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THEORETICAL SOLUTION OF THE MINIMUM CHARGE PROBLEM
FOR GASEOUS DETONATIONS

Raymond W. Ostensen
Containment Modeling Division
Sandia National Laboratories
Albuquerque, NM 87185

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ABSTRACT

A theoretical model was developed for the minimum charge to trigger a gaseous detonation in spherical geometry as a generalization of the Zeldovich model. Careful comparisons were made between the theoretical predictions and experimental data on the minimum charge to trigger detonations in propane-air mixtures. The predictions are an order of magnitude too high, and there is no apparent resolution to the discrepancy. A dynamic model, which takes into account the experimentally observed oscillations in the detonation zone, may be necessary for reliable predictions.

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TABLE OF CONTENTS

<u>Section</u>	<u>Page</u>
Introduction	1
General theoretical solution	3
The shock speed function	4
Comparison to experimental data	10
Summary and Conclusions	12
References	13

ILLUSTRATIONS

<u>Figure</u>		<u>Page</u>
1	Shock speed trajectories for initiation of a gas detonation	15
2	The locus of minimum shock speeds	16
3	Comparison of the Taylor strong blast solution and the fitted correlation to the Baker data	17
4	Correlations for the combined shock speed compared to numerical data	18
5	Correlations for the combined shock speed compared to numerical data	19
6	Minimum charge to trigger a detonation in propane/air mixtures: comparison of theoretical prediction and experiment	20
7	Minimum charge to trigger a detonation in propane/air mixtures: effective energy model vs experiment	21
8	Minimum charge to trigger a detonation in propane/air mixtures: effect of reducing the induction of time	22
9	Minimum charge to trigger a detonation in propane/air mixtures: effect of using the Taylor strong shock model	23

Introduction

The triggering of detonations in gases and combustible aerosols in air has been the subject of extensive theoretical and experimental effort. Gaseous and aerosol detonations develop when a brief, concentrated energy release occurs. When the energy release is too small, or is spread out over too long a time, the detonation fails, and a deflagration may occur [1]. If the trigger energy is obtained from a small high-explosive charge, the release time is effectively instantaneous, and the important unknown is the minimum energy release, or minimum charge, to trigger the detonation. A primary goal of much earlier research has been the prediction of that minimum charge.

The first theoretical model of the initiation of gas detonations was developed by Zeldovich [2,3]. His model relates the minimum charge, E_0 , to the chemical kinetics of the explosive at the temperature and pressure at the head of a sustained detonation (i.e., at the temperature and pressure at the front end of the von Neumann spike in a detonation at the steady-state Chapman-Jouguet, or CJ velocity). The chemical kinetics are summarized by the chemical induction time of the mixture, τ , during which little energy release occurs. Zeldovich assumed that to trigger a detonation, the time available for chemical energy release, must be greater than the induction time, when the speed of a spherically diverging shock drops below the CJ value. The model predicts that E_0 will vary with the cube of the induction time, viz.,

$$E_0 = C\tau^3 \quad (1)$$

where C is an unknown constant, dependent on the fuel. Unfortunately, the theory provides no information about the constant C , so an a priori prediction of E_0 is not possible using chemical kinetic data alone. In addition, it does not enable one to extrapolate reliably from one fuel to another. One can demonstrate this by comparing the minimum charges given by Bull [4], to the induction times given by Westbrook and Urtiew [5], for mixtures in air of hydrogen, methane, acetylene, ethylene, ethane and propane. The ratio of minimum charge to the cube of the induction time varies by a factor of 300, with hydrogen being the largest, and ethane the lowest. Ratios for the three heaviest hydrocarbons, ethylene, ethane and propane, agree within a factor of two, so such extrapolations do provide some guidance for similar fuels when no other data are available.

