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By

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RECONNECTION RATES OF MAGNETIC FIELDS

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ABSTRACT

The Sweet-Parker and Petschek scalings of magnetic reconnection rate are modified to include the effect of the viscosity. The modified scalings show that the viscous effect can be important in high- β plasmas. The theoretical reconnection scalings are compared with numerical-simulation results in a tokamak geometry for three different cases: a forced reconnection driven by external coils, the nonlinear $m = 1$ resistive internal kink, and the nonlinear $m = 2$ tearing mode. In the first two cases, the numerical reconnection rate agrees well with the modified Sweet-Parker scaling, when the viscosity is sufficiently large. When the viscosity is negligible, a steady state which was assumed in the derivation of the reconnection scalings is not reached and the current sheet in the reconnection layer either remains stable through sloshing motions of the plasma or breaks up to higher m modes. When the current sheet remains stable, a rough comparison with the Sweet-Parker scaling is obtained. In the nonlinear $m = 2$ tearing mode case where the instability is purely resistive, the reconnection occurs on the slower dissipation time scale ($\dot{\psi}_g \sim \eta$). In addition, experimental data of the nonlinear $m = 1$ resistive internal kink in tokamak discharges are analyzed and are found to give reasonable agreement with the modified Sweet-Parker scaling.

I. INTRODUCTION

Many important processes in confined plasmas, such as magnetic island growth, Spheromak and Doublet formation, etc., involve changes of the magnetic topology. The rates of such topology changes are determined by the magnetic field reconnection rate at the x-point of the separatrix. The problem of magnetic reconnection has been widely studied during the past twenty years, first triggered by the interest in solar flare phenomena.¹ The reconnection rate is defined here as the time rate of change, $\dot{\psi}_s$, of the magnetic flux at the x-point. There are two widely quoted theoretical scalings, namely, Sweet-Parker scaling,² which gives the reconnection rate of $\dot{\psi}_s \sim \eta^{1/2}$, and Petschek scaling,³ which gives the maximum rate of $\dot{\psi}_s \sim 1$ (i.e., independent of the resistivity η).

Both of these scalings give a reconnection rate faster than $\dot{\psi}_s \sim \eta$ given by the resistive skin time. In general, for the scaling $\dot{\psi}_s \sim \eta^\alpha$, α can have the possible range of $0 \leq \alpha \leq 1$. For a rapid reconnection process (defined here as $\alpha < 1$) to take place, a necessary condition is that a current singularity ($J_s \rightarrow \infty$) develops in the limit of $\eta \rightarrow 0$, since $\dot{\psi}_s = \eta J_s$ where J_s is the current density at the x-point. Such singularities have been found in the numerical simulations of the nonlinear phase of the ideal ($\eta = 0$) internal kink⁴ and ballooning modes⁵ in the tokamak geometry, suggesting that the rapid reconnection process may be a common occurrence in plasma confinement experiments. The actual rates of such rapid reconnection processes are studied in the present paper. In addition, we will also present a case where the reconnection rate is slow, i.e., $\dot{\psi}_s \sim \eta$.

There exist numerical simulation studies indicating either Sweet-Parker⁶ or Petschek⁷ mechanism of the reconnection process. Experiments on laboratory plasmas have been also reported indicating either Sweet-Parker⁸ or Petschek

mechanism.⁹ The present paper aims at accurate quantitative studies of reconnection in tokamak geometry using the magnetohydrodynamic model with scalar resistivity and viscosity. We find that viscous effects can be important in the reconnection process, and in Sec. II we modify the Sweet-Parker and Petschek scalings to include the viscosity. In the subsequent sections, close quantitative agreements between the modified Sweet-Parker scaling and the numerical simulation results are obtained when viscous effects are significant. When the viscosity is negligible, a steady state which was assumed in the derivation of the reconnection scalings is not reached and the current sheet in the reconnection layer either remains stable through sloshing motions of the plasma or breaks up to higher m modes. When the current sheet remains stable, a rough comparison with the Sweet-Parker scaling is obtained.

In Sec. III, the physical and numerical models and basic numerical schemes used are described. In Sec. IV, numerical simulation results are presented for three different cases: a forced reconnection process driven by external coils in Subsection (A), the $m = 1$ nonlinear internal kink¹⁰ in Subsection (B), and the $m = 2$ nonlinear tearing mode in Subsection (C). In cases A and B the numerical results agreed well with the modified Sweet-Parker scaling. In case C, the reconnection rate is found to be slow, ($\propto \sqrt{\eta}$). In Sec. V, the scalings are also compared with the $m = 1$ reconnection period of the experimental sawtooth oscillation¹¹ measurement in the Princeton PDX and ST tokamaks. We find that the Sweet-Parker type scaling gives reasonable agreement with the experimental data in line with the numerical results of Sec. IV(B). Finally, in Sec. VI the results are summarized and their implications for the plasma confinement experiments are discussed.

In all the theoretical studies, we use the single fluid magnetohydrodynamic model with scalar resistivity and viscosity. The MHD

equations are (in rationalized emu units);

$$\rho \frac{d\vec{v}}{dt} = -\nabla p + \vec{J} \times \vec{B} + \rho\mu \left[\nabla^2 \vec{v} + \frac{1}{3} \nabla(\text{div } \vec{v}) \right]$$

$$\frac{\partial p}{\partial t} + \text{div}(\rho \vec{v}) = 0 \quad ,$$

$$\frac{dp}{dt} = \gamma p \text{div } \vec{v} \quad ,$$

$$\frac{\partial \vec{B}}{\partial t} = -\text{curl } \vec{E} \quad , \quad (1)$$

$$\text{div } \vec{B} = 0 \quad ,$$

$$\vec{E} = \eta \vec{J} - \vec{v} \times \vec{B} \quad ,$$

$$\vec{J} = \text{curl } \vec{B} \quad ,$$

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where ρ is the density, $\vec{v}$ the velocity, p the pressure, η the resistivity, μ the kinematic viscosity, and $d/dt = \partial/\partial t + \vec{v} \cdot \nabla$.

II. DERIVATION OF RECONNECTION SCALINGS

In this section, we modify the Sweet-Parker and Petschek scalings to include the effect of the viscosity. Here we note that for a high- β plasma (β : the ratio of the mechanical and magnetic pressures), the classical transport coefficients give a significant value of μ_1/η (in the rationalized emu units), which will be shown later to modify the scalings. From the relations

$$\mu_1 = \frac{k T_i}{m_i \omega_{ci}^2 \tau_i} \text{ and } \eta = \frac{m_e}{n_e e^2 \tau_e},$$

we obtain $\mu_1/\eta = (m_i/m_e)^{1/2} \beta$, with the ion mass m_i and the electron mass m_e . The present day tokamaks have typical β values of ~ 0.02 while space plasmas commonly have $\beta \approx 1$.

Figure 1 illustrates the basic features of the reconnection process. For simplicity, we first consider a case where the magnetic field component perpendicular to the plane of the diagram is zero. When the plasma is squeezed from both sides into the x-point where $B = 0$ by, e.g., nonlinear ideal instabilities, a magnetic field depicted in Fig. 1(a) will be set up. From pressure balance, the plasma pressure p at the x-point will be $B_0^2/2$ higher than the pressure p_0 outside the reconnection layer. The shaded area in Fig. 1(b) represents the reconnection layer. If the downstream pressure is maintained at p_0 , the pressure gradient in the y direction will drive a large plasma velocity V_y . This then further steepens the field gradient until a steady state is reached.

