkov monitor which comprises of the $\gamma \rightarrow e^+ e^-$ pair production probability in the converter plate and the probability of generating Cerenkov photons in the following gas volume is taken into account by a calibration curve, derived in ref. 1.

The utilization of this calibration curve is crucial for the speed of the simulation. In the present code it saves one (iterated) numerical integration compared to the simulation described in ref. 7. Now simulations can be performed interactively on SLAC's IBM 3090 under VM. A scan with 40 points for arbitrary elliptic beams

¹work supported by the Department of Energy Contract DE-AC03-76SF00515

DISCLAIMER

This report was prepared as an account of work sponsored by an agency of the United States Government. Neither the United States Government nor any agency thereof, nor any of their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by trade name, trademark, manufacturer, or otherwise does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof.

MASTER

DISTRIBUTION OF THIS DOCUMENT IS UNLIMITED

takes about 60 seconds. In this time the in and out-of-plane deflection curves and the beamstrahlung fluxes from both beamstrahlung detectors (north and south) are calculated simultaneously.

The program proceeds as follows: First the quantities that characterize the bunches such as bunch length, transverse beam sizes, tilt angle, particle numbers per bunch and the scan path are read from an input file. Then the code calculates the beamstrahlung flux on points along the scan path. In order to correlate observed features in the beamstrahlung data with beam-beam deflection data also the in and out-of-plane deflection curves are calculated. In this way a complete simulation of the currently observed effects from the beam-beam interaction is possible. The input and output files are described briefly in appendix C.

This paper is organized as follows. In the next section we will briefly discuss the generation of beamstrahlung in the beam-beam interaction followed by the discussion of the Cerenkov detector and its response to beamstrahlung. Then the algorithm for simulation code is presented and the final sections are devoted to the discussion of qualitative features of beamstrahlung scans.

2 Generation of Beamstrahlung

At the interaction point of the SLC electron and positron bunches collide head on at energies of about 46 GeV. The typical transverse bunch sizes are a few microns and the bunch length is about 1 mm. Each bunch typically contains a few 10^{10} particles. This large number of particles in that small volume gives rise to large electromagnetic fields that the bunch carries along. Peak values of the magnetic fields in excess of 10 T occur with corresponding electric field strengths. These large fields deflect the particles in the oppositely running bunch with deflection angles of up to 200 μ rad. The effect of both magnetic and electric fields can be characterized by a local bending radius ρ by which the oncoming particles are deflected. It is given by

$$\frac{1}{\rho(x, y, t)} = |\Theta(x, y)| \frac{1}{\sqrt{2\pi}\sigma_z/2} \exp \left[-\frac{(ct)^2}{2(\sigma_z/2)^2} \right] \quad (1)$$

where $\Theta(x, t)$ is the integrated deflection angle on the passage through the target bunch of length σ_z as is obvious from $|\Theta(x, y)| = \int_{-\infty}^{\infty} c dt / \rho(x, y, t)$. Thus the total deflection angle $\Theta(x, y)$ does not depend on the bunch length σ_z but ρ does. This is exemplified in Fig. 1 which shows the transverse trajectories of particles traversing

Figure 1: The trajectories of particles traversing bunches of different length containing the same number of particles. Note the deflection happens over a shorter interval in the short bunches but is more intense. The total deflection angle in all four cases is the same.

a bunch of varying length. In the short bunch the particle is bend sharper over a shorter stretch compared to the cases with longer bunches. However, the total angle after the traversal is the same in all cases. In eq. 1 the factor σ_z always appears in the form of $\sigma_z/2$, because both bunches approach each other with the speed of light and thus the length of the target bunch appears effectively halved.

All transverse dependence of the deflection angle is buried in $\Theta(x, y)$ which for a general elliptic beam is given by [2]

$$\Theta(x, y) = \Delta y' + i\Delta x' = -\frac{2N_t r_e}{\gamma_r} F(x, y, \sigma_{ij}) \quad (2)$$

with

$$F(x_1, x_2, \sigma_{ij}) = \frac{\sqrt{\pi}}{\sqrt{2(\sigma_{11} - \sigma_{22} + 2i\sigma_{12})}} \left\{ w \left[\frac{x_1 + ix_2}{\sqrt{2(\sigma_{11} - \sigma_{22} + 2i\sigma_{12})}} \right] \right\} \quad (3)$$

$$-e^{-\frac{1}{2} \sum_{i,j} (\sigma^{-1})_{i,j} x_i x_j} \left[\frac{(\sigma_{22} - i\sigma_{12})x_1 + i(\sigma_{11} + i\sigma_{12})x_2}{\sqrt{\sigma_{11}\sigma_{22} - \sigma_{12}^2} \sqrt{2(\sigma_{11} - \sigma_{22} + 2i\sigma_{12})}} \right] \Bigg\}$$

