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REMINISCENCE OF A STELLAR PULSATION THEORIST

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ABSTRACT. The author started working in stellar pulsation theory in 1965. Since this time, we have studied the effects of improved radiative transfer and dynamic zoning on models of Cepheids, RR Lyrae, and W Virginis stars. In this paper we discuss the relevant equations and some results in comparisons to observations of W Virginis and long-period Cepheids. Some suggestions for the next generation of stellar pulsation codes is given.

1. Introduction

The first nonlinear stellar pulsation calculations were done in the early 1960s by Christy, Cox, and Hillendahl. These codes were preceded by the development of the method of pseudo-viscosity. The addition of multi-group radiative transfer to the Lagrangian hydrodynamic codes was made in the late 1960s. By the 1970s, a dynamically zoned hydro code was available and it seemed obvious that a more complete code could be developed including dynamic zoning and radiative transfer. The methods to calculate line transfer in a moving media, Ala Mihalas, were developed in the 70s but now with the new computers (i. e. YMPs and connection machines) there is the possibility of solving these equations coupled with the dynamically-zoned hydrodynamic equations.

2. Early Hydro

The early hydro codes were of the Richtmyer-Von Neuman pseudo-viscosity variety. These codes used standard explicit differing schemes Eqs. (1-5) with the possible addition of expansions for time centering. It is interesting to note that the author in his pulsation code (SPECEP) had to add a small amount of linear viscosity to the center zone of the mesh, as did Christy, in order to stabilize the mesh. The standard quadratic pseudo-viscosity term was used in these early codes, with a cutoff added later by Stellingwerf (see Eq. [6]), to reduce unwanted dissipation.

Nonlinear Pulsation Equations

$$\left(\frac{\partial r}{\partial t}\right)_{M_r} = u, \quad (1)$$

$$\left(\frac{\partial u}{\partial t}\right) = -\frac{GM_r}{r^2} - 4\pi r^2 \left(\frac{\partial P}{\partial M_r}\right)_t, \quad (2)$$

$$\left(\frac{\partial E}{\partial t}\right)_{M_r} + P \left(\frac{\partial V}{\partial t}\right)_{M_r} = -\left(\frac{\partial L_r}{\partial M_r}\right)_t, \quad (3)$$

$$V = \left(\frac{\partial}{\partial M_r}\right)_t \left(\frac{4\pi}{3} r^3\right), \quad (4)$$

and

$$L_r = -\frac{(4\pi r^2)^2}{3\kappa} \left(\frac{\partial \tau^4}{\partial M_r}\right)_t. \quad (5)$$

Pseudo Viscosity

$$q = \begin{cases} \epsilon^2 \rho (\partial u / \partial x)^2 \\ \text{or } 0 \text{ if } (\partial u / \partial x) < -bC_0 \end{cases}, \quad (6)$$

$$\epsilon = a\Delta x \quad C_0 = \text{Speed of sound} = \sqrt{\gamma P / \rho}.$$

From a careful study of zoning effects (Davis and Bendt) we found that 72 zones were optimum in resolving the light curves. I believe Christy used 50 zones, because of his limited access to computer time, while the Cox's et al. used 60 zones. In Fig. 1 we make a comparison of an F-Cubed result, for a model due to Christy, with SPECEP in the diffusion approximation and with 50 zones. It is important to test your code against other codes and analytic solutions if available. The same kind of agreement was observed with many other codes as seen in the Goddard Conference Proceedings (1974).

3. Radiative Transfer

The author joined Los Alamos in 1965 to work with A. Cox to add a radiative transfer scheme to the nonlinear pulsation models. At about the same time J. Castor became a graduate student under Christy at Cal.

Fig. 1. Comparisons of radiative flux vs. phase for an early Christy model between F-cubed (—) and SPECEP (----).

Tech. to do approximately the same thing. The author having worked with B. Freeman at General Atomic on various radiative transfer schemes thought it reasonable to apply these approaches to the stellar-pulsation problem. The initial code called SPECEP utilized the variable Eddington method with the Eddington factors obtained by a discrete ordinate approximation in plane geometry. The atmosphere of a red giant is very thin compared to it's radius. The moment equations of radiative transfer are shown in Eq. (7,8). Castor applied the Schwartzchild-Milne

0th Moment

$$\rho \frac{D(E_\nu^0/\rho)}{Dt} + \nabla \cdot (\dot{F}_\nu^0 + \dot{u} \cdot \bar{P}_\nu^0) = c\mu_\nu^0 \left(\frac{4\pi B_\nu^0}{c} - E_\nu^0 \right) - \mu_\nu^0 \frac{\dot{u}}{c} \cdot \dot{F}_\nu^0, \quad (7)$$

1st Moment

$$\rho \frac{D(F_\nu^0/\rho)}{Dt} + c \cdot \nabla \cdot \bar{P}_\nu^0 = - (\mu_\nu^0 + \mu_s) \bar{F}_\nu^0 \quad (8)$$

(Astrophysical Convention: $J = cE$, $H = \bar{F}$, $K = c\bar{P}$)

integral equations in his research. The temperature update for SPECEP is obtained by a partial temperature method as shown in Eqs. (9-11).

$$\frac{\partial E_\nu}{\partial t} + \frac{1}{r^2} \frac{\partial (r^2 F_\nu)}{\partial r} = c \mu_p^\nu \left[\frac{4\pi B_\nu}{c} - E_\nu \right] \quad (9)$$

$$\frac{1}{c} \frac{\partial F_\nu}{\partial t} + c \left[\frac{\partial f^\nu E_\nu}{\partial r} + \frac{3f^\nu - 1}{r} E_\nu \right] = - \mu_R^\nu F_\nu \quad (10)$$

f^ν calculated explicitly by P,Z method (sometimes called Direct integration or Y-line method).

