MASTER

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RF POWER SAVING IN LARGE STORAGE RINGS*

This note considers the application of CARTOP,⁽¹⁾ constant amplitude rf to pulse converter, to obtain average and peak power gains in large e^+ rings. CARTOP is a scheme where energy storage and modulation converts a continuous radio frequency (rf) wave into periodic rf pulses. A system with CARTOP is shown in Fig. 1. The modulated rf goes through an energy storing network (ESN) that contains at least one energy storing cavity (ESC), which acts as a flywheel. Between pulses, it takes energy from the klystron and during the pulse it delivers energy to an accelerating cavity (AC). The control that tells it when to take and when to give up energy is a low power 180° bi-phase modulator. It tells the ESC to empty not by opening a spigot, as does beta switching,⁽²⁾ but by changing the phase of the incident field by 180°, thereby tilting the ESC higher so that energy flows out of it. The periodic, steady state, emitted, and reverse fields of ESC, and AC as a function of time, t ; $E_{e1}(t)$, $E_{r1}(t)$, $E_{e2}(t)$, $E_{r2}(t)$ are shown respectively in Figs. 2a to 2d. They depend on β_1 , T_{co1} , β_2 , T_{co2} , t_b , t_p , C_f where

β_1 , β_2 = coupling coefficient of ESC and AC respectively.

T_{co2} = time constant of unloaded ESC and AC respectively.

t_b = modulation period which is made to equal the bunch period.

t_p = pulse duration.

C_f = compression factor = t_b/t_p

The method by which these functions were obtained is given in Appendix A. The accelerating field is $E_{e2}(t)/\beta_2$. Although the ESN shown in Fig. 1 is most easily understood, an ESN consisting of a 3 dB hybrid and two cavities is more practical.⁽⁸⁾

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SECRET

Start with a cw system feeding a PEP-structure accelerating cavity directly and consider only the power that goes into the AC walls to maintain a voltage V . $V = [2\beta_2^2/(1 + \beta_2)](RP_g)^{1/2}$. R is the shunt resistance of the AC, P_g is the klystron output power. V is a maximum and equals $(RP_g)^{1/2}$ if $\beta_2 = 1$. With CARTOP, V is a function of time. Its peak value, which occurs at the end of the modulation period and coincides with bunch arrival.

Let

f = rf frequency in MHz.

T_{c1} , T_{c2} = Cavity time constant in μs of ESC and AC respectively.

Q_{o1} , Q_{o2} = Unloaded Q of ESC and AC respectively.

t_{bno1} , t_{bno2} = Bunch period normalized to unloaded ESC and AC time constants respectively.

t_{bn1} , t_{bn2} = Bunch period normalized to ESC and AC time constants respectively.

R_s = Surface resistance in ohms.

g = Geometric factor, $Q_o R_s$.

C_m = A constant of cavity material = $R_s/f^{1/2}$

= $2.61 \cdot 10^{-4}$ ohms/MHz^{1/2} for copper.

= $3.26 \cdot 10^{-4}$ ohms/MHz^{1/2} for aluminum.

$T_{col} = Q_{o1}/\pi f = (g/R_s)/\pi f = g/\pi C_m f^{3/2}$; $T_{c1} = T_{col}/(1 + \beta_1)$

$t_{bn1} = t_b/T_{c1} = \pi t_b f(1 + \beta_1)/Q_{o1} = \pi t_b(1 + \beta_1)f^{3/2}C_m/g$.

$t_{co2} = Q_{o2}/\pi f = (g/R_s)/\pi f = g/C_m f^{3/2}$; $T_{co2} = T_{co2}/(1 + \beta_2)$

$t_{bn2} = t_b/T_{c2} = \pi t_b f(1 + \beta_2)/Q_{o2} = \pi t_b(1 + \beta_2)f^{3/2}C_m/g$

The expression for the voltage with CARTOP, V_c , is:

$$V_c = (R_c P_g)^{1/2} [2\beta_2^2/(1 + \beta_2)] f(t_{bn1}, t_{bn2}, C_f)$$

where R_c is the shunt resistance of AC designed for CARTOP. The voltage gain, G_v is:

$$G_v = V_c/V(\beta_2 = 1) = (R_c/R)[2\beta_2^{1/2}/(1 + \beta_2)] f(t_{bn1}, t_{bn2}, C_f) .$$

The square of G_v is the power gain, G_p . It is the cw power without CARTOP divided by the cw power with CARTOP for the same beam voltage. $G_p = G_v^2$.

