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LASER-PLASMA COUPLING

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Laser-Plasma Coupling*

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*** prepared for presentation at the ANL-AUA Faculty Institute on
Laser-Plasma Coupling, Argonne 1978.**

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Laser-Plasma Coupling

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A. Ionization of target

- Target is rapidly ionized.

Example., 1.06 μ light

$$I_L \approx 10^{15} \text{ W/cm}^2 \rightarrow E_L \approx 10^9 \text{ V/cm,}$$

which is sufficient to directly strip an electron
from an atom.

In fact, target ionizes at much lower intensities due to cascade ionization.

always a few free electrons due to impurities,
lattice defects, etc.

these rapidly joule heat, becoming energetic enough
to produce more free electrons

result: a cascade leads to target breakdown

- crude estimate of breakdown intensity

estimate incident light intensity needed to produce and
maintain a plasma with the critical density ($\approx 10^{21} \text{ cm}^{-3}$ for
1.06 μ light) with $\Theta_e \sim 2 \text{ eV}$ (sufficient to ionize).

$I \approx n_{cr} \Theta_e v_e$ incident intensity balances most
pessimistic estimate of heat flux
carried away.

(2)

This gives

$$I \sim 2 \times 10^{10} \text{ W/cm}^2,$$

which is typical of the breakdown intensity for short pulse length 1.06 μ light on glass.

B. EM wave propagation in plasma

[For a general reference, see V. L. Ginzberg, Propagation of Electromagnetic Waves in Plasmas (Gordon and Breach, NY, 1961).]

$$\underline{E} = \underline{E(x)} e^{-i\omega t} \quad \underline{B} = \underline{B(x)} e^{-i\omega t}$$

linearized electron response

$$\frac{\partial \underline{u}}{\partial t} = \frac{-e\underline{E}}{m} - \nu_{ei} \underline{u} \rightarrow \underline{u} = \frac{-ie\underline{E}}{m(\omega + i\nu_{ei})}$$

$$\underline{J} = -n_0 e \underline{u} = \frac{in_0 e^2 \underline{E}}{m(\omega + i\nu_{ei})}$$

$$\underline{J} = \sigma \underline{E}, \text{ where } \sigma = \frac{i\omega_{pe}^2}{4\pi(\omega + i\nu_{ei})}$$

$$\omega_{pe}^2 = \frac{4\pi n_0 e^2}{m}$$

$$\omega_{pe} = 5.65 \times 10^4 \sqrt{n_e}$$

$$\nabla \times \underline{E} = -\frac{1}{c} \frac{\partial \underline{B}}{\partial t} \rightarrow \nabla \times \underline{E} = \frac{i\omega}{c} \underline{B}$$

$$\nabla \times \underline{B} = \frac{4\pi}{c} \underline{J} + \frac{1}{c} \frac{\partial \underline{E}}{\partial t} \rightarrow \nabla \times \underline{B} = \frac{4\pi}{c} \sigma \underline{E} - \frac{i\omega}{c} \underline{E}$$

$$\nabla \times \underline{B} = \frac{-i\omega}{c} \epsilon \underline{E}, \text{ where } \epsilon = 1 - \frac{\omega_{pe}^2}{\omega(\omega + i\nu_{ei})}$$

$$\nabla \times (\nabla \times \underline{E}) = \frac{i\omega}{c} \nabla \times \underline{B}$$

$$= \frac{\omega^2}{c^2} \epsilon \underline{E}$$

em waves $\nabla \cdot \underline{E} = 0$

$$\nabla^2 \underline{E} + \frac{\omega^2}{c^2} \epsilon \underline{E} = 0$$

(3)

Taking $\underline{E} \sim e^{i\mathbf{k} \cdot \underline{x} - i\omega t}$ $\rightarrow \frac{\mathbf{k}^2 c^2}{\omega^2} = \epsilon$ dispersion rel'n for light waves in plasma

With $v_{ei} = 0$, $\omega^2 = \omega_{pe}^2 + \mathbf{k}^2 c^2$

Note that light wave propagates only when $\omega > \omega_{pe}$.

Why? ω_{pe} is the characteristic frequency with which electrons respond. When $\omega < \omega_{pe}$, the electrons can shield out the fields.

