Laboratory for Manufacturing and Productivity

School of Engineering

A STUDY OF REDUCED ORDER MODEL
REFERENCE ADAPTIVE CONTROL SYSTEMS
FOR IMPROVED PROCESS CONTROL

Final Report
Period May 1, 1982 to January 31, 1986

David E. Hardt
Evangelos Papadopoulos
Atsushi Suzuki

DISTRIBUTION OF THIS DOCUMENT IS UNLIMITED

Massachusetts Institute of Technology

DISCLAIMER

This report was prepared as an account of work sponsored by an agency of the United States Government. Neither the United States Government nor any agency Thereof, nor any of their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by trade name, trademark, manufacturer, or otherwise does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof.

DISCLAIMER

Portions of this document may be illegible in electronic image products. Images are produced from the best available original document.

M 86008098

DOE/ER/12073--1

DE86 008098

A STUDY OF REDUCED ORDER MODEL
REFERENCE ADAPTIVE CONTROL SYSTEMS
FOR IMPROVED PROCESS CONTROL

Final Report
Period May 1, 1982 to January 31, 1986

David E. Hardt
Evangelos Papadopoulos
Atsushi Suzuki

DISCLAIMER

This report was prepared as an account of work sponsored by an agency of the United States Government. Neither the United States Government nor any agency thereof, nor any of their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by trade name, trademark, manufacturer, or otherwise does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof.

MASTER

DISTRIBUTION OF THIS DOCUMENT IS UNLIMITED

NOTICE

This report was prepared as an account of work sponsored by the United States Government. Neither the United States nor the Department of the Energy, nor any of their employees, nor any of their contractors, subcontractors, or their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product or process disclosed or represents that its use would not infringe privately-owned rights.

**DO NOT MICROFILM
THIS PAGE**

A STUDY OF REDUCED ORDER MODEL REFERENCE ADAPTIVE
CONTROL SYSTEMS FOR IMPROVED PROCESS CONTROL

Final Report
for the Period May 1, 1982 to January 31, 1986

David E. Hardt
Evangelos Papadopoulos
Atsushi Suzuki

Laboratory for Manufacturing and Productivity
Massachusetts Institute of Technology
Cambridge, Massachusetts 02139

February 1986

Prepared for

THE U.S. DEPARTMENT OF ENERGY
AGREEMENT NO. DE-AC02-82ER12073

ABSTRACT

The objective of this research is to provide a method for design of control systems when the plant involved is highly variable in both structure and parameter values. This typifies many manufacturing processes, which in general are highly sensitive to intrinsic properties of the infeed material, have poorly understood or controlled boundary conditions, and often have highly non-linear characteristics. In addition, the disturbances most frequent in a manufacturing process are best characterized as parameter changes rather than state variable changes. The choice of a parameter adaptive control paradigm is obvious since it has the property of optimizing system response characteristics in the face of wide plant parameter variations. Model Reference Adaptive Systems (MRAS) enjoy an ease of performance specification, and again seem applicable; however, there is a restriction within current theory that the plant be characterized in its structure well enough to have a reference model of identical order. This restriction implies a static plant structure and more a priori information than is often present.

During this contract the basic design characteristics of certain MRAS have been studied and problems with reduced order models identified, such as noise and input magnitude sensitivity and instability caused by high frequency learning signals. This has lead to certain solutions and resulting guidelines for design. This is then followed by design studies for a position servomechanism (which serves as the foundation of many automated manufacturing processes). This study expanded the domain from MRAS to self-tuning controllers (STC) and found that both can deal with reduced order models if the initial parameter values are not too distant from the actual values. However, the STC appears to be the more robust.

Although this contract is being terminated, the application aspect of this work is continuing in the context of automated welding control, which will severely test the capabilities of the MRAS and STC approaches.

INTRODUCTION

The field of adaptive control has emerged strongly in recent years because of a desire to apply analytical control methods to plants that are not well characterized. This poor characterization may include both linear systems with unknown or slowly varying parameters or inherently non-linear plants with both continuous and discontinuous non-linearities. Manufacturing processes, particularly those with a significant thermal component, typify systems that are both inherently non-linear and time or boundary dependent in their dynamics. For example, a welding process has melting dynamics that are non-linear with respect to the size of the weld bead, and for a fixed bead size the model parameters depend strongly on the thermal boundary conditions [1]. Representative of a broad class of manufacturing problems, the forming of continuous shapes from extrusions or tubing has been shown to be controllable in a truly closed-loop sense [2], but the dynamics of the system depend both on the properties of the material involved and on the length of the material being processed [3]. Both of these can change considerably, making high performance control systems difficult to stabilize. More importantly, manufacturing processes have more stringent requirements than just stability in the control system since the actual static and dynamic performance directly affects the quality of the manufactured part.

Because of these growing control applications in manufacturing processes, the model reference adaptive scheme has emerged as a strong candidate since it allows the control designer to explicitly specify a target dynamic model that the overall system should emulate. This implies that MRAS's should be very easy to design and have a highly predictable performance. However, the last two decades of research into MRAS's has concentrated on theoretical issues of global stability (since they are inherently non-linear systems) and specific design issues have not yet been completely addressed.

This research has the broad charge of investigating the applicability of adaptive control schemes to system that are poorly modelled, with an emphasis on reference model mis-match with respect to the plant. However, in the course of this research other problems inherent in the parallel reference model scheme were addressed, and as noted in the second year incremental funding request, it was found necessary to look beyond just the MRAS paradigm and investigate explicit identification methods, in particular the self-tuning control (STC) approach.

SUMMARY OF RESULTS

This investigation has followed two distinct paths, one aimed at general issues in MRAS as applied to reduced order models and one aimed at specific application studies. The latter is not complete and will continue beyond this contract, however initial results are included here.

The details of the results are included in Appendix I "Guidelines for Application of Adaptive Control" and Appendix II "Experimental Study on Adaptive Control of a Position Servo System", however the conclusions of each are summarized here.

In the former study, a linear first-order reference model was used to study the effect of different design parameters on MRAS performance, particularly where a higher order plant was used. Specifically a third order plant with so-called "unmodelled, high order dynamics" was used. The behavior of three different MRAC algorithms are studied, these being the Hyperstability Design, the Augmented Error Design and the Sensitivity Steepest Descent Design. Each is studied assuming (1) that the plant structure is exactly known, (2) that the plant contains unmodeled dynamics. It is shown that under the first assumption the presence of a wide range of reference inputs and the introduction of learning signals improves the adaptation while the output noise merely deteriorates the system response. However, in the presence of unmodeled dynamics, all these situations can cause instabilities. In order to remove the undesirable effects of these algorithms we propose simple application-oriented guidelines that ensure stability. These include normalization of the generalized error by the input and state magnitude, the use of bounded frequency learning signals and the use of model feedback during noise sensitive portions of the adaptation response.

Appendix II addresses a specific application of both MRAC and STC to a real servo mechanism. This was performed to investigate the actual use of adaptive control in a non-ideal environment and to investigate again the effect of reduced order model. The servo used was a large (9000 Watt) DC motor driven system with typical characteristics of backlash, friction and saturation. It was used both with and without inner loop or motor speed compensation to provide two distinctly different situations. The adaptation was forced by initially tuning a position regulator to incorrect values and letting the adaptive mechanism change the control parameters to obtain the desired response.

It was found that both the MRAC hyperstability paradigm and the STC method of Astrom [4] gave good performance under the best servo conditions, but when the initial control parameters were chosen far from the actual (particularly when the plant was assumed to be faster than the actual) the MRAC could take a long time to converge. It was also shown that both methods could adapt to severe changes in the plant dynamics as represented by opening the inner loop of the servo system. This served not only to change the poles of the plant, but also introduced harsh friction and dead-band effects into the system. In general the comparison between MRAC and STC for this system favors the latter since it is faster to adapt and is more robust. However it is more complex in implementation and requires longer computation times than the MRAC.

CONCLUSIONS

From simulation work, several shortcomings of the various candidate MRAC methods have been identified and some design solutions proposed. These have not been exhaustively examined in experimental tests, but initial tests have indicated a greater robustness in actual application than theory would indicate. This work will be continued under different sponsorship and will extend the experimental studies to more severe cases (for example arc welding pool geometry control) and summarize the overall aspects of MRAC vs. STC. It is hoped that this work will spur greater use of adaptive control in essential applications and will provide a basis for comparison of various new methods for non-linear system control that are currently emerging. However, it is clear that with any new control theory the final basis of comparison must be in actual application and that is the basis upon which our conclusions will be drawn.

References

1. Hardt, D.E., Garlow, D.A. and Weinert, J.B., "A Model of Full Penetration Arc-Welding for Control System Design", J. of Dynamic Systems, Measurement and Control, Vol. 107, March 1985, pp. 40-46.
2. Hardt, D.E., Roberts, M.A., and Stelson, K.A., "Closed Loop Shape Control of a Roll Bending Process," Transactions of ASME, Journal of Dynamic Systems, Measurement and Control, 104, no. 4, 1982.
3. Hale, M., Hardt, D.E., Dynamic Analysis and Control of a Roll Bending Process, Proc. Amer. Control Conference, June, 1986.
4. Astrom, K, and Wittenmark, J., " Self-Tuning Controllers based on Pole-Zero Placement", IEE Proceedings, Vol. 127, Pt.D., No.3, 1980.