In the steady state, the reconnection rate defined as the rate of magnetic flux change at the x-point, $\dot{\psi}_s$, is equal to $\dot{\psi}_0$ outside the reconnection layer. One can neglect the resistive effect outside the reconnection layer where the current density remains small. Flux conservation then gives

$$\dot{\psi}_0 = v_x B_0 = \dot{\psi}_s \quad (2)$$

For an incompressible plasma, mass conservation gives

$$lv_x = \delta v_y \quad (3)$$

Another obvious relation is

$$\dot{\phi}_s - \eta J_s = \eta \frac{2B_o}{\delta} \quad (4)$$

Equating Eqs. (2)-(4), we obtain

$$\frac{\delta^2}{x} = \frac{2\eta}{v_y} \quad (5)$$

Up to this point, the derivation is the same as that for the usual Sweet-Parker scaling. The downstream velocity v_y will be determined by the energy conservation and now the viscosity enters into the derivation. The work done by the pressure gradient in the y direction on a unit volume of the plasma moving from the x -point to outside of the reconnection layer is

$$\Delta p = \frac{B_o^2}{2} \quad (6)$$

This is balanced by the kinetic energy increase and the viscous dissipation:

$$\frac{B_o^2}{2} = \frac{\rho}{2} v_y^2 + \rho \mu \frac{(v_y/2)}{(\delta/2)^2} \cdot \frac{l}{2} = \frac{\rho}{2} v_y^2 + \rho \mu \frac{v_y}{\delta^2} l \quad (7)$$

By substituting Eq. (5) to the viscous dissipation term, we obtain

$$\frac{B_o^2}{2} = \rho \frac{v_y^2}{2} \left(1 + \frac{\mu}{\eta} \right) \quad (8)$$

When the viscosity is negligible ($\mu/\eta \ll 1$), Eq. (8) gives $v_y \sim 1$. Note that B_o and ρ are both quantities of ~ 1 (i.e., independent of η and μ).

Substituting $v_y \sim 1$ into Eq. (5) gives

$$\frac{\delta^2}{x} \sim \eta \quad . \quad (9)$$

Scaling the layer length as $l \sim \delta^r$, we obtain $\delta \sim \eta^{1/(2-r)}$. Equation (4) then gives the reconnection rate

$$\dot{\psi}_s \sim \eta^{(1-r)/(2-r)} \quad . \quad (10)$$

The Sweet-Parker model assumes $l \sim 1$ ($r = 0$) while the Petschek model has $l \sim \delta$ ($r = 1$). Thus Eq. (10) produces the Sweet-Parker ($\dot{\psi}_s \sim \eta^{1/2}$) and Petschek ($\dot{\psi}_s \sim 1$) scalings.

When the viscosity is dominant ($\mu/\eta \gg 1$), Eq. (6) gives $v_y \sim (\eta/\mu)^{1/2}$. With $l \sim \delta^r$, Eq. (5) gives $\delta \sim (\eta\mu)^{1/(4-2r)}$. Equation (4) then gives

$$\dot{\psi}_s \sim (\eta\mu)^{-1/(4-2r)} \quad . \quad (11)$$

This equation produces

$$\dot{\psi}_s \sim \eta^{1/2} (\eta/\mu)^{1/4} \quad (12)$$

for the Sweet-Parker type reconnection layer ($l \sim 1$), and

$$\dot{\psi}_s \sim (\eta/\mu)^{1/2} \quad (13)$$

for the Petschek type reconnection layer ($l \sim \delta$). [For the Petschek type layer, one more viscous dissipation term goes into Eq. (7) due to large gradients in the y-direction, but this does not change the scaling.]

Further one can derive scalings which are valid for both regimes, $\mu/\eta \gg 1$ and $\mu/\eta \ll 1$, and also retaining approximate equalities of coefficients. For a Sweet-Parker type layer, Eq. (8) gives

$$v_y = \frac{B_0}{\rho^{1/2}} \left(1 + \frac{\mu}{\eta}\right)^{-1/2}.$$

This equation together with Eqs. (4) and (5) gives the modified Sweet-Parker reconnection scaling,

$$\dot{\psi}_s = \left(\frac{2B_0^{3/2}}{\rho^{1/4}\eta^{1/2}}\right) \eta^{1/2} \left(1 + \frac{\mu}{\eta}\right)^{-1/4}. \quad (14)$$

Likewise for the Petschek type layer one can obtain the modified Petschek scaling,

$$\dot{\psi}_s = \left(\frac{B_0^2}{\rho^{1/2}}\right) \left(1 + \frac{\mu}{\eta}\right)^{-1/2}. \quad (15)$$

The model used for the above derivation does not have a longitudinal magnetic field component. However, it can be easily shown that the same derivation is also applicable to cases with longitudinal field by noting that this field component, due to its topology, can move freely with the fluid without forming a sharp gradient. The reduced tokamak equations given in Sec. III will also show that the only significant effect of the strong longitudinal field is to make the flow nearly incompressible.

In the above derivation, we obtained the reconnection scalings using only the conditions near the reconnection layer. If the reconnection layer solutions are extended outward, rays of discontinuity may enter into the solution as in the case of Petschek's solution with slow shocks.³ It is also

possible that different boundary conditions may be required in extended solutions of different reconnection scalings. Thus, a unique reconnection scaling which satisfies all the cases may not exist. Instead, the reconnection scaling may be determined by the given boundary condition. The cases studied numerically in the following sections have closed boundaries so that the plasma flow across the boundary vanishes.

III. METHODS

A. Equations

In most of the numerical studies presented here, we use the two-dimensional reduced tokamak equations¹² which utilize an inverse aspect ratio (the ratio of the tokamak minor and major radius, $c = a/R_0$) expansion. Here, we represent these equations in helical coordinates (r, ϕ) and unit basis vectors $(\hat{r}, \hat{\phi}, \hat{e})$ where $\hat{e} = \hat{r} \times \hat{\phi}$ and $\hat{\phi} = \hat{\theta} + \alpha \hat{z}$ with the helicity α and with the usual cylindrical coordinates (r, θ, z) . The magnetic field and velocity are represented by a helical flux ψ and a stream function U :

$$\begin{aligned} \vec{B} &= \nabla\psi(r, \phi) \times \hat{e} + B_t^0 \hat{e}, \\ \vec{V} &= \nabla U(r, \phi) \times \hat{e}. \end{aligned} \tag{16}$$

The flow is incompressible to this order due to the strong longitudinal field B_t^0 which is constant to this order. (The toroidal field B_t^0 and the $\hat{e}$ -component B_e are equivalent to this order.) The reduced tokamak equations are (with constant density $\rho_0 = 1$)

$$\frac{d\psi}{dt} = -\eta J \quad (J = -\nabla^2\psi - 2\frac{n}{m}\frac{B_0}{R_0}) \quad , \quad (17)$$

$$\frac{d\nabla^2 U}{dt} = -\vec{B} \cdot \vec{\nabla} J + \mu \nabla^2 \nabla^2 U \quad ,$$

where

$$\vec{\nabla} = \vec{\nabla}_1 = \vec{r} \frac{\partial}{\partial r} + \hat{\phi} \frac{\partial}{r \partial \phi} \quad , \quad \frac{d}{dt} = \frac{\partial}{\partial t} + \vec{v} \cdot \vec{\nabla} \quad ,$$

and n/m the ratio of the initial toroidal and poloidal mode numbers. The helicity α in $\phi = \theta + \alpha z$ is identified to $\alpha = n/(mR_0)$.

