FINAL REPORT

**Impact of Conservation
Measures on Pacific Northwest
Residential Energy
Consumption**

R. J. Moe
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April 1983

**Prepared for the Bonneville Power Administration
under a Related Services Agreement with the
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**Pacific Northwest Laboratory
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SUMMARY

Pacific Northwest Laboratory has been funded by the Division of Power Requirements in the Bonneville Power Administration to analyze the relationship between residential energy conservation and residential electricity consumption in the Pacific Northwest. The focus of this study was the thermal shell efficiency of residential buildings. Thermal efficiency is defined as the ability of a shell to prevent heat generated within a residence from escaping to the area outside the living space. The study had two primary objectives. First, the relationship between the electricity required to maintain a constant internal temperature under controlled conditions and the home's physical characteristics was estimated. These characteristics included climate (heating degree days); square footage; window space and storm window space; number of doors and number of storm doors; the type of foundation; the amount of wall, ceiling, and floor insulation; and the amounts of weatherstripping, caulking, and plastic covering on windows and doors. Resident use conditions, such as thermostat settings and occupant schedules, were controlled in order to isolate the physical relationships. The second objective was to estimate the relationship between actual electricity consumption by individual residences and building thermal efficiency. To fulfill this objective, winter space heating electricity demand functions were estimated. Thermal shell efficiency was included as an explanatory variable. These demand functions were used to estimate the impact of a ten percent increase in building shell efficiency on electricity consumption and on consumer welfare.

Completion of the first objective was performed in two steps. First, data from the Elrick and Lavidge, Inc., Pacific Northwest Residential Energy Survey was obtained. Elrick and Lavidge randomly selected over 4000 individual residences in the Pacific Northwest and surveyed each in great detail. The survey contains information on building characteristics and climate, as well as actual energy use, energy prices, the presence of particular appliances, and demographic/economic information pertaining to each household. For each of three building categories (mobile home, single-family, and multi-family), a subsample of observations was randomly selected. Seventy-five mobile homes, 150 single-

family homes, and 75 multi-family residential units were selected so that the geographical distributions (according to state and climate zone) of the three samples were identical to those of the entire survey. Each of the 300 buildings in the sample was then analyzed in detail. The characteristics of each were extracted, and the DOE II.1 thermal analysis computer program was used to estimate the energy consumption for heating for each building under controlled conditions. The purpose of the DOE II.1 simulations was to describe the building shell or physical energy requirements explicitly void of the occupant requirements. Thus, the thermostat settings, use patterns, and energy consumption by non-heating appliances were held at the same value for all residences. The only variables allowed to differ from one DOE II.1 simulation to the next were climate and building characteristic measures. Hence, the calculated heating energy use varies only because of differences in climate and building characteristics of the residences.

In the second step, the relationship between calculated DOE II.1 heating energy use and the climate and building characteristic variables was statistically estimated. Standard statistical techniques were used to estimate separate equations for each of the three building categories. The most important determinants of calculated heating energy use in each of the three building categories were found to be square feet of heated living space, outside temperature (heating degree days), and square feet of glass area. A number of conservation/policy measures were also found to have an important effect on calculated energy use. Among these were square feet of storm windows; the amount of wall and ceiling insulation; and the amounts of caulking, plastic covering, and weatherstripping on or around windows and doors. These estimated equations can be used in place of DOE II.1 to calculate heating energy use under the same controlled conditions for any residence, given information on the physical characteristics and climate of the home. In addition, the equations can be used to assess the impact of changes in particular building characteristics on building thermal efficiency.

Such a replacement of DOE II.1 by the estimated regression equations could be of great value. Engineering heat-loss models such as DOE II.1 are frequently used by policymakers to assess the effectiveness of alternative

conservation measures. Forecasting energy use under various regulatory/conservation scenarios also entails use of such programs. However, DOE II.1 is a sophisticated program which simulates hourly energy use for an entire year, and takes into account thousands of building characteristics. The program is thus quite expensive to use in a forecasting/policy assessment framework, and much of the detail which it provides is not of much value in this situation. The estimated DOE II.1 summary equations are inexpensive to use in policy situations, yet provide information required by policymakers and consider those building characteristics under the influence of such people.

In fulfilling the second objective, two steps were performed. First, a theoretical model of the relationship between energy consumption for space conditioning and a residence's thermal shell efficiency was developed. The model suggested that increases in thermal efficiency would create incentives for consumers to increase the internal comfort level of their home. Increases in thermal efficiency are equivalent to reductions in the price of internal comfort. Incentives for consumers to increase their consumption of internal comfort are therefore created. Thus, for example, a ten percent increase in the thermal shell efficiency of a residence would have two impacts: 1) holding internal comfort level constant, energy use for space conditioning would decrease ten percent, 2) internal comfort level and thus energy consumption might increase. The net impact on energy consumption of the efficiency improvement is the difference between the two impacts, which push energy consumption in opposite directions.

In the second step, the theoretical model was empirically implemented. The Elrick and Lavidge survey used in the DOE II.1 analysis was again used. All electrically-heated homes in the original sample were analyzed. Data on winter electricity consumption (in KWH) and expenditure, the household's income, size, and age of head, the location of the residence, its appliance stock, and its physical characteristics were extracted from the survey for each electricly-heated household. For each building type, a separate regression equation was estimated. Winter electricity consumption was related to thermal shell efficiency, average and marginal electricity prices, income, number of residents, age of household head, heating degree days, and an appliance

index. The thermal efficiency of each residence was calculated using the thermal equations estimated in the first task. Auxillary price and thermal efficiency equations were included in the estimation to improve the properties of the estimated demand coefficients.

The hypothesis that thermal efficiency improvements create incentives to increase consumption of internal comfort was supported statistically. Internal comfort levels were increased as a result of increases in thermal efficiency; the percentage reduction in space conditioning electricity use was found to be less than the percentage increase in thermal efficiency. For mobile homes, a ten percent thermal improvement led to a 2.19 percent reduction in electricity used for space heating; for single family, a 5.46 percent reduction resulted. The reduction in multi-family electricity consumption for winter space heating was 6.78 percent for a ten percent increase in thermal efficiency. Because of data limitations, the mobile home estimate probably understates the true quantity reduction for a ten percent thermal improvement.

Finally, the benefits to consumers of a ten percent increase in thermal efficiency were calculated. These benefits are the sum of two components: 1) the "windfall" gain associated with paying a lower total amount for the same quantity of internal comfort consumed; 2) the gain associated with the increased consumption of internal comfort. For mobile homes, the mean household benefit per winter due to a ten percent thermal improvement was \$23.81; for single-family, \$21.23; for multi-family, \$12.94. Thus, even small changes in the thermal efficiency of a residence were found to generate measurable benefits to its occupants.

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1.0 INTRODUCTION

This research was performed by Pacific Northwest Laboratory for the Division of Power Requirements in the Bonneville Power Administration. The objective of this study was to estimate the relationship between residential space conditioning energy use and building conservation programs in the Pacific Northwest. The study was divided into two primary tasks. In the first, the thermal relationship between space conditioning energy consumption under controlled conditions and the physical characteristics of the residence was estimated. In this task, behavioral characteristics such as occupant schedules and thermostat settings were controlled in order to isolate the physical relationships. In the second task, work from the first task was used to calculate the thermal efficiency of a residence's shell. Thermal efficiency was defined as the ability of a shell to prevent escapement of heat generated within a building. The relationship between actual space conditioning energy consumption and the shell thermal efficiency was then estimated.

This study was motivated by two policy considerations. First, a large number of residential energy conservation programs have been instituted in recent years. The Bonneville Power Administration has funded a number of such programs. The effect of these programs has not been studied in a detailed manner that considers the economic incentives which the programs created. Second, the draft Regional Power Planning Council's Conservation and Electric Power Plan calls for conservation to play a lead role in the Pacific Northwest's energy future. Thus, programs which provide free or subsidized installation of insulation, weatherstripping, caulking, storm doors, and storm windows to residents can be expected to continue and possibly expand. Effective use of the region's resources requires cost-benefit analysis of each of these programs; however, analytical evaluation of the benefits of such programs which considers the economic incentives which they create has not been seriously undertaken. For example, these programs may create incentives to increase consumption of internal comfort and thus may increase energy consumption by reducing the price of internal comfort. Few estimates of the size of such an energy consumption increase exist in the literature. Information

on the increases in consumer welfare induced by such price reductions is not readily available either. This study was designed to fill these gaps and provide information on the effectiveness and benefits of such programs.

The remainder of this report is divided into three sections. Conclusions and Recommendations are presented in Section 2.0. The first task is discussed in Section 3.0. Separate thermal equations for mobile homes, single-family residences, and multi-family residences are presented. The second task is discussed in Section 4.0. Estimates of the relationship between winter electricity consumption for heating and the building's thermal shell efficiency are presented for each of the three building categories.

2.0 CONCLUSIONS AND RECOMMENDATIONS

In brief, the study yielded the following conclusions:

- The relationship between controlled condition-energy use and a residence's physical characteristics can be efficiently summarized using a single regression equation. "Controlled condition" means behavioral characteristics such as occupant schedules and thermostat settings are held fixed. Approximately 75 percent of the variation in energy use simulated by the thermal analysis program DOE II.1 can be explained using a single equation.
- Increases in a shell's thermal efficiency create an incentive to improve the interior comfort level. Increases in shell thermal efficiency are changes in the building shell which improve the ability of the shell to prevent escapement of heat generated within the building to outside areas.
- Such induced increases in interior comfort levels reduce the fall in energy consumption caused by the efficiency improvement.
- A ten percent increase in the shell thermal efficiency will cause the following reductions in electricity use for winter space heating: mobile homes, 2.19 percent; single-family, 5.46 percent; multi-family, 6.78 percent. Estimates of such impacts vary among households according to the price for electricity paid by the household. Each of these estimates is the reduction calculated at the mean sample prices. Because of data limitations, the mobile home estimate probably underestimates the true reduction.
- Two types of benefits to consumers are produced by increases in thermal efficiency. First, the consumer expends less for a fixed level of internal comfort. Second, the lower effective price of comfort induces him to increase his consumption of comfort.
- Estimates of benefits to a single household of a ten percent increase in the thermal shell efficiency are: mobile home, \$23.81 per winter; single family, \$21.23 per winter; multi-family, \$12.94 per winter.

Based on these conclusions, the following recommendations are made:

- The estimates presented in this report of the effect of particular conservation measures on thermal shell efficiency should be used to analyze the relative cost-effectiveness of alternative conservation programs.
- The estimates of reduced electricity consumption and increased consumer welfare from an increase in thermal efficiency should be used to assess the cost-effectiveness of conservation programs in general.
- The space heating electricity demand elasticities presented in this report should be used to update older estimates of these elasticities now used in residential end-use models.
- More detailed analysis of the thermal relationships in the three building categories is needed to make the equations estimated in this study applicable to a larger number of conservation measures. A more in-depth survey, particularly related to mobile homes and multi-family residences, would be required for such an analysis.
- Utility surveys of electricity consumption by individual households before and after conservation actions should be analyzed to validate the findings of this report.

3.0 DEVELOPMENT OF RESIDENTIAL THERMAL EQUATIONS

The first task of this research involved estimation of the thermal relationship between residential heat load and the physical characteristics of the residence. Residential heat load was defined as the quantity of electricity required to heat a home under controlled use conditions during one calendar year. Use conditions, such as thermostat settings and occupant schedules, were controlled in order to determine the building shell or physical energy requirements explicitly void of the occupant requirements, which are considered in Section 4.0. The physical characteristics considered included climate, as measured by heating degree days, as well as such building characteristics as the presence and quantity of wall and ceiling insulation, the presence and quantity of caulking, weatherstripping, and plastic covering on or around windows and doors, the number and size of windows, storm windows, doors, and storm doors, the type of foundation, and the square footage of the heated living space.

Because residential heat load as defined above is not an observable variable, the DOE II.1 thermal analysis program was used to create the variable. DOE II.1 simulates hourly energy consumption for heating for a residence, given a set of controlled conditions. DOE II.1 was simulated for 75 mobile homes, 150 single-family residences, and 75 multi-family residences. From one simulation to the next, only the residence's physical characteristics were allowed to vary. Thermostat settings, occupant schedules, and equipment schedules were held constant for all simulations. Thus, the DOE II.1 simulated electricity consumption for space heating varied from one residence to the next only because of differences in the physical characteristics of the residences.

Output from the DOE II.1 simulations was then regressed on the physical characteristics of the residence. Separate regression equations were estimated for mobile homes, single-family residences, and multi-family residences. Each of the three equations summarizes the information contained in the DOE II.1 program. These equations can be used in place of the DOE II.1 to calculate heating electricity use for a residence under controlled conditions,

given information on the physical characteristics of the home. By evaluating these equations for different values of the independent (right-hand side) variables, the impact of changes in particular building characteristics on electricity use for heating under controlled conditions can be assessed.

The remainder of this section is divided into two main parts. The first part discusses the DOE II.1 thermal analysis program, the data used in the DOE II.1 simulations, the assumptions used in the simulations, and the simulation procedures. The second part describes the estimation of the DOE II.1 summary equations.

3.1 DOE II.1 ANALYSIS

DOE II.1 is a computer simulation program capable of detailed analysis of energy consumption in buildings. Development of DOE II.1 (and its predecessor, DOE II) was initially sponsored by the U.S. Energy Research and Development Administration and the State of California Energy Resources Conservation and Development Commission. Upon its formation in 1977, the U.S. Department of Energy took over funding. Development of the program was centered at Lawrence Berkeley Laboratory (LBL). Research began in late 1975; the current version was issued in February 1979 and is available for use on the LBL computer.

DOE II.1 was designed primarily for architects and engineers who need to compute building energy consumption in designing fuel-efficient buildings and heating/cooling systems. Building energy consumption in DOE II.1 is a function of a number of variables and parameters, which can be varied by users in order to find the most efficient building and/or system design.

3.1.1 Description of Program

DOE II.1 consists of four major steps: LOADS, SYSTEMS, PLANT, and ECONOMICS. In this study, only the first two steps were used. LOADS calculates hourly space loads imposed by ambient weather conditions; internal occupancy patterns; lighting; equipment; and the size, location, orientation, construction, materials, and shape of the building. LOADS assumes a fixed internal temperature which must be attained each hour, and simulates the

hourly energy required to maintain this temperature. The fixed temperature may vary by time-of-day and day-of-week, but is fixed for a given hour.

A major part of this procedure is calculation of the hourly heat losses and gains through walls, roofs, floors, windows, and doors. Such losses and gains depend, of course, on climatic conditions such as temperature, sun angle, cloud cover, and wind. They are also influenced by such building characteristics as the type, thickness, and placement of windows, doors, walls, and insulation; the type of foundation; the amount and placement of weatherstripping and caulking around windows and doors; and the schedules for occupant use of the home, lighting, and equipment. LOADS accounts for these and other factors in simulating the hourly loads. A library of weather tapes on over 100 U.S. cities is available to provide the required hourly weather information.

The output from LOADS is the hourly Btus required to maintain the desired internal temperature. The SYSTEMS step simulates the operation of the Heating, Ventilation and Air Conditioning (HVAC) system to satisfy these loads demands. SYSTEMS assumes a specific heating/cooling system has been placed in the residence. Output from SYSTEMS depends on the type of system in place. If the system is electric, the SYSTEMS output includes the number of kilowatt hours required to maintain the internal temperature at desired levels. Separate measures of kWh used for heating and for cooling are presented. Energy use is available on an hourly, daily, weekly, monthly, or annual basis.⁽¹⁾

3.1.2 Data Used in DOE II.1 Analysis

In order to generate the regression equations which summarize DOE II.1, a number of DOE II.1 simulations had to be made. For each run, the building characteristics/climate variables had to be varied so that variations in the calculated energy consumption could be attributed to variations in specific

(1) For further information on the DOE II.1 thermal analysis program, see DOE-2 Reference Manual, Version 2.1, Parts 1 and 2, Group WX-4, Program Support, Los Alamos Scientific Laboratory, Los Alamos, New Mexico, May 1980; Lawrence Berkeley Laboratory, Los Alamos Scientific Laboratory. Prepared for U.S. Department of Energy under Contract W-7405-ENG-8.

independent variables. Actual observations on 300 homes in the Pacific Northwest were used in the simulations. Using a sample of actual residences (1) provided sufficient variation in the building/climate characteristics of interest to generate regression equations, and (2) ensured the applicability of the estimated equations to a large number of homes in the Pacific Northwest.