PUBLICATIONS

1. Papadopoulos, E.G. and Hardt, D.E. " Model Reference Adaptive Control: Model Error and Disturbance Effects", DOE Symposium on Non-Linear Problems in Energy Engineering, Argonne, Ill, 1983.
2. Papadopoulos, E.G. "Model Reference Adaptive Control : Some Problems and Solutions", S.M. Thesis, Department of Mechanical Engineering M.I.T., May, 1984.
3. Papadopoulos, E.G. and Hardt, D.E. " A Review of Model Reference Adaptive Control Algorithm : Solutions to Application Problems"
MIT Laboratory for Manufacturing Report No. LMP-FMR-83-13, 1983.
4. Suzuki, A. And Hardt, D.E., "Experimental Study of Adaptive Control of A Position Servo: Comparison of MRAC and STC", MIT Laboratory for Manufacturing Report No. LMP-FMR-86-01, 1986.

Appendix I

APPLICATION GUIDELINES FOR MRAC ALGORITHMS

by

Evangelos G. Papadopoulos
Research Assistant

David E. Hardt
Associate Professor

Laboratory for Manufacturing and Productivity
Massachusetts Institute of Technology

Report LMP - FMR - 83-13

May 1983

ABSTRACT

This paper presents three different MRAC approaches, analyzes the resulting adaptive laws and discusses similarities and differences among them. The behaviour of these algorithms is studied assuming (1) that the plant structure is exactly known, (2) that the plant contains unmodeled dynamics. It is shown that under the first assumption the presence of a wide range of reference inputs and the introduction of learning signals improve the adaptation while the output noise merely deteriorates the system response. However, in the presence of unmodeled dynamics, all these situations can cause instabilities. In order to remove the undesirable effects of these algorithms we propose simple application-oriented guidelines that ensure stability.

1. INTRODUCTION

Since its first introduction by Whittaker (1959) Model Reference Adaptive Control has been a stimulating field of research aiming at a variety of important applications. The works of Monopoli (1974), Narendra (1978), Landau (1979), and Goodwin et al. (1980), have laid the foundations of globally stable algorithms for the deterministic adaptive control problem.

The stability proofs of these algorithms require the knowledge of the structure of the plant, which is assumed to be linear. If this assumption holds, then one can measure the parametric error distance through the conditioning of an error signal which is obtained by comparing the plant output to the output of a model of the same structure as that of the plant.

However, this assumption cannot hold in most cases, because practically all plants cannot be exactly modelled. Recently, much effort has been devoted to the robustness of MRAC algorithms in the presence of unmodelled dynamics. The works of Ioannou (1982), and Rohrs et al. (1982), have shown that if this assumption does not hold, MRAC algorithms may lead to instabilities, making thus MRAC impractical.

The purpose of this paper is to propose algorithm modifications that can overcome the MRAC deficiencies. We first present three different MRAC approaches and compare the adaptive laws that are suggested by each one for the first order plant that was used by Rohrs et al. (1982). We then examine the behavior of these laws in two cases (a) when the plant structure is known and, (b) when the structure contains unmodelled dynamics. The analysis that follows leads to the suggestion of application guidelines. These include the normalization of the generalized error, the restriction of the input frequency content and the use in the MRAC algorithm information extracted from the model.

2. REVIEW OF MRAC ALGORITHMS

The most basic approach to the adaptive control problem has its roots in stability considerations. This is essential because adaptive control systems are highly nonlinear and hard to analyze.

Techniques that ensure global stability are preferable because they remove the burden of stability analysis and allow the designer to concentrate on algorithm parameter selection.

The basic philosophy behind the MRAC designs we are going to consider in the paper is the following: Assume first that the plant is exactly known (both in structure and parameters). Then, if the plant does not contain any non minimum frequency zeros, it is possible to design a compensator that will place all plant poles and zeros. However, if the plant parameters are unknown, this will not be possible. If one uses the same compensator structure but adjusts its parameters until the plant poles and zeros are being placed satisfactorily, then he does not have to estimate the exact plant parameters. A MRAC performs precisely this function.

By placing the poles and zeros, we want to achieve a prescribed performance. This can be described by a model which is set in "parallel" with the closed loop plant. A "satisfactory" performance is then obtained when the generalized error $e = y - y_m$ (see Fig. 2.1) is zero. If it is not zero, it is fed in the MRAC adaptation mechanism which adjusts the compensator gains until e is driven to zero. What remains is to design an algorithm that will drive the compensator parameters towards the right direction.

The hyperstability design is one of the stability driven approaches. It relies on the hyperstability theorem by Popov and is presented by Landau (1975).

Referring to Fig. 2.2, we note that in order to apply the theorem we must meet two requirements:

- (1) The linear forward transfer function must be strictly positive real (i.e. its Nyquist plot must be in the first and fourth quadrants only).
- (2) The nonlinear feedback must satisfy the Popov inequality:

$$\int_{t_0}^t \underline{w}^T \underline{v} dt \geq -\gamma_0^2 \quad \text{for all } t \geq t_0 \quad (2.1)$$

where $\underline{w}$ is the input to the linear forward block, $\underline{v}$ is the output of the same block and γ_0 is a real constant.

We will demonstrate this technique using a first order system with two unknown parameters. Consider the system:

$$\dot{x} = -ax + bu \quad (2.2)$$

where x is the system state, u the control input and a and b the unknown parameters. The control goal is to force (2.2) to behave like a first order model of the form:

$$\dot{x}_m = -a_m x_m + b_m r \quad (2.3)$$

where x is the model state and r the reference input. If we knew what a_m and b were, the following control law would achieve our goal:

$$u = -K_x x - K_r r \quad (2.4)$$

However, a and b are unknown; if K_x and K_r are constant gains, (2.2) will not follow (2.3) in all the cases where a and b have not been exactly estimated. In order to update K_x and K_r we must use some information from the plant behavior. Such an information can be provided by the generalized error e :

$$e = x - x_m \quad (2.5)$$

A basic assumption here is that this error is a function of the parametric distance only, i.e. if $a + bK_x = a_m$ and $bK_r = -b$ then e is identically zero. In the sequel we will show that if this condition is not valid, serious problems may arise.

In order to use the Popov theorem for design purposes, we subtract (2.3) from (2.2) combined with (2.4), and we get:

$$\dot{e} = -a_m e - (a + bK_x - a_m) x - (bK_r + b_m) r \quad (2.6a)$$

Let,

$$w = (a + bK_x - a_m)x + (bK_r + b_m)r \quad (2.6b)$$

$$v = e \quad (2.6c)$$

and after some analysis one can show that if the adaptive laws are of the form:

$$K_x = K_a x e + \frac{d}{dt} K_e x e \quad (2.7a)$$

$$K_r = K_a r e + \frac{d}{dt} K_e r e \quad (2.7b)$$

the Popov theorem is satisfied. K_a and K_e are constant gains. It seems that these forms are the most general ones*. Fig. 2.3 depicts the implementation of the hyperstability design.

In order to analyze this algorithm and select the adaptive gains K_a and K_e we follow the next three steps (see also Rohrs et al (1982), Papadopoulos (1983)).

(1) Shift the origin, using the relations:

$$e = x - x_m \quad (2.8a)$$

$$K_x = K_x - (a_m - a)/b \quad (2.8b)$$

* Different forms can be obtained by replacing K_a and K_e by some time decreasing positive gain or by replacing the integrator in (2.6) by a strictly positive real transfer function with a pole at $s=0$. However, these variations will not affect the arguments that follow.

$$\delta K_r = K_r - (-b_m)/b \quad (2.8c)$$

- (2) Linearize around the adaptation point (the generalized error is small) and get the linear time-variant description:

$$\begin{bmatrix} \dot{e} \\ \delta \dot{K}_x \\ \delta \dot{K}_r \end{bmatrix} = \begin{bmatrix} -(a_m + bK_e (r^2 + x_m^2)) & -bx_m & -br \\ K_a x_m & 0 & 0 \\ K_a r & 0 & 0 \end{bmatrix} \begin{bmatrix} e \\ \delta K_x \\ \delta K_r \end{bmatrix} \quad (2.9)$$

- (3) Assuming that r is a step input and that $x_m = r$, we can find the characteristic equation which is

$$s[s^2 + [bK_e (x_m^2 + r^2) + a_m]s + bK_a [x_m^2 + r^2]] = 0 \quad (2.10)$$

We note that the coefficients of eq. (2.10) depend on the adaptation mechanism gains K_a and K_e and on the magnitude of the reference input r . The adaptation mechanism natural frequency is proportional to r and this is also true for the mechanism damping ratio. Thus, as r increases, we expect the system to follow closer the model without any overshoots or undershoots; in other words we expect to see an improved performance. The gains K_a and K_e can be selected from (2.10) by assuming some nominal b (i.e. $\hat{b}=b_m$) and some nominal r . Then, one can place the (2.10) poles at a distance five times the model natural frequency (a_m). We also note that depends on K_e ; K_e adds proportional action to the integral one (K_a ex) and increases the adaption mechanism damping.

2.2 The Lyapunov Design

Following this approach, we set up a Lyapunov function of the states. Two algorithms will be considered here:

- (a) Monopoli's Algorithm.