For a reader unfamiliar with the tokamak field geometry, a simple analogy of the reduced tokamak equations (17) might be useful. These equations are virtually equivalent to the incompressible MHD equations (A2) with translational symmetry given in the Appendix. This can be easily seen by changing the variables $\psi' = \psi - \eta c_1 t$ and $J' = J + c_1$ where $c_1 = (2n/m)B_0/R_0$,

$$\frac{d\psi'}{dt} = -\eta J' \quad (J' = -\nabla^2\psi') \quad (17')$$

or (A2')

$$\frac{d\nabla^2 U}{dt} = -\vec{B} \cdot \vec{\nabla} J' + \mu \nabla^2 \nabla^2 U \quad .$$

This equivalence allows the reconnection scalings derived in Sec. II assuming translational symmetry to be also applicable to the present helical system. In the remainder of this paper, ψ'_s and J' will be denoted as ψ_s and J for convenience.

In Sec. IV(b), we use the helically symmetric incompressible equations (Eq. A1) given in the Appendix. These are exact for any aspect ratio. These equations were previously used in Ref. 4, and it is shown in the Appendix that these equations can be readily reduced to the translationally symmetric Eq.

(A2) by taking the limit of zero helicity and also can be reduced to the reduced tokamak equations (17) by taking the limit of small aspect ratio.

In all the above equations, the flow is incompressible approximating the nearly incompressible tokamak plasmas. This will certainly be a valid approximation for the Sweet-Parker type reconnection process where no shock structure is involved, while for the Petschek type reconnection process, Ref. 3 shows that a small compressibility changes the reconnection rate only slightly.

B. Numerical methods

The numerical methods used in the two-dimensional code 1MH2D employed in the present study are essentially the same as the ones used in the three-dimensional MHD code MIB which was used to study high- β ballooning modes.¹³ The r -dependence of a variable is represented on a discrete grid in a physical space and the ϕ -dependence in a truncated Fourier space. A simple predictor-corrector type explicit numerical scheme is used, except for the dissipation terms which are solved implicitly. The fact that spatial differentiations in ϕ can be carried out exactly for each harmonic simplifies the implicit solution of the dissipation terms by reducing it to tri-diagonal matrix inversion. Cross product terms are calculated by taking convolutions in Fourier space (i.e., a spectral method not a pseudospectral method), which computationally was the most time consuming part of the simulation.

In the simulation, steep gradients around the x-point lie in the r -direction, giving relatively slowly varying ϕ dependence. Thus the ϕ -dependence can be represented efficiently by a small number of Fourier harmonics, compared to an equivalent representation on a discrete grid in a physical space. The r -dependence is represented in a variable grid dense near

the x-point, which is rezoned automatically as the x-point moves or as gradients become steeper.

The very small r-meshes in the reconnection layer region do not restrict the numerical time steps in the calculation. This can be understood from the Courant condition ($v\Delta t/\Delta x < 1$) for explicit numerical schemes which basically states that for numerical stability the waves should not propagate past the next grid point during one time step. Since the incompressible equations we solve have only the shear Alfvén wave, the component of the wave velocity on the (r, ϕ) computational plane is small in the reconnection layer, reaching zero at the x-point. Moreover, since the shear Alfvén wave propagates on a flux surface which is almost tangential to an $r = \text{constant}$ surface, the possible time step is determined not by the r-mesh size but by the number of ϕ -harmonics (effective ϕ -mesh size: $2\pi/m$). The dissipation terms which do not have the above-mentioned characters are solved implicitly as stated before.

The origin ($r = 0$) values are nonzero for $m = 0$ scalar components and some of these values are needed as extraneous boundary conditions for the difference equations although they are not boundary values for the differential equations. These values are determined by a polynomial extrapolation using the three adjacent grid point values, at each time step. The boundary conditions used for Eq. (17) are $\psi_{\text{wall}} = f(t, \phi)$ representing a given electric field at the boundary, $E_w = \partial f / \partial t$, and $U_{\text{wall}} = \text{constant}$ and $\nabla^2 U - 2/r \cdot \partial U / \partial r|_{\text{wall}} = 0$ representing a rigid inviscid wall. The present numerical schemes produced a good stability character, and no artificial dissipation term is used in the code.

IV. RESULTS OF NUMERICAL SIMULATIONS

The units used in the presentation of the results are the minor radius a as the length unit, $\rho_0 a^3$ as the mass unit, and the poloidal Alfvén time $T_0 = a(\rho_0)^{1/2}/(\epsilon B_t^0)$ as the time unit, where ρ_0 is the mean density, ϵ the inverse aspect ratio, and B_t^0 the mean toroidal field. All the other units are determined by these three units together with dimensionality relations from Eq. (1). For example, the unit of ψ is $a\epsilon B_t^0$ and the unit of both η and μ is a^2/T_0 .

A. External coil driven reconnection

Figure 2 shows the eight external coils which drive $m = 4$ magnetic island growth in the plasma. Constant helical flux, ψ , contours on the (r, ϕ) plane are plotted in Fig. 2(a) showing the magnetic islands, and the B_ϕ profile along the horizontal (on the $\phi = 0$ plane) radii in Fig. 2(b) showing a finite jump of B_ϕ across the x-point.

The external coil currents induce the $m = 4$ component of ψ_{wall} to rise from zero to 5×10^{-3} at $t=50$, and thereafter to remain constant. The total amount of reconnection during the rise time is found to approach zero as the resistivity η becomes smaller, and found to be negligible for the resistivity regime of the present study. In the example shown in Fig. 2, a constant resistivity, $\eta = 10^{-7}$, is used giving the Lundquist number (the ratio of the diffusion and Alfvén time scales) of $S=10^7$. In the following, we will examine the reconnection process which occurs after the external currents reach a steady value. The initial state of the plasma was azimuthally symmetric with the safety factor profile $q = 0.9 (1 + 4r^2)^{1/2}$.

The finite jump in B_ϕ in Fig. 2(b) implies a sharp current spike as seen in Fig. 3. The constant current density J contours are plotted in Fig. 3(a)

and the enlargement of the current spike is shown in Fig. 3(b) which is enlarged 5 times more in the r -direction than in the ϕ -direction, changing the shape of the current sheet curvature. Figure 3(c) plots the J profile along the horizontal radii showing the current spike with height J_s at the x -point. The enlargement of the current spike is shown in Fig. 3(d). There are 200 meshes in the r -direction at this instant with ~ 30 meshes packed inside the current sheet region. About 15 ϕ -harmonics were sufficient. The large current on the boundary of Fig. 3(c) is the skin current induced by external coils. This skin current decays on the skin time ($\sim \eta^{-1}$) which is much longer than the time scale of rapid reconnection.

Figure 4 shows the velocity stream lines (contours of the velocity potential Π) around the x -point enlarged in exactly the same fashion as in Fig. 3(b). Although the r -scale is enlarged 5 times more than the ϕ scale, the long and narrow flow structure is apparent. We also found that the length l of the flow structure is independent of η and μ ($l \sim 1$). Although $l \sim 1$ suggests that the reconnection rate may satisfy the modified Sweet-Parker scaling, Eq. (14), it should be noted that more assumptions were necessary for the derivation of Eq. (14), e.g., the assumption of near equality of the upstream and downstream pressure. Thus, the actual reconnection rate from the simulation should be compared to the theoretical scaling.