The data used in the study was taken from the 1982 updated version of the Pacific Northwest Residential Energy Survey. This comprehensive housing survey was conducted in 1979-1980 by Elrick and Lavidge, Inc., for the Bonneville Power Administration and the Pacific Northwest Utilities Conference Committee. The survey includes detailed descriptions of 4030 homes and their residents in Washington, Oregon, Idaho, and Montana. The survey describes the demographic, economic, and family characteristics of the residents, and the dwelling structure and appliance characteristics of the residences. Data on electricity, natural gas, and fuel oil consumption and prices was appended to the survey data, as was information on climate.⁽¹⁾

The 4030 original observations were split into three categories: mobile homes, single-family residences, and multi-family residences. Single family included all single-family detached homes, duplexes, and two- to four-unit apartments; multi-family included only apartments with five or more units. From each category, a subsample was selected. The selection process involved four steps. First, the proportion of the total number of original observations in each state-climate zone was calculated for each building type. Second, observations with missing values for square footage, number of windows, and the other variables of interest were dropped from the sample. Third, the remaining observations in each building category were sorted by state, and within states by climate zone. Finally, 75 mobile home observations were randomly selected from the mobile home observations that remained after the second step. The proportions of observations selected from each

(1) For further information on this data base, see The Pacific Northwest Residential Survey, Volume 1, Executive Summary, Elrick and Lavidge, Inc., August 1980; prepared for the Bonneville Power Administration and the Pacific Northwest Utilities Conference Committee.

state-climate zone were equal to the proportions in the original sample. One hundred fifty single-family residences and 75 multi-family residences were selected from their respective samples in a similar fashion. Thus, the geographic distribution of the residences used in the analysis replicates the geographic distribution of the original 4030-residence sample.

Ideally, the variables used in the analysis would have been the same for all three building categories, and would have included the following: (a) square footage; (b) numbers of small, medium, and large windows; (c) numbers of small, medium, and large storm windows; (d) number of outside doors; (e) number of storm doors; (f) amount of plastic covering, caulking, and weatherstripping on windows and doors; (g) number of floors (stories) in the home; (h) number of residential units in the building; (i) number of attached walls (to other residences); (j) presence of a unit directly above the residence; (k) presence of a unit directly below the residence; (l) type of foundation; (m) R-values for each outside wall; (n) R-values for ceiling; and (o) R-values for ground (floor). However, for each building category, some of these variables were missing for all but a handful of observations; thus, the set of variables actually allowed to vary in the DOE II.1 simulations differs from one building category to the next.

Mobile Home Variables

In the analysis of mobile home energy consumption, the following building characteristics were allowed to vary: (a) square footage; (b) numbers of small, medium, and large windows; (c) numbers of small, medium, and large storm windows; (d) number of doors; (e) number of storm doors; (f) presence and amount of plastic covering on windows and doors; (g) presence and amount of weatherstripping on windows and doors; and (h) presence and amount of caulking on windows and doors. Information on R-values was not presented in the survey, and values of other variables could not be used to estimate the R-values.

Single-Family Residence Variables

For single-family residences, the building characteristics allowed to vary in the DOE II.1 simulations included all of those that were varied in the mobile home simulations, plus the following: (i) type of foundation; (j) R-values for ceiling; (k) R-values for walls; and (l) number of attached walls. Ceiling R-values appear as a variable in the sample; wall R-values had to be estimated from information in the sample regarding the presence and amount of insulation in the first-floor walls. If a resident said that all walls were insulated, each wall was given an R11 value. If some walls were insulated, then two walls were given R11 values and two were given R0 values. Finally, if the resident said no walls were insulated, all walls were given R0 values. Residences that were part of a duplex or a two-four unit apartment had one attached wall.

Multi-Family Residence Variables

In the multi-family simulations, the building characteristics allowed to vary included all of those varied in the mobile home simulations, plus the following: (i) number of attached walls; (j) presence of unit above the residence; (k) presence of a unit below the residence; and (l) if no unit below, the type of foundation. The position of the residential unit in the larger apartment building was not given in the sample, but "calculated" on the basis of other variables. If the building had one story, the unit had both a ground floor and a roof. If the building had two or three floors, the unit was placed on the ground floor, with a unit directly above it. If the building had four or more floors, the unit was placed on a middle floor, with residences both directly above and below. Regarding the number of attached walls, the following conventions were used: if the building had fewer than three units per floor, the residence had one attached wall; for three to seven units per floor, two attached walls; and for eight or more units per floor, three attached walls.

For all three building categories, only partial information was available in the sample on the amounts of caulking, weatherstripping, and plastic covering. If the respondent said "all" windows/doors had the relevant

characteristics, 100 percent of the windows/doors were endowed with the measure; if he said "some," 50 percent were so endowed; and if he said "none," 0 percent were covered. For single-family and multi-family units, the foundation variable took on three values: (1) concrete slab, (2) basement, and (3) crawl space.

Finally, climatic variables were allowed to vary for all three building categories. DOE II.1 uses hourly weather tapes to feed the simulation of heat losses and gains. Tapes are available for more than 100 U.S. cities. A particular weather tape was assigned to each observation using the following procedures. First, tape cities with latitudes above 42° were selected. For each of the cities in this "pool" of available tapes, the heating degree-days (HDD) and cooling degree-days (CDD) were calculated. Second, for each of the selected 300 observations, the weather tape city with the HDD value closest to the 30-year average HDD from the sample was assigned to that observation. If two cities were equally representative of the actual 30-year average HDD, the one with the CDD value closest to the 30-year average CDD from the sample was assigned to that residence.

3.1.3 Assumptions Used in the DOE II.1 Analysis

DOE II.1 requires the user to assign values to thousands of variables. For example, the program considers the shape of the building (square, rectangular, etc.), the geometry of overhangs, the orientation of the building (windows face north, south, etc.), and the use patterns of the building (residents home from 6 p.m. to 8 a.m. weekdays, etc.). The Elrick and Lavidge data base did not contain information on many of these variables, such as shape of the building. Information on other variables, such as use patterns, was available in the data base but not pertinent to this analysis. Only variables describing the physical characteristics of the residence and its surroundings were allowed to vary; variables describing the economic/demographic condition of the residents or their behavior were held constant for all observations. By holding these variables constant, the effects of a home's physical characteristics on the physical energy requirements of the home could be isolated from

the effects of the residents' behavioral characteristics, which are considered in Section 4.0. Thus, a large number of variables was assigned values which did not vary from one simulation to the next.

For each building category, a prototype residence was "constructed." This prototype was the basis for all simulations in a building category. The three prototypes shared a number of common features, which are presented in the following section. Category-specific prototype characteristics are discussed in the remaining three sections.

General Prototype Description

All three prototypes shared the following characteristics. First, the heated living space of each prototype was square, with total area specified in the data set. The living space had no interior walls. For one-story residences (including one-story units in a multi-story apartment), the interior height of the building was 8 feet. For multi-story residences, the interior height was set at 10 feet per story to account for the ceiling/floor. The residence faced north, and there were no shading surfaces (overhangs, trees, etc.).

Total glass area was defined by

$$\begin{aligned} \text{GAREA} = & 36 * \text{number of large windows} \\ & + 24.8 * \text{number of medium windows} \\ & + 8.7 * \text{number of small windows} \end{aligned}$$

where "*" denotes multiplication. The coefficients 36, 24.8, 8.7 are the mean square footage for each window category in the Elrick and Lavidge sample. The number of windows of each type was taken from the data set. Total glass area was divided by the number of unattached walls to obtain WGAREA. On each unattached wall, one square window of square footage WGAREA was placed. Storm glass area was treated in an analogous manner, with

$$\begin{aligned} \text{SGAREA} = & 36 * \text{number of large windows with storms} \\ & + 24.8 * \text{number of medium windows with storms} \\ & + 8.7 * \text{number of small windows with storms.} \end{aligned}$$

SGAREA was divided by the number of unattached walls to obtain WSGAREA, which was again distributed evenly over the unattached walls. If SGAREA was less than GAREA, each of the windows in the prototype was only partially covered with a storm. In effect, then, two windows were created on each wall. One of these two windows had a storm while the other did not. Windows with storms were assumed to be double-glazed, while those without storms were single-glazed. The presence of plastic covering was treated as an additional glazing.

The total door area (to unheated areas) was assumed to be 16 square feet, multiplied by the number of doors. Total door area (DAREA) was divided by two to obtain WDAREA. One door with square footage WDAREA was placed on each of two walls. In the case of residences with just one unattached wall, one door of area DAREA was placed on that wall. Storm doors were treated analogously.

The heating system was assumed to be forced air electric.⁽¹⁾ The fixed internal temperature which the LOADS step of DOE II.1 satisfies was allowed to vary according to season and hour. During the heating season, the thermostat was set at 70° during the day, with night setback to 55°. During the cooling season, the daytime setting was 78°, with no night setback.

Schedules for occupants were the same for all prototypes. The day was divided into 10 periods, with different levels of activity and different numbers of residents at home during each period. The same day schedule was used for each of the days in the simulation year.

Mobile Home Prototype

In addition to the general prototype features, the mobile home prototype had the following characteristics: the home had metal exterior siding with an

(1) Similar results would have been obtained had a different heating system been assumed. Thus, although the results presented in this section depend on the assumption, they are fairly robust. This is because the largest portion of heating gains/losses are calculated in the LOADS step. At this point, the type of heating system has not even been considered, and calculations are in terms of BTU's. The systems step, and the specification of a heating system required for its use, was performed to more completely analyze the energy requirements of a residence.

R-value of 11. The foundation was crawl space, and the floor/foundation had an R-value of 7. The ceiling/roof was assumed to have an R-value of 19. The roof was horizontal.

Single-Family Residence Prototype

The single-family residence prototype had the following characteristics in addition to the general features. The walls consisted of 5/8 inch plywood and 1/2 inch drywall. The assignment of wall R-values was discussed in Section 3.1.2. All homes had zero or one attached wall. If the residence was single-family detached, there were four exterior walls and zero attached walls. If the residence was part of a duplex or a two- to four-unit building, it was assumed that there was one attached wall and three exterior walls. Attached walls were treated in the following way: a physical wall was removed, so that the unit under analysis was actually two residences. However, an imaginary "wall" replaced the physical wall to create two "zones." Energy consumption in just the zone of interest was then calculated. This is the same procedure typically used to analyze consumption in individual rooms of a home or individual offices within a commercial building.

The ceiling/roof consisted of an attic, 1/2 inch drywall, and insulation R-values as given by the variable RCEIL in the data base. The foundation was one of three types, as specified in the data set: crawl space, basement, or concrete slab. Crawl space foundations were assumed to have R7 insulation over the crawl space. Basement and slab foundations conformed to standards set by the American Society of Heating, Refrigerating, and Air Conditioning Engineers (ASHRAE).

Multi-Family Residence Prototype

The primary difference between the multi-family residence prototype and the general prototype was the treatment of attached walls, ceilings, and floors. Once the location of the unit within the building was determined (discussed in Section 3.1.2), attached walls were removed and replaced by imaginary walls. If there was a unit directly above the residence, the ceiling/roof was replaced by an imaginary ceiling. If there was a unit directly below the residence, the floor/foundation was replaced by an

imaginary floor. Thus, a zone with imaginary boundaries was established. Energy consumption within this zone was analyzed. For exterior walls, ceilings/roofs, and foundations/floors, the following assumptions were made. Exterior walls consisted of 5/8 inch plywood and 1/2 inch drywall, with R11 insulation. The ceiling/roof had R19 insulation. Crawl space foundations had R7 insulation. Basement and slab foundations conformed to ASHRAE standards.

3.1.4 Simulation Procedures

Once the prototypes had been constructed, simulation was quite straightforward. The following steps were generally taken: first, the file containing the DOE II.1 algorithms and the description of the prototype was called. The variables which were allowed to vary were assigned values within this file, and the weather tape city was specified. A batch job was submitted. Upon execution of the job, the output file was called. The output from DOE II.1 contains many reports, since hourly, daily, weekly, monthly, and annual loads on a number of heating/cooling systems are generated by the program. In this study, the total electric heating energy for the simulation year was retained from the monthly loads report. This heating energy variable is measured in kilowatt hours per year for the residence under analysis. Other reports were scanned to check for errors or inconsistencies in the program. The report on the number of underheated hours was checked to ensure that the capacity of the HVAC system was large enough to fulfill all heating demands. In cases where the capacity was not large enough, the HVAC capacity for all residences in the particular building category was increased. This was done to ensure that no house in the sample was underheated, and to ensure that HVAC capacity did not vary from one residence to the next.

3.2 ESTIMATION OF DOE II.1 SUMMARY EQUATIONS

The second step in fulfilling the study's initial objective was to estimate the relationship between electric heating energy calculated in DOE II.1 and the variables that were allowed to vary in the DOE II.1 simulations. Separate regression equations were estimated for each building category.

3.2.1 General Methodology

A similar strategy was used to estimate all three equations. First, the calculated electric heating energy, DOEII, and each of the independent variables were entered into a common data set. The pool of independent variables generally corresponded to the variables allowed to vary in the simulations, which were described in Section 3.1.1. Only those characteristics allowed to vary in the DOE II.1 analysis were used in the regression equations, for the following reasons: first, since the dependent variable DOEII was generated by varying only these characteristics, estimated relationships between DOEII and other variables would obviously have been spurious; second, in order to isolate the relationship between a home's thermal characteristics and its physical energy requirements, behavioral characteristics of the residents had to be ignored.

Second, transformations of these variables were made: in particular, natural logarithms were taken of most of the continuous variables (including the dependent variable DOEII); interaction variables were created (i.e., one of the variables is equal to space multiplied by heating degree days); and dummy variables were created to represent binary variables (i.e., CAKALL=1 if residence had caulking on all windows/doors; CAKALL=0 if residence had some or no caulking). Two dummy variables were created for each binary conservation characteristic (caulking, plastic covering, weatherstripping, wall insulation). The first represents all, the second, some: for example, CAKALL=1 if all windows/doors are caulked, CAKALL=0 if some or none are caulked; CAKSOM=1 if some windows/doors are caulked, CAKSOM=0 if all or none are caulked. In this manner, the effects of both full and partial caulking (or plastic covering, weatherstripping, insulating) can be discerned. Most continuous variables and their logarithms were also squared to create additional variables. Squares, logarithms, and squares of logarithms were included to increase the explanatory power of the equations. Thus, the pool of dependent and independent variables available for the regression analysis was created.

Third, the variables DOEII and LDOEII ($= \log(\text{DOEII})$) were regressed on the complete pool of independent variables. Because no a priori expectations about functional forms were obvious, DOEII was regressed on linear and

logarithmic variables, as was LDOEII. Both squared and non-squared terms were included. A stepwise procedure called MAXR, available on the Statistical Analysis System (SAS), was used to narrow the list of independent variables. MAXR estimates the "best" one-variable equation, two-variable equation, etc., given a dependent variable and a pool of independent variables. "Best" means "best fitting" in the sense of maximizing the coefficient of determination, R^2 .⁽¹⁾ Generally, the best equations with 10-15 variables were compared, and one selected for further investigation. The criteria for selection at this point included the increment to goodness-of-fit by adding an additional variable, as well as the signs of the estimated partial derivatives.⁽²⁾

Fourth, the selected model was adapted by inserting variables of particular interest (i.e., weatherstripping, caulking, and plastic cover dummy variables). Squares, logarithms, and squares of logarithms were included to increase the explanatory power of the equations. Variables such as weatherstripping, which were "forced" into the equation, performed well but had been dropped by MAXR because other variables were more "important" statistically. Variable whose partial derivative had signs which violated a priori expectations were dropped from the equation. A preliminary model for each dependent variable (DOEII and LDOEII) was generated in this manner.