Using the same first order system as before, we set up the function:

$$V = \frac{1}{2} [e^2 + \frac{b}{K_a} K_x^2 + \frac{b}{K_a} K_r^2] \quad (2.11)$$

where e , K_x and K_r are defined by eqs.(2.8). It can be shown that if one chooses:

$$\dot{K}_x = K_a x_e \quad (2.12a)$$

$$\dot{K}_r = K_a r_e \quad (2.12b)$$

the first derivative of V is negative semi-definite.

$$\dot{V} = -a_m e^2 \leq 0 \quad (2.13)$$

This result, along with some additional analysis (that takes care of the equality), proves that the system is uniformly asymptotically stable in the large.

We note that the adaptive laws (2.12) are similar to the (2.7) ones if one sets $K=0$. This means that this algorithm does not incorporate proportional action.

A similar near adaptation technique as before leads to the following characteristic equation:

$$s[s^2 + a_m s + bK_a (x_m^2 + r^2)] = 0 \quad (2.14)$$

It is noted that the adaptation mechanism natural frequency is still proportional to the reference signal r while the damping ratio is now proportional to the inverse of r . Thus, high r 's will lead to a higher bandwidth adaptation mechanism exhibiting poorly damped oscillations.

(b) The Narendra, Lin and Valarani Algorithm (1980)

This is an improved globally stable algorithm. The basic difference with Monopoli's algorithm is that in this case auxiliary signals that improve stability are included. For a first order system, the adaptation laws turn out to be

$$\dot{K}_x = K_a x e \quad (2.15a)$$

$$\dot{K}_r = K_a r e \quad (2.15b)$$

$$\varepsilon = e + e^* \quad (2.15c)$$

$$\dot{e}^* = -[a_m + b_m K_e (r^2 + x^2)]e^* - b_m K_e (r^2 + x^2)e \quad (2.15d)$$

where ε is the "augmented" error. Following the same analysis technique as in section 2.1, we can show that the augmented error is obtained through filtering e with a variable bandwidth filter that attenuates e by r^2 (see Fig. 2.4). Proportional action is again introduced through a feedforward term that appears as the filter zero.

Finally, we present an algorithm which is based on locally valid assumptions.

2.3 The Sensitivity-Steepest Descent Design.

This approach is derived from on-line identification techniques. It is assumed that the plant structure is known and that $\underline{a}$ represents the unknown parameter vector.

$$\dot{\underline{x}} = f(\underline{x}, \underline{a}, \underline{r}) \quad (2.15)$$

A model exhibiting the same (known) structure is considered:

$$\dot{\underline{x}}_m = f_m(\underline{x}_m, \underline{a}_m, \underline{r}) \quad (2.16)$$

We define the generalized error as $\underline{e} = \underline{x} - \underline{x}_m$ and set up the Lyapunov

function:

$$V = \frac{1}{2} \underline{e}^T Q \underline{e} \quad (2.17)$$

where Q is a weighting matrix. If one chooses the adaptation mechanism according to (2.18):

$$\dot{\underline{a}} = -K \Lambda^T Q^T \underline{e} \quad (2.18)$$

where Λ is the sensitivity function:

$$\Lambda = \frac{\partial \underline{e}}{\partial \underline{a}} = \frac{\partial \underline{x}}{\partial \underline{a}} \quad (2.19)$$

and if locally the following approximate relation holds:

$$\Lambda = \frac{\partial \underline{x}}{\partial \underline{a}} \approx \frac{\partial \underline{x}_m}{\partial \underline{a}_m} \quad (2.20)$$

then, it turns out that V is negative semidefinite. Λ is supplied by differentiating (2.16):

$$\dot{\Lambda} = \frac{\partial \underline{f}_m}{\partial \underline{x}_m} \Lambda + \frac{\partial \underline{f}_m}{\partial \underline{a}_m} ; \Lambda(0) = 0 \quad (2.21)$$

This technique does not guarantee a globally stable system but is presented here for completeness and because it behaves well in the presence of noise.

For a first order plant, the above formulation may lead to the following adaptive laws (we also use e):

$$\dot{K}_x = -(q_0 e + q_1 \dot{e})(q_0 \lambda + q_1 \dot{\lambda}) \quad (2.22a)$$

$$\dot{K}_r = (q_0 e + q_1 \dot{e})(q_0 \mu + q_1 \dot{\mu}) \quad (2.22b)$$

$$\dot{\lambda} = -a_m \lambda - x_m ; \lambda(0) = 0 \quad (2.22c)$$

$$\dot{\mu} = -a_m \mu + r ; \mu(0) = 0 \quad (2.22d)$$

Fig. 2.3 depicts the implementation of this adaptive mechanism.

2.4 General Comments

The adaptive laws discussed so far have common elements. A general form for them is:

$$\dot{K} = K_a \{ \text{State or Input Information} \} \{ \text{Generalized Error Info.} \} \quad (2.23)$$

The generalized error information provides a measure of the parametric distance between the plant and the model while the state or input information correlates this distance with the corresponding gain. A high adaptive gain K explores the error information more efficiently. All algorithms employ the error or some filtered version of it. The error derivative may or may not be used. However, while the Hyperstability and the Lyapunov designs use the state x in the adaptation mechanism (2.23), the SSDD design uses information extracted from the model through eqs.(2.21).

Table I. Summary of Adaptive Laws for a First Order Plant

1. HYPERSTABILITY

$$\dot{K}_x = K_a x_e + K_e \frac{d}{dt} x_e$$

$$\dot{K}_r = K_a r_e + K_e \frac{d}{dt} r_e$$

2. MONOPOLI'S ALGORITHM

$$\dot{K}_x = K_a x_e$$

$$\dot{K}_r = K_a r_e$$

3. NLV ALGORITHM

$$\dot{K}_x = K_a x_e$$

$$\dot{K}_r = K_a r_e$$

$$\varepsilon = e + e^*$$

$$\dot{e}^* = -[a_m + b_m K_e (r^2 + x^2)] e^* - b_m K_e (r^2 + x^2) e$$

4. SSDD

$$\dot{K}_x = -(q_0 e + q_1 \dot{e})(q_0 \lambda + q_1 \dot{\lambda})$$

$$\dot{K}_r = (q_0 e + q_1 \dot{e})(q_0 \mu + q_1 \dot{\mu})$$

$$\dot{\lambda} = -a_m \lambda - x_m ; \lambda(0) = 0$$

$$\dot{\mu} = -a_m \mu + r ; \mu(0) = 0$$

In the next section, we evaluate the algorithms performance in two cases: (i) The plant is a first order system (eq. 2.2) with $a=1$, $b=2$, (assumed unknown). The system pole is at -1 . (ii) The plant is of third order but only its low frequency structure is known. The following plant is used:

$$\frac{x}{u} = \frac{y}{u} = \frac{2}{s+1} \quad \frac{229}{s^2 + 30s + 229} \quad (2-24)$$

The above plants are the ones used by Rohrs (1982) and they are used here for comparison.

3. THE BEHAVIOR OF MRAC ALGORITHMS IN VARIOUS OPERATION CONDITIONS

Although a large amount of literature on adaptive algorithms and the associated stability proofs exists, very few studies deal with the behavior of these algorithms in "real world" applications. However, a good understanding of these algorithms is an essential prerequisite for their application. Recent studies Ioannou (1982), Rohrs (1982) revealed that in cases where the algorithm assumptions are violated, poor performance or even instability may occur. In this section we will examine the algorithms performance in the case where the plant is of known structure and in the case where high order unknown dynamics are present. In this last case, the basic assumption of all MRAC algorithms that the error is a measure of the parametric distance only is violated.

3.1 The Case of a Wide Range of Reference Inputs.

We first assume that the plant is of first order (eq. (2.2) with $a=1$, $b=2$ and eq. $a=3$, $b=3$). Using the Hyperstability design to control the first order plant with $K_a=10$, $K_r=3$ and $r=1$, we obtain Fig. (3.1). We note that the state x_a follows the model state x_m . For perfect adaptations ($e=0$ for all t), K_x and K_r should be:

$$K_x = 1.0 \quad (3.1a)$$

$$K_r = -1.5 \quad (3.1b)$$

This does not happen, but the trend is correct and the results can be improved by using higher adaptive gains. Fig. (3.2) is obtained with $K_a=100$, $K_r=10$. K_x and K_r are now closer to (3.1) and will tend to converge to (3.1) after many step input transients. Exactly the same effect is achieved if the size of the step input is increased. This can be explained using the characteristic equation (2.10). Increasing K_a or r increases the bandwidth and effectiveness of the adaptation mechanism. However, this non-linearity may be a problem in the presence of unmodeled high frequency dynamics. Indeed, if we try to control the full order plant and r is greater than 1.35, the plant becomes unstable (Fig. 3.3). This can be explained as follows: A high reference input

leads to a high bandwidth adaptation mechanism. Although this is favorable in the case where the plant is well modeled, it excites any present high frequency (unmodeled) dynamics because there is no way to reduce the part of the error that depends on the structural mismatch between the plant and the model. This leads to instability. If one wants to examine this more quantitatively, then one must use the fifth order characteristic equation which is obtained in the same way as (2.10) by replacing the first order plant with the full order one. It turns out that the equation is unstable if r is greater than 1.35 ($K_a=10$). The error increases, driving K_x and K_r to a level where instability of the controlled plant occurs.

Similar behavior is obtained with Monopoli's algorithm and with the SSDD algorithm, Papadopoulos, (1983). Since Monopoli's algorithm does not contain proportional action ($K_p=0$), high frequency oscillations are present even in the case of a first order plant (3.4). See also section 2.2. However, the NLV algorithm can tolerate high reference signals even in the presence of unmodeled dynamics. This is due to the attenuating error filter that keeps the characteristic equation bandwidth almost independent of r (Figs. 3.5, 3.6).

