MASTER

SU(N) Grand Unification with Several
Quark-Lepton Generations *

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Abstract

Procedures are given for embedding SU(5) into SU(N) such that several SU(5) generations remain as ordinary-mass fermions. Examples with an arbitrary number of generations occur already for SU(7). All models have only quarks and leptons, without quixes or quights.

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The simplest possible grand unification¹ of QCD with the Weinberg-Salam model is based on the group SU(5). Applications of this model to determine the weak mixing angle,² and certain fermion mass ratios,³ are encouraging.

One major shortcoming of the theory is that each generation of quarks and leptons is put into its own irreducible representation in a trivial fashion; different generations are not related by the gauge group. Here we consider embedding $SU(5) \subset G = SU(N)$, $N > 5$, such that a non-trivial flavor group emerges. We choose SU(N) only because Nature appears to have a predilection for special unitary groups. Our results appear more economical than those of Ref. 4.

We make the usual general assumptions. First, we allow only singlets and triplets under SU(3) of color, thus avoiding quixes (6) and quights (8). This requires⁵ that we take only the totally antisymmetric fundamental representations which we designate $[N, m]$, $1 \leq m \leq (N-1)$, for m tensor indices (or boxes in the Young Tableaux). Each generation of left-handed fermions in SU(5) is thus

$$[5, 2] + [5, 4] \equiv 10 + \bar{5}^* \quad (1)$$

and $[N, m]$ decomposes into SU(5) according to

$$[N, m] \rightarrow \binom{N-5}{m} \bar{1} + \binom{N-5}{m-1} \bar{5} + \binom{N-5}{m-2} 10 \\ + \binom{N-5}{m-3} 10^* + \binom{N-5}{m-4} \bar{5}^* + \binom{N-5}{m-5} \bar{1}. \quad (2)$$

Second, we assume the particular set $\{m\}$ we take for the left-handed fermions contains equal number of $\bar{3}_c$ and 3_c^* for the quarks. This ensures non-chiral color. Therefore, if the SU(5) content is

$$[N, \{m\}] \rightarrow v(1) \underline{1} + v(5) \underline{5} + v(10) \underline{10} + v(10^*) \underline{10^*} + v(5^*) \underline{5^*} \quad (3)$$

we require that

$$v(5) + v(10) = v(5^*) + v(10^*). \quad (4)$$

Third, we impose renormalizability in the sense that there are no triangle anomalies. In the SU(5) language, this requirement coincides with Eq.(4): reality under SU(3) color coincides with absence of anomalies. In SU(N) language, one requires

$$\sum_{\{m\}} A[N, m] = 0 \quad (5)$$

with the anomaly number given by⁶

$$A[N, m] = -A[N, N-m] = \frac{(N-2m)(N-3)!}{(N-m-1)!(m-1)!} \quad (6)$$

Fourth, we assume⁴ that the number of ordinary-mass fermion SU(5) families is given by

$$g = v(10) - v(10^*) \quad (7)$$

which also equals $(v(5^*) - v(5))$ by Eq.(4). For each representation one therefore has a generation increment from Eq.(2) of

$$\Delta g[N, m] = -\Delta g[N, N-m] = \binom{N-5}{m-2} - \binom{N-5}{m-3}. \quad (8)$$

We note that if $g \neq 0$, the representation for the left-handed fermions is automatically complex under the flavor $SU(2)_L \otimes U(1)$ subgroup of SU(5).

Fifth, we require⁷ that within the set $\{m\}$ the coefficients C_m in $\{m\} = \sum_m C_m [m]$ do not all have a common factor. This excludes trivial replication of a single generation or of subsets of generations.

