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MASTER

THE SPECIFIC HEAT OF NbTi FROM 0 TO 7 T BETWEEN 4.2 AND 20 K*

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INTRODUCTION

Reasonably reliable zero-field specific heat data for commercial NbTi have been reported.^{1,2} Also reported are specific heat measurements at magnetic field.³ However, existing data are not well known in the technical community, and there appear to be significant uncertainties associated with the measurements at field, as discussed in this paper. We report the results of specific heat measurements made at field on an annealed Nb-44.6% Ti rod. A formula has been derived by thermodynamic arguments for the specific heat c_p of Type II superconductors as a function of temperature T and applied magnetic field H . The formula relates the specific heat in the superconducting state to the Sommerfeld constant γ , the critical temperature T_c , the normal-state phonon contribution BT^3 , and the upper critical magnetic field $H_{c2}(0)$:

$$c_p = \left(\beta + \frac{3\gamma}{T_c^2} \right) T^3 + \frac{\gamma H}{H_{c2}(0)} T.$$

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This formula is used to interpret the data of this paper as well as those previously reported.

THEORY

If one makes the simplifying assumption that hard Type II superconductors exhibit thermodynamic reversibility, one can apply known thermodynamic relationships⁴ to a cylindrical Type II superconductor in a paraxial magnetic field to get:

$$G_s(T,H) = G_n(T) + \frac{\mu_0}{\rho} \int_H^{H_{c2}(T)} M dH. \quad (1)$$

(A table of symbols is given at the end of the paper.) Measured magnetization curves of Type II superconductors look like those sketched in Fig. 1. The upper and lower critical fields as a function of temperature can be very nearly represented by:

$$H_{ci}(T) = H_{ci}(0) \left[1 - \left(\frac{T}{T_c} \right)^2 \right], \quad i = 1, 2. \quad (2)$$

According to (2), the ratio $H_{c1}(T)/H_{c2}(T)$ is independent of temperature.

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Fig. 1. Typical form of magnetization curves for a Type II superconductor.

As the temperature changes from T to T' , both the base and height of the magnetization curve are transformed by the factor $[1 - (T'/T_c)^2]/[1 - (T/T_c)^2]$. As a simplifying approximation, we apply this factor to both ordinate and abscissa of the entire magnetization curve; i.e., we assume that the magnetization curves corresponding to different temperatures are geometrically similar. If so, then

$$\int_0^{H_{c2}} M dH = -a \left(1 - \frac{T^2}{T_c^2}\right)^2 ; \quad a = \left| \int_0^{H_{c2}(0)} M(T=0) dH \right|. \quad (3)$$

Using (1), (3), and the thermodynamic relationship $c_{p,H} = -T(\partial^2 G / \partial T^2)_{p,H}$, one can show that

$$c_s = c_n - \frac{4\mu_0 a T}{\rho T_c^2} \left(1 - \frac{T^2}{T_c^2}\right), \quad (H=0). \quad (4)$$

The constant a can be determined from the empirical observation that the specific heat at zero field in the superconducting phase vanishes faster than linearly with vanishing temperature. Since this is so, the term $-(4\mu_0 a / \rho)(T/T_c^2)$ in (4) must cancel the linear term in $c_n = \gamma T + \beta T^3$, so that $a = \rho \gamma T_c^2 / 4\mu_0$. From this, it follows that

$$c_s = [\beta + (3\gamma/T_c^2)]T^3 ; \quad (c_s - c_n)_{T=T_c} = 2\gamma T_c \quad (5)$$

when $H = 0$. These formulas, derived by Gorter and Casimir using the two-fluid model,⁵ will result from any model in which the magnetization curves at different temperatures are similar to one another and scale according to (2) in M and H .