where N_t is the number of particles in the target beam and γ_r is the energy of the deflected particle in units of the electrons rest mass. The transverse beam sizes are characterized by their covariance matrix σ_{ij} which can be easily related to the principal axes σ_x and σ_y and the tilt angle ϕ by the following relations

$$\begin{aligned} \sigma_{11} &= \sigma_x^2 \cos^2 \phi + \sigma_y^2 \sin^2 \phi \\ \sigma_{12} &= (\sigma_x^2 - \sigma_y^2) \sin \phi \cos \phi \\ \sigma_{22} &= \sigma_x^2 \sin^2 \phi + \sigma_y^2 \cos^2 \phi \end{aligned} \quad (4)$$

The bending radius $\rho(x, y, t)$ a particle experiences on its traversal of the oncoming bunch determines the spectrum of the emitted beamstrahlung which is characterized by its critical energy

$$\epsilon_c = \frac{3 \hbar c}{2 \rho} \gamma^3 \quad (5)$$

Thus the number of beamstrahlung photons emitted per unit time and unit energy interval is given by [5,1]

$$\frac{dN}{d\epsilon dt}(\epsilon, \epsilon_c(\rho(t))) = \frac{1}{\sqrt{3}\pi} \frac{\alpha^2}{r_e m c} \frac{1}{\gamma^2} \int_{\epsilon/\epsilon_c}^{\infty} ds K_{5/3}(s) \quad (6)$$

where $K_{5/3}$ is a Bessel function of fractional order [4]. Clearly the emitted beamstrahlung spectrum depends on the local bending radius the radiating particle experiences through the critical energy ϵ_c . Of course the "hardest" spectrum is emitted when the bending radius is minimum at the core of the target bunch, whereas in the tails only a "soft" spectrum is emitted. Apparently the spectrum varies as the radiating bunch traverses the target bunch. Photon number spectra for different critical energies are shown in Fig. 2 qualitatively for critical energies of 1/2, 1 and 2 times the threshold energy of the Cerenkov detector, discussed below.

Having discussed the radiation that is to be analyzed we will now investigate how this translates into measureable photomultiplier counts. This is the topic of the next section.

Figure 2: The pair production probability, Cerenkov photon generation efficiency and photon number spectra with critical energies of 1/2 (dotdashed), 1 (dashed) and 2 times (solid) the Cerenkov threshold. The vertical bar at 1 marks the Cerenkov threshold.

3 The Cerenkov Detector

At the SLC the beamstrahlung emanating from the interaction point is detected by a Cerenkov monitor about 40 m downstream. The monitor is also exposed to the synchrotron light emitted by the same particles in a strong bending magnet needed to deflect the radiating particles away from the monitor. The Cerenkov monitor is then needed to discriminate the abundantly generated photons in the bending magnet, having a critical energy of about 2 MeV from the fewer photons generated in the beam-beam interaction, having critical energies an order of magnitude larger.

The Cerenkov monitor consists of a converter plate that converts the incident photons into $e^+ e^-$ pairs. This probability is given by the Heitler-Sauter cross section [6] for $\gamma \rightarrow e^+ e^-$ processes which has a logarithmic dependence on the energy of the incoming photon. It is shown by the upper curve in Fig. 2 that starts at the threshold.

The generated $e^+ e^-$ pairs then travel through a gas volume made of ethylene at

Figure 3: I_0 as a function of $\epsilon_c/\epsilon_0 = \rho_0/\rho$.

1/3 atmosphere where they emit Cerenkov light. The Cerenkov photons are subsequently passed through a light channel by mirrors and are then detected by photo multiplier tubes. The threshold energy is determined by the gas and is for the current detector about 25 MeV which leads to a $\gamma_0 \approx 50$. In the gas volume the higher energy pairs can emit more Cerenkov radiation. In ref. 1 it was shown that the number of Cerenkov photons depends on the energy of the incident photon γ_B simply according to $(1/\gamma_0 - 1/\gamma_B)^2$. Here the energy of the photon is given in units of electron rest mass. This dependence is shown in Fig. 2 by the lower curve starting at the threshold.

In order to determine the number of Cerenkov photons N_c we have to integrate the number of incident photons weighted by the e^+e^- pair production probability and the probability of emitting a Cerenkov photon. This calculation has to be done for different critical energies in order to obtain a relation between the critical energy (or the bending radius, see eq. 4) and the number of generated Cerenkov photons per unit time. This task has been achieved in ref. 1 with the result

$$\frac{dN_c}{dt}(x, y, t) = \Omega_0 I_0(\gamma_0, \epsilon_0/\epsilon_c(x, y, t)) \quad (7)$$

where Ω_0 contains all the information about the hardware of the Cerenkov detector.