In Fig. 5 we compare the numerical reconnection rate with the theoretical scalings. Since the reconnection rate satisfies $\dot{\psi}_s = \nabla J_s$ ($\vec{V} \cdot \nabla \psi = 0$ at the x -point), it is convenient to compare scalings of J_s instead of $\dot{\psi}_s$, as will become clear in subsection (C). Figure 5 shows how the height of the current peak J_s changes as the plasma evolves, for three different values of η and μ in $\eta \ll \mu$ regime. The upper and lower solid curves are obtained after

changing η and μ from the middle curve at the branch point. The abscissa is $\Delta\psi_s$, the change of the flux value at the x-point from the initial state. This is used because the value of $\Delta\psi_s$ determines the global plasma state in the small η limit, for the states evolved from the same initial state, independent of the path. The dashed lines in the graph are obtained by applying the modified Sweet-Parker scaling [Eq. (12)] for $\eta \ll \mu$ ($J_s \sim \dot{\psi}_s/\eta \sim \eta^{-1/2}(\eta/\mu)^{1/4}$) to the middle solid line. From this graph, three conclusions can be drawn: (1) the simulation results asymptotically reach the modified Sweet-Parker scaling, (2) the rapidly varying parts of the curves represent the relaxation process toward steady states, (3) the proportionality constant K in $\dot{\psi}_s = K \eta^{1/2} (\eta/\mu)^{1/4}$ is relatively constant in the range of $\Delta\psi_s$ plotted. A further comparison can be made by explicitly evaluating the proportionality constant K from Eq. (14) and from the simulation. Using $B_0 = \Delta B/2 = 0.01$ from Fig. 2(b) and $\ell = 0.4$ from Fig. 4, Eq. (14) produces $K = 2B_0^{3/2}/(\rho^{1/4}\ell^{1/2}) = 0.003$. This value agrees reasonably with $K_{\text{num}} = 0.004$ which can be evaluated from Fig. 5. [In Eq. (14), the proportionality constant is approximate whereas the scaling is exact.]

The fact that the proportionality constant K is as small as 0.003 shows the necessity of using a large Lundquist number, $S = 10^7$ or 10^8 , in the numerical simulation. To obtain a reasonably accurate reconnection scaling, the current density J_s at the x-point should be at least an order of magnitude larger than the background current density. The resistivity $\eta = 10^{-7}$ (i.e., $S = 10^7$) gives $J_s = K\eta^{-1/2} = 10$ for $\mu = \eta$, compared to the background current density, $J = 1$. When the present work was presented in a conference, questions were raised concerning the accuracy of a numerical calculation with such large S values. This question stemmed from the often loosely used

terminology, "magnetic Reynolds number" for the S value which should be properly called the Lundquist number. The actual magnetic Reynolds number, $R_M = aV/\eta$ which determines the resistive layer thickness, is $R_M = 10^3$ for the present case (due to small plasma velocity V perpendicular to the magnetic field) as can be also found from the layer thickness in Fig. 3(b). Thus the mesh density of 2×10^4 per minor radius in the resistive layer region was sufficient.

Simulation results with negligible viscosity ($\eta \gg \mu$) are shown in Fig. 6. The dashed lines are obtained by applying the Sweet-Parker scaling ($\dot{\psi}_s \sim \eta^{1/2}$) to the bottom solid line. The graph shows rough agreement between the solid and the dashed lines. The proportionality constant K_{num} from the graph, $K_{num} = 0.0025$, also agrees reasonably with $K = 0.003$ from Eq. (14). Note that the value of K is the same as that in the $\eta \ll \mu$ case because Eq. (14) applies to both cases and the values of B_0 and λ were found to be the same in both cases. For the present rough comparison, time t is used as abscissa instead of $\Delta\psi_s$.

Only rough comparison with the theoretical scaling was possible for this $\eta \gg \mu$ case because a sloshing motion of the plasma persisted in the simulation without reaching a steady state, which was assumed for the derivation of the theoretical scaling. The fact that a steady state is not reached in the $\eta \gg \mu$ regime can be understood by examining the rates of various energy changes. The Ohmic dissipation rate is $\eta J^2 \delta \sim \eta^{1/2}$ and the rate of kinetic energy increase from the jet flow out of the reconnection layer is $\rho v_y^2 \delta \sim \eta^{1/2}$ while the viscous dissipation rate, $\mu (v_y / \delta^2) \delta \sim \eta^{1/2} (\mu/\eta)$, is much slower. Thus, the free energy released by the reconnection is divided into the heating and kinetic energy on the same order while the viscous dissipation is negligible. Although this by

itself does not prove the nonexistence of a steady state, it indicates the difficulty of reaching a steady state compared to the $\eta \ll \mu$ case. For $\eta \ll \mu$ case, the Ohmic dissipation rate and viscous dissipation rate are both $\sim \eta^{1/2}(\eta/\mu)^{1/4}$ which is the same order as the free energy release from reconnection, while the kinetic energy increase from the jet flow out of the reconnection layer is much slower — at a rate of $\sim \eta^{1/2}(\eta/\mu)^{5/4}$.

B. Nonlinear $m = 1$ resistive internal kink

Figure 7 shows the time history of the nonlinear development of the $m = 1$ resistive internal kink. Plotted are the helical flux ψ contours at various instants of time. The initial concentric equilibrium which is unstable to the $m = 1$ internal kink mode is shown in Fig. 7(a). With finite resistivity, the nonlinear development of this resistive internal kink generates an $m = 1$ magnetic island as seen on the left in Fig. 7(b). This island continues to grow and the original flux lines on the right shrink, accompanied by magnetic reconnection at the x-point on the $q = 1$ surface. The original flux lines are completely replaced by the magnetic island and the final steady Ohmic state is reached in Fig. 7(c). This process of complete reconnection of the flux inside $q = 1$ surface is sometimes called the $m = 1$ flip.¹⁴ The container wall is modeled as a perfectly conducting inviscid wall in the simulation. Thus, there is no energy flow across the wall. For this simulation the helically symmetric incompressible Eq. (A1) in the Appendix are used.

We now examine the reconnection rate during the stage of nonlinear development shown in Fig. 7(b). Figure 8 shows a plasma state during this phase which has evolved from the initial concentric equilibrium with $q = (1.18 - 0.5r^2)^{-1}$. The current density contours in Fig. 8(a) show a current sheet at the $q = 1$ surface; the corresponding finite jump in B_θ across

the x-point is clearly seen in Fig. 8(b) where the B_ϕ profile along the horizontal radii is shown; the stream lines are plotted in Fig. 8(c) and an enlarged picture of the reconnection layer region is shown in Fig. 8(d). Figure 8(d) has been enlarged 5 times more in the r-direction than in the ϕ -direction. Nevertheless, a long and narrow flow channel is still apparent. Together with the finite jump in B_ϕ , this suggests that a similar reconnection mechanism is occurring as in the external coil driven case of subsection (A), which satisfied the modified Sweet-Parker scaling. However, as discussed in subsection (A) other assumptions were also used for the derivation of theoretical scalings and thus an actual comparison between the numerical and theoretical reconnection rates is necessary to establish the reconnection scalings.

The reconnection rates for $\eta \ll \mu$ regime are compared in Fig. 9 in a similar fashion as is done in Fig. 5. As before, we examine the values of J_g . The dashed lines are obtained by applying the modified Sweet-Parker scaling, Eq. (12), to the middle solid line. All the conclusions drawn from Fig. 5 can also be made in this case, except that the proportionality constant K is a more rapidly varying function of $\Delta\psi_g$.