3.2 An MRAC Mechanism for a Wide Range of Inputs

A MRAC algorithm must incorporate a mechanism that will estimate the control gains in such a way as to reduce the generalized error. It is clear that these gains must not depend on the reference input magnitude but must depend on the error vector. However, most MRAC adaptation mechanisms estimate the required control gains by integrating a product comprising the generalized error and some state or input information (eq. 2.23). Since e_2 is of the order of r , the rate $\dot{K}_x$ or $\dot{K}_r$ is proportional to r^2 ; a variable bandwidth mechanism is obtained.

An alternative mechanism was presented by Papadopoulos (1983) where a normalizing factor is introduced, see also Astrom (1983). Consider the laws:

$$\dot{K}_x = K_a x e + \dots \quad (3.2a)$$

$$\dot{K}_r = K_a r e + \dots \quad (3.2b)$$

$$\bar{e} = \frac{e}{\lambda + r^2 + x^2} \quad (3.2c)$$

where λ is a positive constant to avoid division by zero (i.e. $\lambda \neq 0$). If one examines eqs. (3.2), he notes that the gain rate is still proportional to the correlation of e and x (or r) but does not depend on the magnitude of r^2 .

If we introduce this modification to the hyperstability design, we come up with the following characteristic equation (analogous to (2.9):

$$s[s^2 + (bK_e \mu + a_m) s + bK_a \mu] = 0 \quad (3.3a)$$

$$\mu = \frac{x_m^2 + r^2}{\lambda + x_m^2 + r^2} \quad (3.3b)$$

μ is almost constant (for high r 's approaches 1) and the adaptation mechanism bandwidth can be controlled linearly through K_e and K_a . A careful choice of this bandwidth will lead to stable responses even in the presence of unmodeled dynamics. For example, compare Figs. 3.7 and 3.8. The response is strikingly similar (for such a nonlinear system) for $r=10$ and $r=1000$. Also, no instability occurs. This proves that such a normalizing action leads to an algorithm independent of the reference input magnitude.

3.3 Learning Signals

In all the previous simulations, we have applied exclusively step reference inputs and have noted that the gains do not converge to their proper values (3.1). This happens because a step input is not sufficiently "rich", that is it does not sufficiently excite all the system modes; the generalized error can be zero not only when the parameter error distance it is zero, but also when $x_m = x = r$. In order to get better gain estimates, it is a common practice to include in the reference signal low amplitude high frequency sinusoids (learning signals) that excite all the modes and are capable of identifying the system.

We first assume that both the unknown plant and the reference model are of first order. Then by using the following reference signal.

$$r(t) = 1 + 0.2\sin 20t + 0.1\sin 31t \quad (3.4)$$

and the NLV algorithm, we obtain Fig. 3.9, where the gains K_x and K_r do converge to their proper values. From that point on, the generalized error e is zero whatever the reference input is.

However, this case is too idealized. If we next assume that the true plant is third order and we introduce (3.4) again, the system becomes unstable Fig. 3.10. The problem appears because in this case the generalized error is not only a function of the parameter error distance but is a function of the structural mismatch too. This mismatch is worse in higher frequencies; it can be shown Egardt (1983) that instability occurs if the maximum frequency in $r(t)$ exceeds the frequency where the controlled plant has a lag of -180° .

For example, if one uses the signal:

$$r(t) = 1 + 0.2\sin 7t + 0.1\sin 9t \quad (3.5)$$

the obtained response is stable and the gain estimates are realistic, Fig. 3.11. (The controlled plant has -180° lag at $\omega=16.09$ rad/s). The same effects can be observed by using any of

the other discussed algorithms, because all employ similar adaptive laws eq. (2.23).

The above analysis leads to the conclusion that one must restrict the frequency content of the reference signal and avoid any attempt of estimating high order dynamics using a low order model.

3.4 The Effect of Measurement Noise

In all applications, the issue of noise is very important because the available measurements are contaminated by noise. A control algorithm must be able not only to provide good regulation but also to reject noise.

Assuming that $d(t)$ is the noise signal, the measured state y_n and the generalized error e_n are given by:

$$y_n(t) = y(t) + d(t) \quad (3.6a)$$

$$e_n(t) = y_n(t) - y_m(t) \quad (3.6b)$$

Employing the general expression (2.23), we note that the adaptive gain K_y is given by a formula of the following form:

$$K_y(t) = K_a \int_0^T y_n e_n dt + \dots \quad (3.7a)$$

Since both $y_n(t)$ and $e_n(t)$ contain the same noise signal $d(t)$, by taking the mean value of (3.7a) we obtain:

$$E[K_y] = K_a \int_0^t E[d^2(t)] dt + \dots \quad (3.7b)$$

If $d(t)$ is white noise and there is no delay between y_n and e_n , then $E[K_y]$ is drifting away at a rate proportional to the noise covariance σ^2 :

$$E[K_y] = K_a \sigma^2 t \quad (3.8a)$$

On the other hand, if $d(t)$ is sinusoidal, then $E[K_y]$ also drifts away since $E[\sin^2 \omega t] = 1/2$

$$E[K_y] = K_a t/2 \quad (3.8b)$$

Although one is never sure whether such a drift will actually occur (it depends on the actual control structure i.e. on delays and on the noise characteristics), the adaptation mechanism structure is always a candidate for trouble.

In the sequel, we demonstrate the noise effect using the first order plant. We assume that noise is given by:

$$d(t) = .2 \sin 30 t \quad (3.9)$$

and we use the NLV algorithm. In Fig. 3.12 we note that the control gains drift away with decreasing rate while the output x is actually

following the noise pattern. The plant is transformed to a high pass filter and thus its noise rejection characteristics are very poor. We also note that the response is stable since in this case the stability proof requirements (well modelled plant) are satisfied.

However, if the plant contains high frequency unmodeled dynamics, it can not tolerate high gains. There will be a point (Fig. 3.13) where the drift of the control gains will drive the system to the unstable region! In other words, a persisting noise signal can destabilize the system. Another interesting point is that this problem does not depend on the frequency content of noise; instability can occur whatever the noise frequency is.

3.5 A New Formulation to Tackle the Noise Problem

Recently, several attempts have been made to ensure signal boundedness in the presence of disturbances. It was argued that one can replace the adaptation mechanism integrator by a first-order block that approximates the integral action Ioannou (1982). However, such a modification does not allow a zero steady state error and there is a bias in the estimation of the controller gains even in the case where the plant matches the model structure. Another approach suggests that all parameter adaptation be stopped when the generalized error is smaller than the maximum magnitude of the disturbance Egardt (1980), Samson (1983). Such an approach is more rational but its applicability depends on the disturbance characteristics. Other approaches suggest the use of a time decreasing adaptation gain K_a , Landau (1983), but in such a case the algorithm must be able to re-initialize the adaptive gains for each transient and provide a method to adjust the speed of the time decreasing action. Another approach concentrates on incorporating noise models by using shaping filters and trying to estimate not only the plant parameters, but also the parameters of the noise model Landau (1983), Astrom (1970). However, it is uncertain what will happen if the assumed shaping filter structure does not match with the real "filter" or noise dynamics. It is conjectured that problems similar to the ones presented so far would appear.

A new way to tackle the noise problem is presented here. In order to find a way to avoid integrating the squared noise, we examine again expression (2.23). The error term should not be modified since this is the basic driving force of the adaptation mechanism. However, the state information function is used to correlate the error magnitude to the corresponding gain; one could try to replace this information with one obtained through the noise free model state x_m . Note that this characteristic is inherent in the SSDD method presented in section 2.3 where the sensitivity functions are provided by differentiating the model equation.

The proposed modification can be shown to preserve the adaptive qualities that characterize the discussed MRAC algorithms. For example, if one uses the learning signal (3.4), similar results to those depicted in Fig. 3.14 where the response of the full order plant under the noisy signal (3.9) has been simulated.

We must note that this modification has not been proved to be "globally" stable. If large parametric deviations are projected, one can use plant state information during the beginning of the

transient (where the error is large) and switch to the model state information when this error is comparable to noise. Thus, one will obtain improved stability characteristics and drift free gain estimates.

4. CONCLUSIONS

Three representative MRAC approaches have been presented. Although these algorithms are highly nonlinear and difficult to analyze, some results can be obtained by using an approximate linearization technique. This technique can also provide the means to choose the adaptation mechanism gains K_a , K_e . It has been shown that all MRAC algorithms employ similar adaptation laws, i.e. the control gains are obtained by integrating a product comprising some generalized error information, some state or reference input information and a constant gain. A basic assumption around which the stability proofs hinge is that the plant structure is known; from this it follows that the generalized error is a function of the parametric distance only.

In the case where this last assumption is valid, then it has been shown that high reference input signals and learning signals improve the adaptation process. Measurement noise is not rejected; it degrades system performance without destabilizing it. However, in the case where the plant and model structures do not match (unmodeled dynamics), then high inputs, learning signals and measurement noise can cause instabilities.

In this paper, some application-orientation guidelines were presented. These include the use of a normalized error (insensitivity to high r 's), restriction of the frequency content of the learning signals (insensitivity to sinusoidal reference inputs) and the replacement of the noise-contaminated plant state information of the adaptation mechanism by its equivalent provided by the model. It is hoped that the introduction of these modifications will lead to more reliable and robust MRAC algorithms.