Fifth, this preliminary model was subjected to a number of tests. Partial derivatives with respect to all continuous independent variables were evaluated for all values of the variables within the sample, as well as reasonable values outside of the data space. The predicted dependent variable (DOEIIHAT or LDOEIIHAT) was calculated for each observation. These values were checked for reasonableness; i.e., negative values or very large positive values were noted. The regression errors were calculated (DOEII minus DOEIIHAT, LDOEII minus LDOEIIHAT) for each observation. The square of these

(1) This statistic measures the proportion of the total variation of DOEII (or LDOEII) explained by the equation.

(2) Since the equations are nonlinear in the explanatory variables, the sign of the partial derivative with respect to a particular variable is not merely that variable's coefficient, but an algebraic expression involving explanatory variables. In evaluating these derivatives, and testing their statistical significance, mean sample values of the explanatory variables were used.

errors was also computed (ERRORSQ) and the correlation between ERRORSQ and each of the independent variables (as well as the square of each independent variable) calculated. These calculations provided a simple test against heteroscedasticity. Models that failed any of these "tests" were modified or discarded. Modification included testing new explanatory variables and other estimation techniques (in the case of heteroscedasticity, weighted least squares was used). Once "final" DOEII and LDOEII equations were estimated, the two equations were compared on the basis of goodness-of-fit and the structure of the equations to choose the "best" of the two equations.

3.2.2 Mobile Home Equation

The final mobile home equation is presented in Table 3.1, along with a description of the final independent variables. The dependent variable is DOEII. The unadjusted coefficient of determination (R^2) is .8297, and the adjusted (for degrees of freedom) R^2 is .8031. The equation is quadratic in the logarithms of glass, space, and heating degree days; includes a linear interaction term between space and heating degree days; and is linear in the percent of glass area with storms and the plastic covering dummy variable. Five of the nine variables are significantly different from zero at the 5 percent level (the test on COVSOM is one-sided). Both glass variables are significantly different from zero at the 15 percent level. The individual square footage variables are not significant at reasonable levels, but the overall effect of square footage is significant due to the interaction term, INACT3.

The derivatives of this equation are quite reasonable. The glass derivative is positive for values of GAREA less than 170 square feet; for GAREA above 170 square feet, the derivative is effectively zero. The derivative with respect to space is positive for all positive values of space when evaluated using any sample value of heating degree days. The derivative with respect to heating degree days is positive for all but one observation in the sample. The derivatives with respect to the other independent variables are the same sign as their coefficients, and all have the expected sign.

Finally, the correlations between the squared residual and SPACH, SPACH2, LSPACH, LSPACH2, CALHDD, CALHDD2, LHDD, and LHDD2 (defined in Table 3.1) were

TABLE 3.1. Estimated Mobile Home Thermal Equation

$$\begin{aligned}
 \text{DOEII} = & -1367643 + 22408.24 \text{ LGAREA} \\
 & (-2.989) \quad (1.5356) \\
 & -2188.46 \text{ LGAREA}^2 + 17545.13 \text{ LSPACH} \\
 & (-1.5114) \quad (1.0357) \\
 & -1303.59 \text{ LSPACH}^2 + 284320.5 \text{ LHDD} \\
 & (-0.8707) \quad (2.1157) \\
 & -16101 \text{ LHDD}^2 + .00162 \text{ INACT3} \\
 & (-2.0781) \quad (2.2277) \\
 & -17.8778 \text{ PSGAREA} - 1238.41 \text{ COVSOM} \\
 & (-2.3230) \quad (-1.7037)
 \end{aligned}$$

t - statistics in ()

$$R^2 = .8297 \quad \bar{R}^2 = .8031 \quad \text{DF} = 65$$

Key to Regression Equation

DOEII = Output from DOE II.1 simulations (KWH/year)
 LGAREA = LOG(GAREA)
 LGAREH2 = LGAREA**2
 LSPACH = LOG(SPACH)
 LSPACH2 = LSPACH**2
 LHDD = LOG(CALHDD)
 LHDD2 = LHDD**2
 INACT3 = CALHDD*SPACH
 PSGAREA = 100 * SGAREA/GAREA
 COVSOM = dummy, =1 if plastic covering on some windows/doors
 (=0 for all, none)
 GAREA = square feet of glass area
 SPACH = square feet of heated living space
 CALHDD = heating degree days, 65° base, from DOE II.1 weather tape
 SGAREA = square feet of storm glass area
 SPACH2 = SPACH**2
 GAREA2 = GAREA**2
 CALHDD2 = CALHDD**2

computed. None were statistically significant at reasonable significance levels. Thus, the null hypothesis of homoscedasticity was not rejected.

3.2.3 Single Family Equation

The final estimated equation is presented in Table 3.2. The dependent variable is LDOEII, the natural logarithm of DOEII. The unadjusted R^2 in logarithmic units is .7287, and the adjusted (for degrees of freedom) R^2 in logarithmic units is .7092. The predicted LDOEII from the estimated equation was calculated for each observation. The predicted DOEII was also calculated. The R^2 in natural units was then calculated (this represents the percentage of the variation in DOEII captured by the equation). The natural-unit unadjusted R^2 value is .7230, and the natural-unit adjusted R^2 is .7031. All coefficients are significant at the 5 percent level (tests on CAKSOM and WSTALL are one-sided).

The derivatives of LDOEII with respect to all independent variables are of the expected sign when evaluated at all reasonable values of the independent variables, as are the signs of the coefficients of dummy variables. All expected signs are obvious except that on FOUNDBAS. Since a basement provides more thermal protection than a concrete slab (the default), the expected sign of this variable is negative. Because the logarithmic function is monotonic, the derivatives of DOEII with respect to the independent variables are also of the expected sign.

The correlations between the squared residual (in logarithmic units) and a number of independent variables were computed. The correlations between the squared residual and SPACH, SPACH2, LSPACH, LSPACH2, HDD, HDD2, LHDD, and LHDD2 (defined in Table 3.2) are statistically insignificant from zero at all reasonable significance levels; those between the squared residual and GAREA, GAREA2, LGAREA, and LGAREA2 are not significant at the 10 percent level, although each is significant at the 15 percent level. Given the fact that glass area is a relatively minor determinant of LDOEII, and the insignificance of these correlations at the 10 percent level, the null hypothesis of homoscedasticity was not rejected.

TABLE 3.2. Estimated Single-Family Thermal Equation

$$LDOEII = -4.95542 + 0.00051 \text{ GAREA}$$

$$(-3.3669) \quad (2.4906)$$

$$+0.00045 \text{ SPACH} + 1.70335 \text{ LHDD}$$

$$(9.5088) \quad (9.7867)$$

$$-0.00122 \text{ PSDAREA} - 0.00109 \text{ PSGAREA}$$

$$(-2.1663) \quad (-2.0246)$$

$$-0.0000026 \text{ RINACTIC} - 0.000106 \text{ RINACT2W}$$

$$(-2.3269) \quad (-11.4299)$$

$$-0.21451 \text{ FOUNDBAS} - 0.10767 \text{ CAKSOM}$$

$$(-4.3109) \quad (-1.7911)$$

$$-0.08036 \text{ WSTALL}$$

$$(-1.7548)$$

t - statistics in ()

$$R^2 = .7287 \quad \bar{R}^2 = .7092 \quad R^2 \text{ (natural units)} = .7230$$

$$\bar{R}^2 \text{ (natural units)} = .7031 \quad DF = 139$$

Key to Regression Equation

LDOEII = LOG(DOEII)

DOEII = Output from DOE II.1 simulations (KWH/year)

GAREA = Square feet of glass area

SPACH = Square feet of heated living space

LHDD = LOG(CALHDD)

CALHDD = Heating degree days, 65° base, from DOE II.1 weather tape

PSDAREA = 100 * NSDOOR/NDOOR

NSDOOR = Number of storm doors

NDOOR = Number of doors to unheated area

PSGAREA = 100 * SGAREA/GAREA

SGAREA = Square feet of storm glass area

TABLE 3.2. (contd)

Key to Regression Equation (contd)

RINACT3W = RWALLALL * CALHDD
 RWALLALL = dummy, =1 if all walls insulated
 CAKSOM = dummy, =1 if caulking on some walls
 FOUNDBAS = dummy, =1 if foundation is basement
 RINACT1C = RCEIL * SPACH
 WSTALL = dummy, =1 if weatherstripping on all windows/doors
 RCEIL = Ceiling R-value
 GAREA2 = GAREA**2
 LGAREA = LOG(GAREA)
 LGAREA2 = LGAREA**2
 SPACH2 = SPACH**2
 LSPACH = LOG(SPACH)
 LSPACH2 = LSPACH**2
 CALHDD2 = CALHDD*2
 LHDD2 = LHDD**2

3.2.4 Multi-Family Equation

Initial attempts to estimate a multi-family residence thermal equation were plagued by heteroscedasticity. In the "best" equation estimated using ordinary least squares, the correlation between SPACH2 and the squared residual was .291, significant at the 1.1 percent level. To correct the heteroscedasticity problem, the weighted least squares procedure was used.⁽¹⁾ All variables, both dependent and independent, were divided by SPACH. The general method of estimation described in Section 3.2.1 was then applied to the transformed variables.

The final estimated equation is presented in Table 3.3. The dependent variable is XLDOEII, which equals LDOEII divided by SPACH (not LOG(DOEII/SPACH)). The untransformed, unadjusted R^2 is .9984; the untransformed, adjusted (for degrees of freedom) R^2 is .9982. All but one of the coefficients are significant at the 10 percent level (test for XWALLAT3 is

(1) For a discussion of heteroscedasticity and the weighted least squares procedure, see H. Theil, Principles of Econometrics, John Wiley and Sons, New York, 1971.

TABLE 3.3. Estimated Multi-Family Thermal Equation
(Scaled Logarithmic Units)

$$\begin{aligned}
 \text{XLDOEII} = & -8.6324 \text{ XINT} + 0.1233 \text{ XLSPACH2} \\
 & (-2.5609) \quad (4.9547) \\
 & +0.2321 \text{ XLGAREA} + 0.1621 \text{ XLHDD2} \\
 & (1.8824) \quad (4.2622) \\
 & -0.00000015 \text{ XINACT3} - 0.00289 \text{ XPSGAREA} \\
 & (-1.3872) \quad (-1.6988) \\
 & -0.3325 \text{ XFNDCRW} - 0.1272 \text{ XWALLAT3} \\
 & (-2.2412) \quad (-1.3795) \\
 & -0.3000 \text{ XWSTALL} \\
 & (-2.5533)
 \end{aligned}$$

t - statistics in ()

$$R^2 = .9984 \quad \bar{R}^2 = .9982 \quad \text{DF} = 66$$

Key to Regression Equation

See Table 3.4 for key to roots of variable names.

"X" at beginning of variable name denotes the root variable is divided by SPACH. For example, XLDOEII = LDOEII/SPACH. Also, XINT = 1/SPACH.

one-sided). By multiplying both sides of the XLDOEII equation by SPACH, the equation presented in Table 3.4 is obtained. T-statistics and coefficients are the same as for the XLDOEII equation, but a new R^2 , in LDOEII units, can be calculated. This R^2 is equal to .5826, and the R^2 adjusted for degrees of freedom is .5320. The R^2 for this equation measured in natural (DOEII) units instead of logarithmic units is .6063, and the natural unit adjusted R^2 is .5585. Thus, nearly 56 percent of the variation in DOEII is captured by this equation.

TABLE 3.4. Estimated Multi-Family Thermal Equation
(Logarithmic Units)

$$\begin{aligned}
 \text{LDOEII} = & -8.6324 + 0.1233 \text{ LSPACH2} \\
 & (-2.5609) \quad (4.9547) \\
 & +0.2321 \text{ LGAREA} + 0.1621 \text{ LHDD2} \\
 & (1.8824) \quad (4.2622) \\
 & -0.00000015 \text{ INACT3} - 0.00289 \text{ PSGAREA} \\
 & (-1.3872) \quad (-1.6988) \\
 & -0.3325 \text{ FOUNDCRW} - 0.1272 \text{ WALLAT3} \\
 & (-2.2412) \quad (-1.3795) \\
 & -0.3000 \text{ WSTALL} \\
 & (-2.5533)
 \end{aligned}$$

t - statistics in ()

$$\begin{aligned}
 R &= .5826 \quad \bar{R}^2 = .5320 \quad R^2 \text{ (natural units)} = .6063 \\
 \bar{R}^2 \text{ (natural units)} &= .5585 \quad DF = 66
 \end{aligned}$$

Key to Regression Equation

LDOEII = LOG(DOEII)
 DOEII = Output from DOE II.1 simulations, (KWH/year)
 LSPACH2 = LSPACH**2
 LSPACH = LOG(SPACH)
 SPACH = Square feet of heated living space
 LGAREA = LOG(GAREA)
 GAREA = Square feet of glass area
 LHDD2 = LHDD**2
 LHDD = LOG(CALHDD)
 CALHDD = Heating degree days, 65° degree base, from DOE II.1
 weather tape
 INACT3 = CALHDD * SPACH
 PSGAREA = 100 * SGAREA/GAREA
 SGAREA = Square feet of storm glass area
 FOUNDCRW = dummy, =1 if crawl space foundation
 WALLAT3 = dummy, =1 if three attached walls
 WSTALL = dummy, =1 if weatherstripping on all windows/doors
 SPACH2 = SPACH**2
 GAREA2 = GAREA**2
 LGAREA2 = LGAREA**2
 CALHDD2 = CALHDD**2

The implied derivatives of LDOEII with respect to each of the independent variables are all of the expected sign for all observations in the sample. Coefficients on dummy variables are also of the expected sign. However, the SPACH derivative becomes zero for reasonable values of SPACH and CALHDD not found in the sample.

Finally, the correlations between the squared residual term and various independent variables was calculated. The squared residual term is equal to the square of LDOEII minus LDOEIIHAT, measured in logarithmic units, not scaled logarithmic units. None of the correlations are significant at the 20 percent level. Thus, the null hypothesis of homoscedasticity was not rejected for the weighted least squares model.

4.0 THE IMPACT OF IMPROVED THERMAL SHELL EFFICIENCY ON RESIDENTIAL ENERGY CONSUMPTION

In this section, the relationship between space heating energy consumption and residential thermal efficiency is statistically estimated. The results from this estimation can be used to assess the impacts of conservation programs on energy consumption. Consumer benefits from such programs can also be calculated from the estimated relationships.

The section is divided into four parts. The problem under consideration is analyzed from a policy perspective in Section 4.1. The theory of household demand for space conditioning energy is discussed in Section 4.2. Empirical implementation of the theory is described in Section 4.3. Results of the statistical analysis are presented in Section 4.4.

4.1 THE PROBLEM FROM A POLICY PERSPECTIVE

Policymakers in the energy field are frequently presented with proposed new programs which will reduce energy consumption. About such a proposal, two primary questions must be asked: First, how much energy will actually be saved by the proposed program? Second, how much will consumers benefit from the program? The two questions are intricately related, and with each come subsidiary questions: for a given expenditure, does program A save more or less energy than program B? For a given expenditure, do consumers benefit more from program A or program B?

This general problem was analyzed in a partial way in the previous section. The three estimated thermal equations can be used to assess the reductions in residential space conditioning energy use due to a particular conservation program. These estimated reductions are applicable only under controlled conditions. In that analysis, consumer behavior was not considered. In all DOE II.1 simulations, thermostat settings and occupant/equipment schedules were identical. Energy savings calculated using these equations thus do not represent the total energy savings that a particular conservation program will yield.