Since ω_{pe} depends on density, there is a "critical density" above which light will not propagate.

$$\frac{4\pi e^2 n_{cr}}{m} = \omega^2 \rightarrow n_{cr} = 10^{21} \left(\frac{1.06\mu}{\lambda_0} \right)^2$$

$\lambda_0 \equiv$ free space wavelength of light

Note that the laser light couples then in a rather low density plasma which surrounds the target.

Fig. 1

C. Collisional absorption

Collisions introduce a damping of the light wave.

$$\frac{\mathbf{k}^2 c^2}{\omega^2} = 1 - \frac{\omega_{pe}^2}{\omega(\omega + i\nu_{ei})} \rightarrow \omega^2 = \mathbf{k}^2 c^2 + \omega_{pe}^2 \left(1 - \frac{i\nu_{ei}}{\omega} \right)$$

$$\nu_{ei} \ll \omega$$

(4)

$$\omega = \omega_r - \frac{1}{2}\nu + \nu \text{ is energy damping rate}$$

$$\omega_r = \sqrt{\omega_{pe}^2 + \kappa^2 c^2}, \quad \nu = \frac{\omega_{pe}^2}{\omega_r} \nu_{e1}$$

$$\nu = \frac{n}{n_{cr}} \nu_{e1}, \text{ where } \nu_{e1} \approx 3 \times 10^{-6} \frac{n_i z^2 \ln \Lambda}{\Theta_e^{3/2}}$$

Physically the coherent energy of oscillation of electrons in the light wave is converted into random energy in a collision time:

$$\frac{\nu E_L^2}{8\pi} = \nu_{e1} \frac{nm V_{os}^2}{2}, \text{ where } V_{os} = \frac{e E_L}{m \omega}$$

Example:

$$\Theta_e = 1 \text{ keV} \quad \lambda_0 = 1.06 \mu$$

$$n = n_{cr} = 10^{21} \text{ cm}^{-3} \quad z = 1$$

$$\nu = 10^{12} \text{ sec}^{-1}$$

$$l_{abs} \approx \frac{c}{\nu} \approx 300 \mu$$

Note that $\propto \frac{1}{\Theta_e^{3/2}}$

Thus as the plasma becomes hot, it becomes nearly collisionless.

Reference: Phys. Fluids 12, 875 (1969)

D. Light absorption in collisionless plasma

Plasma supports waves - collective modes of oscillation.

These are charge density fluctuations.

Laser light $\xrightarrow{\text{excites}}$ plasma waves $\xrightarrow[\text{into}]{\text{which damp}}$ plasma particles

Plasma waves - principally two are of interest for laser plasma interactions.

For a general reference, see F. F. Chen

Introduction to Plasma Physics (Plenum Press, NY, 1974)

(5)

- electron plasma wave: a high frequency charge density oscillation with a frequency characteristic of the electron motion.

Use a 2-fluid description of the plasma

$$\text{electron fluid} \quad \frac{\partial n_e}{\partial t} + \frac{\partial}{\partial x} n_e u_e = 0 \quad \text{continuity eq'n}$$

$$\frac{\partial u_e}{\partial t} + u_e \frac{\partial u_e}{\partial x} = -\frac{e}{m} E - \frac{1}{m} \frac{\partial P_e}{\partial x} \quad \text{force eq'n}$$

$$\frac{\partial E}{\partial x} = 4\pi e(n_i - n_e) \quad \text{Poisson's eq'n}$$

Electron oscillations

Treat massive ions as fixed neutralizing background, neglect electron pressure, and linearize.

$$n_e = n_0 + \tilde{n} \quad E = \tilde{E}$$

$$u_e = \tilde{u}$$

$$\textcircled{1} \quad \frac{\partial \tilde{n}}{\partial t} + n_0 \frac{\partial \tilde{u}}{\partial x} = 0 \quad \textcircled{3} \quad \frac{\partial \tilde{E}}{\partial x} = -4\pi e \tilde{n}$$

$$\textcircled{2} \quad \frac{\partial \tilde{u}}{\partial t} = -\frac{e}{m} \tilde{E}$$

Take $\frac{\partial}{\partial t}$ of $\textcircled{1}$, $\frac{\partial}{\partial x}$ of $\textcircled{2}$, combine using $\textcircled{3}$.

$$\left(\frac{\partial^2}{\partial t^2} + \omega_{pe}^2\right) \tilde{n} = 0 \quad \text{electron plasma wave}$$

density fluctuations with frequency ω_{pe} .