To study the field dependence of c_p , one must go further, approximating the curve PQ in Fig. 1 by a straight line. The slope of segment PQ then equals

$$\left(\frac{dM}{dH}\right)_{H_{c2}} = \frac{H_{c1}}{H_{c2} - H_{c1}} = \frac{H_{c1}(0)}{H_{c2}(0) - H_{c1}(0)} \approx \frac{H_{c1}(0)}{H_{c2}(0)}, \quad (6)$$

the second equality following from (2). Furthermore,

$$\frac{1}{2} H_{c1}(0) H_{c2}(0) = a. \quad (7)$$

Then $(dM/dH)_{H_{c2}} = \rho \gamma T_c^2 / 2 \mu_0 H_{c2}^2(0)$, and it follows from (1) that

$$c_s = \left[\beta + \left(\frac{3\gamma}{T_c^2} \right) \right] T^3 + [\gamma T H_{c2}(0)]. \quad (8)$$

Thus, the presence of a magnetic field is expected to increase c_s by roughly $[\gamma H_{c2}(0)]T$ over its zero-field value. This result, derived in the microscopic theory of superconductivity, has been cited in Refs. 4 and 5.

EXPERIMENTAL PROCEDURE

The low temperature section of the cryostat is shown diagrammatically in Fig. 2. The sample, a 10-mm-diam, 75-mm-long, annealed NbTi rod weighing 35 g, is hung by nylon threads inside an evacuated, temperature-controlled copper can. An evacuated outer can isolates the copper can from the bath. Table I summarizes the characteristics of our sample and of samples used in Refs. 1 and 2. The value of T_c for our sample is taken from recent data of Larbalestier⁶ for alloys of this composition.

A 42-gage manganin wire, tightly wrapped around the sample, served as the sample heater. A thin layer (≈ 0.15 g) of GE 7031 varnish bonded the heater to the sample.

Pulses of 1- to 100- μ s duration ($E \approx 0.1$ -1.0 mJ) to the heater caused temperature excursions of 1-10 mK. The record of a particular pulse is shown in Fig. 3. The temperature excursion was measured by a carbon resistor embedded in the sample (Allen-Bradley, 1/8 W, 430 Ω at 300 K, 11 k Ω at 4.2 K). An ac bridge technique (15 Hz) was used to monitor the change in resistance resulting from the temperature excursions.

Fig. 2. Schematic of the cryostat used for the calorimetric measurements of the specific heat of the NbTi sample at field.

Table I. Sample specifications

Measurement	Composition	T_c
Corsan, Ref. 1	Nb-44% Ti	9.2 ^a
Zbasnik, Ref. 2	Nb-48.8% Ti	9.0 ^a
Elrod, Miller, and Dresner, this paper	Nb-44.6% Ti ^b	9.1

^aMidpoint of transition (determined graphically from experimental data).

^bProvided by Kawicki Berylco Industries; 270 ppm Fe, 400 ppm Ta.

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Fig. 3. An example of the transient record of the response to a particular heat pulse. This record provides one data point.

Heater 2 (see Fig. 2) controlled the temperature of the inner can to within ≤ 5 mK/min. A calibrated germanium resistance thermometer (GRT) (Lake Shore Cryotronics GR-200A-2500) gave the temperature of the inner can to within ± 5 mK. Electrical leads running to the sample and to the GRT were singly heat sunk to the outer can and doubly heat sunk to the inner can.

After a series of data points was taken at field, it was necessary to transfer the GRT factory calibration to the carbon resistor. The GRT's highly field-dependent resistance complicated the situation. In order to transfer the zero-field GRT calibration to the carbon resistor at field, we filled the inner can with 2.66×10^3 Pa (20 torr) of helium gas at 4.2 K and used its gas pressure as an intermediate temperature scale. A pressure transducer (sensitivity = 3×10^{-5} V output/mm Hg) communicated with the inner can via a capillary (see Fig. 2).

The following polynomials are least squares fitted to the calibrations, along with the maximum estimated errors associated with each procedure.

$$\text{GRT vs } T \text{ (factory calibration): } \log T = \sum_{n=1}^5 A_n (\log R)^n,$$

$$\text{error } \Delta T_{\max} = 2 \text{ mK} \quad (9)$$

$$\text{Carbon resistor vs } T: \frac{1}{T} = \frac{A}{\log R} + B + C \log R,$$

$$\text{error } \Delta T_{\max} = 10 \text{ mK} \quad (10)$$

In order to obtain ΔT corresponding to a change in the resistance of the carbon resistor, (10) is explicitly differentiated to give dT/dR . The error involved in this procedure is estimated both graphically and by comparing dT/dR for (10) with

that for $1/T = \sum_{n=2}^2 A_n (\log R)^n$. The error appears to be no greater than 2%.