Similarly, the benefits of any particular conservation program to consumers were only partially considered in the previous section. Benefits of such programs are typically calculated by measuring the increase in consumer surplus for the good affected by the program. In this case, the good in question is internal comfort. Changes in consumer surplus are the sum of two components. First, the consumer experiences a "windfall" gain equal to the change in the price of comfort multiplied by the initial quantity. This component was considered in Section 3.0. The second component is related to the induced increase in consumption of internal comfort. It was implicitly disregarded in the previous analysis.

4.1.1 Estimation of Program Impacts

In the present section, these restrictions on consumer behavior are relaxed. When the thermal efficiency of a residence is increased, the effective price paid by the consumer for internal comfort falls.⁽¹⁾ This price reduction occurs because fewer Btus, and thus fewer dollars, are required to increase the level of internal comfort by one unit. Incentives are thus created for the consumer to purchase more internal comfort. Energy consumption may increase relative to the level purchased if there were no consumer response. Of course, the net impact may still be a reduction in energy consumption, but not as large a reduction as implied by a non-behavioral engineering analysis. A 1 percent increase in the thermal shell efficiency causes a 1 percent decrease in space conditioning energy use due to the engineering effect; by the price effect, space conditioning energy use increases X percent. The net reduction in space conditioning energy use is thus one minus X percent.

The equations estimated in this section permit computation of X and thus $Y = 1 - X$, which represents the percentage reduction in space conditioning energy use associated with a 1 percent increase in a building's thermal shell

(1) Increases in shell thermal efficiency are changes in the building shell which improve the ability of the shell to prevent escape of heat generated within the building to outside areas.

efficiency. Combined with the thermal equations estimated in Section 3.0 and the formal definition of shell thermal efficiency given in Section 4.3, the equations presented in this chapter can be used to estimate the energy savings of particular conservation programs.

4.1.2 Estimation of Program Benefits

Programs which improve the thermal efficiency of a residence produce a benefit which has two components. Both result from the reduced price of internal comfort caused by the thermal improvement. First, holding the quantity consumed of internal comfort fixed, the consumer receives a "wind-fall" gain equal to the initial quantity consumed multiplied by the change in the price of internal comfort. This represents the reduction in total expenditure required to maintain the previous level of consumer well-being; it can be treated analogously to an increase in consumer income. The second component is related to the induced increase in consumption of internal comfort due to the fall in the price of internal comfort.

The two components are displayed in Figure 4.1. Before the thermal improvement, a quantity Q_0 of internal comfort was purchased. At this quantity, the value to the consumer of purchasing an additional unit (its marginal value) is just equal to the price P_0 he must pay to obtain it; for the following unit of comfort, the price exceeds the marginal value. As price falls from P_0 to P_1 , quantity consumed increases from Q_0 to Q_1 . Assume for simplicity that Q_1 minus Q_0 equals three. The first component of benefits is thus equal to three (Q_1 minus Q_0) times the price change (P_0 minus P_1). The second component of the benefit to the consumer can then be calculated as follows: for the $Q_0 + 1$ unit, the consumer was willing to pay some price called P' , just below P_0 , greater than P_1 , to obtain the unit. This price he was willing to pay represents the marginal value to him of obtaining the $Q_0 + 1$ unit. However, he only pays P_1 . Thus, he receives a "benefit" equal to P' minus P_1 from buying the $Q_0 + 1$ unit. Similarly, for the $Q_0 + 2$ unit, he is willing to pay some price P'' slightly less than P' but still greater than P_1 . Hence, the benefits to the consumer of buying the $Q_0 + 2$ unit at price P_1 are equal to P'' minus P_1 . The process stops at unit $Q_0 + 3$, which

FIGURE 4.1. Calculation of Consumer Surplus

equals Q_1 ; here the price he is willing to pay for the unit (its marginal value) just equals the price he must pay for it. Thus, the second component of consumer benefits in this example is equal to the sum of the benefits on the two additional units purchased: 1) P' minus P_1 ; and 2) P'' minus P_1 . The two components are more clearly shown in Figure 4.2; area A represents the first benefit component, the "windfall", area B the second. This argument generalizes to any combination of Q_0 and Q_1 , with Q_1 greater than Q_0 . The sum of the two components represents the total change in consumer surplus associated with the fall in the price of internal comfort; this change in consumer surplus in turn represents the change in consumer welfare from the price reduction or, alternatively, from the efficiency improvement.

4.1.3 Price Elasticity Estimates

A third use in policymaking of the statistical analysis presented in this section concerns space conditioning energy demand price elasticities. The analysis of the impacts of changes in thermal shell efficiency on energy use is performed by estimating residential space conditioning energy demand functions. Price elasticities for space conditioning energy demand are generated in the process.

These price elasticities can be used in the energy demand forecasting process. If thermal shell efficiency is a determinant of residential space conditioning energy consumption, as hypothesized in this section, price elasticities estimated in equations from which the shell thermal efficiency is excluded are biased. Thus, the demand price elasticities presented in this section, which reflect thermal shell efficiencies, may be superior to elasticities which do not reflect thermal efficiency.

4.2 THEORY OF RESIDENTIAL DEMAND FOR SPACE CONDITIONING ENERGY

The economic theory of household demand for space conditioning energy, although not well-documented in the literature, is but a special case of

FIGURE 4.2. Consumer Surplus

general consumer demand theory.⁽¹⁾ In particular, the consumer is assumed to derive utility (satisfaction) from three goods: space conditioning (R_1), other residential services (R_2), and a composite commodity (Z) that represents all other goods and services. The utility function relating consumption of these three goods to consumer satisfaction is assumed to be homothetic and separable between the two residential services (R_1 and R_2) and the composite commodity. Households produce space conditioning by combining energy (Q_1), the building's thermal efficiency (θ), and a set of environmental variables (N_1). The thermal efficiency of a building is the ability of the building's shell to prevent escapement to outside areas of heat generated within the building. The set of environmental variables include climate, size of home, and the number of residents. These variables are either not under the control of the consumer or can be considered exogenous to the problem at hand. Other residential services are also produced by the household, using energy (Q_2), the stock of appliances and the efficiency of this stock (S), and a set of environmental variables (N_2). Summarizing,

$$U = U(R_1, R_2; Z) \quad (4.1)$$

$$R_1 = F_1(Q_1, \theta, N_1) \quad (4.2)$$

$$R_2 = F_2(Q_2, S, N_2) \quad (4.3)$$

4.2.1 Long-Run Utility Maximization

The household utility maximization problem can be considered in two parts. First, each of the two "produced" goods R_1 and R_2 will be produced at minimum cost, for any level of R_1 , R_2 . Thus, cost functions for these two production processes can be developed. Second, consumers will choose R_1 , R_2

(1) This part relies heavily on work performed earlier at PNL. See R. C. Adams and A. D. Rockwood, Impact of Improved Building Thermal Efficiency on Residential Energy Demand, prepared for U.S. Department of Energy under contract DE-AC06-76RLO 1830, Pacific Northwest Laboratory, Richland, Washington, 1981.

and Z to maximize their utility, subject to a budget constraint which includes the two cost functions. Thus, in the long-run, three optimization processes occur:

A) Minimize the cost (C_1) of producing R_1

$$\min C_1 = P_Q Q_1 + P_\theta \theta \quad (4.4)$$

$$\text{s.t. } R_1 = F_1(Q_1, \theta, N_1) \quad (4.5)$$

where

P_Q = price of energy

P_θ = price of shell efficiency.

This yields the conditional (upon R_1) demands

$$Q_1 = Q_1^*(P_Q, P_\theta, R_1, N_1) \quad (4.6)$$

$$\theta = \theta^*(P_Q, P_\theta, R_1, N_1) \quad (4.7)$$

and the cost function

$$C_1 = C_1^*(P_Q, P_\theta, R_1, N_1) = P_Q Q_1^*(P_Q, P_\theta, R_1, N_1) + P_\theta \theta^*(P_Q, P_\theta, R_1, N_1) \quad (4.8)$$

B) Minimize the cost (C_2) of producing R_2

$$\min C_2 = P_Q Q_2 + P_S S \quad (4.9)$$

$$\text{s.t. } R_2 = F_2(Q_2, S, N_2) \quad (4.10)$$

where

P_S = the price of appliances.

This yields the conditional (upon R_2) demands

$$Q_2 = Q_2^*(P_Q, P_S, R_2, N_2) \quad (4.11)$$

$$S = S^*(P_Q, P_S, R_2, N_2) \quad (4.12)$$

and the cost function

$$C_2 = C_2^*(P_Q, P_S, R_2, N_2) = P_Q Q_2^*(P_Q, P_S, R_2, N_2) + P_S S^*(P_Q, P_S, R_2, N_2) \quad (4.13)$$

C) Maximize utility subject to the budget constraint

$$\max U = U(R_1, R_2, Z) \quad (4.14)$$

$$\text{s.t. } C_1^*(P_Q, P_\theta, R_1, N_1) + C_2^*(P_Q, P_S, R_2, N_2) + P_Z Z = Y \quad (4.15)$$

where

P_Z = the price of Z

Y = income.

This yields the service demands

$$R_1 = R_1^*(P_Q, P_\theta, P_S, P_Z, N_1, N_2, Y) \quad (4.16)$$

$$R_2 = R_2^*(P_Q, P_\theta, P_S, P_Z, N_1, N_2, Y) \quad (4.17)$$

The long-run unconditional demand functions for Q_1 and θ are obtained by inserting equation 4.16 into 4.6, 4.7:

$$Q_1 = Q_1^{**}(P_Q, P_\theta, P_S, N_1, N_2, Y) = Q_1^*(P_Q, P_\theta, R_1^*(P_Q, P_\theta, P_S, N_1, N_2, Y), N_1) \quad (4.18)$$

$$\theta = \theta^{**}(P_Q, P_\theta, P_S, N_1, N_2, Y) = \theta^*(P_Q, P_\theta, R_1^*(P_Q, P_\theta, P_S, N_1, N_2, Y), N_1). \quad (4.19)$$

4.2.2 Short-Run Utility Maximization

In the long-run, θ does not determine Q_1 ; instead, both are endogenous, jointly determined by $P_Q, P_\theta, P_S, N_1, N_2$, and Y . Thus, the long-run equations derived in Section 4.2.1 are not of assistance in assessing the impacts of changes in thermal efficiency on space conditioning energy consumption, when such changes are exogenously caused by conservation programs. Such assessments require construction of short- or intermediate-run models, in which thermal efficiency is not a choice variable of the consumer but a parameter to be varied outside of the consumer's control. Construction of a short-run model in which theta (θ) is a parameter merely recognizes that in the short run the thermal efficiency of a home, like the appliance stock, is fixed. It does not imply that the cost of thermal efficiency improvements are not borne by the consumer. Nor does it imply that the equations estimated below are irrelevant in the long run, for in any single period theta still remains fixed, although it may vary between periods. Construction of such a model is merely a tool to analytically derive a relationship between energy use and thermal efficiency. Moreover, in the short- or intermediate run, optimal quantities of theta and energy are not necessarily selected by consumers. Thus, policies which alter theta or energy consumption may be appropriate in this time framework.

In the short run, θ and s are considered fixed, determined by past levels of $P_Q, P_\theta, P_S, N_1, N_2$, and Y . A two-step maximization procedure similar to that used in the long-run can be used to derive the consumer's demand functions for Q_1 and Q_2 for given levels of θ and s ($\bar{\theta}, \bar{s}$):

$$A') \quad \min C_1 = P_Q Q_1 + P_\theta \theta \quad (4.20)$$

$$s. t. R_1 = F_1(Q_1, \theta, N_1) \quad (4.21)$$

$$\theta = \bar{\theta} \quad (4.22)$$

yields

$$Q_1 = Q_1^S(\bar{\theta}, R_1, N_1) \quad (4.23)$$

and

$$C_1 = C_1^S(P_Q, \bar{\theta}, R_1, N_1) = P_Q Q_1^S(\bar{\theta}, R_1, N_1) \quad (4.24)$$

B')

$$C_2 = P_Q Q_2 + P_S S \quad (4.25)$$

$$\text{s.t. } R_2 = F_2(Q_2, S, N_2) \quad (4.26)$$

$$S = \bar{S} \quad (4.27)$$

yields

$$Q_2 = Q_2^S(\bar{S}, R_2, N_2) \quad (4.28)$$

$$C_2 = C_2^S(P_Q, \bar{S}, R_2, N_2) = P_Q Q_2^S(\bar{S}, R_2, N_2) \quad (4.29)$$

C')

$$\max U(R_1, R_2; Z) \quad (4.30)$$

$$\text{s.t. } C_1^S(P_Q, \bar{\theta}, R_1, N_1) + C_2^S(P_Q, \bar{S}, R_2, N_2) + P_Z Z = Y \quad (4.31)$$

which, after insertion into equations 4.23 and 4.28, yields

$$Q_1 = Q_1''(P_Q, \bar{\theta}, \bar{S}, P_Z, N_1, N_2, Y) \quad (4.32)$$

$$Q_2 = Q_2''(P_Q, \bar{\theta}, \bar{S}, P_Z, N_1, N_2, Y)$$

which are analogous to equations 4.18 and 4.19.

Equations 4.32 and 4.33 represent the short-run demand functions for space conditioning energy and other residential energy. These functions, the former in particular, are of interest to the policymaker. A policy

instrument, $\bar{\theta}$, appears as a determinant of household energy consumption. Thus, estimates of this function will yield estimates of the impacts of programs which affect θ .

4.2.3 Constraints on Demand Functions Imposed by Thermal Relationship

Economic theory provides very little information about the space conditioning energy demand function presented in equation 4.32. About all that is known without further information is the sign of the partial derivative of Q_1 with respect to P_Q . However, information on the relationship between R_1 , and Q_1 , θ , N_1 can be used to place more restrictions on the Q_1 function.

First, suppose the production function for R_1 (space conditioning services) can be separated into two multiplicative components:

$$R_1 = G(Q_1, \theta) \cdot H(N_1) \quad (4.34)$$

where

$\hat{Q}_1 = G(Q_1, \theta)$ is energy consumption measured in efficiency units. Thus, space conditioning services produced equals energy consumption in efficiency units multiplied by some measure of how much conditioning (heating in winter, cooling in summer) needs to be done to obtain a particular comfort level. What is the appropriate form of $G(Q_1, \theta)$? Although restrictions on the space conditioning energy demand function are attainable using more general functional forms, the following attributes of G seem to be desirable. First, a 1% increase in the quantity of energy measured in physical units, holding shell efficiency constant, should lead to a 1% increase in the quantity of energy measured in efficiency units. Thus, G is multiplicative in Q_1 :

$$\hat{Q}_1 = G(Q_1, \theta) = Q_1 \cdot g(\theta) \quad (4.35)$$

where " $\cdot$ " denotes multiplication. Second, a 1 percent increase in the thermal efficiency of the shell should be associated with a 1 percent increase in the quantity of energy in efficiency units, holding quantity in physical units constant. This attribute seems less apparent than the first, and is in large

part determined by how θ is empirically defined. In the empirical analysis to be discussed below, θ was defined as square footage divided by calculated energy use, DOEIIHAT (see Section 3.0). This definition appears to be consistent with this desired attribute of $G(Q_1, \theta)$.⁽¹⁾ Thus, $G(Q_1, \theta)$ is multiplicative in θ as well:

$$\hat{Q}_1 = G(Q_1, \theta) = Q_1 \cdot \theta \quad (4.36)$$

What restrictions on the demand function are implied by equation 4.36? First, define the price of energy in efficiency units, $\hat{P}_Q$, such that

$$\hat{P}_Q \hat{Q}_1 = P_Q Q_1 \quad (4.37)$$

so

$$\hat{P}_Q = \frac{P_Q Q_1}{\hat{Q}_1} = \frac{P_Q}{\theta} \quad (4.38)$$

Second, reformulate the consumer's utility maximization problem in terms of efficiency units; the unconditional demand functions derived from this problem have the general form

$$\hat{Q}_1 = \hat{Q}_1^* (\hat{P}_{Q1}, \hat{P}_{Q2}, P_Z, N_1, N_2, Y, \bar{s}) \quad (4.39)$$

$$\hat{Q}_2 = \hat{Q}_2^* (\hat{P}_{Q1}, \hat{P}_{Q2}, P_Z, N_1, N_2, Y, \bar{s}) \quad (4.40)$$

where $\hat{P}_{Q1}$ represents the price of space conditioning energy in efficiency units, $\hat{P}_{Q2}$ the price of other residential energy in efficiency units, and $\hat{Q}_1$, $\hat{Q}_2$ are quantities in efficiency units. Different efficiency indices are used