Partial Temperature

$$\frac{D[b_j \phi]}{Dt} + \frac{4a\phi^3}{\rho C_v} \left\{ b_j \left[\left(P_m + \frac{\partial E}{\partial r} \right) \rho \frac{dr}{dt} \right] + \left[c \mu_{pj} (b_j \phi - E_j) \right] \right\} \quad (11)$$

The $1/c \partial F_\nu / \partial t$ term is usually dropped and the velocity terms ($\dot{a}/c$) are small. A flux limiter is then applied.

Later, we developed a multi-frequency grey approach in order to speed up the calculation (see Eqs. [12-17]). A discussion of the effects of multi-group radiative transfer on models of "bump" Cepheids is also given in the 1974 Goddard Conference Proceedings. Probably the most successful application of our radiative-transfer code was to a model of

Multi-Frequency Grey

$$\frac{\partial E_R}{\partial t} + \frac{1}{r^2} \frac{\partial (r^2 F_R)}{\partial r} = c \mu_a \phi^4 - c \mu^e E_R \quad (12)$$

$$\frac{1}{c} \frac{\partial F_R}{\partial r} + c \left[\frac{f(f^0 E_R)}{\partial r} + \frac{(3f^0 - 1)}{r} E_R \right] = - \mu^0 F_R \quad (13)$$

$$\mu^b = \int_0^\infty \mu^\nu B_\nu d\nu / \int_0^\infty B_\nu d\nu \quad (14)$$

$$\mu^e = \int_0^\infty \mu^\nu E_\nu d\nu / \int_0^\infty E_\nu d\nu \quad (15)$$

$$f^o = \int_0^\infty F^\nu E_\nu d\nu / \int_0^\infty E_\nu d\nu \quad (16)$$

$$\mu^e = \int_0^\infty \mu_\nu F_\nu d\nu / \int_0^\infty F_\nu d\nu \quad (17)$$

Fig. 2. The calculated light curve for W Virginis using a multi-group radiative-transfer code (SPECER).

W Virginis first proposed by Christy in 1966. The calculated light curve is shown in Fig. 2 with an estimate of the shock location vs. phase in Fig. 3. The shoulder, as observed in W Virginis (Fig. 4), did not appear in Christy's diffusion model.

W VIRGINIS PHOTOSPHERIC RADIUS AND SHOCK LOCATION

Fig. 3. The photospheric and shock radii locations from the W Virginis model.

LIGHT CURVE W VIRGINIS

Fig. 4. The observations of the light curve of W Virginis.

4. Dynamic Zones

In association with John Castor, we developed a dynamically-zoned stellar pulsation code (DYN) in 1977. The idea of resolving the ionization driving region by adapting zones, in a non-Lagrangian mesh had been studied by Castor in his thesis at Cal Tech (see Fig. 5). The code is implicit because of restricted Courant time steps that occur in these thin zones. An idea of the improvement obtained with dynamic zoning is shown in Fig. 6. Castor suggests that further improvements can be obtained by using the more robust adaptive mesh-algorithms of Winkler. The code DYN has been used extensively by Takeuti's group in Tokyo. Recently we have applied DYN to questions of model content (Buchler and Kovacs) [1], resonance in Cepheids (Simon) [2] and long-period variables with Barnes and Moffet [3]. I will only show results from the later problems to emphasize the importance of dynamic zoning. A particular model of X Cygni was studied because of the careful observations of it's light curve. Using an evolutionary mass with an appropriate luminosity and effective temperature, we produced the model which is compared to the observations in Fig. 7. The "dip" observed before light maximum appears real and was observed in other stars.

Fig. 5. Dynamic Zoning: note the location of zones in the ionization region.

Fig. 6. Calculated light curves with and without dynamic zoning.

The dip in the calculation is due to the transit of the shock through the atmosphere and it is therefore effected by the pseudo-viscosity.

Fig. 7. A model of a long-period cepheid (X Cygni) using dynamic zoning (DYN) as compared to the observations.

5. Conclusions

From these reminiscence, it is obvious that a lot more could be done in modeling stellar pulsations. The addition of radiative transfer to a dynamically-zoned code (DYN for instance) would enhance our ability to study high luminosity to mass stars. A treatment of line transfer in the moving atmosphere (Ala Mihalas) would allow us to make a more direct comparison to observations. I have not mentioned improved opacities nor equations of state that are being developed; these should be included in

future codes. There is a lot to be done but only a few new codes have appeared in recent years (we should not ignore the contributions of Von Sengbush and Stellingwerf using relaxation methods).

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