- ion acoustic wave: a low frequency density oscillation which is the plasma analogue of a sound wave.

low frequency ($\omega \ll \omega_{pe}$), neglect electron inertia

$$\text{force eq'n for electron fluid} \quad 0 = -eE - \frac{1}{n_e} \frac{\partial P_e}{\partial x} \quad P_e = n_e \Theta_e$$

$$\text{linearizing} \quad n_e = n_0 + \tilde{n}_e, \quad E = \tilde{E}$$

$$\textcircled{1} \quad e\tilde{E} = -\frac{\Theta_e}{n_0} \frac{\partial \tilde{n}_e}{\partial x}$$

ion fluid

continuity $\frac{\partial n_1}{\partial t} + \frac{\partial}{\partial x} n_1 u_1 = 0$

force eq'n $\frac{\partial u_1}{\partial t} + u_1 \frac{\partial u_1}{\partial x} = \frac{ZeE}{M}$

linearizing $n_1 = n_0 + \tilde{n}_1 \rightarrow \textcircled{2} \frac{\partial \tilde{n}_1}{\partial t} + n_0 \frac{\partial \tilde{u}_1}{\partial x} = 0$
 $u_1 = \tilde{u}_1 \quad \textcircled{3} \frac{\partial \tilde{u}_1}{\partial t} = \frac{Ze\tilde{E}}{M}$

Taking $\frac{\partial}{\partial t}$ of $\textcircled{2}$, $\frac{\partial}{\partial x}$ of $\textcircled{3}$, and combining using $\textcircled{1}$:

$$\left(\frac{\partial^2}{\partial t^2} - c_s^2 \frac{\partial^2}{\partial x^2}\right) \tilde{n}_1 = 0 \quad \text{ion acoustic wave}$$

$$c_s = \sqrt{\frac{ZeE}{M}}, \quad \omega_{fa} = kc_s$$

Charge density waves driven by electron pressure which is transmitted to the ions via self-consistent electric field.

These are the modes of oscillation. How does the laser light excite them?

The oscillation of electrons across a density gradient by the electric field of the laser light drives a density fluctuation.

$$\delta n_e = n_e(x + x_{os}) - n_e(x) \approx x_{os} \cdot \nabla n_e(x).$$

$$\text{where } x_{os} = \frac{eE_L}{m\omega^2}$$

Hence if $\omega = \omega_{pe}$ (that is, if the laser light frequency is properly matched to the electron plasma frequency), electron plasma waves are resonantly excited.

The variation in density can be due to the overall density gradient due to plasma expansion (resonance absorption) or even to the density fluctuations associated with other plasma waves.

E. Resonance Absorption

The next process we consider is resonance absorption. In this process, a light wave is partially converted into a high frequency electrostatic oscillation (electron plasma wave) near the critical density. First we will give a physical model for resonant coupling into a plasma wave and then show how an obliquely incident light wave can provide the coupling. This simple treatment is sufficient to both elucidate the physics and to exhibit the important dependences of resonance absorption.

To investigate the resonant coupling, consider a one-dimensional capacitor problem: a nonuniform plasma externally driven by a spatially uniform electric field with amplitude E_d and frequency ω_0 . From Maxwellian equations we have

$$\frac{\partial E}{\partial t} + 4\pi J = \left\langle \frac{\partial E}{\partial t} + 4\pi J \right\rangle \quad (1)$$

where E is the electrostatic field, J is the current density and the brackets denote the spatially independent component. If we neglect ion motion and linearize, the current density is

$$J = -en_0(x)v. \quad (2)$$

Here v is the oscillation velocity of electrons in both the imposed field ($E_d \cos \omega_0 t$) and the self-consistent field (E) due to the plasma. Taking time derivatives of equations (1) and (2) and using the linearized equation of motion (see Section B), we then obtain

$$\frac{\partial^2 E}{\partial t^2} + \omega_{pe}^2(x)E + v_{ei} \frac{\partial E}{\partial t} = - \left[\omega_{pe}^2(x) - \langle \omega_{pe}^2(x) \rangle \right] E_d \cos \omega_0 t \quad (3)$$

Taking $E \sim e^{i\omega_0 t}$, assuming a linear density profile, and solving for the driven response gives

$$E = \frac{\omega_{pe}^2(x) E_d}{\omega_0^2 - \omega_{pe}^2(x) + i\nu_{ei}\omega_0} \quad (4)$$

Note the resonant response when $\omega_0 \approx \omega_{pe}$.