(1) In the empirical definition, holding square footage constant, a one percent increase in theta is associated with a one percent decrease in DOEIIHAT, the quantity of energy required to maintain a constant internal temperature (comfort level). Thus, if the quantity of energy was fixed, internal comfort level (temperature) would increase one percent if theta rose one percent.

in the two production processes, so the two prices are not necessarily equal. Note that neither θ nor γ , the "other" efficiency index, determine quantity consumed in efficiency units. Thus, the demand for space conditioning energy can be represented as

$$\hat{Q}_1 = \hat{Q}_1^* (\hat{P}_Q, X) \quad (4.41)$$

where $\hat{P}_Q$ represents $\hat{P}_{Q_1}$ and X represents $\hat{P}_{Q_2}$, P_Z , N_1 , N_2 , Y , and $\bar{s}$. Converting to physical units,

$$Q_1^* (\hat{P}, X, \theta) \equiv \frac{\hat{Q}_1^* (\hat{P}_Q, X)}{\theta} \quad (4.42)$$

Taking the derivative of this demand function with respect to θ ,

$$\frac{\partial Q_1^*}{\partial \theta} = - \frac{\hat{Q}_1 (\hat{P}_Q, X)}{\theta^2} + \frac{1}{\theta} \frac{\partial \hat{Q}_1}{\partial \hat{P}_Q} \frac{\partial \hat{P}_Q}{\partial \theta} \quad (4.43)$$

But $\frac{\partial \hat{P}_Q}{\partial \theta} = - \frac{P_Q}{\theta^2}$ by Equation 4.38, so

$$\frac{\partial Q_1^*}{\partial \theta} = \frac{-\hat{Q}_1}{\theta^2} - \frac{1}{\theta^2} \frac{\partial \hat{Q}_1}{\partial \hat{P}_Q} \left(\frac{P_Q}{\theta} \right) \quad (4.44)$$

However, $\frac{P_Q}{\theta} = \hat{P}_Q$, so (4.38)

$$\frac{\partial Q_1^*}{\partial \theta} = - \left(\frac{\hat{Q}_1}{\theta} \right) \left(\frac{1}{\theta} \right) \left\{ 1 + \frac{\partial \hat{Q}_1}{\partial \hat{P}_Q} \frac{\hat{P}_Q}{\hat{Q}_1} \right\} \quad (4.45)$$

and $\left(\frac{\hat{Q}_1}{\theta} \right) = Q_1$ (4.36)

$$\frac{\partial Q_1^*}{\partial \theta} = - \frac{Q_1}{\theta} \{1 + \hat{\eta}\} \quad (4.46)$$

where $\hat{\eta} = \frac{\partial Q_1}{\partial \hat{P}} \frac{\hat{P}}{\hat{Q}_1}$ is

$$\hat{\eta} = \frac{\partial Q_1}{\partial \hat{P}_Q} \frac{\hat{P}_Q}{\hat{Q}} = \left[\frac{\partial Q_1}{\partial Q_1} \frac{\partial Q_1}{\partial P_Q} \frac{\partial P_Q}{\partial \hat{P}_Q} \right] \left[\frac{P_Q / \theta}{Q \theta} \right] = \frac{\partial Q_1^*}{\partial P_Q} \frac{P_Q}{Q} = \eta \quad (4.47)$$

by equations 4.36 and 4.38 and the chain rule, where η is the price elasticity of demand when measured in physical units. Thus,

$$\frac{\partial Q_1^*}{\partial Q} = - \frac{Q_1}{\theta} \{1 + \eta\} \quad (4.48)$$

and

$$\frac{\partial Q_1^*}{\partial \theta} \frac{\theta}{Q_1} = - \{1 + \eta\} \quad (4.49)$$

This leads, by substitution, to

$$\epsilon = -1 - \eta \quad \text{or} \quad (4.50)$$

$$\epsilon + \eta = -1, \quad (4.51)$$

where ϵ is the thermal efficiency elasticity of space conditioning energy demand and η is the price elasticity of space conditioning energy demand.

Thus, the sum of the price and thermal efficiency elasticities of the space conditioning demand function must equal negative one. This restriction,

of course, depends on the special functional form given the relationship among $\hat{Q}_1$, Q_1 , and θ given in equation 4.36. However, the result is consistent with the intuitive argument presented in Section 4.1. It was there stated that a one percent thermal efficiency improvement would cause space conditioning energy consumption to fall one percent, holding the internal comfort level fixed. However, the efficiency improvement is equivalent to a one percent reduction in the price of internal comfort, causing an η percent increase in the consumption of internal comfort, where η is the price elasticity of the demand for internal comfort. This in turn causes an η percent increase in space conditioning energy consumption compared to the case where internal comfort level was held constant. Thus, the net percentage reduction in space conditioning energy consumption due to a one percent increase in thermal efficiency is $\epsilon = -1 - \eta$. Without a behavioral response to the change in the effective price of energy, $\epsilon = -1$; ϵ is reduced in absolute value by a price response, in magnitude equal to the price elasticity.

4.3 IMPLEMENTATION OF THEORY

Quantity, price, and thermal efficiency equations were estimated for each of three building types. In this section, the data base used to estimate these equations is described; the construction of a number of variables is also discussed. The problems of simultaneity and the proper price to include in the demand (quantity) functions are described. Finally, the functional forms and final specifications used are presented.

4.3.1 Data Used in Analysis

The data base used for estimation was the same as in the DOE II.1 simulations, as described in Section 3. The Elrick and Lavidge survey includes monthly information for each residence regarding quantities consumed of and expenditures for electricity, natural gas, and oil, as well as building characteristics of the household's residence, its climate, and the household's income. Data on number of residents, age of head of household, age of home, location of home (state and climate zone) and type (public, private) of electric utility serving the residence was also available.

The 4030 residences in the sample were split into three categories: mobile home, single-family, and multi-family. Within each building category, residences were further divided into groups according to type of main heating system. Only electrically-heated homes were analyzed. Monthly consumption and expenditure data for the months November through April was aggregated into total winter consumption and expenditure. The determination of the season was made on the basis of mean monthly heating degree days for the entire sample; large month-to-month changes occurred in the monthly means between October and November (increase) and April and May (decrease). Because most of the Pacific Northwest is winter peaking, estimation of summer demand equations did not appear to be of much value. Thus, quantity, price, and thermal efficiency equations were only estimated for winter electricity.

4.3.2 Construction of Shell Thermal Efficiency Index

The variable θ (theta) was constructed for each residence as a proxy for the thermal shell efficiency. Theta was calculated in the following manner. First, for each building category, the space conditioning energy consumption under controlled conditions (DOEIIHAT) was calculated for each residence in the category using the estimated thermal equations presented in Section 3. Theta was then defined as the ratio of square footage of heated living space to calculated energy use. This definition of theta is consistent with that proposed in Section 4.2; a 1 percent increase in θ leads to a 1 percent decrease in the calculated heating energy use required to maintain a particular comfort level. Thus, if actual energy use remained constant, energy use measured in efficiency units would increase 1 percent.

4.3.3 Construction of Appliance Index

The quantity of electricity consumed for space conditioning is not available in the data set. The variable that is available, total electricity consumption, includes electricity for appliances and other non-heating uses. To estimate space conditioning electricity demand functions using total electricity consumption as the dependent variable, the electricity used for these other purposes must be accounted for. Because individual appliances are not a concern of this analysis, an index of total electricity use by appliances in the household was constructed. Information on the presence of particular

appliances was taken from the data base. Weights for the index were taken from an internal Bonneville Power Administration study performed in 1981 using the Elrick and Lavidge data base. These weights represent the average electricity consumption per year for each appliance in the household; weights were scaled to reflect winter-only use.

4.3.4 Functional Forms of Estimated Equations

Two alternative general functional forms were used in this analysis. The first form was the log-linear or double-logarithmic, in which the natural logarithm of the dependent variable Y is regressed on the natural logarithms of the independent variables X_1 , X_2 , etc. This form has been used in many studies of electricity demand in particular and economic relations in general. The form is relatively easy to estimate and the coefficients of the individual explanatory variables represent elasticities. Furthermore, under certain assumptions about the price schedule facing consumers of electricity, the distinction between average and marginal prices becomes irrelevant (see Section 4.3.5). The log-linear form does, however, have limitations. The most significant shortcoming is that the elasticity of the dependent variable Y with respect to an explanatory variable X_1 is a constant. For example, the price elasticity of demand at high prices is restricted to be identical to the price elasticity at low prices. This is potentially a fairly restrictive assumption.

To overcome this limitation, each of the equations was also estimated using a variable-elasticity functional form. This form, first proposed by Mount, Chapman, and Tyrrell,⁽¹⁾ uses the log-linear form as a foundation, and adds reciprocal terms. For example, the natural logarithm of Y would be regressed on the natural logarithm of X as well as the reciprocal of X :

(1) T. D. Mount, L. D. Chapman, and T. J. Tyrrell, Electricity Demand in the United States: An Econometric Analysis, ORNL-NSF-3P-49, Oak Ridge National Laboratory, Oak Ridge, Tennessee, 1973.

$$\ln Y = a + b \ln X_1 + c_1 1/X_1 + b_2 \ln X_2 + c_2 1/X_2 + e. \quad (4.52)$$

With this functional form, the elasticity of Y with respect to X_1 is equal to $b_1 - c_1/X_1$. Thus, the elasticity takes on different values as X_1 changes.

4.3.5 Average Vs. Marginal Price

In standard consumer theory, the consumer takes the product price as given. This price does not change as quantity purchased changes. Marginal price and average price are identical. In the case of electricity, the consumer typically faces a tiered price structure; the marginal price at low quantities is different (typically, higher) than the marginal price at high quantities. Average and marginal prices are thus no longer equal.⁽¹⁾

Consumer demand is no longer related just to marginal price, but to the entire price schedule facing the consumer. Both prices should be included in the demand equation. Two price elasticities of demand would emerge from such an equation. However, as Taylor shows, only the marginal price elasticity is a true price elasticity. The elasticity with respect to average price, in fact, is closely related to the income elasticity of demand.

Average price, equal to total expenditure divided by total consumption, has frequently been used to summarize the information contained in the price schedule, for three reasons. First, marginal prices are not available in most data bases, and are difficult to calculate. Second, average price clearly plays some role, even if only through an income effect. Third, average price and marginal price are quite correlated, and thus serve as proxies for one another. In fact, attempts to include both variables in the estimation process has brought unsatisfactory results because of collinearity between the two variables. Thus, most researchers have used average price despite warnings that elasticities resulting from its use are overstated because of left-out-variable bias.

(1) See L. D. Taylor, "The Demand for Electricity: A Survey," Bell Journal of Economics, Spring 1975 for a more thorough discussion of this problem.

Halvorsen⁽¹⁾ has shown that the bias inherent in elasticities estimated using average price disappears under the following conditions. First, log-linear forms must be used for both equations; the natural logarithm of one variable appears in the equation explaining the natural logarithm of the other. Second, the price schedule must be assumed continuous. Then

$$\ln Q = a_0 + a_1 \ln P_m + a_2 \ln Z + e_1 \quad (4.53)$$

$$\ln P_m = b_0 + b_1 \ln Q + b_2 \ln X + e_2 \quad (4.54)$$

where "ln" denotes natural logarithm, $a_0, a_1, a_2, b_0, b_1, b_2$ are coefficients, Z and X are exogenous variables, and e_1 and e_2 are the error terms. Q represents quantity and P_m represents marginal price. Use of two-stage least squares on equations 4-53 and 4-54 would result in consistent estimates of the price elasticity a_1 . However, the same price elasticity results when using average price in two-stage least squares estimation: Assume

$$\ln P_a = b_0 + b_1 \ln Q + b_2 \ln X + e_2 \quad (4.55)$$

or

$$P_a = e^{b_0} Q^{b_1} X^{b_2} e^{e_2} \quad (4.55')$$

by using equation 4.54, replacing marginal with average price (P_a), where "e" represents Euler's constant. Assuming a continuous price schedule,

$$P_a Q = \int_{q=0}^{q=Q} P_m dq \quad (4.56)$$

Substituting 4.55' into 4.56, and differentiating both sides with respect to Q ,

(1) R. Halvorsen, Econometric Models of U.S. Energy Demand, Lexington Books, Lexington, Massachusetts, 1978.

$$\frac{d}{dQ} \left(e^{b_0 Q^{b_1+1}} \chi^{b_2} e^{e_2} \right) = \frac{d}{dQ} \int_{q=0}^{q=Q} P_m dq . \quad (4.57)$$

Differentiation yields

$$(b_1 + 1) e^{b_0 Q^{b_1+1}} \chi^{b_2} e^{e_2} = P_m \quad (4.58)$$

$$(b_1 + 1) P_a = P_m \quad (4.59)$$

or

$$P_a = P_m / (b_1 + 1) . \quad (4.60)$$

Substituting average for marginal price in the estimated quantity equation, the demand equation is

$$\ln Q = a_0 + a_1 \ln P_a + a_2 \ln Z + e_2 . \quad (4.61)$$

Substituting 4.60 into 4.61 yields

$$\ln Q = \{a_0 + a_1 \ln (1 + b_1)\} + a_1 \ln P_m + a_2 \ln Z + e_1 . \quad (4.62)$$

Thus, in a regression of quantity on price, only the intercept term is affected by using the average price instead of the marginal price. Each of the log-linear demand equations was estimated using average price, appealing to this technique. The result of zero bias does not apply to the variable elasticity functional forms. In these equations, marginal price was included as an explanatory variable in place of the average price. The marginal price was calculated in a manner similar to the procedure used in Halvorsen's proof. The estimated average price equation is of the form

$$\ln P_a = a + b_1 \ln Q + c_1 1/Q + d_1 (\ln Q \cdot \text{DUM1}) + e_1 (\ln Q \cdot \text{DUM2}) + f_1 \ln X + U \quad (4.63)$$

where DUM1 and DUM2 represent state dummy variables and/or utility-type dummy variables, and X represents a general exogenous variable. Then

$$P_a = e^{a} Q^{b_1 + d_1 \text{ DUM1} + e_1 \text{ DUM2}} e^{c_1/Q} X^{f_1} e^U \quad (4.64)$$

By definition,

$$P_a Q = \int_{q=0}^{q=Q} P_m dq \quad (4.65)$$

so

$$\frac{d}{dQ} \left(e^{a} Q^{b_1 + 1 + d_1 \text{ DUM1} + e_1 \text{ DUM2}} e^{c_1/Q} X^{f_1} e^U \right) = \frac{d}{dQ} \int_{q=0}^{q=Q} P_m dq \quad (4.66)$$

This yields

$$P_a [b_1 + 1 + d_1 \text{ DUM1} + e_1 \text{ DUM2} - c_1/Q] = P_m \quad (4.67)$$

The marginal price was calculated using this formula for each observation in the sample. The parameters b_1 , d_1 , e_1 , c_1 were taken from the final two-stage least squares price equation. By using marginal price in the variable elasticity models, the calculated price elasticities remained bias-free. Attempts were also made to include both average and marginal prices in the same equation to improve the price and income elasticity estimates but again failed due to collinearity.

4.3.6 Simultaneity of Quantity, Price, and Thermal Efficiency

Because the consumer faces a price schedule instead of a single price, price determines quantity while quantity also determines price. The price-quantity system is simultaneous. In the present case a third variable, the thermal efficiency index, enters the system of equations simultaneously. In

the short run, θ cannot vary, but the level of θ is determined (theoretically, at least) by past prices, which in a cross-section sample are reflected in the current price. Thus, the simultaneity problem is relevant in consideration of θ .