For given t_{bn1} , t_{bn2} , $G_p = f_1(\beta_2) \times f_2(\beta_2, C_f)$. On one hand, if C_f is too large then t_p is too small, and the AC has no time to charge. On the other hand, if C_f decreases toward 2, then the pulse amplitude approaches its cw value and the gain decreases toward 1, no matter how much time there is to charge the AC. The normalized pulse amplitude starts at 1 and increases to 9 as C_f increases from 2 to infinity. Hence, for a given t_b , t_{bn1} , β_2 there is a value of C_f for which G_p is maximum.

Three plots of G_p as a function of t_{bn2} , maximized with respect to C_f , are shown in Fig. 3. The optimum β_2 depends weakly on t_{bn2} . The reason is as follows: $f_1(\beta_2)$ has a maximum at $\beta_2 = 1$. As β_2 increases above 1, $f_1(\beta_2)$ decreases and $f_2(\beta_2)$ increases. The rate of increase of $f_2(\beta_2)$ is high for low T_{bn2} and approaches zero as T_{bn2} increases. The most likely gains attainable with CARTOP are $2 < G_p < 5$, therefore, as is apparent from Fig. 3, G_p is maximized with respect to β_2 , if $\beta_2 = 2.7$.

Using $\beta_2 = 2.7$, $g = 170$, $\beta_1 = 10^4$, $t_{bn1} = 1$, we obtain G_p as a function of t_b for several rf frequencies. Plots of these functions are shown in Fig. 4.

Plots that take into account the increase in power gain due to the increase in shunt resistance with the square-root of frequency are shown in Fig. 5. G_p of Fig. 4 is multiplied by $(f/353)^{1/2}$. In both figures, the left axis and the first vertical line corresponds to PEP three and one bunch

operation respectively; the next three vertical lines and the right axis correspond to LEP-8 4, 3, 2, 1 bunch operation respectively. The numbers along the right axis represent the beam energy gain, G_e , which is the eighth root of the power gain.

The geometric factor used for these plots is 170, the PEP structure g . To obtain the gain at a different g , transform t_b to $t_b' = (170/g)t_b$. For example, if $g = 125$, then at 1-bunch PEP, $t_b' = (170/125) 7.2 = 9.8$ micro-second. At 1300 MHz, G_p is 3 and circulating beam energy increases 15%.

CARTOP can also be used with a traveling-wave accelerating structure. In fact, for a given bunch period to filling time ratio, the power gain is greater, because with CARTOP, as with any pulsed system, a standing-wave structure reflects all the power at the instant of turn-on.

ESC Parameters

In calculating the power gain we have neglected the ESC losses. It will be shown that this is justified. β_1 has to be large enough and t_{bnl} small enough to make the loss in gain due to their finite values negligible. Knowing β_1 and $T_{cl} = t_b/t_{bnl}$ we obtain the unloaded ESC Q, $Q_{ol} = (1 + \beta_1) \pi f T_{cl} = (1 + \beta_1) \pi f t_b/t_{bnl}$. If $t_b = 7.2 \mu s$, $f = 1300$ MHz, $\beta_1 = 100$ then $Q_{ol} = 6 \times 10^6$. This Q_{ol} is difficult to realize at room temperatures but is easily realized with niobium cavities at liquid helium temperature, 4.22°K. The size of the refrigerator, S_r , is calculated as follows. The external Q, Q_{el} , is: $Q_{el} = \pi f / T_{cl} = \pi f t_b/t_{bnl}$. The room temperature unloaded Q of a copper right circular cylinder T_{oll} cavity, Q_{ort} , is: $Q_{ort} = 3 \times 10^6 / f^{1/2}$. Its diameter $D(\text{cm}) = 3 \times 10^4 [0.25 + (3.83/\pi)^2]^{1/2} / f$. The superconducting improvement factor at 4/2°K, I_f is: (3)