Predictions based on Equation (1) agree fairly well with experimental data that compare minimum charges for the same fuel at various concentrations in air [4]. In addition, the model predicts that when the initiating charge is barely adequate to trigger a detonation, the shock speed will drop below the CJ velocity and then reaccelerate back up to it. This also is confirmed by experimental data [6,7,8]. A notable contradiction with experiment is the implied assumption that the speed of the shock is a state variable of the initial energy release, the chemical energy density,

chemical induction time and the shock radius, and is insensitive to the history of the shock front and variations in the pressure field behind it. The experiments [6,7] show that the shock speed strongly oscillates prior to final reacceleration. One-dimensional numerical simulations of detonations [9] have also demonstrated oscillations prior to a steady detonation wave. Thus the experimentally observed oscillations are not necessarily a precursor to a multi-dimensional cellular structure for the detonation, but may be strictly longitudinal oscillations, triggered by the rapid deceleration of the shock and the nonlinear coupling between the gasdynamics and the chemical kinetics. These oscillations will certainly have an effect on the minimum charge, but the successes of the Zeldovich approach suggest its further use.

A number of theories have been developed as extensions or variations of the Zeldovich theory [6,10-14]. Most of these use the Zeldovich approach to develop a relationship between the minimum charge and other detonation properties, such as the detonation cell size. Since these require experimental data on detonations with the explosive in question, they offer no predictive capability for new or modified fuels. Lee and Ramamurthi [11] developed a variation of Zeldovich's theory designed to predict the minimum charge from chemical kinetic data alone. However, their predictions are highly sensitive to the choice of a certain minimum shock speed, corresponding to the ill-defined autoignition limit of the gas, so their model has little predictive capability.

In contrast to these models, Ul'yanitskii [15] has developed a theory that incorporates longitudinal shock front oscillations. Predictions of the theory agree fairly well (within a factor of three) with some minimum charge data, for some values of a parameter that is somewhat adjustable. The theory, at its present stage, is clearly not adequate for predictions on untested fuels.

The work in this report uses Zeldovich's implied assumption that the shock speed is a state variable, and does not undergo oscillations. In addition, the chemical energy release is assumed to occur instantaneously, after a temperature and density-dependent induction time. With these assumptions, an exact analytic solution is found for the minimum energy of a point trigger charge. The solution developed in this work is for spherical geometry, but the results can be generalized for cylindrical or Cartesian geometry.

General theoretical solution

The analysis in this work assumes the detonation wave follows the classic, one-dimensional ZND model [16]. The shock wave compresses and heats the explosive, starting the chemical reactions. A short time, called the induction time, is required before significant energy release occurs. In a detonating hydrocarbon-air mixture, the induction time is characterized by breakup, or pyrolyzation of the fuel molecules, and formation of a pool of

free radicals [16]. Since these are almost thermally neutral processes, energy release occurs in a narrow zone, called the fire, that trails some distance behind the shock. In this work, the fire is treated as an infinitely thin sheet, and no energy release is assumed to occur ahead of it. One can think of this as a square wave ZND model of the detonation.

When a gas phase detonation is triggered by a small explosive charge, the shock speed slows from a very high value near the charge, to a sub-CJ value, and then reaccelerates back to the steady CJ value. The classic picture of detonation failure is that the fire falls too far behind the shock to support it, so the detonation is quenched. Since the detonation is spherically diverging, an increase in the induction length between the shock and the fire leads to a reduction in effective energy supplied to the shock front. This reduces the shock speed, which reduces the shock temperature, which in turn reduces the induction length even further. Below some initial charge energy, this process quenches the fire.

The critical assumption of the present theory is that the shock speed can be expressed as a function of the initial charge energy, E , the radius of the shock front, R , and the induction time, τ . That is,

$$S = f(E, R, \tau(S)) \quad (2)$$

We assume that the induction time is a function only of the shock speed. That is, it is independent of the rate of change of shock speed, and is unaffected by variations in the flow field behind the shock front. Equation (2) then implies that the shock speed is a state variable of the shock radius and the initiating energy. The equation defines a set of shock front trajectories in the phase space R - S , with E as the parameter defining separate trajectories. Figure 1 illustrates these trajectories. If we focus on the point at which the shock speed goes through a minimum prior to reaccelerating, we can apply to Equation (2) the fact that

$$\frac{dS}{dt} = 0 \quad (3)$$

This yields

$$f_R S + f_S \frac{dS}{dt} = 0 \quad (4)$$

where the subscripts imply the partial derivative with respect to that variable, and the state variable assumption, that τ is a function only of S , has been used. Since the shock speed must be nonzero, while its derivative is zero, this becomes

$$f_R = 0 \quad (5)$$

A potential criticism of the state variable approach is that the induction time in Equation (2) should be based on the shock speed at the time $t-\tau$ instead of t . That is, it should be based on conditions at the fire instead of the shock. However, the analysis focuses on the point of minimum shock speed, for which the rate of change of induction time is zero, so conditions at the shock and fire should be almost identical, and the error is negligible.