$I_0(\gamma_0, \epsilon_0/\epsilon_c)$ is given by a complicated integral that is numerically evaluated in ref. 1. It is illuminating to plot I_0 versus ϵ_c/ϵ_0 as is done in Fig. 3. Recalling that $\epsilon_c \propto 1/\rho$ this gives the sought relation among the local bending radius and the number of Cerenkov photons. The vanishing slope of I_0 around the origin corresponds the response of the monitor to the exponentially decreasing tail of the "soft" spectrum emitted by the particles while they traverse the tail of the target beam where the bending radius is large. As this particle traverses the target bunch it experiences smaller bending radii and correspondingly moves up the I_0 -curve. The total number of Cerenkov photons generated by an individual particle while traversing the target bunch is then given by the time-integral over $dN_c/dt \propto I_0$.

To assess the transverse beam sizes one has to average over the transverse profile of the radiating beam. This will be the subject of the next section.

4 The Simulation Algorithm

Using eq. 6 it is easy to calculate the total number of Cerenkov photons generated during a collision by integrating dN_c/dt over t and averaging over the transverse dimensions x and y

$$N_c = \int_{-\infty}^{\infty} dx \int_{-\infty}^{\infty} dy \psi_r(x, y) \int_{-\infty}^{\infty} dt \frac{dN_c}{dt}(x, y, t) \quad (8)$$

where $\psi_r(x, y)$ is the transverse particle distribution of the radiating beam, given by

$$\psi_r(x_1, x_2) = \frac{N_r}{2\pi\sqrt{\det \sigma}} \exp \left[-\frac{1}{2} \sum_{i,j=1}^2 (\sigma^{-1})_{ij} (x_i - X_i)(x_j - X_j) \right] \quad (9)$$

where N_r is the number of particles in the radiating beam. σ_{ij} is the covariance matrix of the radiating beam and X_i is the position of its centroid. dN_c/dt depends on the target beam distribution through the dependence of the critical energy ϵ_c on the bending radius $\rho(x, y, t)$ as given by eq. 1.

Apparently three iterated integrations are necessary to calculate N_c . However, this can be reduced to two integrations by expanding I_0 into a power series in ρ_0/ρ . namely $I_0 = \sum_{n=1} a_n(\rho_0/\rho)^n$. The coefficients a_n can be determined by a least square fit from the data plotted in Fig. 3. Using this expansion the integral over t can be

performed and we obtain

$$\begin{aligned} \int_{-\infty}^{\infty} dt \frac{dN_c}{dt} &= \Omega_0 \int_{-\infty}^{\infty} dt \sum_{n=1}^{\infty} a_n \left(\frac{\rho_0}{\rho(x, y, t)} \right)^n \\ &= \sqrt{\frac{\pi}{2}} \frac{\Omega_0 \sigma_z}{c} \sum_{n=1}^{\infty} \frac{a_n}{\sqrt{n}} \left[\sqrt{\frac{2}{\pi}} \frac{\rho_0}{\sigma_z} |\Theta(x, y)| \right]^n. \end{aligned} \quad (10)$$

After inserting this expression in eq. 8 we see that only the two spatial integrations remain which then have to be evaluated numerically. In the code the numerical integrations are performed using the Romberg integration routines in ref. 8, because they require the least number of function evaluations of the integrand [8].

The beam-beam deflection angle for the centroid kick is given by the average of the deflection angle over the distribution of the kicked particles. It was evaluated in ref. 2 in closed form and can be written as

$$\langle \Theta \rangle = -\frac{2N_t r_e}{\gamma_r} F(X_1, X_2, \Sigma_{ij}) \quad (11)$$

where N_t is the number of particles in the target beam and γ_r the energy of the radiating beam in units of the electron rest mass. X_1 and X_2 are the relative offsets in x and y and Σ is the sum of the covariance matrices of the target and the deflected beam.

The simulation code evaluates the detected Cerenkov monitor fluxes emanating from both beams and the deflection angles for both beams and displays these quantities along any path. Currently only straight paths are supported, but any extension is trivially accomplished.

Fig. 4 shows a typical output of this code. In the upper left corner the input data are echoed. The top right displays the flux from the radiating positrons on the north BSMD 400 monitor (solid line) and the flux from the radiating electrons on the south BSMD 700 monitor (dashed line). In the lower left frame the scan path and both bunches are shown. The positron bunch is located in the center and the electron bunch is scanned over it. The lower right frame shows the electron beam deflection angle.

