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TITLE: NEW METHOD FOR INVERTING THE CLOSED-ORBIT DISTORTION PROBLEM

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Summary

A certain class of magnet misalignments in storage rings and other accelerators produces closed orbit distortions (CODs). Often the CODs are measured at a fewer number of locations (N) than the number of misalignment parameters (M). There is a linear relation between the measurements, \( y_j \), \( j = 1, 2, \ldots, N \) and the misalignment parameters \( \theta_1, \theta_2, \ldots, \theta_M \), where the \( \theta_i \)'s are underdetermined. If \( N < M \), one usually obtains an overdetermined set of equations by measuring the COD at two quadrupole settings. There are several ways of inverting the COD measurements to get the misalignment parameters that are fairly insensitive to errors in the measured CODs. A regular procedure is used and is written in test form of the equations. The scheme gives fairly good results, and similar to the version of the Los Alamos Euler equations (AE 5). The first step requires measurements of the closed orbit and the use of simultaneous least squares method. The second scheme requires measurements of the orbit in both horizontal and vertical detectors, but is more mathematically.

The problem of approximating \( y_j \) in terms of real numbers, has been solved for a long time. The approach of approximating the misalignment parameters in terms of closed orbit measurements a few times in the ring or along the accelerator is a potential one. If we know that small misalignments lead to the centroids, the orbit of the transverse coordinate y, and by fluctuations of the closed orbit, there are distortions, lower the machine acceptance and are of primary concern when the system is turned on. It may well be that, if one ignores variations or incorrectness of the quadrupoles, then there are only these small misalignments that can cause such distortions. By the above, the horizontal displacement of the horizontal detector, then the alignment parameters, in turn, must be determined. The equation of the horizontal axis for one detector, with one quadrupole of fixed strength and with each. It is possible that the horizontal displacement of one in a horizontal detector is a small displacement of a single horizontal detector. For another detector, horizontal displacement affects the horizontal and vertical displacement affects vertical. All other displacements of these magnets either have no effect on the beam, or else they are not in the quadrupole and do not affect the orbit. We may choose to use the quadrupole and the magnetic field, N to M = N = M. However, the usual case is that N (the number of beam monitors) is less than M (the number of magnets). When \( N > M \), one says that the system of equations is underdetermined. There are least squares methods for dealing with these systems, but they always involve imposing additional constraints. The system such as simultaneously minimizing the sum of squares.

\[
y = \sum_{i=1}^{M} a_i \theta_i + \epsilon
\]

One can also impose a physical constraint by ignoring misalignments of the dipole and by fitting the COD using quadrupole parameters only. The result of these procedures often be unrealistic in terms of measured and estimated magnet displacements and sensitivity to slight changes in the measured COD.

One would like to increase the number of equations by increasing the number of points at which the COD is measured. Often it is experimentally impossible to increase the number of beam-positions, because the desirable locations are taken up with electron beam, halo monitors, or other equipment. The alternative being suggested in this paper is to make some change in machine, which will lead to changes in the matrix elements of \( [A(k, M)] \) and in measurement of the COD measuring the closed orbit.

The set of equations

\[
\begin{bmatrix}
    [A(k, M)] & \epsilon \\
\end{bmatrix}
\]

which can be written as one double-indexed equation

\[
y = \sum_{i=1}^{M} a_i \theta_i + \epsilon
\]

where \( \theta_i \) horizontal, and \( \epsilon_i \) vertical. The linear set of equations could be solved exactly for \( \theta_i \) if \( N > M \) or could be solved in a least squares sense if \( N < M \). However, the usual case is that \( N < M \).
We have investigated two other ways of producing matrices $T_0$ for the PSR. One can change the sign of the quadrupole gradients, or one can interchange the quadrupoles to make OFFO cells out of FODO cells. The latter change has the advantage of not changing the tunes. This probably will make it easier to get a beam through the lattice. In both of these cases, the $T_0$'s are well-conditioned matrices that give stable least-square solutions even when random errors are introduced into the COINFs, $\psi_i(s)$. The computer program COINF, which we have written to study the stability of these inversion methods, also produces beamline plots of the COINFs. Although the program was written specifically for the PSR lattice, it should be easily adaptable to any other Ring or even to a linear transport system.

The next section outlines how one goes from the equations of motion to the matrix equation, Eq. (1). The final section gives some applications to the PSR.

**Matrix Equations**

Let us assume that we have a storage ring with separate function magnets. Ignore the synchrotron motion caused by bunches. The equations for the transverse motion relative to the ideal orbit are

$$\mathbf{u}_1(s) = \mathbf{P}_1^{-1}(s) \mathbf{u}_0(s) - \mathbf{P}_1^{-1}(s) \mathbf{v}_0; \quad i = 1, 2; \quad s > 0$$

where $v_0$ is the particle velocity along the direction of the orbit, and $\mathbf{P}_1$ is the magnetic field parallel to $v_0$. Because $\mathbf{u}_0$ and $\mathbf{v}_0$ are usually small, one neglects the $\mathbf{u}_0$ terms. It is customary to expand $\mathbf{P}_1$ in powers of $\mathbf{u}_0$, to use path length $s - v_0 t$ as the independent variable, and to move the linear terms to the left side of the equation. The coupling terms and the nonlinear term are neglected. The result is

$$\mathbf{u}_1(s) = (-1)^i \left( \mathbf{P}_1^{-1}(s) \mathbf{v}_0 \right) \mathbf{u}_0(s) - \mathbf{P}_1^{-1}(s) \mathbf{v}_0,$$

The double prime is a second derivative with respect to $s$; $\mathbf{u}_{1''}(s)$ is proportional to the field gradients in the quadrupoles. Also, there is a $(s')^i$ term from the centrifugal force in the dipoles. The components $a_{ij}(s)$ are the field errors caused by misalignments. Because one is looking for the closed orbit, periodic boundary conditions are imposed on the Green's function solution

$$\mathbf{u}_i(s) = (-1)^i \left( \mathbf{P}_1^{-1}(s) \mathbf{v}_0 \right) \mathbf{u}_0(s) - \mathbf{P}_1^{-1}(s) \mathbf{v}_0,$$

where $sR$ is the ideal path length around the Ring. The Green's function can be expressed in terms of the derivative function $\mathbf{D}(s)$ of the perfect Ring

$$\mathbf{u}_i(s, \Delta s) = \sum_{n=1}^{\infty} \int [0, \infty] \mathbf{D}_n(s, \Delta s) \psi_i(s) \psi_i(s) \Delta s.$$
Fig. 1. Horizontal COD produced by random magnet displacements.

Fig. 2. Residual horizontal COD after fitting with program (11) INV.

Table I

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Acknowledgments

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References