Equation (5) defines a locus of minimum shock speeds in the phase space, R - S . The locus is illustrated by the plot in Figure 2, which is based on the shock trajectories in Figure 1. To find the minimum charge, we need to locate the point on this locus for which E is a minimum. Since this implies $dE=0$, we take the differential of Equation (2), viz.,

$$dS - f_R dR - f_S dS - f_E dE = 0, \quad (6)$$

and apply the condition $dE=0$, and the locus condition, $f_R=0$, to obtain

$$dS - f_S dS = 0, \quad (7)$$

or

$$f_S = 1. \quad (8)$$

We have assumed here that dS is not zero anywhere along the minimum speed locus. If it is, the locus is tangent to the shock trajectory intersecting it there, and another extremum condition is obtained: the second partial derivative of f with respect to R and S equals zero. This improbable event was not explored in the present study. Since Equation (8) defines the point on the minimum speed locus for which the charge energy is a local extremum, the simultaneous solution of Equations (2), (5) and (8) will yield the three unknowns R , S , and E at the minimum point. This is the general solution of the minimum charge problem, given 1) the function, f , that defines the shock speed, and 2) the chemical kinetics of the fuel, to define $\tau(S)$.

The shock speed function

Three things are needed to define the shock speed relation, f : (1) a function for the speed of a shock from a condensed high explosive charge, (2) a function for the speed of gaseous detonation in a diverging environment, and (3) a function that defines the combined shock speed, given the shock speeds from the charge alone and the gaseous detonation alone. In all previous work, it has been assumed that the initiating shock can be approximated by Taylor's solution [17] for a strong blast wave, viz.,

$$M = 0.36 R^{*-1.5}, \quad (9)$$

where M is the shock Mach number, and R^* is the shock radius normalized to the classic blast radius, R_B , given by

$$R_B = (E/p)^{1/3} \quad (10)$$

Here, E is the trigger charge energy, and p is the ambient pressure. Figure 3 compares the Taylor solution to Baker's [18] tabulation of experimental data. The Taylor solution disagrees significantly with the data in the range of importance for triggering gaseous detonations in air: Mach 2 to Mach 4. This error in shock speed was found to cause substantial errors in the predicted minimum charge. The alternative approximation shown in the figure was found by fitting to the Baker data. It is given by

$$M = 1 + 0.2 R^{*-1.75} \quad (11)$$

This correlation was used for all subsequent calculations in this study.

The effect of divergence on the velocity of a spherical detonation can be approximated by using the average density of chemical energy deposited behind the shock front instead of the instantaneous value [2,15]. The ratio of that energy density to the ideal value varies as the volume contained within a spherical volume bounded by the fire, divided by the spherical volume bounded by the shock front. Since the shock speed varies roughly as the square root of the energy density, we can write

$$S^2 = S_0^2 (1 - S_0 r / R)^3, \quad (12)$$

where S_0 is the ideal, CJ detonation velocity. This approximation was used in the present work.

With the exception of the work of Ul'yanitskii [14,15], all previous work has estimated the combined shock speed by adding to the initiating charge energy, the chemical energy released in the gas behind the shock. This approach, called here the effective energy model, was first used by Zeldovich [1,2], and has been widely copied [9,10,19]. Ul'yanitskii [15], however, has criticized the effective energy model, charging it gives shock speeds that are far too high. From comparison to numerical calculations [15] and experimental data [14], he arrived at an approximation that can be written in the form

$$M^8 = M_p^2 M^6 + M_g^8, \quad (13)$$

where M is the Mach number of the combined shock, M_p is the Mach number from the point charge alone, and M_g is the Mach number for the gas detonation alone. The latter includes the effect of the shock speed deficit due to divergence, viz.,

$$M_g = M_o (1 - Sr/R)^{3/2} \quad (14)$$

where M_o is the Mach number for Chapman-Jouget conditions.