When the viscosity is negligible ($\eta \gg \mu$), the nonlinear development of the plasma is found to be more complicated. The current sheet with single extremum at the x-point shown in Fig. 8(a) now becomes unstable to the $m > 1$ tearing modes, whereas it remains stable in the $\eta \lesssim \mu$ regime. The single current sheet then breaks up into multiple current sheets generating increasingly higher m components. Since substantial computing time is required to follow this high m development, we plan to complete this study in the future. One might suspect, however, that the higher m development may eventually lead to a turbulent region around the reconnection layer, thereby

increasing the effective resistivity. Finally, it is noted that all the thin current sheets found in the present study would be unstable to a wide range of m values if the current sheets were in a static equilibrium state.¹⁵ The stabilizing effects of the plasma flow and viscosity are probably not enough to stabilize $m > 1$ tearing modes in the present case.

C. $m = 2$ tearing modes

In this section, the reconnection rate of the nonlinear $m = 2$ tearing mode is studied. Unlike the $m = 1$ mode studied in the previous section, the $m = 2$ mode is purely resistive in that it is stable for an ideal ($\eta = 0$) plasma. Here we recall from the introduction that for a rapid reconnection to occur, the plasma should develop a current singularity ($J \rightarrow \infty$) when η is set to zero, as seen in the nonlinear development of ideal instabilities studied in Refs. (4) and (5). For the present $m = 2$ case, where the ideal mode is stable, one can see the possibility that the plasma may reach an equilibrium without a current singularity when η is set to zero during the nonlinear phase, which indicates a slow reconnection, $\dot{\psi}_B \sim \eta$, for finite η . The following shows this conjecture to be correct.

Figure 10 shows a nonlinear state of an $m = 2$ tearing mode with $\eta = 10^{-7}$ at the x-point, which is evolved from an initial concentric equilibrium with $q = 1.4 [1 + (r/0.6)^4]^{1/2}$. For the present study, a resistivity profile $\eta = c_1/J_0$, where J_0 is the current profile of the initial concentric equilibrium, is used together with a constant electric field at the wall, $E_w = c_1$. This is to maintain a final steady state, since as shown below the time scale of interest turns out to be comparable to the plasma skin time. The helical flux ψ contours in Fig. 10(a) show $m = 2$ magnetic islands. The B_ϕ profile in Fig. 10(b) does not show a finite jump of B_ϕ across the x-point, unlike the

previous cases in subsections (A) and (B). However, the current density J profile in Fig. 10(c) still shows a small current spike indicating a tiny jump in B_ϕ . For the reconnection scaling, we must determine how the current peak J_s scales with η .

Figure 11 plots J_s versus $\Delta\psi_s$ in a similar fashion as in Figs. 5 and 9, with three η values, 10^{-7} , 10^{-8} , and 10^{-9} . The value of viscosity used here is 10^{-5} , but it was found that the viscosity does not play a major role for the present case. One can readily conclude from Fig. 11 that J_s reaches a finite value as $\eta \rightarrow 0$, unlike the previous cases where $J_s \rightarrow \infty$ as $\eta \rightarrow 0$. The absence of a current singularity in the $\eta \rightarrow 0$ limit requires the reconnection rate to be slow ($\dot{\psi}_s = \eta J_s \sim \eta$). The growth of $m = 2$ magnetic islands occurs also on the slow resistive time scale since $\dot{w}(\psi_s) = w' \dot{\psi}_s$ where w is the width of the island.

This result is consistent with Ref. 16 which found analytically that the magnetic island growth of the tearing mode makes a transition from an exponential growth (the growth rate $\gamma \sim \eta^{3/5}$) to an algebraic growth on the resistive time scale when the island size is still much smaller than the shear length of the magnetic field. This so-called Rutherford behavior has been previously confirmed in a numerical simulation¹⁷ and we include this section for completeness.

V. COMPARISON WITH TOKAMAK EXPERIMENTS

Figure 12(a) depicts a center channel soft X-ray signal measured in the Princeton PDX tokamak discharge, showing a sawtooth oscillation.¹¹ It is generally believed that the steep drop phase (more accurately the drop phase of the lower envelope) of the sawtooth corresponds to an $m = 1$ flip which was studied in Sec. IV(B).¹⁸ Figure 12(b) shows soft X-ray signals including the

center channel signal, B_0 , shown in Fig. 12(a) on an expanded time scale. The lower envelope of B_0 starts to decrease [this corresponds to the beginning of the $m = 1$ flip shown in Fig. 7(b)] at around 400.5 msec and completes its descent [corresponding to the completion of the $m = 1$ flip shown in Fig. 7(c)] in about 1 msec. In this section, we compare the theoretical scaling with the time scale of the $m = 1$ flip.

First, we perform a computer simulation using the experimental q profile, and obtain a plasma state shown in Fig. 13. This simulation follows only the initial part of the $m = 1$ flip process using a value of η larger than the experimental value. The helical flux ψ profile along the horizontal radii in Fig. 13(a) gives the amount of flux ($\Delta\psi = 0.015$) to be reconnected during the $m = 1$ flip; the B_ϕ profile in Fig. 13(b) gives the jump $2B_0$ ($2B_0 = 0.1$) across the x-point; the stream lines in Fig. 13(c) give the reconnection layer length, $\lambda = 0.5$. Using these values and recalling that $\rho = 1$ in our units, the proportionality constant $K = 2B_0^{3/2}/(\rho^{1/4}\lambda^{1/2})$ of the modified Sweet-Parker scaling [Eq. (14)], is obtained as $K = 0.03$. The experimental resistivity is $\eta = 0.5 \times 10^{-7}$ and the ratio $\mu/\eta = 0.6$ can be obtained from $\mu/\eta = (m_i/m_e)^{1/2}\beta$ derived in Sec. II, for a deuterium plasma with $\beta = 0.01$. Then from Eq. (14), we find the reconnection rate, $\dot{\psi}_g = 7 \times 10^{-6}$. The viscosity effect was not important for this case. The theoretical duration of the $m = 1$ flip is then estimated as $T_f = \Delta\psi/\dot{\psi}_g = 2000$. The time unit $T_0 = \tau_A = 0.2 \mu\text{sec}$ gives $T_f = 0.4 \text{ msec}$. This value is roughly comparable to the experimental value of $\sim 1 \text{ msec}$. A similar procedure assuming the Petschek scaling, Eq. (15), gives $T_f^1 = 1 \mu\text{sec}$. This is clearly too short.