$$I_f = 10^9 f^{-3/2}$$

Thus,

$$Q_{ol} = Q_{ort} I_f^2 = 3 \times 10^{15} / f^2 \text{ and } \beta_1 = Q_{ol} / Q_{el} = 3 \times 10^{15} t_{bnl} / t_b \pi f^3.$$

The fraction of rf power dissipated in the ESC, P_c , is: (1)

$$P_c = (1/\beta_1) \int_0^{t_{bnl}} E_{el}^2(t_{nl}) dt_{nl} \quad \text{where } t_{nl} = t/T_{cl}$$

For a given t_{bnl} the value of the integral depends on C_f only. The refrigerator size, S_r , and the line power required by the refrigerator P_{lr} are:

$S_r = P/P_g$; $P_{lr} = R_f P/P_g$ where R_f is the inverse of the overall refrigeration efficiency. For large refrigerators it is on the order of 350. (3)

Define the average power gain G_{ap} as the ac line power without CARTOP divided by the ac line power with CARTOP, for the same beam voltage. The peak and average power gains are equal, i.e., $G_{ap} = G_p$, except when the ESC is superconducting. Then, we must add to the ac line power, the power required by the refrigerator, and $G_{ap} = L_r G_p P_g / (L_r P_g + P_g P_c R_f) = G_p / (1 + P_c R_f / L_r)$ where L_r is the ratio of line ot rf power.

Design for PEP and LEP-8 (4)

If we cannot design a structure with an R/Q higher than that of the PEP structure, and cannot design a structure with an elliptical aperture whose shunt resistance for a fixed aperture size increases as $f^{1/2}$, then from Fig. 4 for 1-bunch PEP at 1500 MHz, $G_p = 2.8$, $G_e = 1.14$; for 2-bunch LEP-8 at 500 MHz, $G_p = 3.3$, $G_e = 1.16$. If we can design a structure whose shunt resistance, for the same aperture, is proportional to $f^{1/2}$, then, from Fig. 5, for PEP $G_p = 5.9$, $G_e = 1.25$; for LEP-8 $G_p = 4$, $G_e = 1.19$. If also the structure R/Q is increased a factor of 2, then substituting $t_b' = 2t_b$ in Fig. 5, we

obtain for PEP: $G_p = 8.1$, $G_e = 1.31$; for LEP-8: $G_p = 5.3$, $G_e = 1.25$. If the last condition is achieved, the beam energy increases from 18 to 35 GeV for PEP, at $P_g = 6$ MW, $f = 1500$ MHz, $t_b = 7.2$ μ s; and from 85.5 to 105 GeV for LEP-8 at $P_g = 96$ MW, $f = 500$ MHz, $t_b = 50$ μ s. The ESC and refrigerator parameters are:

	$10^{-3}Q_{el}$	D	$10^{-3}Q_{ort}$	$10^{-9}Q_{ol}$	$10^{-3}\beta_1$	$\beta_1 P_c$	$\frac{S_r}{\text{watts}}$	$\frac{P_{lr}}{\text{kW}}$
PEP	34	26	77	1.3	39	2.1	323	113
LEP-8	78	79	134	1.2	158	2.3	1400	489

Improvements

As both t_p/T_c and t_b/t_p increase to values much greater than 1, G_p approaches the limiting value of 9. This barrier can be overcome if more than one energy storing network is used. If two are used then the power gain limit is 36.