In the present work, Lagrangian hydrocode calculations were run to evaluate this question. The calculations used 5000, equal-mass nodes in spherical geometry. The detonation energy release was triggered by the pressure rise in the cell at the shock front, and the time span of the energy release was preset to create a fire zone two nodes wide. Figure 3 compares the calculated combined shock speed to predictions using separately computed shocks from a point charge high explosive (HE) alone, and a gas detonation alone. Mach numbers from the HE and gas detonations were combined according to

$$M^N = M_p^N + M_g^N \quad (15)$$

with N an adjustable parameter. The case $N=2$, corresponding to the effective energy model, is a very poor approximation. The best fit was obtained with $N=4$. The root mean square (RMS) error of the fit was computed, using data at seven radii, normalized to the point charge blast radius: $R/R_B = .2, .225, .25, .275, .3, .325, \text{ and } .35$. The RMS error for case $N=4$ was found to be slightly better than for Ul'yanitskii's approximation: 0.020 vs. 0.046. A search was conducted to find the best fit of the polynomial form like Ul'yanitskii's:

$$M^N = M^{N-P}_p M^P_p + M^B_g \quad (16)$$

The best fit was

$$M^6 = M^6_p + M^4_p M^2_g \quad (17)$$

which had an RMS error of 0.010, or half the value found using $N=4$. These two approximations are compared in Figure 5. The improvement here was judged to be too small to warrant the complexity of the polynomial form. Furthermore, the RMS error is sensitive to the choice of fit radii, and if one varies these, one could arrive at different choices for the best fit. The $N=4$ approximation was used in further work. It should be noted that no approximation of these types can be exact. Chernyi [9] has shown that the Mach number for the combined shock goes exactly to the value for the gas detonation at some normalized radius, R/R_B , and is independent of the point charge energy, beyond that point. Nevertheless, the approximation used here should be adequate for an estimate of minimum charge.

Particular solutions

By using Equation (15) to approximate the combined shock speed, one finds

$$S = \left[g^N + S_o^N (1 - S\tau/R)^{3N/2} \right]^{1/N} \quad (18)$$

where g defines the speed of a shock from a point charge, as a function of scaled radius. By applying the locus condition, Equation (5), one finds

$$Rg_R g^{N-1} + \frac{3}{2} S_o^N \left(1 - \frac{S\tau}{R} \right)^{\frac{3N}{2} - 1} \left(\frac{S\tau}{R} \right) = 0 \quad (19)$$

Then, by using the minimum energy condition, Equation (8), one finds

$$1 + \frac{S\tau'}{\tau} - \left(\frac{S}{g} \right)^N \left(\frac{g}{Rg_R} \right) = 0 \quad (20)$$

where τ' is the derivative of the induction time with shock speed. These three equations must be solved simultaneously to find the minimum charge. The validity of this solution was verified by comparing to results from a series of direct numerical integrations of the equation of motion, Equation (18), using trigger charge energies that bracketed the predicted minimum critical charge. The agreement was exact. For all runs with energies above the predicted charge, a stable detonation formed, while for all runs with triggers below it, the detonation was extinguished.

The solution is simplified if we change variables. Define

$$Z = S\tau/R \quad (21)$$

Note that Z is the Lagrangian width of the induction zone, relative to the shock radius. Equations (18) to (20) can be expressed as a relationship between S , Z and R^* (the energy-scaled shock radius) at the minimum energy point. For example, in Equation (18), the term $S\tau'/\tau$ is the logarithmic derivative of the induction time with shock speed, and is a function of shock speed only. The term Rg_R/g is the logarithmic derivative of the point charge shock speed with radius, and is a function of scaled radius only. If one specifies Z , the scaled radius is fixed by Equation (19), and the shock speed is then determined from Equation (18). Equation (20) will

only be satisfied if Z corresponds to the value for the minimum charge. Thus, the equations may be solved using a root finding procedure on Equation (18), with Z as the independent parameter.