The flux of energy (I_{ABS}) which must be supplied by the externally imposed driver field is now readily obtained by computing the energy lost by the pump field:

$$I_{ABS} = \int \nu_{ei} \frac{|E|^2}{8\pi} dx ,$$

If we again assume a linear density gradient ($n = n_{cr} \frac{x}{L}$).

$$I_{ABS} = \frac{\omega_0 L E_d^2}{8} \quad (5)$$

Note that ν_{ei} cancels out, and so the energy flow is independent of the detailed mechanism which provides the damping of the electrostatic field.

Physically the height of the resonance is $\propto \frac{1}{\nu_{ei}}$ and the width of the resonance is $\propto \nu_{ei}$; hence the cancellation.

Equations (4) and (5) show that whenever a driver field oscillates electrons across a density gradient, an electrostatic oscillation is resonantly driven near the critical density (i.e., when $\omega_0 \approx \omega_{pe}$). Such a driver field is provided by a light wave obliquely incident onto an inhomogeneous plasma if a component of its electric field vector is along the direction of the density gradient.

Consider now a light wave obliquely incident onto an inhomogeneous plasma slab. As shown in Fig. 2, we take the propagation vector of the wave to be

in the Y-Z plane (without loss of generality), the density gradient to be in the Z direction, and the angle between ∇n and k in vacuum to be θ .

Since the density is only a function of Z, k_y is conserved and so equals $\frac{\omega_0}{c} \cos \theta$. The dispersion relation for the light wave then becomes

$$\omega_0^2 = \omega_{pe}^2 + \omega_0^2 \sin^2 \theta + k_z^2 c^2. \quad (6)$$

The maximum density which the wave reaches (n_t) is given by the cutoff condition $k_z = 0$:

$$n_t = n_{cr} \cos^2 \theta. \quad (7)$$

Note that the wave reflects below the critical density, although of course the fields do penetrate roughly a few skin depths into the higher density region.

The occurrence of resonance absorption depends on the orientation of the electric vector (E_L) of the light. If E_L is in the X-direction (i.e., out of the plane of incidence which is the plane defined by ∇n and k), the electric field has no component along the density gradient and so does not excite the resonance. This is called s-polarized light. If E_L is in the plane of incidence (i.e., in the Y-Z plane), it does have a component along the density gradient. This is called p-polarized light. Although the obliquely incident light reflects at a density lower than critical, its fields will still tunnel to the critical density region and so excite the resonance.

A very simple estimate can be given for the resonance absorption of p-polarized light. Basically we need only to estimate the component of the electric field which drives the resonance (E_d) and then use equation (5). To do this it is most convenient to work in terms of the magnetic field of the light wave. If we assume p-polarized light and the geometry depicted in

Fig. 2,

$$\underline{B} = \hat{x}B(z) \exp(-i\omega t + \frac{i\omega}{c} \sin\theta y).$$

From Ampere's law we then obtain

$$E_z = \frac{\sin\theta B(z)}{\epsilon(z)}$$

where ϵ is the dielectric function of the plasma. By analogy (4),

$$E_d = \sin\theta B(n = n_{cr}).$$

The magnetic field at the critical density $[B(n = n_{cr})]$ is estimated as its value at the turning point density times the decay of the field into the critical density region. If we have a density ramp and then use the well-known Airy function solution for wave propagation in a linear profile,

$$B(n = n_{cr}) \approx \frac{.92E_{FS}}{\left(\frac{\omega_0 L}{c}\right)^{1/6}} \exp\left(-\frac{2}{3} \frac{\omega_0 L}{c} \sin^3\theta\right),$$

where E_{FS} is the free-space value of the electric field of the wave. The $(k_0 L)^{1/6}$ term comes from the decrease of the magnetic field from the turning point as the group velocity of the wave becomes small. Inserting this into equation (9) then gives

$$E_d \approx \frac{.92E_{FS}}{\left(\frac{\omega_0 L}{c}\right)^{1/6}} \sin\theta \exp\left(-\frac{2}{3} \frac{\omega_0 L}{c} \sin^3\theta\right).$$