It is possible that rf energy reflected by a standing wave structure and the rf energy not used by the beam in both a traveling and standing wave structure, can be redirected to flow back into the ESC which already exists as part of the CARTOP system (see Figs. 1, 6 and 7). The transient reflections from a standing wave structure can be reduced by starting with a high coupling factor and gradually reducing it as the cavity fills. The coupling factor can be varied with time using the formula presented by R. Sundelin,⁽⁷⁾ for shaping the incident power. (See Appendix B.)

At least, initially, the power gain varies as $\omega R/Q$, which is proportional to volume/meter (v/m) of the AC. Therefore, to maximize gain at a given frequency we have to minimize v/m. Unfortunately, the Jungle Jim structure,

whose R/Q is twice that of the PEP structure, cannot be used in the standing wave mode because its R/Q degrades as the phase/cell approaches π . It should be possible to come up with a π -structure whose v/m is significantly less than that of the PEP structure. Perhaps at the expense of a smaller percentage increase of the shunt resistance which is proportional to surface area/unit length. The effect of increasing the surface area (or surface resistance) does not affect the gain: although it increases the loss it also increases the rate of energy flow into the AC.

Why CARTOP

CARTOP has some of the features of the pulsed rf system described by Wilson⁽⁴⁾ and of the coupled cavity system described by Schnell.⁽⁵⁾ With pulsed rf the energy is stored in a capacitor, and a switch allows the energy to flow into a klystron, where it is converted to rf. With the Schness system, the rf is modulated and flows into the coupled cavity system where it oscillates between the ESC and AC.

The advantages of CARTOP over a DC pulsed system are:

1. It can be installed where a cw system already exists. Even if the operating frequency is changed, only the klystrons and cavities have to be replaced (since the frequency is increased there is plenty of room for them) but not the klystron power supplies. The already existing waveguides can be fitted with mode suppressors to enable them to carry the higher frequency power.
2. Peak power gain. High power amplifiers are peak power limited. Instead of interrupting the power, you squeeze it into the pulse.
3. High power amplifiers are more efficient than pulsed amplifiers.

4. It is easy to switch between pulsed and cw operation.
5. The pulse width has to change to accommodate different number of bunches. This is easier to do than changing klystron voltage or current. Moreover, klystrons are most efficient at a specific voltage and current.

The advantages of CARTOP over the coupled cavity system are:

1. Higher power gain.
2. The coupling is not critical.
3. The storage and accelerating cavities can be thermally insulated from each other.
4. Only one klystron per cavity, instead of two, is needed.
5. The ESC is not in the beam tunnel.
6. CARTOP can be implemented, and therefore the energy of the storage ring can be increased, in installments over an extended time interval.

Like the DC pulsed rf, CARTOP requires a high R/Q accelerating cavity; and like the coupled cavity system it requires a high-Q storage cavity. Thus, the AC and ESC development for these systems can be adapted to CARTOP.

Conclusion

If we are willing to increase frequency, even if we do not change the accelerating structure and use only presently available components, significant power gains are obtained--greater than two for one bunch PEP at frequencies greater than 1.1 GHz and for four bunch LEP-8 at frequencies greater than 500 MHz. These might be sufficient to reduce the rf power and its cost to a small fraction of the storage ring total power and its cost. We may decrease the frequency and increase the number of bunches and still have the same gain, if we develop a high R/Q standing wave structure. Further work should be done on high R/Q structures, on using energy feedback and on using

tandem cavities using both computer and hardware models. As a compromise between computer and hardware models, the low frequency analog of an energy storing network shown in Fig. 8, can be used.

If the development program is successful and we can decrease Q of a structure (without changing its R) by a factor of 2, then we can obtain power gains greater than 2 for 1-bunch PEP at frequencies greater than 700 MHz and for 4-bunch LEP-8 at frequencies greater than 350 MHz. For 1-bunch LEP-8, using two ESN's in tandem power gains greater than 10 and circulating beam energy gains greater than 33% can be achieved.

CARTOP is also useful for saving bunch-lengthener rf power.