The qualitative investigation of the influence of the beam sizes on the beamstrahlung profiles is the subject of the next sections.

5 Simulation Results for Round Beams

The most striking feature of the beamstrahlung scans is the existence of the dip at the center of some of the scans. It is known that this dip is related to the transverse electric field of the target bunch [3]. To visualize this, imagine a round beam of finite size, say $5\text{ }\mu\text{m}$ transverse size and an infinitely thin beam. We know that the response of the Cerenkov detector mainly reflects the electric field (or equivalently the bending radius) a radiating particle experiences as it traverses the target bunch [1]. The local bending radius is given by eq. 1 which implies that on a transverse scan the experienced electric field is determined by $\Theta(x, y)$. This, on the other hand, describes the deflection angle of a particle with transverse offset (x, y) . Typical deflection curves for the above mentioned configuration are depicted in the lower right frame of Fig. 4. From the above argument it should be clear that the maxima of the beamstrahlung flux are at the locations where the beam-beam deflection curve assumes its extrema. This happens approximately at $x = \pm 1.6\sigma$, where σ is the transverse size of the beam as can be seen by comparing the solid curve in the upper right frame with the dashed deflection curve in the lower right frame in Fig. 4.

Another consequence is that the flux should vanish when the collision is head on. However, this argument is only strictly true for an infinitely thin beam. If the radiating beam has a finite transverse beam size the characteristics are smeared out. Then there are always particles in the radiating beam that experience larger deflection angles and therefore radiate more. However, the dip remains as a signature that the centroid of the radiating beam sits in a low field region of the target beam.

Conversely, the beamstrahlung flux generated by the transversely large beam in the field of the thin beam should not show a dip. In this configuration the electric field of the thin beam serves as a window over which the transverse dimension of the large radiating beam is averaged. Therefore, the flux generated by the radiating beam reflects the transverse profile of the large beam. This can be seen in the upper right frame of Fig. 4. The dashed line shows the beamstrahlung flux from the radiating electrons which are much bigger in this case. Also compare the truncated width W_2 from the radiating electrons on the BSMD 400 monitor ($4.91\text{ }\mu\text{m}$) with the width of the electron distribution ($5\text{ }\mu\text{m}$). They agree fairly well. W_2 is slightly smaller because it pays less attention to the tails and therefore under estimates the real width of a gaussian (see Appendix B for a discussion of the width measure W_2).

To substantiate this claim a set of simulations with round beams with transverse

Figure 4: A typical output from the simulation code. In the upper left the input data are echoed. In the upper right the beamstrahlung fluxes are shown in arbitrary units. The solid curve is the flux from the radiating positrons on the north detector. In the lower left depicts the path on which the scan was taken and the lower right shows the electron deflection. Here the solid curve shows the horizontal deflection and the dashed curve the vertical.

dimensions of 3,4 and 5 μm was done. the results are shown in Fig. 5. The bunch length of both beams was chosen to be 1 mm and the energy to be 46 GeV. The number of electrons per bunch was set to $2.5 \cdot 10^{10}$ particles and for positrons to $1.5 \cdot 10^{10}$ particles. In this simulation the electrons were passed vertically over the positron beam. The number of Cerenkov photons (in arbitrary units) generated in the detector is calculated on a vertical path $\pm 20 \mu\text{m}$ in steps of $2 \mu\text{m}$ and plotted. The solid line is the flux from the radiating positrons on the north BSMD 400 detector and the dotdashed line from the radiating electrons onto the south BSMD 700 detector. The number in the upper left corner is a code number that represents the transverse size of the electron and positron beam, thus 35 means $3 \mu\text{m}$ electron spot size and $5 \mu\text{m}$ positron spot size. The positron beam size increases from left to right in Fig. 5 and the electron beam size increases from top to bottom. The numbers W2 on every

Figure 5: Beamstrahlung fluxes (in arbitrary units) for round electron and positron beams with sizes of 3, 4 and 5 μm from vertical scans.

small plot denote the two sigma truncated widths introduced by W. Kozanecki and E. Gero [3,7]. They are discussed in more detail in appendix B. These quantities will be used in the future to determine individual spot sizes from the beamstrahlung scans. R is the ratio of the peak values of both curves (with pedestal subtracted).

From these plots it is obvious that the flux from the radiating positrons (solid line) at the north detector is larger than the flux from the radiating electrons, even though there are more electrons. This is, because the approximate scaling relation for the beamstrahlung flux, derived in ref. 1

$$Flux \propto \frac{N_{target}^4 N_{source}}{\sigma_z^3 \gamma_{source}^4} \quad (12)$$

implies that the flux is strongly determined by the number of particles in the target beam. The target beam for the heavily radiating positron beam is the heavily populated electron beam.