Coefficients estimated using ordinary least squares are biased in such a situation. To overcome this problem, two-stage least squares (2SLS) was used to estimate all of the quantity and price equations. In this procedure, the predicted values of price and θ , both from the first-stage estimates, are used in the demand equation instead of the actual, observed values of price and θ . Predicted quantity is used in the price equation, and predicted price in the θ equation. The θ equation was estimated using 2SLS for single-family residences; for the other two building categories the influence of price on θ was so weak that simultaneity did not appear to be a problem. Even for the single-family this relationship is weak; however, specifying θ as an endogenous variable improved the coefficient estimates of the quantity and price equations dramatically. Thus, the hypothesis of joint determination was retained.

4.3.7 Final Specification of Equations

Three sets of equations were estimated, one for each building category: mobile home, single-family, and multi-family. Each set contained three equations: one explaining the logarithm of quantity consumed, one explaining the logarithm of average price, and one explaining the logarithm of θ , the index of shell efficiency. For each equation, two forms were estimated: log-linear and variable-elasticity. Thus, a total of 18 equations were estimated. All quantity (demand) equations were estimated with the restrictions derived in Section 4.2.3 imposed on the coefficients. Thus, in all demand equations, the elasticities with respect to price and θ sum to negative one. It will be recalled that this result depended on a rather restrictive assumption about the relationship between energy in efficiency units, energy in physical units, and the thermal shell efficiency. For less restrictive assumptions about this relationship, other, less stringent restrictions on the demand elasticities can be derived. A large number of alternative specifications of the thermal relationship restrict the elasticities to sum to negative

one at the mean value of θ .⁽¹⁾ In all equations, the coefficient of $1/\theta$ was very insignificant and thus dropped. The elasticity of demand with respect to θ therefore does not vary with the level of θ , only with the level of price. Thus, the estimated elasticity of demand with respect to θ equals $-1 - b_1 + c_1/P$ where b_1 is the coefficient of $\log(P)$ and c_1 is the coefficient of $1/P$.

The following variables formed the pool of explanatory variables for the quantity equations (demand for space conditioning-heating electricity function). Each appeared in logarithmic form as well as in reciprocal form in the variable-elasticity formulations. Average and marginal prices were expected to have a negative influence on quantity consumed; the thermal efficiency of the residence was also expected to have a negative impact. The elasticity with respect to θ was expected to be different from negative one. Income and heating degree days were expected to have positive effects on quantity consumed, as was the number of residents. Age of head of household was included with an unexpected sign. The appliance index discussed in Section 4.3.3 was also included in each quantity equation, to account for the electricity used for non-heating purposes. Interaction terms between the index and both price and income were statistically insignificant in the equations estimated at early stages of the analysis. Although the successful inclusion of the price interaction term would have strengthened the claim that the estimated price elasticities do fairly represent space conditioning energy demand price elasticities, failure to include these interaction terms does not seriously weaken the analysis. The interaction terms were always insignificant from zero, implying that electricity demand for aggregate appliance use is price inelastic. Such a result does not seem implausible.

The following variables were included in the pool of explanatory variables for the price equation. Again, each appeared in logarithmic and/or

(1) The restrictions were imposed by substitution. In the equation $\log Q = a + b_1 \log P + b_2 \log \theta + c_1 1/P + c_2 1/\theta + d_1 X + e$, $\epsilon + \eta = -1$ implies $b_1 + b_2 - c_1/P - c_2/\theta = -1$, or $b_2 = -1 - b_1 + c_1/P + c_2/\theta$. Substituting this expression in for b_2 and rearranging, one obtains the estimating equation: $\log Q + \log \theta = a + b_1 (\log P - \log \theta) + c_1 1/P (1 + \log \theta) + c_2 1/\theta (1 + \log \theta) + d_1 X + e$.

reciprocal form, except for dummy variables. The primary explanatory variable was quantity purchased. The expected sign of the elasticity of price with respect to quantity was negative to reflect declining block structures prevalent throughout the region. Heating degree days and dummy variables for state, climate zone, and public utility type (public, private) were also included in the price equations. The state and utility-type dummies appeared as both intercept adjusters and slope adjusters.

The primary determinants of residential shell efficiency considered were average and marginal prices of electricity, heating degree days, income, and number of residents. Each of these variables was expected to have a positive influence on shell efficiency. Other determinants considered include age of head of household and age of home.

4.4 STATISTICAL RESULTS

In this section, the final estimated equations are presented and analyzed. The policy implications of the results are discussed as well.

4.4.1 Mobile Home Equations

Demand and price equations were estimated for mobile homes using two-stage least squares. A shell efficiency equation could not be estimated; although θ varies in the sample, these variations did not appear to be related to any economic or demographic variables. Both constant elasticity (log-linear) and variable elasticity (Mount, Chapman, Tyrrell) models were estimated. The sample used for estimation contained 166 observations; from the original survey of mobile homes, a number of observations were dropped because 1) natural gas was the primary source of heating; 2) quantity consumed was missing for one or more months. Mobile homes which were billed on a bi-monthly basis were included in the sample.

Constant Elasticity Equations

The constant elasticity demand for winter electricity equation is presented in Table 4.1. Since the appliance index variable accounts for non-heating uses of electricity, the remaining variables in the equation represent

TABLE 4.1. Mobile Home Electricity Demand Equation
Constant Elasticity Form

$$\begin{aligned} \text{LKWH} = & - 6.829 - 0.889 \cdot \text{LPRICE} - 0.111 \cdot \text{LTHETA} + 0.207 \cdot \text{LHDDDB} \\ & (1.824) \quad (0.037) \quad (0.037) \quad (0.182) \\ & + 0.120 \cdot \text{LINCOME} + 1.12 \cdot \text{LAPP} \\ & (0.057) \quad (0.150) \end{aligned}$$

$$\begin{aligned} \bar{R}^2 &= 0.4757 \\ \text{d.f.} &= 161 \end{aligned}$$

Asymptotic standard errors in parentheses.

Key to Equation

LKWH = log (kilowatts used from November through April)
LPRICE = log (average price of electricity for November through April,
\$/kWh)
LTHETA = log (shell efficiency)
LHDDDB = log (average monthly heating degree days for November through
April)
LINCOME = log (gross yearly income)
LAPP = log (calculated total appliance electricity use, November
through April)

an electricity demand for heating equation. Nearly half of the variation in the logarithm of electricity consumption is accounted for by the equation; $\bar{R}^2 = .4757$.⁽¹⁾ All coefficients are of the expected sign and the price, thermal efficiency, income, and appliance coefficients (which represent heating demand elasticities) are statistically significant from zero. In addition, the thermal efficiency elasticity is statistically significant from negative one; the null hypothesis that a one percent increase in thermal efficiency causes a one-percent decrease in heating electricity use is thus rejected.

(1) The R^2 used to calculate the $\bar{R}^2$ is equal to the square of the Pearson correlation coefficient for this and all other equations in this report estimated using two-stage least squares.

The constant-elasticity average winter electricity price equation for mobile homes is presented in Table 4.2. Over three-fourths of the variation in the logarithm of the average price is accounted for by the equation. As expected, average price falls as quantity consumed increases; the rate of price reduction per unit increase in quantity is higher for consumers served by public utilities, and less for consumers residing in Oregon than in the default states, Idaho and Montana. The level of average prices is lower in Washington than in the default states.⁽¹⁾

Variable Elasticity Equations

The variable elasticity winter electricity demand equation for mobile homes is presented in Table 4.3. The $\bar{R}^2$ is .4556, so almost half of the variation in the dependent variable is explained by the equation. The equation includes the marginal price, which was calculated using the procedure outlined in Section 4.3.5. The elasticities of heating electricity demand with respect to price, thermal efficiency, and income are given at the bottom of the table. The first two of these elasticities are variable, and here evaluated at the means of the data. Both the price and thermal efficiency elasticity are significantly different from zero; the income elasticity is not. The thermal efficiency elasticity is also statistically significant from negative one, supporting the hypothesis proposed in this paper. An increase in thermal efficiency of a particular percentage leads to a smaller percentage reduction in the quantity of space heating electricity consumption. As the marginal price increases, the price elasticity becomes less negative (less elastic);⁽²⁾ thus, the thermal efficiency elasticity moves further from zero, since the two elasticities sum to negative one.

-
- (1) It is likely that state of residence and type of utility affects both the level and the rate of reduction per unit quantity increase of average prices. Individual variables should thus be used both as intercept and slope adjusters. Inclusion of both types of shifters was plagued by multicollinearity.
 - (2) The elasticity of Y with respect to X in the equation $\log Y = a + b \log X + C 1/X$ is equal to $b - C/X$; thus, $\partial \epsilon / \partial X = C/X^2$, which is the same sign as C.

TABLE 4.2. Mobile Home Electricity Price Equation
Constant Elasticity Form

$$\begin{aligned} \text{LPRICE} = & - 3.874 - 0.217 \cdot \text{LKWH} + 0.224 \cdot \text{LHDD} \\ & (0.705) \quad (0.029) \quad (0.076) \\ & - 0.065 \cdot \text{WASHTN} - 0.043 \cdot \text{LQPUBLIC} + 0.035 \cdot \text{LQOREGON} \\ & (0.039) \quad (0.003) \quad (0.004) \end{aligned}$$

$$\begin{aligned} \bar{R}^2 &= 0.7850 \\ \text{d.f.} &= 160 \end{aligned}$$

Asymptotic standard errors in parentheses.

Key to Equation

LPRICE = log (average price from November through April, \$/kWh)
 LKWH = log (kilowatts used from November through April)
 LHDD = log (yearly heating degree days [30-year average])
 WASHTN = dummy, = 1 if in Washington State
 LQPUBLIC = LKWH · PUBLIC
 LQOREGON = LKWH · OREGON
 PUBLIC = dummy, = 1 if in a Public Utility District
 OREGON = dummy, = 1 if in Oregon State

TABLE 4.3. Mobile Home Electricity Demand Equation
Variable Elasticity Form

$$\begin{aligned} \text{LKWHTH} = & 18.513 - 0.510 \cdot \text{LPRICETH} + 0.004 \cdot \frac{(\text{LTHETA}+1)}{\text{PRICE}} \\ & (4.693) \quad (0.054) \quad (0.0005) \\ & + 0.063 \cdot \text{LINCOME} + 2.69 \cdot \text{LAPP} + 4598.52 \cdot \frac{1}{\text{APP}} \\ & (0.059) \quad (0.52) \quad (2015.34) \end{aligned}$$

$$\begin{aligned} \bar{R}^2 &= 0.4770 \\ \text{d.f.} &= 160 \end{aligned}$$

Elasticities
(calculated at mean values)

MARGINAL PRICE - 0.781 (0.036)
THETA - 0.219 (0.036)
*INCOME 0.063 (0.057)
APP 1.933 (0.783)

*Constant elasticity

Asymptotic standard errors in parentheses.

Key to Equation

LKWHTH = LKWH + LTHETA
LKWH = log (kilowatts used from November through April)
LPRICE = log (PRICE)
PRICE = marginal price of electricity from November through April,
\$/kWh
LTHETA = log (THETA)
THETA = shell efficiency
LINCOME = log (yearly gross income)
LAPP = log (APP)
APP = calculated total appliance electricity use, November through
April
LPRICETH = LPRICE - LTHETA

The variable elasticity price equation for mobile homes is presented in Table 4.4. The equation is similar to the constant elasticity price equation presented in Table 4.2. The exceptions are 1) the Washington dummy variable has been dropped due to lack of statistical significance and 2) reciprocal terms for quantity and heating degree days have been included. The coefficient on 1/KWH implies that the elasticity of price with respect to quantity becomes more negative as quantity increases.

Analysis of Equations

The price elasticities of demand appear to be very large in absolute value in both the constant and variable elasticity models. This is associated with a thermal efficiency elasticity much closer to zero than was expected. The primary cause of this aberration, and a weakness of the mobile home analysis in general, is lack of data on the thermal characteristics of mobile homes. The simulated energy use for heating, DOEII, varied only a small amount because only a small number of variables were available for the thermal analysis. Thus, a rather weak thermal equation was estimated, and the theta variable calculated for each residence displays little variation across the residences. Without the restrictions on the price and efficiency elasticities, in fact, the efficiency elasticity was not statistically significant from zero. This general problem also explains why a reasonable thermal efficiency equation could not be estimated for mobile homes. Further research using a more in-depth survey of mobile homes is required to effectively answer some of the questions raised by this report.

4.4.2 Single-Family Equations

The problems encountered in estimation of the mobile home equations were not seen in the single-family sample. Reasonable thermal equations were estimated for single-family residences, and the theta variable displayed significant variation. In fact, in both the constant- and variable-elasticity models, the thermal efficiency elasticity exceeded the price elasticity in absolute value. In unrestricted estimates, the theta elasticity was statistically significant from both zero and negative one; thus, the variable showed enough variation that its effect on consumption was identifiable.

TABLE 4.4. Mobile Home Electricity Price Equation
Variable Elasticity Form

$$\begin{aligned} \text{LPRICE} = & - 20.707 - 0.336 \cdot \text{LKWH} - 1014.9 \cdot \frac{1}{\text{KWH}} + 2.079 \cdot \text{LHDD} \\ & (9.813) \quad (0.071) \quad (424.5) \quad (0.989) \\ & + 11211.1 \cdot \frac{1}{\text{HDD}} - 0.046 \cdot \text{LQPUBLIC} + 0.038 \cdot \text{LQOREGON} \\ & (6159.64) \quad (0.004) \quad (0.004) \end{aligned}$$

$$\begin{aligned} \bar{R}^2 &= 0.7030 \\ \text{d.f.} &= 159 \end{aligned}$$

Asymptotic standard errors in parentheses.

Key to Equation

LPRICE = log (average price for November through April, \$/kWh)
 LKWH = log (KWH)
 KWH = kilowatt hours used from November through April
 LHDD = log (HDD)
 HDD = average yearly heating degree days (30-year average)
 LQPUBLIC = LKWH • PUBLIC
 LQOREGON = LKWH • OREGON
 PUBLIC = dummy, = 1 if in a Public Utility District
 OREGON = dummy, = 1 if in Oregon State

Constant Elasticity Equations

The single-family winter demand for electricity equation is presented in Table 4.5. The $\bar{R}^2$ is .3592, so that over one-third of the variation in the dependent variable is explained by the equation. The primary determinants of non-appliance electricity use are price and thermal efficiency. Climate, income, age of household head, and number of residents also affect winter heating electricity use.

The corresponding average price equation is presented in Table 4.6. Almost two-thirds of the variation in the logarithm of the average price can be explained by the five variables included in the equation. As expected, the average price falls as quantity consumed increases. The rate of price reduction is faster for Washington residents than for Idaho or Montana residents, and faster for consumers served by public utilities than for those serviced by private utilities. In Oregon the price does not fall as much for a unit change in quantity as in the default states.

The constant-elasticity thermal efficiency equation is presented in Table 4.7. Very little of the variation in theta is explained by the equation: $\bar{R}^2 = .0700$. The effect of price on thermal efficiency is not statistically significant from zero; it is large enough in the ordinary least squares formulation that simultaneity had to be considered a problem. The behavioral variables income, age of household head, and number of residents all had coefficients that are 1) of the expected sign and 2) statistically significant from zero.

Variable Elasticity Equations

The single-family variable elasticity winter demand for electricity equation is presented in Table 4.8. The table also includes estimates of the elasticities and their standard errors, each calculated at the means of the data. The $\bar{R}^2$ is 0.3913. All of the elasticities are significantly different from zero; furthermore, all of the elasticities except for income are variable in the sense of statistical significance. This follows from the fact that all

TABLE 4.5. Single-Family Electricity Demand Equation
Constant Elasticity Form

$$\begin{aligned}
 \text{LKWH} = & - 10.051 - 0.424 \cdot \text{LPRICE} - 0.576 \cdot \text{LTHETA} \\
 & (0.897) \quad (0.051) \quad (0.051) \\
 & + 0.288 \cdot \text{LHDDDB} + 0.114 \cdot \text{LINCOME} + 1.48 \cdot \text{LAPP} \\
 & (0.078) \quad (0.026) \quad (0.073) \\
 & - 0.181 \cdot \text{LHEADAGE} + 0.229 \cdot \text{LNUMRES} \\
 & (0.053) \quad (0.030)
 \end{aligned}$$

$$\begin{aligned}
 \bar{R}^2 &= 0.3592 \\
 \text{d.f.} &= 1430
 \end{aligned}$$

Asymptotic standard errors in parentheses.