While the above study compared the absolute magnitude of the reconnection rate, one could also compare the reconnection scalings using experimental data with different temperatures. For this purpose, data of discharges with large

differences in temperature, and with all other parameters fixed, would be useful. Without such data, we will instead use data of an ST tokamak discharge reported in Ref. 19 to compare with the PDX discharge studied above, since these two cases have similar q profiles. Although the two tokamaks, PDX and ST, are of different dimensions, the similarity character of the reduced tokamak equations [Eq. (17)] can be used to scale the results appropriately. Expressing Eq. (17) in the units defined in the beginning of Sec. IV (which we will call GU) preserves the form of the equations and set $B_0^0/R_0 = 1$. Thus, if the initial ψ values, η , and μ values in GU are the same (boundary conditions: $\psi_w = U_w = 0$) in two discharges, the subsequent plasma development according to Eq. (17) will be also the same as long as all the quantities are measured in GU. The initial ψ values in GU are determined by the initial q profile from $q = r/(\partial\psi/\partial r + (n/m)r)$. Then, the duration T_f of the $m = 1$ flip according to the Sweet-Parker scaling and Eq. (17) is scaled as (for discharges with the same q profile)

$$T_f = T_0 (T_f)_{GU} \sim T_0 (\eta^{-1/2})_{GU} = T_0 \eta_0^{1/2} \eta^{-1/2} \sim T_0 \eta_0^{1/2} \text{Temp}^{3/4}$$

$$= [a^{3/2} \rho_0^{1/4} (\epsilon_B^0)^{-1/2}] \text{Temp}^{3/4} ,$$

where T_0 is the time unit and $\eta_0 = a^2/T_0$ the resistivity unit in GU. (From the parameters given below, the viscous effect can be found to be small enough to be neglected in the above scaling, but large enough to probably prevent the breaking up of the current sheet.) Parameters for the PDX discharge (subscript 1) and the ST discharge (subscript 2) are the plasma minor radius $a_1 = 44$ cm, $a_2 = 10$ cm; the inverse aspect ratio $\epsilon_1 = 0.3$, $\epsilon_2 = 0.1$; fill density $n_1 = 3 \times 10^{13}$ cm⁻³ deuterium atoms, $n_2 = 6 \times 10^{13}$ cm⁻³ hydrogen atoms;

toroidal field $B_{t1}^0 \approx 22$ KG, $B_{t2}^0 \approx 21.5$ KG; $Temp_1 \approx 2$ keV, $Temp_2 \approx 0.5$ keV. The ratio of the durations of the $m = 1$ flip is then found as $T_{f2}/T_{f1} \approx 0.07$. This value is in a rough agreement with the experimental value, $(T_{f2}/T_{f1})_{exp.} \approx 100 \text{ } \mu\text{sec}/1 \text{ msec} \approx 0.1$. The results obtained so far are consistent with that of Sec. IV(B) where the numerical simulation results also agreed with the modified Sweet-Parker scaling.

Although the above results are promising, we have to examine whether the plasma in the experiment actually satisfied the assumptions made in the theoretical modeling. The above experimental values for the PDX discharge (which gives a more stringent test than the ST discharge) give the electron gyro-radius $r_e \approx 0.005$ cm, ion gyro-radius $r_i \approx 0.5$ cm, and collisionless skin depth $d \approx 0.06$ cm. The resistive layer thickness at the x-point can be estimated from $\dot{\psi}_s \approx \eta 2B_0/\delta$ [Eq. (4)] to be $\delta \approx 0.05$ cm. The fact that the collisionless skin depth is comparable to the layer thickness δ would, by itself, only make the effective layer thickness slightly larger than δ , thereby slowing down the reconnection a little.¹ The electron gyro-radius is sufficiently small but the ion gyro-radius r_i is an order of magnitude larger than δ . Thus, the theoretical model should be modified to include the effect of the large ion gyro-radius. Here, we will simply speculate about the main effect of the large r_i on reconnection. Due to the large r_i , the actual jet flow out of the reconnection layer will be much wider than the theoretical model predicts, but the net flow in the jet will not be drastically different, because the effective driving magnetic pressure will be proportionately smaller. Remembering that the total flow in the jet is what enters in the derivation of the reconnection rate in Sec. II and assuming that the effective layer thickness δ of the current channel does not change much (since this is due to the electron flow) one can expect that the reconnection rate itself

would not be affected strongly by the large r_i . Then, this may explain the rough agreement between theory and experiment obtained above.

VI. SUMMARY AND DISCUSSIONS

A. Summary of the results

In Sec. II, we have modified the Sweet-Parker and Petschek scalings of magnetic reconnection rate $\dot{\psi}_S$ to include the effect of the viscosity μ . The modified Sweet-Parker scaling is $\dot{\psi}_S = \kappa \eta^{1/2} (1 + \mu/\eta)^{-1/4}$ with the proportionality constant $\kappa = [2R_0^{3/2}/(\rho^{1/4} k^{1/2})]$. The modified Petschek scaling is $\dot{\psi}_S = \kappa' (1 + (\mu/\eta))^{-1/2}$ with $\kappa' = (R_0^2/\rho^{1/2})$. These scalings are valid for both regimes, $\eta \gg \mu$ and $\eta \ll \mu$.

From the general form of the reconnection scaling with the resistivity, $\dot{\psi}_S = \eta J_S \sim \eta^\alpha$, we defined a slow reconnection as the $\alpha = 1$ case (i.e., on the usual resistive time scale) and a rapid reconnection as $0 \leq \alpha < 1$ cases. All the scalings in the above paragraph then fall into the rapid reconnection category. A rapid reconnection requires a singular current density ($J_S \rightarrow \infty$) at the x-point in the limit of $\eta \rightarrow 0$. If a current singularity does not develop as in the case of the nonlinear $m = 2$ resistive tearing mode studied in Sec. IV(C), the reconnection rate is slow. This is probably related to the fact that the $m = 2$ mode is ideally ($\eta = 0$) stable.

The two cases of rapid reconnection studied numerically, i.e., the external current driven reconnection [Sec. IV(A)] and the $m = 1$ nonlinear internal kink [Sec. IV(B)], are both found to satisfy the modified Sweet-Parker scaling. In the $\eta \ll \mu$ regime, accurate agreement is obtained in both cases. However in the $\eta \gg \mu$ regime, only a rough agreement could be obtained

in the externally driven reconnection case, due to the nonexistence of a quasi-steady state which was assumed in the theoretical derivation. Further, in the $\eta \gg \mu$ regime, the current sheet in the nonlinear $m = 1$ kink is found to be unstable to higher m tearing modes and breaks up into many separate current sheets probably leading to a turbulent region.

In Sec. V, we compared the experimental sawtooth oscillation (a phenomenon due to the nonlinear $m = 1$ resistive internal kink) measurements in the PDX and ST tokamaks with the theoretical scalings. The duration of the $m = 1$ flip process in the experimental sawtooth oscillation roughly agreed with the modified Sweet-Parker scaling. This result is consistent with the numerical result of Sec. IV.B). However, the physical parameters of the experiment show that the theoretical model is not strictly applicable to this case due to the large ion gyro-radius r_i . Simple reasoning suggests that the effect of the large r_i may not be crucial for the reconnection rate, thus accounting for the rough agreement between the theory and experiment.

In the present study, we did not find any case where the Petschek scaling is applicable, although there exist numerical⁷ and experimental⁹ studies of reconnection which reported finding Petschek scaling. However, the argument given at the end of Sec. II shows that this discrepancy is not necessarily a contradiction, since the boundary conditions for the two groups of studies are quite different. For example in the numerical studies of Ref. 7, there is plasma flow across the boundary, whereas all cases studied here are closed systems. Perhaps this question can be answered more definitively by studying whether the open boundary cases satisfy the modified Petschek scaling in the $\eta \ll \mu$ regime where an accurate comparison could be made.

B. Implications for the experiments

The results of the present study give the following picture. During a nonlinear development of highly conducting confined plasmas, current singularities may or may not develop. (The phrase, current singularity here should be interpreted as $J \rightarrow \infty$ as $\eta \rightarrow 0$.) If no current singularity develops, the reconnection will take place slowly ($\dot{\psi}_g \sim \eta$) and the magnetic topology will change in the resistive skin time scale. One example of this is the nonlinear tearing mode. On the other hand, if a current singularity develops, the reconnection process can take place in a faster time scale than the resistive time scale (i.e., a rapid reconnection). According to the present results, the value of the reconnection rate will follow the modified Sweet-Parker scaling, Eq. (14), in a tokamak plasma or similarly confined plasmas. It is also possible that a current sheet may become unstable to higher mode number tearing and break up into separate sheets of current, for example, in the $\eta \gg \mu$ regime of the $m = 1$ internal kink. In this case, a turbulent region around the reconnection layer with anomalous resistivity may eventually develop.