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APPENDIX A

Field Solutions of Tandem Energy Storing Network (ESN)

With Power Incident Upon First ESN a Piece-Wise

Function of Time

Let the cavity parameters of the nth ESN (see Fig. A1) be α_n , T_{cn} , and the initial emitted field at the beginning of an interval, which is proportional to the initial energy stored and which is also the emitted field at the end of the previous interval, be E_{eon} . Let the driving function consist of constant term a_0 and exponential terms $a_1 e^{-t/T_1}$. This is the case if the field incident upon the first cavity contains only constants and exponentials and only the last two ESN's are allowed to be identical.

Fig. A1. Input and output fields of the nth ESN.

The output fields are obtained by solving, simultaneously, the two equations:

$$T_{cn} \frac{dE_{en}}{dt} + E_{en} = \alpha_n E_{r(n-1)} \quad A-1$$

$$E_{rn} = E_{en} - E_{r(n-1)} \quad A-2$$

The solution for E_{en} is obtained by using the following simple rules:

1. Multiply E_{eon} by $e^{-t/T_{cn}}$.
2. Multiply a_o by $\alpha_n (1 - e^{-t/T_{cn}})$.
3. For each exponential $a_i e^{-t/T_i}$ substitute $\alpha_n a_i (e^{-t/T_i} - e^{-t/T_{cn}})$.
4. If $T_i = T_{cn}$, then for $a_i e^{-t/T_i} = a_i e^{-t/T_{cn}}$ substitute $\alpha_n t e^{-t/T_{cn}}$.

E_{ro} is the input to the first cavity and is the generator field.

Thus, if we know the piece-wise variable first cavity input field at all time intervals and the initial fields in all cavities at the beginning of the first time interval (usually zero) then we can find the field of all cavities at all time intervals.

APPENDIX B

Minimizing Cavity Reflection by Varying β

After an initial energy U_0 has been introduced into a cavity, the variation of power incident upon it that results in no reflections was derived by Sundelin⁽⁷⁾ and is:

$$P_i = \beta U_0 / \tau_0 e^{(\beta - 1)t / \tau_0}; \quad \text{where } \tau_0 = Q_0 / \omega$$

Rewriting the Sundelin equation we have:

$$P_i / (U_0 / \tau_0) = \beta e^{(\beta - 1)t / \tau_0}$$

At

$$t = 0, \quad \beta = P_i U_0 / \tau_0$$

Choose the initial energy such that $\beta = 100$ at $t = 0$. Then, β as a function of time for no further reflection is obtained from the solution of:

$$\beta e^{(\beta - 1)t / \tau_0} - 100 = 0$$

This transcendental equation solved numerically yields the following values of β as a function of normalized time:

t / τ_0	0	0.001	0.002	0.004	0.006	0.01	0.02
β	100	92	85	74.5	67	57	43
t / τ_0	0.04	0.08	0.1	0.5	0.8	1	2
β	30.8	20.6	18	6.5	4.8	4.18	2.79

Fig. 1. CARTOP System

FIG. 2A. FIRST CAVITY EMITTED FIELD, E_{e1} VS. TIME

FIG. 2B FIRST CAVITY REFLECTED FIELD VS. TIME

FIG 2c. SECOND CAVITY EMITTED FIELD VS. TIME

FIG 2D. SECOND CAVITY REFLECTED FIELD, E_{r2} VS. TIME.

Fig. 3. Power gain, G_p , as a function of unloaded accelerating cavity time constant T_{bno2} .

Fig. 4. POWER AND CIRCULATING BEAM ENERGY GAIN VS BUNCH PERIOD, T_b $\gamma_0 = 170$.

FIG. 5. POWER AND CIRCULATING BEAM ENERGY GAIN VS BUNCH PERIOD, $T_b, g = 170$.

The increase in power gain due to increase of shunt resistance with the square root of frequency is taken into account.

Fig. 6. CARTOP with SW load and feedback.

Fig. 7. CARTOP with TW load and feedback

Fig. 8. CARTOP low frequency analog.

FIG. 2A. FIRST CAVITY EMITTED FIELD, E_{e1} VS. TIME