REFERENCES

- Astrom, K.J., 1970, Introduction to Stochastic Control Theory, Academic Press, New York.
- Astrom, K.J., 1983, "Analysis of Rohrs Counterexamples to Adaptive Control," 22nd IEEE Conference on Decision and Control, San Antonio, TX.
- Egardt, B., 1980, "Stability Analysis of Continuous Time Adaptive Control Systems," SIAM Journal of Control and Optimization, Vol. 18, No. 5.
- Goodwin, G.C., Ramadge, P.L., and Caines, P.E., 1980, "Discrete Time Multivariable Adaptive Control," IEEE Transactions on Adaptive Control, Vol. AC-25.
- Ioannou, P.A., and Kokotovic, P.V., 1980, "Singular Perturbations and Robust Redesign of Adaptive Control," Proceedings 21st IEEE Conference on Decision and Control, Orlando.
- Landau, Y.D., 1979, Adaptive Control Systems, Marcel Dekker, Inc., New York.
- Landau, Y.D., 1983, Lecture Notes on Adaptive Control, Notes from an Intensive Course on Adaptive Control, Berkeley.
- Monopoli, R.V., 1974, "Model Reference Adaptive Control with an Augmented Error Signal," IEEE Transactions on Adaptive Control, AC-19.
- Narendra, K.S. and Valavani, L.S., 1978, "Stable Adaptive Controller Design-Direct Control," IEEE Transactions on Adaptive Control, Vol. AC-23.
- Narendra, K.S., Liu, Y.H., Valavani, L.S., 1980, "Stable Adaptive Controller Design, Part II: Proof of Stability," IEEE Transactions on Adaptive Control, Vol. AC-25.
- Papadopoulos, E.G., 1983, An Investigation of MRAC Algorithms for Manufacturing Processes: Problems and Potentials, S.M. Thesis, Department of Mechanical Engineering, M.I.T.
- Papadopoulos E.G., and Hardt, D.E., 1983, "Model Reference Adaptive Control: Model Error and Disturbance Effects," Proceedings of the Symposium on Nonlinear Problems in Energy Engineering, Argonne, Illinois.
- Rohrs, C.E., Valavani, L.S., and Athans, M., 1982, "Robustness of Adaptive Control Algorithms in the Presence of Unmodeled Dynamics," Proceedings 21st IEEE Conference on Decision and Control, Orlando, Florida.
- Rohrs, C.E., 1982, Adaptive Control in the Presence of Unmodeled Dynamics, Ph.D. Thesis, Department of Electrical Engineering, M.I.T.
- Samson, C. 1983, "Stability Analysis of Adaptively Controlled Systems Subject to Bounded Disturbances," Automatica.
- Whittaker, H.P., Yamron, J., Keezer, A., 1959, "Design of Model Reference Control Systems for Aircraft," MIT Instruments Lab Report, R-164.

Fig. 2.1 The Parallel MRAC structure.

Figure 2.2 The Hyperstability Theorem Feedback Configuration

Figure 23

Implementation of the *Hyperstability Algorithm*
For a First Order Plant.

Figure 2.5 Implementation of the Sensitivity-Steepest Descent Design (SSDD) Adaptation Mechanism for a First Order Plant

(a)

(b)

(c)

Figure 3.1

Simulation of the Hyperstability Design for a First Order Plant with $K_e = 3$ and $K_a = 10$; $r=1$.
 (a) the Plant (x) and Model (x_m) Outputs; (b) the Adjustable Control Gains K_x and K_r ; (c) the Plant Control Input (u)

(a)

(b)

(c)

Figure 3.2

Simulation of the Hyperstability Design for a First Order Plant with $K_e = 10$ and $K_a = 100$; $r=1$.
 (a) the Plant (x) and Model (x_m) Outputs; (b) the Adjustable Control Gains K_x and K_r ; (c) the Plant Control Input (u)

(a)

(b)

(c)

Figure 3.3

Simulation of the Hyperstability Design for a Plant with Unmodeled Dynamics with $K_e = 3$, $K_a = 10$, $r = 1.5$ (unstable response). (a) the Plant (y) and Model (y_m) Outputs; (b) the Adjustable Control Gains K_y and K_r ; (c) the Plant Control Input (u)

(a)

(b)

(c)

Figure 3.4

Simulation of Monopoli's Algorithm for a First Order Plant with $K_a = 100$, $r = 1$. (a) the Plant (x) and Model (x_m) Outputs; (b) the Adjustable Control Gains K_x and K_r ; (c) the Plant Control Input (u)

(a)

(b)

(c)

Figure 3.5

Simulation of the NLV Algorithm for a First Order Plant with $K_e = 3$, $K_a = 10$, $r = 10$.
 (a) the Plant (x) and Model (x_m) Outputs;
 (b) the Adjustable Control Gains K_x and K_r ;
 (c) the Plant Control Input (u)

(a)

(b)

(c)

Figure 3.6

Simulation of the NLV Algorithm for a Plant with Unmodeled Dynamics with $K_a = 10$, $K_e = 3$, $r = 10$.

- (a) The Plant (y) and the Model (y_m) Outputs;
- (b) the Adjustable Control Gains K_y and K_r ;
- (c) the Plant Control Input (u)

(a)

(b)

(c)

Figure 3.7

Simulation of the Hyperstability Design with Normalizing Action, Unmodeled Dynamics and $K_a = 10$, $K_e = 3$, $r = 10$.

- (a) The Plant (y) and Model (y_m) Outputs;
- (b) the Adjustable Control Gains K_y and K_r ;
- (c) the Plant Control Input (u)

(a)

(b)

(c)

Figure 3.8

Simulation of the Hyperstability Design with Normalizing Action, Unmodeled Dynamics and $K_a = 10$, $K_e = 3$, $r = 1000$.

- (a) The Plant (y) and Model (y_m) Outputs;
- (b) the Adjustable Control Gains K_y and K_r ;
- (c) the Plant Control Input (u)

(a)

(b)

(c)

Figure 3.9

Simulation of the NLV Algorithm for a First Order Plant and with $K_a = 100$, $K_e = 10$ and $r(t)$ given by (5.17).

- (a) The Plant (x) and the Model (x_m) Outputs;
- (b) the Adjustable Control Gains K_x and K_r (they converge to their proper values);
- (c) the Plant Control Input (u)

(a)

(b)

(c)

Figure 3.10

Simulation of the NLV Algorithm for a Plant with Unmodeled Dynamics with $K_a = 100$, $K_e = 10$, $r(t)$ given by (5.17) (system becomes unstable).
 (a) The Plant (y) and the Model (y_m) Outputs;
 (b) the Adjustable Control Gains K_y and K_r ;
 (c) the Plant Control Input (u)

(a)

(b)

(c)

Figure 3.11

Simulation of the NLV Algorithm for a Plant with Unmodeled Dynamics with $K_a = 100$, $K_e = 10$, $r(t)$ given by (5.18).

- (a) The plant (y) and the Model (y_m) Outputs;
- (b) the Adjustable Control Gains K_y and K_r ;
- (c) the Plant Control Input (u)

(a)

(b)

(c)

Figure 3.12

Simulation of the NLV Algorithm for a First Order Plant with Noisy Measurement given by (5.25) and with $K_a = 100$, $K_e = 10$, $r = 2$.

- (a) The Plant (x) and the Model (x_m) Outputs;
- (b) the Adjustable Gains K_x and K_r ;
- (c) the Plant Control Input (u)

(a)

(b)

(c)

Figure 3.13

Simulation of the NLV Algorithm for a Plant with Unmodeled Dynamics with Noisy Measurement given by (5.25) and with $K_a = 100$, $K_e = 10$, $r = 2$ (instability occurs).
 (a) The Plant (y) and the Model (y_m) Outputs;
 (b) the Adjustable Gains K_y and K_r ;
 (c) the Plant Control Input (u)

(a)

(b)

(c)

Figure 3.14

Simulation of the Modified NLV Algorithm for a Plant with Unmodeled Dynamics with Measurement Noise Given by (5.25) and with $K_a = 100$, $K_e = 10$, $n = 2$.

- (a) The Plant (y) and the Model (y_m) Outputs;
- (b) the Adjustable Control Gains K_y and K_r ;
- (c) the Plant Control Input (u)

Appendix II

AN EXPERIMENTAL STUDY ON ADAPTIVE CONTROL OF
A POSITION SERVO SYSTEM

by

A. Suzuki
Research Assistant

D. E. Hardt
Associate Professor

Laboratory for Manufacturing and Productivity
Massachusetts Institute of Technology

Report LMP - FMR - 86-01

February, 1986

Table of Contents

ABSTRACT

I. Introduction

II. Algorithms and Computer Simulations

1. A Zero of a Sampled Second Order System

2. STC/Pole Placement

3. State Feedback MRAC

III. Experimental Studies

1. System

2. STC/Pole Placement

2.1 Case 1) ("Ideal" Servo)

2.2 Case 2) ("Non-Ideal" Servo)

3. State Feedback MRAC

3.1 Case 1) ("Ideal" Servo)

3.2 Case 2) ("Non-Ideal" Servo)

IV. Conclusions and Future Work

V. References

ABSTRACT

The performance of both Self-Tuning Controller (STC) with Pole Placement and State Feedback Model Reference Adaptive Controller (MRAC) are investigated in a position servo system. The open loop plant was set-up in two different conditions: an "ideal" servo and a "non-ideal" servo with a large dead-zone.