Key to Equation

LKWH	= log (kilowatt hours used from November through April)
LPRICE	= log (average price of electricity for November through April, \$/kWh)
LTHETA	= log (shell efficiency)
LINCOME	= log (yearly gross income)
LHDDDB	= log (average monthly heating degrees for November through April)
LAPP	= log (calculated total appliance electricity use for November through April)
LHEADAGE	= log (age of head of household)
LNUMRES	= log (number of residents)

TABLE 4.6. Single-Family Electricity Price Equation
Constant Elasticity Form

$$\begin{aligned} \text{LPRICE} = & - 6.073 - 0.136 \cdot \text{LKWH} + 0.400 \cdot \text{LHDD} \\ & (0.341) \quad (0.011) \quad (0.036) \\ & - 0.127 \cdot \text{LQWASHTN} - 0.043 \cdot \text{LQPUBLIC} + 0.030 \cdot \text{LQOREGON} \\ & (0.014) \quad (0.001) \quad (0.002) \end{aligned}$$

$$\begin{aligned} \bar{R}^2 &= 0.6419 \\ \text{d.f.} &= 1430 \end{aligned}$$

Asymptotic standard errors in parentheses.

Key to Equation

LPRICE = log (average electricity price for November through April,
\$/kWh)
LKWH = log (kilowatts used during November through April)
LHDD = log (heating degree days per year [30-year average])
LQWASHTN = LKWH • WASHTN
WASHTN = a dummy, = 1 if in Washington State
PUBLIC = a dummy, = 1 if in Public Utility District
LQPUBLIC = LKWH • PUBLIC
LQOREGON = LKWH • OREGON
OREGON = a dummy, = 1 if in Oregon State

TABLE 4.7. Single-Family Theta Equation
Constant Elasticity Form

$$\begin{aligned} \text{LCTHETA} = & - 2.795 - 0.018 \cdot \text{LPRICE} + 0.332 \cdot \text{LHDD} \\ & (0.676) \quad (0.034) \quad (0.065) \\ & + 0.116 \cdot \text{LINCOME} + 0.092 \cdot \text{LHEADAGE} + 0.045 \cdot \text{LNUMRES} - 0.055 \cdot \text{LHOMEAGE} \\ & (0.018) \quad (0.038) \quad (0.021) \quad (0.013) \end{aligned}$$

$$\begin{aligned} \bar{R}^2 &= 0.0700 \\ \text{d.f.} &= 1430 \end{aligned}$$

Asymptotic standard errors in parentheses.

Key to Equation

LCTHETA = log (100 • shell efficiency)
 LPRICE = log (average price of electricity from November through April,
 \$/kWh)
 LHDD = log (heating degree days per year [30-year average])
 LINCOME = log (yearly gross income)
 LHEADAGE = log (age of head of household)
 LNUMRES = log (number of residents)
 LHOMEAGE = log (age of home residence)

TABLE 4.8. Single-Family Electricity Demand Equation
Variable Elasticity Form

$$\begin{aligned}
 \text{LKWHTH} = & - 8.199 - 0.439 \cdot \text{LPRICETH} \\
 & (8.533) (0.041) \\
 & + 0.00028 \cdot \frac{(\text{LTHETA}+1)}{\text{PRICE}} \\
 & (0.001122) \\
 & + 0.111 \cdot \text{LINCOME} + 328.23 \cdot \frac{1}{\text{INCOME}} - 2.762 \cdot \text{LHddb} \\
 & (0.056) (518.05) (0.963) \\
 & - 2718.06 \cdot \frac{1}{\text{Hddb}} + 2.383 \cdot \text{LAPP} + 4474.94 \cdot \frac{1}{\text{APP}} \\
 & (865.14) (0.292) (1402.55) \\
 & - 0.820 \cdot \text{LHEADAGE} - 29.086 \cdot \frac{1}{\text{HEADAGE}} + 0.325 \cdot \text{LNUMRES} \\
 & (0.350) (14.339) (0.037) \\
 & + 0.00031 \cdot \frac{1}{\text{NUMRES}} \\
 & (0.00007) \\
 \bar{R}^2 = & 0.3913 \\
 \text{d.f.} = & 1424
 \end{aligned}$$

Elasticities

(calculated at mean values)

MARGINAL PRICE	-	0.454	(0.042)
THETA	-	0.546	(0.042)
INCOME		0.095	(0.036)
Hddb		0.373	(0.086)
APP		1.682	(0.095)
HEADAGE	-	0.245	(0.082)
NUMRES		0.325	(0.037)

Asymptotic standard errors in parentheses.

Key to Equation

LKWHTH	=	LKWH + LTHETA
LKWH	=	log (KWH)
KWH	=	kilowatt hours used from November through April
LPRICE	=	LOG (PRICE)
PRICE	=	Marginal price of electricity from November through April, \$/kWh
LTHETA	=	LOG (THETA)
THETA	=	shell efficiency
LINCOME	=	LOG (INCOME)
INCOME	=	yearly gross income
LHddb	=	LOG (Hddb)

TABLE 4.8. (contd)

Key to Regression Equation (condt)

HDDB = average monthly heating degree days for November through April
LAPP = LOG (APP)
APP = calculated total appliance electricity use for November through April
LHEADAGE = LOG (HEADAGE)
HEADAGE = age of head of household
LNUMRES = LOG (NUMRES)
NUMRES = Number of residents
LPRICETH = LPRICE - LTHETA

of the reciprocal terms except $1/\text{INCOME}$ are statistically different from zero. As with the mobile home equation, the price elasticity of heating electricity demand for single-family residences moves toward zero as price increases. Thus, as price increases, the thermal efficiency elasticity increases in absolute value. The thermal elasticity is statistically different from both zero and negative one (separate tests), supporting the proposed hypothesis that increases in thermal efficiency cause increases in consumption of internal comfort.

The single-family winter average electricity price equation is given in Table 4.9. The $\bar{R}^2$ for this equation is .7886. The equation is similar in form to that presented in Table 4.7. This equation, however, includes reciprocal terms for quantity and climate. The equation is generally similar to the variable-elasticity mobile home price equation presented in Table 4.4 with one curious exception: here, as quantity consumed increases, the price/quantity elasticity falls in absolute value, opposite in direction to the mobile home case. Greater variability of quantity consumed in the single-family sample may account for this inconsistency.

The corresponding thermal efficiency equation is presented in Table 4.10. Again, the fit is rather poor, and the only statistically significant variables involve income, the number of residents and the age of the home.

Analysis of Equations

Both of the demand equations appear quite reasonable. Signs of elasticities are consistent with economic theory, and the magnitudes of the price, thermal efficiency, and income elasticities are plausible. One puzzling characteristic of the variable elasticity equation is the change in the price elasticity as the price increases. For linear demand curves, the elasticity increases in absolute value as price increases. Although not necessitated by theory, this general result has been found by a number of researchers. The typical rationalization runs along the following lines: as price increases, it really starts to "hurt", as expenditures on the good in question potentially account for a larger share of the consumer's budget. The result estimated here (for all three building categories) runs counter to this argument. A plausible rationalization of the result is that at some

TABLE 4.9. Single-Family Electricity Price Equation
Variable Elasticity Form

$$\begin{aligned} \text{LPRICE} = & - 11.473 - 0.155 \cdot \text{LKWH} + 231.84 \cdot \frac{1}{\text{KWH}} \\ & (3.056) \quad (0.015) \quad (47.32) \\ & + 0.963 \cdot \text{LHDD} - 3780.36 \cdot \frac{1}{\text{HDD}} - 0.043 \cdot \text{LQPUBLIC} \\ & (0.312) \quad (1919.51) \quad (0.001) \\ & + 0.028 \cdot \text{LQOREGON} - 0.014 \cdot \text{LQWASHTN} \\ & (0.002) \quad (0.002) \end{aligned}$$

$$\begin{aligned} \bar{R}^2 &= 0.7886 \\ \text{d.f.} &= 1429 \end{aligned}$$

Asymptotic standard errors in parentheses.

Key to Equation

LPRICE = log (average price of electricity for November through April
[\$/kWh])
LKWH = log (kWh)
KWH = kilowatt hours used from November through April
LHDD = log (HDD)
HDD = heating degree days per year (30-year average)
LQPUBLIC = LKWH • PUBLIC
PUBLIC = dummy, = 1 if in a Public Utility District
LQOREGON = LKWH • OREGON
OREGON = dummy, = 1 if in Oregon State
LQWASHTN = LKWH • WASHTN
WASHTN = dummy, = 1 if in Washington State

TABLE 4.10. Single-Family Theta Equation
Variable Elasticity Form

$$\begin{aligned}
 \text{LCTHETA} = & - 3.224 + 0.002 \cdot \text{LPRICE} - 6.31 \times 10^{-6} \cdot \frac{1}{\text{PRICE}} \\
 & \quad (6.969) \quad (0.030) \quad (0.00003) \\
 & + 0.346 \cdot \text{LHDD} + 117.42 \cdot \frac{1}{(4203.24) \text{ HDD}} + 0.164 \cdot \text{LINCOME} + 585.28 \cdot \frac{1}{(371.28) \text{ INCOME}} \\
 & \quad (0.697) \quad (0.049) \\
 & + 0.075 \cdot \text{LHEADAGE} - 1.109 \cdot \frac{1}{(10.317) \text{ HEADAGE}} + 0.066 \cdot \text{LNUMRES} + 0.000063 \cdot \frac{1}{(0.000049) \text{ NUMRES}} \\
 & \quad (0.252) \quad (0.026) \\
 & - 0.074 \cdot \text{LHOMEAGE} - 0.190 \cdot \frac{1}{(0.206) \text{ HOMEAGE}} \\
 & \quad (0.024)
 \end{aligned}$$

$$\begin{aligned}
 \bar{R}^2 &= 0.0694 \\
 \text{d.f.} &= 1424
 \end{aligned}$$

Asymptotic standard errors in parentheses.

Key to Equation

LCTHETA	= log (CTHETA)
CTHETA	= 100 • shell efficiency
LPRICE	= log (PRICE)
PRICE	= marginal price of electricity from November through April, \$/kWh
LHDD	= log (HDD)
HDD	= heating degree days per year (30-year average)
LINCOME	= log (INCOME)
INCOME	= yearly gross income
LHEADAGE	= log (HEADAGE)
HEADAGE	= age of head of household
LNUMRES	= log (NUMRES)
NUMRES	= number of residents
LHOMEAGE	= log (HOMEAGE)
HOMEAGE	= age of home

"high" price a consumer has cut all the "fat" from his consumption: he has turned the thermostat down, he has insulated, etc. Thus, very little is left to cut without slicing into the near-necessary component of consumption.

4.4.3 Multi-Family Equations

The quality of the multi-family residence behavioral equations appears to lie somewhere between that of the mobile home and single-family equations. The predictive power of the equations is equal to or greater than that of the single-family equations; the statistical significance of most variables, however, is much lower. This suggests that there is not as much variation in the dependent variables as there was for the single-family sample, and that multicollinearity is potentially a problem. Searches for correlations among the explanatory variables were fruitless.

Constant Elasticity Equations

The constant elasticity multi-family winter electricity demand equation is shown in Table 4.11. The price elasticity is small in absolute value, and insignificant from zero. The thermal elasticity is thus insignificant from negative one. It is statistically significant from zero. The only significant variables in the heating component of winter electricity demand are heating degree days and income. The latter variable has a negative coefficient.

The corresponding price equation is presented in Table 4.12. Its explanatory power and characteristics are similar to those estimated for mobile homes and single-family dwellings. The constant elasticity thermal efficiency equation is shown in Table 4.13. The equation was estimated using ordinary least squares. Although the coefficient on price is large enough to create concern about simultaneity, attempts to estimate the equation using 2SLS resulted in unsatisfactory estimates for all three equations. The only statistically significant coefficient is on age of the home, although both price and age of household head have t-ratios exceeding one. All coefficients are of the expected sign.

TABLE 4.11. Multi-Family Electricity Demand Equation
Constant Elasticity Form

$$\begin{aligned} \text{LKWH} = & -12.915 - 0.276 \cdot \text{LPRICE} - 0.724 \cdot \text{LTHETA} + 0.945 \cdot \text{LHddb} \\ & (2.05) \quad (0.176) \quad (0.176) \quad (0.212) \\ & - 0.176 \cdot \text{LINCOME} + 1.701 \cdot \text{LAPP} - 0.125 \cdot \text{LHEADAGE} + 0.151 \cdot \text{LNUMRES} \\ & (0.095) \quad (0.169) \quad (0.149) \quad (0.133) \end{aligned}$$

$$\begin{aligned} \bar{R}^2 &= 0.5304 \\ \text{d.f.} &= 99 \end{aligned}$$

Asymptotic standard errors in parentheses.

Key to Equation

LKWH = log (kilowatt hours used from November through April)
 LPRICE = log (average price of electricity from November through April,
 \$/kWh)
 LTHETA = log (shell efficiency)
 LHddb = log (average monthly heating degree days for November through
 April)
 LINCOME = log (yearly gross income)
 LAPP = log (calculated appliance electricity use for November through
 April)
 LHEADAGE = log (age of head of household)
 LNUMRES = log (number of residents)

TABLE 4.12. Multi-Family Electricity Price Equation
Constant Elasticity Form

$$\begin{aligned}
 \text{LPRICE} = & - 6.01 - 0.199 \cdot \text{LKWH} + 0.608 \cdot \text{LHDDDB} - 0.054 \cdot \text{LQPUBLIC} \\
 & (0.476) \quad (0.032) \quad (0.074) \quad (0.007) \\
 & + 0.015 \cdot \text{LQOREGON} - 0.018 \cdot \text{LQWASHTN} - 0.239 \cdot \text{CLIMATE2} \\
 & (0.009) \quad (0.066) \\
 & - 0.355 \cdot \text{CLIMATE3} \\
 & (0.150)
 \end{aligned}$$

$$\begin{aligned}
 \bar{R}^2 &= 0.7869 \\
 \text{d.f.} &= 98
 \end{aligned}$$

Asymptotic standard errors in parentheses.

Key to Equation

LPRICE = log (average price of electricity for November through April, \$/kWh)
 LKWH = log (kilowatt hours used from November through April)
 LQPUBLIC = LKWH • PUBLIC
 LQOREGON = LKWH • OREGON
 LQWASHTN = LKWH • WASHTN
 PUBLIC = a dummy, = 1 if in Public Utility District
 OREGON = a dummy, = 1 if in Oregon State
 WASHTN = a dummy, = 1 if in Washington State
 CLIMATE2 = a dummy, = 1 if in Climate zone 2
 CLIMATE3 = a dummy, = 1 if in Climate zone 3

TABLE 4.13. Multi-Family Shell Efficiency
Equation Constant Elasticity Form

$$\text{LCTHETA} = - 2.512 - 0.079 \cdot \text{LPRICE} + 0.062 \cdot \text{LHEADAGE}$$

(0.261) (0.051) (0.046)

$$- 0.067 \cdot \text{LHOMEAGE} + 0.093 \cdot \text{PAYTYPE}$$

(0.023) (0.090)

$$\bar{R}^2 = 0.1024$$

d.f. = 101

Asymptotic standard errors in parentheses.