The absorption is now readily estimated using equation (11) in equation (5) noting that $I_{\text{ABS}} = \frac{CE_{\text{FS}}^2}{8\pi}$, and defining $\tau = (k_0 L)^{1/2} \sin \theta$ gives

$$f = \frac{\phi^2(\tau)}{2},$$

where f is the fractional absorption and

$$\phi(\tau) = 2.31 \tau \exp\left(-\frac{2}{3} \tau^3\right) \quad (12)$$

The important features of resonance absorption can be deduced from the function $\phi(\tau)$, which is plotted in Fig. 2. Note that the absorption, which is equal to $\phi^2(\tau)/2$, vanishes at $\theta = 0$ (no component of the electric field along the direction of the density gradient). The absorption is very small for large angles such that $\tau \gg 1$ (the fields have to tunnel through too large a distance). Optimum absorption occurs for an angle θ_{MAX} , defined by $\tau = .8$, and the range of angles for which the absorption is sizeable is $\Delta\theta \approx \theta_{\text{MAX}}$. Hence the significance of resonance absorption depends strongly on the value of the density gradient length near the critical density. If $\frac{\omega_0 L}{c}$ is very large, this absorption is effective for only a narrow range of angles. If $\frac{\omega_0 L}{c} \lesssim 10$, resonance absorption occurs for a broad range of angles. The optimum absorption in a linear profile is somewhat over-estimated by this simple model and is actually $\approx .5$, rather than $.85$.

(12)

Fig. 2

Fig. 3

F. Instabilities

So a variation in plasma density can couple light into plasma waves:

$$E_L \frac{\delta n}{n} \rightarrow E_{pw}$$

- The density variation can itself be due to an electrostatic wave.

$$n = n_0 + \delta n$$

$$\omega_{pe}^2(x) - \langle \omega_{pe}^2 \rangle = \frac{4\pi e^2}{m} \delta n$$

and

$$\left(\frac{\partial^2}{\partial t^2} + \omega_{pe}^2 + v_e \frac{\partial}{\partial t} \right) E = -\frac{4\pi e^2}{m} \delta n E_L$$

where δn is the density fluctuation associated with either an ion acoustic wave or even another electron plasma wave.

- In turn, a density fluctuation can be produced by gradients in the electric field pressure.

To see this, we compute the low frequency electron response to a high frequency field

$$\underline{E} = \underline{E}(x) \sin \omega_0 t.$$

Again describe electrons as a fluid.

$$\frac{\partial n_e}{\partial t} + \frac{\partial}{\partial x} (n_e \underline{u}_e) = 0$$

$$m \left(\frac{\partial}{\partial t} + \underline{u}_e \cdot \frac{\partial}{\partial x} \right) \underline{u}_e = -e \underline{E} - \frac{1}{n_e} \frac{\partial p_e}{\partial x}$$

Decompose into high and low frequency components

$$\underline{u}_e = \underline{u}_e^h + \underline{u}_e^l$$

Then

$$m \frac{\partial u_e^L}{\partial t} = -eE^L - \frac{1}{n_e} \frac{\partial p_e^L}{\partial x} - m \langle u_e^h \cdot \frac{\partial}{\partial x} u_e^h \rangle t.$$

To compute the low frequency response to second order (E^2), we need only determine the high frequency response to first order:

$$\frac{\partial u_e^h}{\partial t} = \frac{eE(x)}{m} \sin \omega_0 t$$

$$u_e^h = \frac{eE(x)}{m\omega_0} \cos \omega_0 t$$

Substituting gives

$$m \frac{\partial u_e^L}{\partial t} = -eE^L - \frac{\nabla p_e^L}{n_e} - \frac{e^2}{4m\omega_0} \nabla E^2(x)$$

Hence there is a low frequency force due to the gradient of the electric field pressure.

$$F_p = - \frac{\omega_{pe}^2}{\omega_0^2} \nabla \langle \frac{E^2}{8\pi} \rangle$$

ponderomotive
force
density

Physically, electric field pressure is analogous to a particle pressure. The gradient gives a force pushing electrons away from regions of high field intensity.

Note that the ponderomotive force acts directly on the electrons - it is transmitted to the ions via electric fields.

Now we can understand why instabilities are possible.