Another observation confirms the above made statement that the dip reflects a larger target beam. For instance in the curve in the lower left corner with an electron beam of $5\text{ }\mu\text{m}$ and a positron beam of $3\text{ }\mu\text{m}$ the dip is in the solid curve from the radiating positrons and the radiating electrons produce a round curve. Generally, as one progresses from the bottom left to the top right the ratio of electron beam size to positron beam size decreases and the dip in the solid curve gradually vanishes. At the same time the dip in the dotdashed curve gradually appears.

Note that equal beamsizes (the major diagonal with the scans labelled 33, 44 and 55) produce dips in both scans. this fact can be used as a tuning tool to optimize the transverse beam sizes, because the goal of the optimization is the appearance of dips (hopefully discerable in real scans) in the beamstrahlung scans on both sides. The information which side is worse comes from the fact which side shows a dip and which does not. **The side without a dip has the smaller beam size.**

As an example imagine the Cerenkov flux in the detector on the north side has a dip and the flux on the south side has a round appearance. According to the above argument this would hint at a larger electron beam size.

6 Simulation Results for Elliptic Beams

In the previous section the discussion was based entirely on round beams. We will now relax this constraint and will first discuss the effect of ellipticity and will then turn to the signature of a tilted beam and how to diagnose it.

First let us look at the effect of elliptic beams. To this end we simulate vertical scans of $\pm 20\text{ }\mu\text{m}$ in steps of $2\text{ }\mu\text{m}$ where the electron beam size is kept fixed at $4\text{ }\mu\text{m}$ horizontally and vertically. Then the horizontal and vertical beam size of the positrons is varied to 3, 4 and $5\text{ }\mu\text{m}$. All other quantities are the same as depicted in the upper left of Fig. 4. The simulated fluxes are shown in Fig. 6. The code number in the upper right of each frame displays the beam sizes in the simulation, i.e. 4435 means: electron beam sizes of $4 \times 4\text{ }\mu\text{m}$ and positron beam sizes of $3 \times 5\text{ }\mu\text{m}$. Thus the horizontal positron beam size increases from left to right and the vertical positron beam size increases from bottom to top.

Since all the scans are vertical it is no surprise that the dip in the solid curve is

Figure 6: Beamstrahlung fluxes (in arbitrary units) for a round electron beam with $4\text{ }\mu\text{m}$ beam size and of positron beams with sizes of 3, 4 and $5\text{ }\mu\text{m}$. The horizontal beam size of the positrons increases from left to right and the vertical beam size from bottom to top.

biggest at the bottom for the smallest vertical extension of the radiating positrons and gradually decreases as the vertical beam size increases towards the top. The flux from the radiating electrons shown by the dashed line shows the expected converse behaviour. They develop a dip as the vertical positron beam size increases.

A little less expected is the effect of the variation in the horizontal beam size going from left to right. Clearly, there is an effect of the horizontal beam size on the vertical beamstrahlung scans. The general rule that smaller beam sizes of the radiating beam leads to a deeper dip still holds true.

However, this statement that *the dip is deeper* is not formulated entirely correct.

because comparing the small frames labelled 4434 and 4443 which differ only in the orientation of the elliptic positron beam ($\sigma_x \times \sigma_y = 3 \times 4 \mu\text{m}$ in the first and $\sigma_x \times \sigma_y = 4 \times 3 \mu\text{m}$ in the second case). The flux at the center for both cases is the same, as should be expected from symmetry considerations (the electron beam is round) but the 4443 frame shows higher peaks. This observation can be readily explained by the following consideration. For the round electron beam the lines of equal deflection angle are circles. The circle with radius ≈ 1.6 times their beam size marks the maximum deflection angle. Positrons sitting in the vicinity of this maximum deflection circle will contribute most to the flux. However, if the positron beam is elliptic with the major axis tangential to this line, the positrons will on the average experience a bigger deflection compared to the situation where the major axis is perpendicular. Since Fig. 6 comes from a vertical scan and the frame labelled 4443 has its major axis horizontally the peaks in this frame are elevated more. We conclude that not always *the dip is deeper* but sometimes *the peaks are higher*.

7 Simulation Results for Tilted Beams

Finally, we want to consider the signature of tilted beams. So far all beamstrahlung scans were symmetric and at first sight it looks intuitively right to associate tilted beams with asymmetric scans. This intuition is not entirely right.

Consider two tilted beams that are scanned across each other well centered. Assume a vertical scan with the major axis of the target beam tilted by -45 degree to the horizontal axis and a radiating beam with its major axis tilted by $+45$ degrees. The configurations shortly before the radiating beam enters the target beam and shortly after it exits are point symmetric images (with respect to the center of the target beam) of each other. Consequently, since this argument holds for every point on the scan path, the beamstrahlung scans are symmetric. In order to see the tilt directly we have to break this symmetry. The easiest way to do so is to offset the scan path from the center of the target beam.