The plant is modeled as a second order system although the real system has an additional pole due to a built-in regulator. In general, each algorithm shows good adaptation for different set-ups and for different incorrect initial plant parameter values in spite of the lower order model, large velocity measurement noise (for MRAC) and a plant dead-zone nonlinearity.

However, unstable responses are observed under certain conditions where the initial plant parameter values are far from the appropriate ones, and at the same time, a large reference command is applied.

I. Introduction

Considerable progress has been made in the analysis and design of Model Reference Adaptive Control (MRAC) (eg. [1]-[5]) and Self Tuning Regulator (STR) systems (eg. [6]).

One of the fields where adaptive control schemes will play an increasingly important role is a manufacturing system. However, there have not been as many applications of an adaptive control design to manufacturing systems as would be expected because of the complexity of the associated algorithms and their robustness questions. The ultimate goal of our project is to propose a practical design methodology of adaptive control systems for manufacturing processes and to contribute to bridging the gap between adaptive control theory and its practice.

The purpose of the present paper is to experimentally study the performance of typical adaptive control designs when applied to a position servo system.

Two adaptive schemes have been chosen, namely STC combined with pole placement technique [6] and state feedback MRAC [3]. The reason for these choices, and brief explanations of each algorithm as well as computer simulations are given in section II. In the experiments, these adaptive controllers are applied to a servo system of two different set-ups (ideal servo and non-ideal servo) and the results are discussed in section III. Finally, conclusions and future work are described in section IV.

II. Algorithms and Computer Simulations

1. A Zero of a Sampled Second Order System

We are interested in the second (or higher) order model for the servo system. Suppose the plant to be controlled is modeled by the following second order system:

$$G_p(s) = \frac{\omega_p^2}{s^2 + 2 \xi_p \omega_p s + \omega_p^2} K_p \quad \dots(1)$$

the corresponding pulse transfer function with a zero order hold is (e.g. [10]) given by

$$G_p(z) = \frac{b_{p0} z^{-1} + b_{p1} z^{-2}}{1 + a_{p1} z^{-1} + a_{p2} z^{-2}} \quad \dots(2)$$

where $a_{p1} = -2 \alpha \beta$

$$a_{p2} = \alpha^2$$

$$b_{p0} = 1 - \alpha \left(\beta + \frac{\xi_p \omega_p}{\omega'} \gamma \right)$$

$$b_{p1} = \alpha^2 + \alpha \left(-\beta + \frac{\xi_p \omega_p}{\omega'} \gamma \right) \quad \dots(3)$$

$$\alpha = \exp(-\xi_p \omega_p T_s)$$

$$\beta = \cos(\omega' T_s)$$

$$\gamma = \sin(\omega' T_s)$$

$$\omega' = \omega_p \sqrt{1 - \xi_p^2}$$

$$T_s = \text{sampling time}$$

Figure 1a shows the open loop poles and a zero on the z-plane. The important point is that a zero appears on the negative real axis, in other words, we have a poorly damped open loop zero in the sampled second order system.

Figure 1a: pole/zero configuration of a sampled second order system in the z-plane

Most of the adaptive control algorithms utilizing only input and output measurements (e.g. [1], [2], [4], [5], [11]) are based on exact pole/zero cancellations. If one of those schemes is directly applied to the present case, a controller pole must cancel the poorly damped plant zero, which as a result causes undesirable ripples in the control input.

Figures 2 - Figure 5 are the results of computer simulations where Landau's algorithm based on pole/zero cancellation [11] is applied to control the plant (1) to follow the model $G_m(s)$:

$$G_m(s) = \frac{\omega_m^2}{s^2 + 2 \zeta_m \omega_m s + \omega_m^2} \quad \dots(4)$$

$$\text{with } \begin{matrix} \omega_p = 0.2, & \zeta_p = 0.2, & K_p = 5 \\ \omega_m = 3, & \zeta_m = 0.7 \\ T_s = 0.1 \end{matrix} \quad \dots(5)$$

At the beginning, the plant parameter values are incorrectly assumed as:

$$\begin{matrix} \omega_p = 1 \\ \zeta_p = 0.2 \\ K_p = 1 \end{matrix}$$

Though the plant outputs follow the model output (Figure 4) and controller gains are adjusted in a few sampling steps (Figure 2,3), significant ripples are observed in the control input (Figure 5) because of cancellation of the poorly damped zero. Accordingly, STC combined with pole placement and state feedback MRAC were chosen for this study since these controllers leave this plant zero uncanceled.

2 STC/Pole Placement

This design method is a heuristic combination of parameter estimation and a pole placement design technique using only input and output measurements. More details are found in [6] and are summarized below.

i) Pole Placement (for a known plant)

$$\text{Plant: } H(z) = B(z) / A(z) \quad \dots(6)$$

$$\text{Model: } H_m(z) = B_m(z) / A_m(z) \quad \dots(7)$$

$$\text{Control Law: } U(z) = \frac{T(z)}{R(z)} r(z) - \frac{S(z)}{R(z)} y(z) \quad \dots(8)$$

Figure 1b: Pole Placement Controller

$$\text{Goal: } y(z)/r(z) = \frac{B(z) T(z)}{A(z)R(z) + B(z)S(z)} \Rightarrow \frac{B_m(z)}{A_m(z)} \quad \dots(9)$$

Design Procedure:

$$1) \quad B = B^+ B^- \quad \dots(10)$$

where B^+ = plant zeros to be cancelled
 B^- = plant zeros not to be cancelled

$$2) \quad B_m = B^- B_m' \quad \dots(11)$$

$$3) \quad R = B^+ R' \quad \dots(12)$$

$$4) \quad \text{Solve : } AR' + B^- S = A_o A_m \quad \dots(13)$$

$$T = B_m' A_o \quad \dots(14)$$

where A_o is referred to as an observer polynomial and is chosen to satisfy.

$$\deg A_0 > 2 \deg A - \deg A_m - \deg B^T - 1 \quad \dots(15)$$

These equations ((13)-(15)) have a causal solution if

$$\deg A_m - \deg B_m > \deg A - \deg B \quad \dots(16)$$

ii) Parameter Estimations

When the plant parameter values are not known, they are estimated at each sampling step and control gains are updated accordingly.

Among many estimation schemes, Recursive Least Square (RLS) method is used because of its simplicity (see for example [12]) as summarized below.

For an auto-regressive system:

$$\begin{aligned} y &= -a_1 y_{k-1} - \dots - a_n y_{k-n} \\ &= [a_1 \dots a_n] \begin{bmatrix} -y_{k-1} \\ \vdots \\ -y_{k-n} \end{bmatrix} \\ &= \eta^T \delta_k \end{aligned} \quad \dots(17)$$

the estimate of the parameter vector η is given by:

$$\eta_k = \eta_{k-1} + L_k (y_k - \eta_{k-1}^T \delta_k) \quad \dots(18)$$

where

$$L_k = \frac{P_{k-1} \delta_k}{\lambda + \delta_k^T P_{k-1} \delta_k} \quad \dots(19)$$

$$P_k = \frac{1}{\lambda} \left(P_{k-1} - \frac{P_{k-1} \delta_k \delta_k^T P_{k-1}}{\lambda + \delta_k^T P_{k-1} \delta_k} \right) \quad \dots(20)$$

o Computer Simulations

The same plant example in equation (1) is used for the simulation. The model is chosen as

$$H_m(z) = \frac{b_{p0} z^{-1} + b_{p1} z^{-2}}{1 + a_{m1} z^{-1} + a_{m2} z^{-2}} \frac{1 + a_{m1} + a_{m2}}{b_{p0} + b_{p1}} \quad \dots(21)$$

Note the model zero is the same as the plant zero. In other words, the model zero changes as the parameter identification proceeds. The constant term in (21) is required to have zero steady state error. By following the design procedure with

$$B^+ = 1 \quad \dots(22)$$

$$B^- = b_{p0} z + b_{p1} \quad \dots(23)$$

the control law is obtained as

$$U_k = C_m r_k - S_0 y_k - S_1 y_{k-1} - r_1 U_{k-1} \quad \dots(24)$$

where

$$C_m = \frac{1 + a_{m1} + a_{m2}}{b_{p0} + b_{p1}} \quad \dots(25)$$

$$r_1 = \frac{c_1^2 (a_{m1} - a_{p1}) - c_1 (a_{m2} - a_{p2})}{c_1^2 - a_{p1} c_1 + a_{p2}} \quad \dots(26)$$

$$C_1 = b_{p1} / b_{p0} \quad \dots(27)$$

$$S_0 = \frac{a_{m1} - a_{p1} - r_1}{b_{p0}} \quad \dots(28)$$

$$S_1 = - \frac{a_{p2} r_1}{b_{p1}} \quad \dots(29)$$

In the estimation, the parameter vector η and regression vector δ for this case are defined as:

$$\eta = [a_{p1} \ a_{p2} \ b_{p0} \ b_{p1}]^T \quad \dots(30)$$

$$\delta = [-y_{k-1} \ -y_{k-2} \ U_{k-1} \ U_{k-2}]^T \quad \dots(31)$$

Figure 6 and Figure 7 are the simulation results for a step reference command with the constant gain (no adaptation) controller computed from the "incorrect" plant parameter values. The plant output does not follow the model output.

Figures 8 - Figure 11 are the case with the adaptive controller. The plant parameter values are adjusted towards the correct ones after a few sample steps (Figure 8,9) and the desired plant response is obtained (Figure 10).