Fig. 4. Experimental results for c_p/T vs T^2 for Nb-44.6% Ti at fields from 0 to 7 T.

Fig. 5. Comparison of results from this paper with others from the literature for c_p/T vs T^2 at zero field.

ANALYSIS

Shown in Fig. 4 are the experimental results of this paper. Addenda corrections, involving $\leq 8\%$ of the total specific heat, are made based on specific heat values reported in the literature.⁷ The error bars shown in Fig. 4 represent the maximum random errors due to resolution limits in nulling the bridge and in reading the transient recorder. Error bars for $T < T_c$ are smaller than 2%.

The solid lines are constructed by first drawing a best-fit line to the superconducting-state specific heat. The best-fit normal-state line must, according to the simple theory outlined above, have an intercept of γ and result in a sharp increase of 2γ in the specific heat at T_c . Such a line can be drawn through the experimental points, suggesting that the assumption of self-similarity of magnetization curves was a fairly good one.

In Fig. 5, c_p/T versus T^2 is plotted for what we believe to be the best zero-field data from the literature (Refs. 1, 2, and 8) and for data from this paper. Table II summarizes the numerical results of this best-fit procedure.

Table II. Experimental values of γ and β

	γ (mJ·g ⁻¹ ·K ⁻²)	β (mJ·g ⁻¹ ·K ⁻⁴)	$\beta + (3\gamma/T_c^2)$ (mJ·g ⁻¹ ·K ⁻⁴)
Elrod, Miller, and Dresner, this paper	0.145	2.3×10^{-3}	7.5×10^{-3}
Corsan, Ref. 1 and Zbasnik, Ref. 2	0.175	2.6×10^{-3}	9.0×10^{-3}
Schmidt, Ref. 8 ^a			8.3×10^{-3}

^aComposition of sample not given.

By requiring that c_p/T versus T^2 at field be parallel to the zero-field line in Fig. 4, we construct the 4-T and 7-T lines shown in the figure. A comparison of the theoretical and experimental values for the field-dependent c_p/T intercept is given in Table III.

Table III. Comparison of theory and experiment

$\mu_0 H$	Value of c_p/T intercept ($\text{mJ}\cdot\text{g}^{-1}\cdot\text{K}^{-2}$)	
	Theory [$\gamma H/H_{c2}(0)^a$]	Experiment
4 T	0.041	0.053
7 T	0.073	0.105

$a_{\mu_0 H_{c2}(0)} = 14 \text{ T}$ (paramagnetically limited value; see Ref. 6, p. 58).

The discrepancies most likely result from the assumed linearity of the magnetization from H_{c1} to H_{c2} .

We do not include the data of Turck and Duchateau (Ref. 3) in Table I, as they appear to involve significant uncertainties. The authors do not give the composition of NbTi; more seriously, their field-dependent curves do not show differences proportional to temperature.

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SYMBOLS

c_p	Constant pressure specific heat ($\text{J} \cdot \text{kg}^{-1} \cdot \text{K}^{-1}$)
c_s, c_n	Superconducting and normal state specific heats ($\text{J} \cdot \text{kg}^{-1} \cdot \text{K}^{-1}$)
G_s, G_n	Superconducting and normal state Gibbs free energy per unit mass ($\text{J} \cdot \text{kg}^{-1}$)
H	Magnetic field strength ($\text{A} \cdot \text{m}^{-1}$)
H_{c1}, H_{c2}	Lower and upper critical field strengths ($\text{A} \cdot \text{m}^{-1}$)
M	Magnetization ($\text{A} \cdot \text{m}^{-1}$)
T	Temperature (K)
T_c	Critical temperature (K)
β	Proportionality constant in Debye T^3 law of phonon contribution to specific heat ($\text{J} \cdot \text{kg}^{-1} \cdot \text{K}^{-4}$)
γ	Sommerfeld constant ($\text{J} \cdot \text{kg}^{-1} \cdot \text{K}^{-2}$)
μ_0	Permittivity of free space ($= 4\pi \times 10^{-7} \text{ H} \cdot \text{m}^{-1}$)
ρ	Density ($\text{kg} \cdot \text{m}^{-3}$)