The current singularities found in the present study are all associated with a finite jump of the poloidal magnetic field (i.e., $\int_0 J dr$ across the reconnection layer is integrable and nonzero in the $\eta \rightarrow 0$ limit). In a helically symmetric system, this jump of magnetic field can only occur on the resonant magnetic surface where the gradient of the helical flux ψ vanishes. This constraint is relaxed in a general three-dimensional plasma evolution, and current singularities can occur at unpredictable locations as seen in the three-dimensional simulations of nonlinear ideal ballooning modes in Ref. 5. These singularities then probably cause a rapid reconnection similar to that in the two-dimensional cases, rapidly changing the magnetic topology. In such

a rapid relaxation process, the magnetic helicity will change slowly on the resistive time scale since

$$\frac{d}{dt} \left(\int \vec{A} \cdot \vec{B} d\tau \right) \sim \int \vec{\nabla} \times \vec{B} \cdot \vec{B} d\tau \sim \eta \int \delta J d\tau \sim \eta,$$

where δ is the thickness of a reconnection layer. Thus, magnetic helicity is "conserved" on the time scale of the topology change, as assumed in Ref. 20.

If we apply the rapid reconnection rate scaling found in the present study to plasma formation schemes where the magnetic topology change is necessary, such as a Spheromak or a Doublet, the formation time scale can be estimated from Eq. (14) as $T_f \sim [\eta^{1/2}(1 + \mu/\eta)^{-1/4}]^{-1}$. This is much slower than an often predicted formation time scale of $T_f \sim 1$ from assuming a Petschek scaling. One may also utilize the rapid reconnection mechanism to release the magnetic energy to heat a confined plasma on a rate of $\sim \eta^{1/2}(1 + \mu/\eta)^{-1/4}$.

Finally, the present results support the feasibility of some feedback stabilization schemes. In one example of such schemes, ($m = 7$, $n = 4$) magnetic islands are formed driven by external coils to stabilize the dangerous $m = 2$ tearing mode.²¹ One requirement for doing this is that the stabilizing islands should be formed before the $m = 2$ islands become too large. According to the results in Sec. IV(A) and (C), the driven islands can be formed at a rate $[\sim \eta^{1/2}(1 + \mu/\eta)^{-1/4}]$ much faster than the growth of the $m = 2$ islands $[\sim \eta]$.

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APPENDIX: TWO-DIMENSIONAL INCOMPRESSIBLE MAGNETOHYDRODYNAMIC EQUATIONS

HELICAL INCOMPRESSIBLE MHD EQUATIONS

In helical symmetry, all variables are functions of (r, ϕ) where the helical angle $\phi = \theta + \alpha z$ with helicity α and with the usual cylindrical coordinates (r, θ, z) . We use basis vectors $(\hat{r}, \hat{\phi}, \hat{e})$ where $\hat{r} = \nabla r$, $\hat{\phi} = \nabla \phi / |\nabla \phi|$, and $\hat{e} = \hat{r} \times \hat{\phi}$. Then, the magnetic field and the velocity can be represented as

$$\vec{B} = g \nabla \psi \times \hat{e} + B_e \hat{e} ,$$

$$\vec{V} = g \nabla U \times \hat{e} + v_e \hat{e} ,$$

$$\text{where } g \equiv (1 + \alpha^2 r^2)^{-1/2} .$$

The helical incompressible MHD equations⁴ with a scalar resistivity η and viscosity μ and homogeneous density $\rho_0 = 1$ are

$$\frac{d\psi}{dt} = - \frac{\eta J_e}{g} ,$$

$$\left(\frac{d}{dt}\right)(\Delta^\dagger U) = -\vec{B} \cdot \nabla (g J_e) + \alpha^2 g^2 \left[\frac{\partial (v_e^2 - B_e^2)}{\partial \phi} \right] - 2\alpha \vec{V} \cdot \nabla (g^3 v_e) + \mu \nabla^2 \left(\frac{\Delta^\dagger U}{g} + 2\alpha g v_e \right) , \quad (A1)$$

$$\left(\frac{d}{dt}\right)(g B_e) = \vec{B} \cdot \nabla (g v_e) + 2\alpha g^4 \vec{V} \cdot \nabla \psi + \left\{ \nabla \cdot \left[g^2 \eta \nabla \left(\frac{B_e}{g} \right) \right] + \eta (2\alpha g^3) J_e \right\} ,$$

$$\left(\frac{d}{dt}\right)\left(\frac{v_e}{g}\right) = \vec{B} \cdot \nabla \left(\frac{B_e}{g}\right) + \mu \left[\left(\frac{\Delta^\dagger (v_e/g)}{g^2}\right) - 4\alpha^2 g^3 v_e - 2\alpha \Delta^\dagger U \right] .$$

Here,

$$\nabla f = \left(\frac{\partial f}{\partial r}\right) \nabla r + \left(\frac{\partial f}{\partial \phi}\right) \nabla \phi = \left(\frac{\partial f}{\partial r}\right) \hat{r} + \left(\frac{1}{rg}\right) \left(\frac{\partial f}{\partial \phi}\right) \hat{\phi} ,$$

$$\Delta^* f \equiv \nabla \cdot (g^2 \nabla f) = \left(\frac{1}{r}\right) \left[\left(\frac{\partial}{\partial r}\right) \left\{ r g^2 \frac{\partial f}{\partial r} \right\} \right] + \left(\frac{1}{r^2}\right) \left(\frac{\partial^2 f}{\partial \phi^2}\right) ,$$

$$J_e = -(\Delta^* \psi / g + 2\alpha g^2 B_e), \quad \vec{V} \cdot \nabla f = (\nabla f \times g \nabla U) \cdot \hat{e}, \quad \vec{B} \cdot \nabla f = (\nabla f \times g \nabla \psi) \cdot \hat{e} \text{ and}$$

$$d/dt = \partial/\partial t + \vec{V} \cdot \nabla ,$$

INCOMPRESSIBLE MHD EQUATIONS WITH TRANSLATIONAL SYMMETRY

Since helical symmetry is the most general two-dimensional symmetry, other two-dimensionally symmetric cases can be easily obtained by taking limits $\alpha \rightarrow 0$ or $\alpha \rightarrow \infty$. When $\alpha \rightarrow 0$ in Eq. (A1), $g \rightarrow 1$, $\hat{\phi} \rightarrow \hat{\theta}$, and $\hat{e} \rightarrow \hat{z}$, and Eq. (A1) reduces to the following incompressible MHD equations with translational symmetry:

$$\left(\frac{d}{dt}\right) \psi = \eta \nabla^2 \psi ,$$

$$\left(\frac{d}{dt}\right) \nabla^2 U = \vec{B} \cdot \vec{\nabla} \nabla^2 \psi + \mu \nabla^2 \nabla^2 U , \quad (A2)$$

$$\left(\frac{d}{dt}\right) B_z = \vec{B} \cdot \vec{\nabla} V_z + \nabla \cdot (\eta \vec{\nabla} B_z) ,$$

$$\left(\frac{d}{dt}\right) V_z = \vec{B} \cdot \vec{\nabla} B_z + \mu \nabla^2 V_z ,$$

$$\text{with } \vec{B} = \nabla \psi \times \hat{z} + B_z \hat{z} \text{ and } \vec{V} = \nabla U \times \hat{z} + V_z \hat{z} .$$

One can readily see that the equations for ψ and $\nabla^2 U$ are decoupled from

the rest of the equations. These decoupled equations were used as Eq. (A2') in Sec. III.