Through these simulations, it has been shown that STC is capable of quick parameter estimation and gain adjustment, at least in these ideal situations.

3. State Feedback MRAC

This design technique utilizes state feedback and the control gains are updated directly in such a way as to guarantee hyperstability of the system as well as to bring the error between the model and the plant state variables to zero. Details are given

in [3].

$$\text{Plant: } \underline{X}_p(k+1) = A_p \underline{X}_p(k) + B_p \underline{U}_p(k) \quad \dots(32)$$

$$\text{Model: } \underline{X}_m(k+1) = A_m \underline{X}_m(k) + B_m \underline{r}(k) \quad \dots(33)$$

i) Linear Model Following Control (LMFC) (for a known plant)

When a plant parameter values are well known, a so-called Linear Model Following Control (LMFC) design is one of the design methods achieving desired transient characteristics of the system and is recapitulated as follows.

$$\text{Control Law: } \underline{U}_p(k) = K_r \underline{r}(k) - K_x \underline{X}_p(k) \quad \dots(34)$$

where

$$K_r = B_p^+ B_m \quad \dots(35)$$

$$K_x = B_p^+ (A_p - A_m) \quad \dots(36)$$

$$B_p^+ = (B_p^T B_p)^{-1} B_p^T \quad \dots(37)$$

ii) Adaptive Model Following Control (AMFC) (for a unknown plant)

For a plant with unknown parameter values, a natural extension of the LMFC design is to adjust the control gains K_x and K_r in an aforementioned way and it is called Adaptive Model Following Control (AMFC) design. Below is summarized the adaptation mechanism when hyperstability theorem is applied.

$$K_x(k+1) = - V_{i+1} [L' \underline{X}_p(i)]^T - V_{k+1} [L \underline{X}_p(k)]^T + K_x(0) \quad \dots(38)$$

$$K_r(k+1) = V_{i+1} [M' \underline{r}(i)]^T - V_{k+1} [M \underline{r}(k)]^T + K_r(0) \quad \dots(39)$$

The first term is called an integral adaptation term (L' , M' : integral adaptation gains) and the second term is called a proportional adaptation gain (L , M : proportional adaptation gains). V_k is a filtered generalized error given by

$$V_i = D e_i \quad i=1, \dots, k \quad \dots(40)$$

The linear filter D is required in order to obtain a strictly positive real (s.p.r.) transfer function $T(z)$:

$$T(z) = D (I - A_m z^{-1})^{-1} B_p \quad \dots(41)$$

e_i is a generalized error vector between a model and a plant state variables.

Finally, one step ahead prediction of V_k is computed by

$$V_{k+1} = \{ I + DB_p(0) [\underline{x}^T(k)(M+M')\underline{x}(k) + \underline{x}_p^T(k)(L+L')\underline{x}_p(k)] \}^{-1} V_k \quad \dots(42)$$

o Computer Simulations

The same example as before is used for the simulations.

Figures 12 and Figure 13 are the simulation results for a step command with the linear controller computed from the incorrect initial parameter values. The plant response is far from the desired (model) response.

Figures 14 and Figure 15 are the results with the adaptive controller showing the gain adjustment (Figure 14) and the desirable performance.

Next two figures depict the influence of the linear filter $D=[1,\varepsilon]$ which plays an important role in the adaptation process, that is,

$$\text{where } V_k = e_{1k} + \varepsilon e_{2k} \quad \dots(43)$$

$$e_{1k} = (\text{plant position}) - (\text{model position}) \quad \dots(44)$$

$$e_{2k} = (\text{plant velocity}) - (\text{model velocity}) \quad \dots(45)$$

In Figure 16, with $\varepsilon=0.1$, a poorly damped and oscillatory response is observed, while in Figure 17, with $\varepsilon=10$, a good transient response but slow settling to the steady state are obtained.

III. Experimental Studies

1. System

In this experiment, the position of a DC motor driven milling machine table (Bridgeport) is controlled through a PDP 11/03 micro-computer in real-time, given incorrect initial plant parameter values by adjusting the controller gains. Figure 18 shows the schematic diagram of the system.

The plant is assumed as the first order system + integrator (an encoder) although the real plant has both a built-in lead-lag regulator in the feed forward loop and a velocity feedback as shown in the same figure.

The control objective is to move the table in one direction to a desired position in a desired manner specified by the model without knowing the correct plant parameter values, τ_p and K_p .

Two set-ups are considered: 1) with an internal velocity feedback ($k_\omega = 0$) and 2) without an internal velocity feedback ($k_\omega = 0$). Case 1) is thought of as an ideal servo, while case 2) has a dead-zone of considerable width due to static friction.

Figures 19 and Figure 20 show the open loop velocity responses to a step input for the two cases respectively. In 1), the velocity response is adequately damped while, in 2) the overdamped response is observed.

2. STC/Pole Placement

2.1 Case 1) (an "Ideal" Servo)

o Linear Design (Pole Placement)

First, linear controllers are designed using the pole placement technique from several sets of incorrect plant parameter values in an "ideal" servo. The incorrect parameter values yield inadequate controller gains and desired plant outputs are not obtained. In figure 21, four examples are shown where the initial parameter values are set as follows.

	τ_p	K_p	Results
A)	0.02	2.00	slow response
B)	0.02	0.05	fast response with overshoot
C)	0.02	0.01	marginally stable response
D)	0.02	0.10	close to a desired response

o Adaptive Controller (STC/PP)

Adaptive controllers are designed using the STC algorithm and applied to the above four cases in an "ideal" servo. The STC/PP algorithm updates the controller gains at each sampling based on the estimated plant parameter values from the current and past input/output data. The subsequent figures show that the system responses are drastically improved by these adaptive schemes and the plant parameter values are adjusted in several time steps.

- Figure 22 - Figure 25... adaptive control for case A
- Figure 26 ... adaptive control for case B
- Figure 27 ... adaptive control for case C
- Figure 28 ... adaptive control for case C with a larger reference command($r=1,2,...,5$ compared to $r=0.5$ otherwise)
- Figure 29 ... adaptive control for case D

2.2 Case 2) (a "Non-Ideal" Servo)

o Linear Design (Pole Placement)

Once again, linear controllers are designed using the pole placement technique for a "non-ideal" servo for the following three sets of different initial plant parameter values. Results of all these cases are shown in Figure 30.

	τ_p	K_p	Results
A)	0.03	0.9	slow response
B)	0.05	0.17	fast response
C)	0.045	0.25	close to desired response

o Adaptive Controller (STC/PP)

Adaptive controllers using the STC/PP algorithm are also applied to the above cases in a "non-ideal" servo. Again the performance in all the cases is drastically improved as shown in the subsequent figures.

Figure 31	... adaptive control for case A
Figure 32	... adaptive control for case B
Figure 33	... adaptive control for marginally stable case ($\tau_p=0.05$, $K_p=0.13$)
Figure 34	... adaptive control ^P for the same case as in Figure 33 with a larger reference command ($r=1,2$ and 3 compared to $r=.5$ in the other figures)

However, notice the steady state error in each case caused by the plant dead-zone (a static friction). This non-linear element in the plant may bring parameter estimation drift when left operated for long periods of time as depicted in Figures 35 - Figure 37, indicating potential instability problems.

The algorithm requires 67 multiplications, 18 divisions and 51 additions and subtractions. Sampling time is 0.09 seconds.

In general, for both cases with different set-ups, a good adaptation and a desired performance are obtained by STC. However, it is also pointed out that without a proper care, a plant dead-zone misleads the parameter estimation in the steady state, which may cause an instability problem in the long run.

3. State Feedback MRAC

3.1 Case 1) (an "Ideal" Servo)

o Linear Design (LMFC)

In the same manner as before, linear controllers for three different incorrect initial parameter values in an "ideal" servo are designed using the LMFC algorithm and the results are shown in Figure 38, where the initial plant parameter values are assumed as follows.

	τ_p	K_p	Results
A)	0.02	0.4	slow response
B)	0.02	0.03	marginally stable
C)	0.02	0.055	close to a desired response

Again, the desirable responses are not obtained in either case A or B by the linear controllers.

o Adaptive Controller (MRAC)

In the MRAC algorithm the controller gains are directly updated based on the generalized error. It should be also noted that in this specific MRAC design, the state feedback control is utilized.

Adaptive controllers are designed for the above cases in an "ideal" servo using the MRAC algorithm and the results are shown in the following figures.

Figure 39 - Figure 42 ... adaptive control for case A
Figure 43 ... adaptive control for case B

In both cases, though the performance is much improved compared to the linear design, it is not as good as in STC. The reason for this is considered to be the ample noise in the velocity signal which this specific MRAC design uses in the control law as well as in the adaptation mechanism. Partly because of this noise, when a larger reference command is applied to case B, the adaptive design goes to unstable as witnessed in Figures 44 - Figure 46 ($r=1.0$ and $r=2.0$ in Figure 44-46 compared to $r=0.5$ in Figure 43).

3.2 Case 2) (a "Non-Ideal" Servo)

o Linear Design (LMFC)

Linear controllers are designed using the LMFC design method for the following two sets of different parameter values in a "non-ideal" servo and Figure 47 shows the responses of the system for each case.

	τ_p	K_p	Results
A)	0.03	0.45	slow response
B)	0.05	0.19	fast response, overshoot

o Adaptive Controller (MRAC)

In the same fashion, adaptive controllers are designed using the MRAC design method for the above two cases in a "non-ideal" servo. In spite of the large dead-zone in the plant, MRAC still achieves good adaptation for these cases as demonstrated in the following figures.