In order to construct models, our procedure is now to tabulate the $A[N,m]$ and $\Delta g[N,m]$ for $1 \leq m \leq (N-1)$ with a given N . We then look for linearly independent combinations $\{m\}'$ satisfying

$$\sum_{\{m\}'} A[N,m] = 0 \quad (9)$$

$$\sum_{\{m\}'} \Delta g[N,m] \neq 0. \quad (10)$$

The number of such linearly-independent combinations is given by $(N-3)$ for odd N , $(N-4)$ for even N since for even N , $A[N,N/2] = \Delta g[N,N/2] = 0$. This number falls into $[\frac{1}{2}(N-3)]$ ($[x] = \text{integer part of } x$) pairs of complex conjugates having $\Delta g > 0$ and $\Delta g < 0$, respectively. These combinations, each of particular Δg , are then readily assembled to make models of definite total generation number, g .

The procedure is best illustrated by the simplest examples $N=6$ and 7.

For $SU(6)$:

$m = 1$	2	3	4	5
$A = +1$	+2	0	-2	-1
$\Delta g = 0$	1	0	-1	0.

we have the anomaly-free combination ($D = \text{dimension}$; $D[N,m] = \binom{N}{m}$)

$$\{m\} = ([2] + 2[5]); \quad \Delta g = 1; \quad D = 27 \quad (11)$$

and its complex conjugate. The only model is uninteresting since it has $g=1$.

For $SU(7)$:

$m = 1$	2	3	4	5	6
$A = +1$	+3	+2	-2	-3	-1
$\Delta g = 0$	1	1	-1	-1	0.

There are now two independent combinations

$$\{m\}_1 = ([2] + 3[6]); \quad \Delta g = 1; \quad D = 42 \quad (12)$$

$$\{m\}_2 = ([3] + 2[6]); \quad \Delta g = 1; \quad D = 49 \quad (13)$$

and this complex conjugates. Here we can make models for all g , including the six interesting values $8 \geq g \geq 3$, respecting asymptotic freedom and accomodating the known fermions. For $g=3$, say, there are two models with dimension 133 and 140. The lower dimension is explicitly

$$\{m\} = (2[7,2] + [7,3] + 8[7,6]); \quad g = 3; \quad D = 133. \quad (14)$$

The second model corresponds to the alternative partitioning of $3 = 2 + 1 = 1 + 2$. Models for $g=4,5$. . . are similarly possible. Thus, already in $SU(7)$ we have models with arbitrary generation number. The same procedure can be followed for $SU(N)$, $N > 7$, quite easily.

At this point, we should indicate the slight difference between this and Ref. 4 which finds in $SU(11)$ the candidate model

$$\{m\} = ([11,4] + [11,8] + [11,9] + [11,10]); \quad g = 3; \quad D = 561 \quad (15)$$

and, no others for $3 \leq g \leq 8$ based on the assumption that any given irreducible representation $[m]$ can occur in the set $\{m\}$ not more than once. We have relaxed this assumption, regarding it as more advantageous to minimize D than the number of irreducible representations.

There now seems to appear much arbitrariness but for given g , and the minimal value of $N=7$, we achieve uniqueness by minimizing the number D of left-handed fermions. Thus one finds the sequence $g=3$, $D=133$; $g=4$, $D=175$; $g=5$, $D=217$; $g=6$, $D=259$; $g=7$, $D=301$; $g=8$, $D=343$. In each case (15g) fermions are the

residual light $SU(5)$ generations and the remainder are expected to have super-heavy mass comparable to the grand unification scale.

Of course, the mystery of the gauge heirarchy remains. Also the uneasy feeling remains concerning the fact that ordinary matter is comprised only of the minority fraction debris, while most quarks and leptons have completely inaccessible masses. Nevertheless, a study of the symmetry breaking in a model like my Eq.(14) should lead to new testable relations between generations; the difficulty here is first that the $SU(7)$ group appears to mix the light fermions with the superheavy ones of unknown experimental properties, but perhaps the latter can be somehow factored out? Second, one must invent a plausible simplicity criterion for the breaking pattern itself given the plethora of possible irreducible representations? Our purpose here was only to note a simplifying possibility for the matter fields.

References

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7. This is where we differ from Ref. 4; see later in the text.