Fig. 7 shows a simulation where a $5 \times 3 \mu\text{m}$ electron beam is passed over a $5 \times 3 \mu\text{m}$ positron beam with $3 \mu\text{m}$ offset. The code numbers on the top left of each small frame encode the tilt angle. A 1 stands for -45 degree tilt angle, 2 for 0 degree and 3 for $+45$ degree with respect to the horizontal axis. The first number refers to the electron beam and the second to the positron beam. The paths and the respective one sigma ellipses are shown in Fig. 8 to visualize the configurations that give rise to the fluxes

Figure 7: Vertical beamstrahlung scans for tilted beams that are offset with each other. The beamsizes for both beams are $5 \times 3 \mu\text{m}$. The tilt angle is -45 , 0 and $+45$ degree with respect to the horizontal axis from left to right for the positrons and top to bottom for electrons.

shown in Fig. 7.

Observe that tilted beams passed over each other with an offset actually give rise to asymmetric beamstrahlung scans. The solid curves in Fig. 7, stemming from the radiating positrons, show a clear correlation with the tilt angle of the positrons. Within the columns the symmetry of the solid curves are the same. The first column has a hump on the right (outgoing) side (here the positrons are tilted -45 degree). In the center column the peaks are of equal height (e^+ : 0 degree tilt) and in the right column the hump is on the left side (e^+ : $+45$ degree tilt). The same observation holds for the fluxes from the radiating electrons, shown as dashed lines in Fig. 7. The top row indicates a hump on the right (e^- : -45 degree tilt) and the center row is symmetric (e^- : 0 degrees tilt). The bottom row shows a hump on the left side (e^- : $+45$ degrees tilt). These observations seem to indicate that negative tilt angles

Figure 8: The setup used for the scans in Fig. 7.

of the radiating beams produce humps on the outgoing side and positive tilt angles produce humps on the incoming side. However, this conclusion has to be taken with a caveat, because it is only true elliptic beams of approximately equal size. The problems will become apparent in the following paragraphs, in which we explain the beamstrahlung scans.

It is instructive to plot the deflection angle $|\Theta(x, y)|$ produced by the target beam as a function of x and y in a 3-D plot, shown in Fig. 9. In this plot $-15 \leq x \leq 15 \mu\text{m}$ goes from left to right and $-15 \leq y \leq 15 \mu\text{m}$ goes from front to back. The major axis of the target beam is oriented from the rear left to the front right. A further visualization comes from Fig. 10 which shows a contour plot of lines of equal deflection angle in μrad . On both plots it is clearly visible that the points of maximum deflection angle lie on a rim around the origin with the peak deflection angles situated along the minor axis.

We will now use these plots to explain the humps in the dashed lines from the

Figure 9: A 3-D plot of the deflection angle $|\Theta(x,y)|$ for $-15 \leq x, y \leq 15 \mu\text{m}$ due to the positron beam. The path of the electron beam is indicated by a thickened line.

radiating electrons in Fig. 7. The scan path used in Fig. 7 is indicated by a thick line in Figs. 9 and 10. In the plot labelled 11 both beams are tilted by -45 degrees and the hump in the dashed curve is on the outgoing side. Two facts contribute to this behaviour. First, the peak field is on the outgoing side as can be seen on Fig. 10. Second, the major axis of the radiating beam is parallel to the rim on the outgoing side, whereas it is perpendicular to the rim on the incoming side. In the frame labelled 31 the electron beam is tilted $+45$ degrees. Here the maximum flux is recorded on the incoming side, even though the peak deflection occurs on the outgoing side. However, here the major axis of the radiating beam is parallel to the rim on the incoming side. On the outgoing side the center of the radiating beam goes through the peak on the rim, but large portions of the beam fall into the low deflection valley. Therefore, the flux is lower on the outgoing side. The same type of argument can be used to explain all other beamstrahlung scans in Fig. 7.

From the previous paragraph it should have become clear that the flux depends very well on the shape of the radiating beam and that the above arguments do not hold if the radiating beam were either round or very small.

Figure 10: Contour plot of the deflection angle $|\Theta(x, y)|$ for $-15 \leq x, y \leq 15 \mu\text{m}$ from the positron beam. The straight vertical line shows the path of the electron bunch.

8 Conclusion

A fast algorithm to calculate beam-beam deflection angles and beamstrahlung fluxes taking the response of the Cerenkov monitor into account is presented. A code in which this algorithm is implemented was used to interpret typical beamstrahlung scans qualitatively.