Before proceeding with the numerical solutions, it is worthwhile to examine a simplified case. If we approximate the flow divergence term by

$$\left(1 - \frac{S\tau}{R}\right)^3 = 1 - \frac{3S\tau}{R}, \quad (22)$$

use Taylor's strong blast approximation for the point charge, and use the energy addition model, the shock speed equation becomes

$$S = \left[\frac{aE}{R^3} + S_o^2 \left(1 - \frac{3S\tau}{R}\right) \right]^{1/2}, \quad (23)$$

The locus condition becomes

$$aE/R^2 = S_o^2 S\tau \quad (24)$$

which can be written as

$$S_p^2 = S_o^2 \left(\frac{S\tau}{R} \right) \quad (25)$$

which relates the relative Lagrangian induction zone width to the point charge shock speed, at the point of reacceleration of the combined shock. By using the shock speed equation, this may be rewritten as

$$1 - S^2/S_o^2 = 3S\tau/R \quad (26)$$

The minimum energy condition applied to Equation (23) yields,

$$3(S_o^2 - S^2) \left(\tau + S\tau' \right) + 4S^2\tau = 0, \quad (27)$$

which may be combined with Equation (26) to yield

$$E_o = \frac{3a S_o^6 \tau^3}{4S} \left(1 + \frac{S\tau'}{\tau} \right)^2 \quad (28)$$

It is interesting to compare this result with that of Zeldovich, given in Equation (1). First, the induction time in Equation (28) must be evaluated at the minimum speed point instead of the CJ speed, so the two equations are not fully comparable, even though the forms are similar. Second, we now have a strong explicit dependence on the CJ detonation velocity. If we assume S is some fixed fraction of S_o , we obtain a factor of S_o^5 , which does not appear in Zeldovich's result. This factor does appear in a minimum charge expression developed by Lee et al [20], in an attempt to make the Zeldovich approach fully quantitative. That approach, which was based on a combination of the Zeldovich criterion and the Taylor strong blast model, proved to be highly inaccurate. Finally, Equation (28) includes a parameter which does not appear in previous solutions, the rate of change of induction time with shock speed. One can demonstrate the importance of this parameter on physical grounds: an explosive with an induction time independent of shock speed would be detonable by an arbitrarily small charge, since an arbitrarily small shock would drive the chemical reactions to completion. Thus, the derivative of induction time with shock speed is a critical parameter, and any model that predicts minimum charge must include it.

An interesting result is obtained if one assumes the induction time can be varied by some multiplicative factor, x . This is roughly what one obtains when varying the concentration of a given fuel in air, since the expression for the induction time is then usually given as some concentration term multiplying a temperature term. In that case, the logarithmic derivative of induction time with shock speed in Equation (28) is unaffected, so the minimum charge varies approximately as $S_o^5 \tau^3$. This is probably a better correlating approximation for experimental data than the commonly used τ^3 [4].

Equation (28) is not useful from a predictive standpoint. The predicted minimum charge is very sensitive to the approximations used for the combined shock speed and the point charge shock speed; the approximations in this model are simply too inaccurate.

Unfortunately, more accurate approximations for the shock speeds do not allow the minimum charge to be expressed in a simple form like Equation (28). If we use the $N=4$ approximation for the combined shock speed, and the fit in Equation (11) for the point charge blast, the locus condition becomes

$$C_o^3 \left(1 + 0.2 R^{*-1.75} \right) \left(0.35 C_o R^{*-2.75} \right) =$$

(29)

$$\frac{3}{2} S_o^4 \left(1 - \frac{S\tau}{R} \right)^5 \left(\frac{S\tau}{R} \right),$$

where c_o is the ambient speed of sound. The minimum charge condition becomes

$$\left(1 + \frac{S_r}{\tau}\right) = \left(\frac{E}{R E_R}\right) \left[1 + \left(\frac{S_o}{E}\right)^4 \left(1 - \frac{S_r}{R}\right)^6\right] \quad (30)$$

These clearly offer no simple expression for E_o .