The light couples with a density fluctuation to produce an electric field. In turn, the electric field couples with the light wave field to produce a fluctuation in field pressure which enhances the density fluctuation.

There are many different instabilities driven by laser light, but the physics is basically the same.

$E_L \delta n \rightarrow$ electrostatic
or
transverse wave

$$\frac{\partial E^2}{\partial x} \rightarrow \delta n$$

E can be due to an
electrostatic or transverse
wave.

For a detailed reference, see Advances in Plasma Physics, Vol. 6, edited by A. Simon and W. Thompson (Wiley, NY, 1976)

Various combinations

$$\omega_0 \rightarrow \omega_{pe} + \omega_{ia}$$

Ion acoustic decay

$$\omega_0 \rightarrow \omega_{pe} + \omega_{pe}$$

2 plasmon decay

$$\omega_0 \rightarrow \omega_t + \omega_{ia}$$

Brillouin instability

$$\omega_0 \rightarrow \omega_t + \omega_{pe}$$

Raman instability

And there are other instabilities in which one of the waves is not a "normal mode" of an undriven plasma but rather a zero frequency, purely growing mode (oscillating two stream instability, filamentation,...).

G. Brillouin instability

As a specific example, we consider the Brillouin instability.

light wave $\rightarrow$ scattered light wave and ion acoustic wave

$$\nabla \times \underline{B} = \frac{4\pi}{c} \underline{j} + \frac{1}{c} \frac{\partial \underline{E}}{\partial t} \quad \rightarrow \quad \frac{\partial^2 \underline{E}}{\partial t^2} - c^2 \nabla^2 \underline{E} = -4\pi \frac{\partial \underline{j}}{\partial t}$$

$$\nabla \times \underline{E} = \frac{1}{c} \frac{\partial \underline{B}}{\partial t}$$

$$\underline{j} = -n(x)e \underline{u}_e$$

$$\frac{\partial \underline{j}}{\partial t} = -n(x)e \frac{\partial \underline{u}_e}{\partial t} = \frac{ne^2}{m} \underline{E}$$

using the linearized electron
force eq'n

$$\left[\frac{\partial^2}{\partial t^2} - c^2 \nabla^2 + \omega_{pe}^2(x) \right] \underline{E} = 0$$

Consider 1-d (back-scatter), uniform plasma with

$$n = n_0 + \delta n \leftarrow \text{ion density fluctuation}$$

$$\left[\frac{\partial^2}{\partial t^2} - c^2 \frac{\partial^2}{\partial x^2} + \omega_{pe}^2 \right] E = -\omega_{pe}^2 \frac{\delta n}{n} E$$

$$E = E_L + \delta E \leftarrow \text{scattered light wave}$$

$$\left[\frac{\partial^2}{\partial t^2} - c^2 \frac{\partial^2}{\partial x^2} + \omega_{pe}^2 \right] \delta E = -\omega_{pe}^2 \frac{\delta n}{n} E_L$$

To derive an eq'n for the evolution of δn , use the 2-fluid description for the plasma and linearize

electrons $u_e = u_e^h + u_e^l$

$$n_e = n_e^l \leftarrow \text{low frequency electron density fluctuation in the ion wave } (n_e^l \approx \delta n)$$

low frequency
component
of electron
force eq'n

$$m \frac{\partial u_e^l}{\partial t} = -eE^l - \frac{\theta_e}{m_0} \frac{\partial n_e^l}{\partial x} - \frac{e^2}{2m\omega_0^2} \frac{\partial}{\partial x} (2E_L \delta E)$$

Neglecting electron inertia gives

$$eE^l = \frac{\theta_e}{m_0} \frac{\partial n_e^l}{\partial x} - \frac{e^2}{m\omega_0^2} \frac{\partial}{\partial x} (E_L \delta E)$$

ion fluid

$$n_i = n_0 + \delta n \quad u_i = u_i$$

$$\frac{\partial}{\partial t} \delta n + n_0 \frac{\partial u_i}{\partial x} = 0$$

linearized continuity
eq'n

$$\frac{\partial u_i}{\partial t} = \frac{Ze}{M} E^l$$

linearized force
eq'n

ion pressure is neglected for simplicity.