The dip in the center of some scans was shown to be correlated with a big target beam size. Approximately equal beam sizes were shown to occur when both beamstrahlung fluxes (north and south monitor) show small dips simultaneously.

The beamstrahlung fluxes from elliptic beams showed considerable influence of the beam sizes in the direction perpendicular to the scan direction. However, the

general feature that the bigger target beam is responsible for a more pronounced dip is still valid.

Asymmetric beamstrahlung scans were related to tilted beams passing through each other with a transverse offset. This phenomena might be used to eliminate the tilt of the beams empirically. However, the direction of the tilt of the individual beams can only be determined under the assumption that both target beam and radiating beam are elliptic and of approximately equal size.

9 References

1. V. Ziemann, SLAC Collider Note CN-379, 1990.
2. V. Ziemann, SLAC Collider Note CN-382, 1990.
3. E. Gero, W. Kozanecki, private communication.
4. M. Abramowitz, I. Stegun, *Handbook of Mathematical Functions*, Dover, New York, 1972.
5. A. Sokolov, I. Ternov, *Synchrotron Radiation*, Pergamon Press, New York, 1968.
6. J. Motz, H. Olsen, H. Koch, *Rev. Mod. Phys.* **41**, 581, 1969.
7. E. Gero, PhD Thesis, Univ. of Michigan, in preparation
8. W. Press, et.al., *Numerical Recipes*, Cambridge University Press, Cambridge, 1987.

10 Acknowledgements

Discussions with N. Toge, SLAC and E. Gero, SLAC are appreciated.

11 Appendix A: Subroutine KICK

At the heart of this computer code lies the subroutine that calculates the electric field or the deflection angle of an electron due to an elliptic and tilted gaussian beam.

This routine is used in two ways. First, it is used in the calculation of the kick of the centroid in beam-beam deflection scans. Second, it is used in the integration routine that calculates the beamstrahlung fluxes. Especially in the second application it is necessary that the routine is fast and robust.

From eq. 3 it is clear that a fast and robust evaluation of the complex error function $w(z)$ [4] is of paramount importance. This is achieved by using a Padé approximation that approximates the complex error function in the upper complex half plane by a rational function with polynomials of order 10 in the numerator and order 11 in the denominator. Since the complex error function grows exponentially along the negative imaginary axis the values in the lower half plane have to be computed according to

$$w(x - iy) = 2e^{-(x-iy)^2} - \overline{w(x + iy)}. \quad (13)$$

The exponential growth along the negative imaginary axis is thus explicitly taken care of. The routine in which the above algorithms are coded calculates the complex error function in the entire complex plane to about 5 significant digits and is about 3 times faster than the corresponding IMSL routine.

Eq. 3 also shows two potential problematic points in the evaluation of the right hand side. First, $\sqrt{\sigma_{11} - \sigma_{22} + 2i\sigma_{12}}$ can become zero. This blows up the forefactor and the arguments of the complex error functions. However, using the asymptotic expansion for the complex error function $w(z) \approx i/\sqrt{\pi}z$. This reduces $F(x, y, \sigma_{ij})$ to the round beam case, namely

$$F(x, y, \sigma_{ij}) = i \frac{1 - e^{-(x^2+y^2)/2\sigma_{11}}}{x + iy} \quad (14)$$

The second problematic point is the evaluation of the complex error function itself if their arguments have a negative imaginary part. Then both complex error functions become very large and $F(x, y, \sigma_{ij})$ has to be calculated from a difference of large expressions, which leads to numerical problems. This problem can be overcome by using the identity

$$w(z_1) - e^g w(z_2) = -\overline{[w(\bar{z}_1) - e^g w(\bar{z}_2)]} \quad (15)$$

with

$$z_1 = \frac{X_1 + iX_2}{\sqrt{2(\sigma_{11} - \sigma_{22} + 2i\sigma_{12})}}$$

TRUNCATED WIDTH FOR LORENTZIAN DISTRIBUTION

Figure 11: σ_1 (dashed) and W_2 (solid) as a function of the initial interval size R where the interval is $[-R, R]$ for a lorentzian profile given by $\psi(x) = 1/(1+x^2)$.

$$z_2 = \frac{(\sigma_{22} - i\sigma_{12})X_1 + i(\sigma_{11} + i\sigma_{12})X_2}{\sqrt{\sigma_{11}\sigma_{22} - \sigma_{12}^2}\sqrt{2(\sigma_{11} - \sigma_{22} + 2i\sigma_{12})}}$$

$$g = -\frac{\sigma_{22}X_1^2 - 2\sigma_{12}X_1X_2 + \sigma_{11}X_2^2}{2(\sigma_{11}\sigma_{22} - \sigma_{12}^2)}$$

which relates the evaluation of the complex error functions to those with complex conjugate arguments. In the proof of eq. 15 the identity $g + z_1^2 - z_2^2 = 0$ was used.