INCOMPRESSIBLE MHD EQUATIONS WITH TOROIDAL SYMMETRY

When $\alpha \rightarrow \infty$ in Eq. A1, $g \rightarrow (\alpha r)^{-1}$, $\hat{\phi} \rightarrow \alpha z$, $\hat{\phi} \rightarrow \hat{z}$, and $\hat{\theta} \rightarrow -\hat{\theta}$. By denoting ψ/α as ψ and U/α as U , we obtain the toroidally symmetric incompressible MHD equations:

$$\begin{aligned} \left(\frac{d}{dt}\right)\psi &= \eta(r^2 \tilde{\Delta}\psi) \quad , \\ \frac{d\tilde{\Delta}U}{dt} &= \vec{B} \cdot \nabla(\tilde{\Delta}\psi) + \frac{1}{r^2} \frac{\partial}{\partial z} (v_\theta^2 - B_\theta^2) + \mu \tilde{\Delta}(r^2 \tilde{\Delta}U) \quad , \\ \frac{d}{dt}\left(\frac{B_\theta}{r}\right) &= \vec{B} \cdot \nabla\left(\frac{v_\theta}{r}\right) + \nabla \cdot \left[\frac{\eta}{r^2} \nabla(r v_\theta)\right] \quad , \\ \frac{d}{dt}(r v_\theta) &= \vec{B} \cdot \nabla(r B_\theta) + \mu[r^2 \tilde{\Delta}(r v_\theta)] \quad , \end{aligned} \tag{A3}$$

where

$$\begin{aligned} \nabla f &= \frac{\partial f}{\partial r} \hat{r} + \frac{\partial f}{\partial z} \hat{z} \quad , \\ \tilde{\Delta} f &= \frac{1}{r^2} \Delta^* f = \nabla \cdot \left(\frac{1}{r^2} \vec{\nabla} f\right) \quad , \\ \vec{B} &= -\frac{1}{r} \nabla \psi \times \hat{\theta} + B_\theta \hat{\theta} \quad , \\ \vec{v} &= -\frac{1}{r} \nabla U \times \hat{\theta} + v_\theta \hat{\theta} \quad , \\ \vec{B} \cdot \nabla f &= -\nabla f \times \frac{1}{r} \nabla \psi \cdot \hat{\theta} \quad , \end{aligned}$$

and

$$\vec{v} \cdot \nabla f = -\nabla f \times \frac{1}{r} \nabla U \cdot \hat{\theta} .$$

TWO-DIMENSIONAL REDUCED TOKAMAK EQUATIONS

The reduced tokamak equations, Eq. (17), can be also readily derived from Eq. (A1). Here, a tokamak is approximated as a straight cylinder (minor radius a) with periodic boundary conditions imposed at each end. Consistent orderings, $\nabla \psi / B_e \sim v_e / \nabla U \sim (\alpha_a)$ and $(a \nabla B_e) / B_e \sim (\alpha_a)^2$, are applied and the lowest order terms in (α_a) are taken. The equations for ψ and $\nabla^2 U$ [Eq. (17)] are decoupled from the rest of the equations as in the translationally symmetric case, Eq. (A2). An analogy between Eqs. (17) and (A2') is made in Sec. III(A).

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FIGURE CAPTIONS

- FIG. 1. Schematic diagrams showing a reconnection process. (a) Oppositely directed magnetic fields are compressed to produce a sharp gradient across the x-point. (b) The shaded region represents the reconnection layer of a size $\delta \times l$.
- FIG. 2. Profiles during reconnection driven by external coils. (a) Helical flux ψ contours show $m = 4$ magnetic islands driven by eight external coils. (b) The B_ϕ profile along the horizontal radii shows a finite jump ΔB across the x-point.
- FIG. 3. Current density J profiles showing current sheets. (a) Current density contour plots. (b) Enlarged plots of (a) in the reconnection layer region. The r -scale is enlarged 5 times more than the ϕ -scale distorting the shape of the current sheet. (c) Current density profile along the horizontal radii. (d) Enlarged picture of the current spike in (c).
- FIG. 4. Velocity stream lines around the reconnection layer. The r -scale is enlarged 5 times more than the ϕ -scale.
- FIG. 5. The current density at the x-point, J_g , is plotted versus $\Delta\psi_g$, the total change of the flux at the x-point, in $\eta \ll \mu$ regime. $\Delta\psi_g$ defines the plasma state while J_g determines the reconnection rate ($\dot{\psi}_g = \eta J_g$). The solid lines are numerical results and the dashed ones correspond to the modified Sweet-Parker scaling.

FIG. 6. J_s versus time in $\eta \gg \mu$ regime. No steady state is found in the simulation, hence the solid lines wiggle. The dashed lines correspond to the Sweet-Parker scaling.

FIG. 7. Helical flux ψ contours show the time history of the nonlinear development of an $m = 1$ internal kink unstable plasma. This process of complete reconnection of the flux inside the $q = 1$ surface is sometimes called the $m = 1$ flip.

FIG. 8. Profiles of the plasma state during the reconnection phase shown in Fig. 7(b). (a) Current density J contours. (b) B_ϕ profile along the horizontal radii. (c) Velocity stream lines. (d) An enlarged picture of the reconnection layer region of (c).

FIG. 9. The current density, J_s , at the x-point versus $\Delta\psi_s$, in $\eta \ll \mu$ regime. As in Fig. 5, the solid lines are from the calculation and the dashed ones from the modified Sweet-Parker scaling.

FIG. 10. Profiles during the nonlinear development of an $m = 2$ tearing mode unstable plasma. (a) Helical flux ψ contours show $m = 2$ magnetic islands. (b) No jump of B_ϕ across the x-point is detectable in the B_ϕ profile along the horizontal radii. (c) Current density J profile along the horizontal radii. (d) Velocity stream lines.

FIG. 11. The current density, J_s , at the x-point versus $\Delta\psi_s$. These numerical results show that J_s reaches a finite value as $\eta \rightarrow 0$.

FIG. 12. Experimental soft X-ray measurement in a PDX tokamak discharge.

(a) Center channel signal showing a sawtooth oscillation. The drop phase of the lower envelope of the signal corresponds to the reconnection phase of the $m = 1$ internal kink shown in Fig. 7(b).

(b) One such drop phase is shown enlarged. Multichannel data are shown including the center channel signal BO.

FIG. 13. Profiles of a plasma state in a simulation corresponding to the drop

phase of Fig. 12. (a) ψ profile along the horizontal radii giving the total amount of flux to be reconnected. (b) B_ϕ profile giving the jump $2B_\phi$ across the x-point. (c) Stream lines giving the reconnection layer length l .

#83T0053

(a)

(b)

Fig. 1

#6310077

(a)

(b)

Fig. 2

*83T0061

Fig. 3

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Fig. 4

*BIT0285

Fig. 5

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Fig. 6

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Fig. 7

Fig. 8

*81T0286

Fig. 9

Fig. 10

#81T0284

Fig. 11

#83T0056

Fig. 12

83T0054

Fig. 13

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