Figure 48 ... adaptive control for case A
 Figure 49 ... adaptive control for case B

Figure 50 depicts the system response with the adaptive controller for the case where the incorrect initial plant parameter values are such that the linear controller computed from them would result in marginally stable system ($\tau_p=.045$, $K_p=.18$). The system remains stable in Figure 50 but if a larger reference command is given, the system goes unstable as shown in Figures 51 - Figure 52 ($r=1.0$ and 2.0 in Figure 51 and 52 compared to $r=0.5$ in Figure 50) as having been observed by, for example, Papadopoulos [8].

In Figure 53, the effect of the adaptation gains on the system performance in case A is demonstrated. It can be seen that the integral adaptation gains (L', M') play a more important role in the adaptation process than the proportional adaptation gains (L, M).

Figure 54 shows the effect of the linear filter $D=[1, \varepsilon]$, where $\varepsilon=0$ causes instability, in other words, the velocity error term can not be neglected in generating the generalized error in the adaptation mechanism since the s.p.r. condition in (41) would otherwise be lost. In most experiments, ε is set to 1.

Finally, the effect of dead zone in the steady state is examined in Figures 55 - Figure 56. The system stays stable at least in this duration of time.

The algorithm requires 37 multiplications, 2 divisions, and 29 additions and subtractions. Sampling time is 0.03 seconds.

In general, the system performance is much improved by MRAC compared to the linear design although it is not as good as in STC partly because of the presence of noise in velocity measurement. Instability is observed when the incorrect initial plant parameter values are so chosen that the resulting linear controller would bring about marginally stable system and at the same time the larger reference command is applied.

IV. Conclusions and Future Work

Both STC using pole placement design and state feedback MRAC have been implemented in an LSI 11/03 microcomputer driving a D.C. motor milling machine servo system in order to investigate the performance of the associated algorithms. The open loop plant was set up in two different conditions:

- 1) with no dead-zone and an adequately damped velocity response - an "ideal" servo
- 2) with a large dead-zone caused by static friction and an overdamped velocity response

The plant is modeled as a second order system in both cases although the real system has an additional pole due to a built-in regulator. A number of interesting results have been observed.

- Each control design shows good adaptation for each set-up for different incorrect initial plant parameter values in spite of the lower order model, velocity measurement noise (for state feedback MRAC) and a plant dead-zone.
- However, the plant dead-zone causes parameter estimation drift in STC, indicating potential hazard in the system stability.
- Also, apparent unstable phenomena are observed in state feedback MRAC under certain conditions where the incorrect initial plant parameter values are far from the appropriate ones together with a relatively large reference command and noise. This has been predicted by simulations, for instance, by Papadopoulos [8] and specific methods to avoid this problem have been derived in [8].

This series of experiments clarified that even under the "real world" environment, the adaptive control design is able to compensate for the plant uncertainties but at the same time that it is also vulnerable to modeling errors, measurement noise and non-linear plant elements.

One of the central issues concerning the stability of the adaptive control design focuses on modeling errors and measurements noise as being addressed in current papers (e.g. [7]). Some of the modifications on the algorithms to guarantee stability robustness under these conditions are suggested in, for example, [8] [9]. A systematic procedure of designing a practical adaptive control system in manufacturing process environments as to when adaptive control is preferable as well as how to apply it is of great interests and is an important issue of the future work.

References

- [1] Monopoli, R.V. "Model Reference Adaptive Control with an Augmented Error Signal," IEEE Trans.A.C., Vol. AC-19, No5, Oct. 1974.
- [2] Narendra, K.S. and Valavani, L.S. "Stable Adaptive Controller Design--Direct Control," IEEE Trans.A.C., Vol.AC-23, No4, Aug. 1978.
- [3] Landau, Y.D. Adaptive Control, The Model Reference Approach, Marcel Dekker Inc., N.Y., 1979.
- [4] Goodwin, G.C., Ramadge, D.J. and Caines, P.E. "Discrete Time Multivariable Adaptive Control," IEEE Trans.A.C., Vol. AC-25, No3, June 1980.
- [5] Egardt, B. "Unification of Some Discrete Time Adaptive Control Schemes," IEEE Trans.A.C., Vol. AC-25, No4, Aug. 1980.
- [6] Astrom, K.J. and Wittenmark, B. "Self Tuning Controllers based on Pole-Zero Placement," IEE Proc., Vol. 127, Pt.D, No3, May 1980.
- [7] Rohrs, E.C., Valavani, L., Athans, M. and Stein, G. "Robustness of Continuous-Time Adaptive Control Algorithms in the Presence of Unmodeled Dynamics," IEEE Trans.A.C., Vol. AC-30, No9, Sept. 1985.
- [8] Papadopoulos, E.G. An Investigation of Model Reference Adaptive Control Algorithms for Manufacturing Processes: Problems and Potentials, M.S. Thesis, Dept. of Mech. Eng., MIT 1983.
- [9] Orlicki, D.M. Model Reference Adaptive Control Systems Using A Dead Zone Non-Linearity, Ph.D. Thesis, Dept. of EECS, MIT, 1985.
- [10] Franklin, G.F. and Powell, J.D. Digital Control of Dynamic Systems, Addison Wesley Pub. Comp., M.A., 1980.
- [11] Landau, Y.D. and Lozano, R. "Unification of Discrete Time Explicit Model Reference Adaptive Control Design," Automatica, Vol.17, No4, pp 593-611, 1981.
- [12] Ljung, L and Soderstrom, T. Theory and Practice of Recursive Identification, M.I.T. Press, Cambridge, M.A., 1983.

apr 4/
10 apr

$\lambda=0.2, \mu=0, \tau_0=0.1, \tau_1=100$

Fig 2 ~ Fig 5 MRAC / 10 approach

Fig 6~7 Pole Placement (Linear control)
 $K = 0 \quad \tau = 1 \quad T = 1$

Fig 8 ~ Fig 11 STC/Pole Placement ($T_s = .1$)

Fig 12-13 LMFC (state feed back)

$$\Delta T = 0.002$$

Fig 14,15 State Feed Back MRAC ($T_s=0.02$)

Fig 16 MRAC ($\epsilon=0.1$)

$\epsilon = 10$

$\sigma = 100$ s: 0.1, 0-10

Fig 17

MRAC ($\epsilon=10$)

Fig 18 Schematic Diagram of the System

IR=500, 1000, 2000

Velocity A/D
Offset ≈ 2030

Fig 19 Open Loop Velocity Response with a velocity Fixed Back loop

W/O VEL. F.B.

Fig 20 open Loop Velocity Response without Velocity F.B. (case 2)

LINEAR DESIGN: PP

Fig 21 Pole Placement (Linear) for case 1)

10/.02/2.

Fig 22 STC/PP for case 1)- A

10/.02/2.

Fig 23 STC/PP case 1) -A

10/.02/2.

Fig 24 STC/PP case 1) - A

10/.02/2.

Fig 25 STC/PP case 1) - A control u

10/.02/.05

Fig 26 STC/PP case 1) - B output y

10/.02/.01

10/.02/.01

Fig 28 STC/PP case 1)-C for large reference command

10/.02/.1

Fig 29 STC/PP case 1) - D

LINEAR DESIGN

EXMRA5
T_s = .25

Fig 30 Pole Placement (Linear) case 2)

10/.03/.9

Fig 31 STC/PP case 2)- A

10/.05/.17

Fig 32 STC/PP case 2)-B

10/.05/.13

Fig 33 STC/PP case 2)

10/.05/.13

ε: 0.34 STC/PP case 2) - marginal case + large reference command

1E3/.03/.9

Fig 35 STC/PP case 2) - A long time scale

1E3/.03/.9

I: 0 22 STC/PP case 2) - A long time scale

1E3/.03/.9

Fig 37 STC/PP case 2) - A long time scale

LINEAR DESIGN

Fig 38 LMFC (Linear) case 1)

$1E3/.02/.4$

Fig 39 MRAC case 1) - A output Y

.02/.4

Fig 40 MRAC case 1) - A velocity v

.02/.4

Fig 41 MRAC case 17-A control gains

.02/.4

Fig 42 MRAC case 17-A control input u

.02/.03

Fig 43 MRAC case 1)-B output y

.02/.03

Fig 44 MRAC case 1) - B w/larger reference command

.02/.03

r. 45 MRAC case 1) - B + larger reference command

.02/.03

Fig 46 MRAC case 1) - 8 w/larger ref. command

NO VEL. F.B. /TS=.03

Fig 47 LMFC (Linear) case 2)

$$1E3/.03/.45/E=1$$

Fig 48 MRAC case 2) - A output y

1E3/.05/.19/E=1

Fig 49 MRAC case 2)-B

$$1E3/.045/.18/E=1$$

Fig 50 MRAC case 2) - marginal case

$$1E3/.045/.18/E=1$$

Fig 5/ MRAC case 2) - marginal case w/ larger command

$$1E3/.045/.18/E=1$$

Fig 52 MRAC case 2) - marginal case w/larger command

.03/.45/E=1

Fig 53 MRAC case 2) - A effect of adaptation gains

Fig 54 MRAC case 2) - A effect of ϵ

$1E3/.03/.45/E=1$

Fig 55 MRAC case 2) - A long time scale

1E3/.03/.45/E=1

Fig 56 MRAC case 2)-A long time scale