All of the presented problematic cases can be handled by a little if-then-else-ing which is implemented in the subroutine KICK.

12 Appendix B: Two Sigma Truncated Widths

In the attempt to characterize the beamstrahlung scans W. Kozanecki and E. Gero first introduced the two sigma truncated width W_2 [3,7]. W_2 is defined for a distribution $\psi(x)$ defined on a finite interval $[R_1, R_2]$ by the following algorithm.

1. Subtract the pedestal: $\psi_1(x) = \psi(x) - \frac{1}{2}(\psi(R_1) + \psi(R_2))$
2. Calculate the mean m_1 and width σ_1 of $\psi_1(x)$.
3. Calculate the two sigma truncated width $W2$ of $\psi_1(x)$ as the rms of $\psi_1(x)$ in the interval $[m_1 - 2\sigma_1, m_1 + 2\sigma_1]$.

This measure $W2$ has a few striking features. First, it largely eliminates the influence of the initial size on the measure of the width. σ_1 is much more sensitive on R_1 and R_2 than $W2$ as can be seen from Fig. 11. Fig. 11 depicts σ_1 (dashed) and $W2$ for a lorentzian distribution $\psi(x) = 1/(1+x^2)$. A lorentzian was chosen because the tails are heavily populated and we are mainly interested at the features of the core. In fact the rms of a lorentzian (σ_1) diverges for increasing interval size. However, Fig. 11 shows that $W2$ is much less dependent in the interval size than σ_1 .

Second, since $W2$ measures mainly the features of the core, it carries some information about the dip in the sense that a dip in the center leads to an artificially increased rms. If the interval on which $W2$ is calculated were larger, the tails would determine the rms rather than the core.

13 Appendix C: Input and Output of the Code

The simulation code is written in standard FORTRAN77 in order to facilitate portability. It requires as input a file named BSM INPUT which contains all the information about the beam sizes and the scan path. The input file organization is shown below:

```

ICODE #####
1  ESIGX    ESIGY    EPHI
   EPART    EGAMMA   ESIGZ
1  PSIGX    PSIGY    PPHI
   PPART    PGAMMA   PSIGZ
ISCAN
  OFFSET   TMIN      TMAX    TSTEP
0 #####

```

All quantities starting with an I are required to be integer, all the others are real variables. ICODE is a code number to label individual scans. It appears in the upper left corner of the beamstrahlung flux frames in Figs. 4, 5, 6 and 7. The next line contains an integer 1 (for debugging reasons) and the electron beam sizes σ_x and σ_y in microns and the tilt angle in degrees (σ_x versus the horizontal x -axis). The third line contains the number of electrons in one bunch N , the energy of the electron bunch in units of the electron rest mass γ and the bunch length σ_z of the electron bunch in mm. The fourth and fifth line contain the same information for the positron bunch. The following line containing ISCAN encodes the type of scan (ISCAN = 1: vertical, ISCAN = 2: horizontal, ISCAN = 3: diagonal) and the next line contains the information about the scan. If a vertical or horizontal scan are chosen the 4 numbers correspond to the offset perpendicular to the path, lower limit, upper limit and the step size in microns. If a diagonal scan is chosen the sixth and seventh line have to be substituted by

```

ISCAN
XINI  YINI  ALPHA  TMIN  TMAX  TSTEP

```

where XINI and YINI denote the origin of the path in microns with respect to the center of the positron beam. ALPHA is the tilt angle of the path with respect to the horizontal axis and the remaining quantities have the same meaning as before.

As many sets of the above described seven lines as desired can be stacked in the input file. This is particularly convenient for scans in which correlations are investigated such as presented in Figs. 5, 6 and 7. A run is finally terminated by a line containing a single zero after 7 valid input lines. Note that the zero appears where the ICODE of the following run were expected. Therefore a code of 0 is impossible. The cross hatching following ICODE serves to easily distinguish individual runs and is not required by the code.

The code produces 5 different output files. The BSM NUMBERS file is an ASCII file that contains the input data and the truncated widths $W/2$ for easy input to other programs. All other output files are TOPDRAW files. The BSM SUMMARY file contains the output shown in Fig. 4 for every run. The BSM PLOT BYTHREE file contains the figures like the ones shown in Fig. 5, 6 and 7. BSMPATH BYTHREE contains the corresponding paths and BSMDEFL BYTHREE contains the electron deflection data in the 3×3 format.

END

DATE FILMED

01 / 22 / 91