Key to Equation

LCTHETA = log (CTHETA)

CTHETA = 100 • shell efficiency

LPRICE = log (average price of electricity for November through April,
\$/kWh)

LHEADAGE = log (age of head of household)

LHOMEAGE = log (age of residence)

PAYTYPE = dummy, = 1 if electricity included in rent

Variable Elasticity Equations

The three multi-family variable-elasticity equations are presented in Tables 4.14 - 4.16. The theta equation was estimated using OLS. The explanatory power of the demand equation is reasonable, but only two of the elasticities are significantly different from zero. Non-economic variables (climate, appliance stock) account for much of the equation's predictive power. The price equation is similar to the single-family variable-elasticity price equation. In particular, as quantity increases the price-quantity elasticity decreases in absolute value. Finally, the price and income coefficients in the theta equation are insignificant, as are most of the other variables.

Analysis of Equations

Further analysis, again using a more in-depth data base, would be helpful regarding multi-family residences. The data used in this analysis suffers from the same problems as described in Section 4.4.1 in discussion of mobile homes.

The low price elasticity and associated large thermal efficiency elasticity for this building category may be reasonable. Although statistical studies which distinguish between building types are not readily available, casual empiricism suggests the following. First, in the intermediate- or short-run under analysis here, residents of multi-family residences have only one instrument at their disposal to reduce electricity consumption when the price increases. Since many of these people are renters, they cannot install weatherstripping or make structural changes to the residence that owners of the single-family homes can. Renters can only lower the thermostat. Second, for many of these people the price paid for electricity is "high" relative to their income; thus, a large number of them may fall along the high price - low elasticity section of the demand function which was estimated for single-family residences. Low income elasticities, even negative estimates, may also be related to these and other general characteristics of the residents. For example, for the elderly, increases in income may be dedicated to activities which are substitutes for staying home, and thus for electricity consumption. Travel, eating out, and other recreational activities come to mind.

TABLE 4.14. Multi-Family Electricity Demand Equation
Variable Elasticity Form

$$\begin{aligned} \text{LKWHTH} = & 46.437 - 0.225 \cdot \text{LPRICETH} + 0.0016 \cdot \frac{(\text{LTHETA}+1)}{\text{PRICE}} \\ & (49.327) \quad (0.173) \quad (0.0006) \\ & - 0.158 \cdot \text{LINCOME} + 180.15 \cdot \frac{1}{\text{INCOME}} - 6.732 \cdot \text{LHDDDB} - 6520.4 \cdot \frac{1}{\text{HDDDB}} \\ & (0.254) \quad (1699.43) \quad (6.006) \quad (5233.4) \\ & + 1.619 \cdot \text{LAPP} + 62.659 \cdot \frac{1}{\text{APP}} + 0.481 \cdot \text{LNUMBRES} + 0.361 \cdot \frac{1}{\text{NUMRES}} \\ & (1.008) \quad (3299.6) \quad (0.712) \quad (1.16) \end{aligned}$$

$$\begin{aligned} \bar{R}^2 &= 0.498 \\ \text{d.f.} &= 94 \end{aligned}$$

Elasticities

(calculated at mean values)

MARGINAL PRICE	- 0.322 (0.189)
THETA	- 0.678 (0.189)
INCOME	- 0.173 (0.135)
HDDDB	1.358 (0.676)
APP	1.605 (0.311)
HEADAGE	0.277 (0.146)

Key to Equation

(calculated at mean values)

LKWHTH	= LKWH + LTHETA
LKWH	= log (kilowatt hours used during November through April)
LPRICE	= log (PRICE)
PRICE	= marginal price of electricity for November through April, \$/kWh
LTHETA	= log (THETA)
THETA	= shell efficiency
LINCOME	= log (INCOME)
INCOME	= yearly gross income
LHDDDB	= log (average monthly heating degree days for November through April)
LAPP	= log (APP)
APP	= calculated total appliance electricity use for November through April
LHEADAGE	= log (HEADAGE)
HEADAGE	= age of head of household
LPRICETH	= LPRICE - LTHETA

TABLE 4.15. Multi-Family Electricity Price Equation
Variable Elasticity Form

$$\begin{aligned} \text{LPRICE} = & -74.413 - 0.167 \cdot \text{LKWH} + 214.60 \cdot \frac{1}{\text{KWH}} + 7.509 \cdot \text{LHDD} \\ & (15.401) \quad (0.006) \quad (85.24) \quad \text{KWH} \quad (1.591) \\ & + 39714.6 \cdot \frac{1}{\text{HDD}} - 0.059 \cdot \text{LQPUBLIC} + 0.035 \cdot \text{LQOREGON} \\ & (9584.71) \quad \text{HDD} \quad (0.006) \quad (0.006) \end{aligned}$$

$$\begin{aligned} \bar{R}^2 &= 0.6368 \\ \text{d.f.} &= 98 \end{aligned}$$

Asymptotic standard errors in parentheses.

Key to Equation

LPRICE = log (average price for electricity for November through April,
\$/kWh)
 LKWH = log (kWh)
 KWH = kilowatt hours used from November through April
 LHDD = log (HDD)
 HDD = average yearly heating degree days (30-year average)
 LQPUBLIC = LKWH • PUBLIC
 LQOREGON = LKWH • OREGON
 LQWASHTN = LKWH • WASHTN
 PUBLIC = dummy, = 1 if in Public Utility District
 OREGON = dummy, = 1 if in Oregon State
 WASHTN = dummy, = 1 if in Washington State

TABLE 4.16. Multi-Family Shell Efficiency
Equation Variable Elasticity Form

$$\begin{aligned}
 \text{LCTHETA} = & - 18.063 + 0.001 \cdot \text{LPRICE} - 0.00095 \cdot \frac{1}{\text{PRICE}} \\
 & (22.190) \quad (0.225) \quad (0.0042) \\
 & + 2.195 \cdot \text{LHDD} + 11257.55 \cdot \frac{1}{\text{HDD}} - 0.0141 \cdot \text{LINCOME} - 102.89 \cdot \frac{1}{\text{INCOME}} \\
 & (2.329) \quad (13444.97) \quad (0.093) \quad (596.57) \\
 & + 0.105 \cdot \text{LHEADAGE} + 2.383 \cdot \frac{1}{\text{HEADAGE}} - 0.431 \cdot \text{LNUMRES} - 0.617 \cdot \frac{1}{\text{NUMRES}} \\
 & (0.367) \quad (14.27) \quad (0.268) \quad (0.429) \\
 & - 0.094 \cdot \text{LHOMEAGE} - 0.134 \cdot \frac{1}{\text{HEADAGE}} + 0.055 \cdot \text{PAYTYPE} \\
 & (0.047) \quad (0.247) \quad (0.105)
 \end{aligned}$$

$$\begin{aligned}
 \bar{R}^2 &= 0.0539 \\
 \text{d.f.} &= 92
 \end{aligned}$$

Asymptotic standard errors in parentheses.

Key to Equation

LCTHETA = log (100 • shell efficiency)
 PRICE = (average price of electricity for November through April,
 \$/KWH)
 LPRICE = log (PRICE)
 LHDD = log (HDD)
 HDD = average yearly heating degree days, (30-year average)
 LINCOME = log (INCOME)
 INCOME = gross yearly income
 LHEADAGE = log (HEADAGE)
 HEADAGE = age of head of household
 LNUMRES = log (NUMRES)
 NUMRES = number of residents
 LHOMEAGE = log (HOMEAGE)
 HOMAGE = age of residence
 PAYTYPE = dummy, = 1 if electricity included in rent

4.4.4 Estimated Energy Savings and Consumer Benefits From Conservation Programs

Table 4.17 presents, in summary form, 95 percent confidence intervals for projected energy savings caused by a one percent increase in the thermal shell efficiency. The "center" of each interval is also shown. This represents the point estimate of percentage energy savings. Three confidence intervals are given for each of the building categories: one with the thermal efficiency elasticity evaluated at the minimum sample price, one the mean sample price, and one the maximum sample price. Approximate standard errors calculated at the means of the data are multiplied by 1.96 to construct the interval. Thus, each triplet of numbers (l,m,n) shows, for a given building category and price assumption, the following: the lower bound (l), the mean value (m), and the upper bound (n), in absolute values, of the percentage reduction in electricity used for space heating.⁽¹⁾ In each of the minimum price scenarios, the estimated price elasticity is so large that the price effect of an efficiency increase dominates the engineering effect. In these situations, electricity consumption for heating increases as a result of an increase in thermal efficiency. The table shows that in the first two building categories, the maximum decrease in consumption associated with a one percent increase in shell efficiency is less than .65 percent. For multi-family residences, the standard errors are so large that nearly anything could happen: for the mean price scenario, consumption could decrease as little as .308 percent or as much as one percent when efficiency increases one percent.

The benefits to consumers of improvements in thermal efficiency were discussed generally in Section 4.1.2. As shown in Figure 4.3, these benefits are equal to the sum of two components. The first, labeled C in the graph, represents the "windfall gain" to consumers associated with a reduction in the price of internal comfort from P_0' to P_1' , holding the quantity consumed of comfort constant at Q_0' . The second component, labeled D, corresponds to the increase from Q_0' to Q_1' of the consumption of internal comfort associated with the

(1) Values of b greater than one were set equal to one, causing a truncated confidence interval.

TABLE 4.17. Estimates of Percent Reduction in Space Heating
Electricity Use - 95 Percent Confidence Intervals

	<u>Lower Bound</u>	<u>Center</u>	<u>Upper Bound</u>
<u>Mobile Home</u>			
Minimum Price	(+).518	(+).448	(+).377
Mean Price	.148	.219	.290
Maximum Price	.271	.342	.412
<u>Single Family</u>			
Minimum Price ⁽¹⁾	(+).482	(+).400	(+).318
Mean Price	.464	.546	.628
Maximum Price	.471	.553	.635
<u>Multi-Family</u>			
Minimum Price	(+)1.200	(+).830	(+).460
Mean Price	.308	.678	1.0 ⁽²⁾
Maximum Price	.388	.758	1.0 ⁽²⁾

(+) denotes electricity consumption increase caused by one percent decrease in thermal efficiency.

(1) next-to-minimum price used because of presence of outlier.

(2) truncated

FIGURE 4.3. Increase in Consumer Surplus

reduction in price of internal comfort from P_0' to P_1' . The sum of areas C and D represents the increase in consumer surplus or, alternatively, the benefits to consumers due to a fall in the price of internal comfort from P_0' to P_1' .

The axes in Figure 4.1 are the price of comfort in dollars per unit and the quantity consumed of comfort. Holding the environmental variables N_1 fixed, there is a one-to-one relationship between 1) internal comfort and energy in efficiency units and 2) price of internal comfort in dollars per unit of comfort and dollars per unit of energy measured in efficiency units. Thus, the analysis of benefits of conservation programs could have been performed in terms of energy use measured in efficiency units. In this case, the drop in price measured in efficiency units is more clearly associated with the increase in thermal shell efficiency which caused it; in fact, the percentage drop in price, $(P_1' - P_0')/P_0'$, is exactly equal in magnitude to the percentage increase in thermal efficiency, $(\theta_1 - \theta_0)/\theta_0$.

The area C in Figure 4.1 can be approximated in energy efficiency units by $Q_0' (P_0' - P_1')$. The area D can be represented by $1/2 (Q_1' - Q_0')(P_0' - P_1')$. Thus, the increase in total consumer surplus due to the increase in thermal efficiency is given by

$$\Delta C.S. = -Q_0' (P_1' - P_0') - 1/2 (Q_1' - Q_0')(P_1' - P_0') \quad (4.68)$$

or

$$\Delta C.S. = -Q_0' \Delta P' - 1/2 \Delta Q' \Delta P' \quad (4.68^1)$$

where

$$\Delta P' = P_1' - P_0' \text{ and } \Delta Q' = Q_1' - Q_0'.$$

Of course, ΔQ is not an arbitrary quantity, but related to the price elasticity of demand measured in efficiency units ϵ' (which equals the price elasticity of demand measured in physical units, ϵ), and the percentage change in the price, measured in efficiency units. By definition,

$$\epsilon' = \frac{\Delta Q'}{Q_0'} \frac{P_0'}{\Delta P'} , \quad (4.69)$$

so

$$\Delta Q' = \epsilon' \frac{\Delta P'}{P_0'} Q_0' . \quad (4.70)$$

Thus,

$$\Delta C.S. = -Q_0' \Delta P' - \frac{1}{2} \frac{Q_0'}{P_0'} (\Delta P')^2 \epsilon' \quad (4.71)$$

Furthermore, a given percentage change in shell efficiency causes an equal percentage change in the price of energy measured in efficiency units, but of the opposite size. Suppose thermal efficiency increases K percent; then P' falls K percent, or $\Delta P' = -P_0' K$. Thus, for an increase in thermal efficiency of K percent, by substitution

$$\Delta C.S. = K P_0' Q_0' - 1/2 \epsilon' K^2 Q_0' P_0' \quad (4.72)$$

or

$$\Delta C.S. = K P_0' Q_0' (1 - 1/2 \epsilon' K) . \quad (4.73)$$

The above calculations are in terms of efficiency units. However, given the definitions of Q' and P' presented in Section 4.2.3, similar formulas can be obtained involving Q and P , energy consumption and the price of energy measured in physical units. Thus, the sum of the areas C and D can be calculated in observable, physical units, using estimates of the space conditioning electricity consumption price elasticity, ϵ . In particular,

$$P' = P/\theta$$

so

$$\Delta P' = 1/\theta \Delta P - P/\theta^2 \Delta \theta \quad (4.74)$$

and

$$P_0' = P_0/\theta \quad (4.75)$$

where P_0 is the initial price of energy measured in physical units, ΔP is the change in the price measured in physical units, and $\Delta\theta$ is the change in the shell efficiency. Furthermore,

$$Q' = Q\theta$$

so

$$\Delta Q' = \Delta Q \cdot \theta + \Delta\theta \cdot Q \quad (4.76)$$

and

$$Q_0' = Q_0 \cdot \theta \quad (4.77)$$

By substitution of these identities into equation 4.68

$$\Delta C.S. = Q_0 (\Delta P - P_0/\theta_0 \Delta\theta) - 1/2 [Q_0 \Delta\theta + \theta_0 \Delta Q](1/\theta_0)(\Delta P - P_0/\theta_0 \Delta\theta). \quad (4.78)$$

But P in physical units is held constant; thus, $\Delta P = 0$. Hence,

$$\Delta C.S. = \frac{Q_0 P_0}{\theta_0} \Delta\theta + \frac{1}{2} \left[-\frac{Q_0 P_0}{\theta_0^2} (\Delta\theta)^2 + \frac{P_0}{\theta_0} \Delta Q \Delta\theta \right] \quad (4.79)$$

But

$$\Delta Q = Q_0 \frac{\Delta\theta}{\theta_0} \eta, \quad (4.80)$$

where

η is the elasticity of space conditioning electricity consumption with respect to shell efficiency. Also, for a K percent increase in shell efficiency, $\Delta\theta = K \theta_0$. Thus, by substitution,

$$\Delta C.S. = Q_0 P_0 K + \frac{1}{2} Q_0 P_0 K^2 (1 + \eta) \quad (4.81)$$

or

$$\Delta C.S. = Q_0 P_0 K - \frac{1}{2} Q_0 P_0 K^2 \epsilon \quad (4.82)$$

where ϵ is the price elasticity of space conditioning electricity demand in physical units. Equation 4.82 defines the increase in consumer surplus for a single household due to a K percent increase in thermal efficiency of the residence. Since the quantities Q_0 and P_0 are observable, and estimates of ϵ were presented above, the equation can be used to calculate the benefits to an individual consumer of a particular percentage change in thermal efficiency.

Equation 4.82 was evaluated for each of the three building categories, using the following data: 1) sample mean winter dollar per kilowatt prices for each building category; 2) sample mean winter kilowatt consumption of electricity for each building category; 3) price elasticities estimated using the variable-elasticity functional form, evaluated at the sample means: 4) a ten percent increase in thermal shell efficiency. The estimated increases in consumer surplus per winter or, alternatively, the benefits per winter to individual households of a ten percent increase in thermal efficiency, were: 1) mobile homes, \$23.81; 2) single-family residences, \$21.23; 3) multi-family residences, \$12.94.

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