Taking derivatives of these eq'ns and combining

$$\left(\frac{\partial^2}{\partial t^2} - c_s^2 \frac{\partial^2}{\partial x^2} \right) \delta n = \frac{Zn_0 e^2}{mM\omega_0^2} \frac{\partial^2}{\partial x^2} (E_L \delta E)$$

Dispersion rel'n

$$E_L = E_L \cos(\kappa_0 x - \omega_0 t) = \frac{E_L}{2} [e^{i\kappa_0 x - i\omega_0 t} + e^{-i\kappa_0 x + i\omega_0 t}]$$

Fourier-analyze

$$[-\omega^2 + \kappa^2 c^2 + \omega_{pe}^2] \delta E(\kappa, \omega) = \frac{-4\pi e^2}{m} \frac{E_L}{2} [\delta n(\kappa - \kappa_0, \omega - \omega_0) + \delta n(\kappa + \kappa_0, \omega + \omega_0)]$$

$$[-\omega^2 + \kappa^2 c^2] \delta n(\kappa, \omega) = \frac{-Z n_0 e^2}{m m \omega_0^2} \kappa^2 \frac{E_L}{2} [\delta E(\kappa - \kappa_0, \omega - \omega_0) + \delta E(\kappa + \kappa_0, \omega + \omega_0)]$$

Choose ω low frequency ($\omega \ll \omega_0$), substitute and neglect responses with frequency $n\omega_0$ which have $|n| > 1$ as off-resonant.

$$\omega^2 - \kappa^2 c^2 = \frac{\omega_{pe}^2}{\omega_0^2} \frac{Z e^2}{m M} \frac{\kappa^2}{\omega_0^2} \frac{E_L^2}{4} \left[\frac{1}{(\omega - \omega_0)^2 - (\kappa - \kappa_0)^2 c^2} - \omega_{pe}^2 + \frac{1}{(\omega + \omega_0)^2 - (\kappa + \kappa_0)^2 c^2} \right]$$

Solutions

$$\kappa = 2\kappa_0$$

$$\gamma \gg \kappa v_s \quad \omega = \kappa v_s + i\gamma$$

$$\gamma = \frac{1}{2\sqrt{2}} \sqrt{\frac{Zm}{M}} \kappa_0 v_s \sqrt{\frac{\omega_{pe}^2}{\omega_0 \kappa_0 v_s}}$$

$$\gamma \gg \kappa v_s \quad \omega = \left(\frac{\kappa_0^2 v_s^2}{2} \frac{Zm}{M} \frac{\omega_{pe}^2}{\omega_0} \right)^{1/2} \left(\frac{1}{2} \pm i \frac{\sqrt{3}}{2} \right)$$

Example

$$I_L = 10^{16} \text{ W/cm}^2 \quad (\text{Nd light})$$

$$\frac{v_{os}}{c} = .1 \quad \frac{Zm}{M} = \frac{1}{1800} \quad \frac{n}{n_{cr}} = \frac{1}{2}$$

$$\gamma = .01 \omega_0 \quad \omega_0 = 2 \times 10^{13} \text{ sec}^{-1}$$

$$t \sim \frac{1}{\gamma} = \frac{1}{20} \text{ ps}$$

H. Some other effects

• Self-generated magnetic fields

Expanding plasma

$$ne\underline{E} = -\frac{\partial p_e}{\partial x}$$

$$\underline{E} = -\frac{1}{ne} \frac{\partial p_e}{\partial x}$$

$$\frac{1}{c} \frac{\partial B}{\partial t} = \nabla \times \underline{E}$$

$$\frac{\partial B}{\partial t} = \frac{c}{e} \nabla \times \frac{1}{n} \nabla p_e$$

$$p_e = n\theta_e \quad \nabla \times \frac{1}{n} \nabla p_e = \frac{-\nabla n}{n^2} \times \nabla p$$

$$\frac{\partial B}{\partial t} = -\frac{c}{e} \frac{\theta_e}{n} \nabla n \times \nabla \theta_e$$

B fields are generated - these can affect the absorption and heat transport.

See Phys. Fluids 20, 1688 (1977) and references therein.

• Ion turbulence driven by heat flow

Electron heat flow → net drift in velocity between the ions and the electrons

Fig. 4

ion acoustic waves are driven unstable by the relative drift. These grow up and can affect the absorption and transport.

see Phys. Fluids 20, 55 (1977) and references therein.

. . . . others

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