Comparison to experimental data

The theory was tested by comparing its predictions to experimental data on the minimum charge for the detonation of propane in air. Propane was chosen because of the accurate data on propane kinetics available from the work of Burcat, Lifshitz, Scheller and Skinner [21]. They performed shock tube experiments to measure the ignition delay of propane, in mixtures of oxygen and argon, in reflected shocks with temperatures from 1250 to 1600 K. They defined ignition as occurring when the temperature and pressure just started to rise on their transducers. They developed the correlation

$$\tau = 4.4 \times 10^{-14} \exp(42200/RT) [C_3 H_8]^{0.57} [O_2]^{-1.22} \quad (31)$$

where the concentrations are given in moles/cm³. The appropriate time delay for predicting detonability is the time necessary for half the chemical energy to be released. The difference between these times should be relatively small, and is neglected here.

The density and temperature in the induction zone were determined from TIGER[22] calculations of the shock Hugoniot for propane in air at various concentrations from 0.6 to 1.8 of stoichiometric, using the BKW equation of state. For these calculations, propane was maintained as a separate species, which did not dissociate. Alternative TIGER calculations with an ideal gas equation of state gave temperatures within about 10K of the BKW calculations so shock temperatures are accurately known. By using results for two shock speeds at each propane concentration, the temperature data were fit to an expression of the form

$$T = A S^B \quad (32)$$

where A and B were adjustable constants. Similarly, the results for relative specific volume were fit using the expression

$$v/v_o = 1 - \left[A' + B'/S^2\right] \quad (33)$$

where A' and B' were adjustable constants. These fits were programmed, along with the correlation of Burcat et al, into a root-finding algorithm, containing Equations (18) and (19). The algorithm searched directly for the minimum charge, rather than using Equation (20). The computed minimum charge was converted to an equivalent minimum mass of Tetryl, high explosive, using an explosive energy yield of 4.77×10^6 J/Kg [23].

Predicted minimum charges were compared to experimental data of Benedick [24] and of Bull, Elsworth and Hooper [7]. The data of Benedick are for 0.8 kg charges of sheet explosive, and yield detonation limits of 3 and 7 volume percent propane. These yield equivalence ratio limits of 0.74 and 1.79 for the 0.8 kg initiator. Because of the planar geometry, one would expect these data to underestimate the critical charge for spherical initiators. These data are included here because of a lack of data on spherical charges at non-stoichiometric concentrations. The data of Bull et al [7] provide a minimum charge, for a stoichiometric concentration, of 50 to 80 gm of Tetryl, in an almost spherical geometry, based on the charge weights for their GO and NO-GO shots that bracketed the minimum charge. These values are consistent with their minimum charge data for several other hydrocarbons: ethane (35 gm); n-butane (65 gm); i-butane (90 gm); and ethylene (12 gm). As Bull et al [7] point out, the pattern of minimum charges from fuel to fuel is consistent with high temperature kinetic data. Further, Kogarko et al [25] provide an upper bound of about 140 gm of Tetryl, and Pfoertner [26] provides an upper bound of 80 gm of unspecified explosive. Elsworth and Eyre [27] bracketed the critical charge for stoichiometric propane in air with charges of 53 and 45 gm of the explosive PE4, which their tests indicate is very close to Tetryl in its triggering effects. The charge ranges of Bull et al [7] and of Elsworth and Eyre [27] overlap slightly. We can choose a value of 50 gm, within the overlap range, to obtain a value with an estimated error of only + or - 5 gm, or only 10%, neglecting other experimental uncertainties. Thus, the minimum charge for propane in air is known roughly at equivalence ratios of 0.74 and 1.79, and it is known quite accurately at the stoichiometric concentration.

The predictions are compared to experimental data in Figure 5. At the lower equivalence ratios, ϕ , the predicted charge is a factor of ten to twenty too high, rising to a factor of fifty too high at $\phi=1.8$. Thus, the predictions are very inaccurate.

Some parametric variations were used to explore the sensitivity of the predictions to the assumptions. Predictions obtained using the effective energy model (i.e., $N=2$) are compared in Figure 7. These agree with the data for stoichiometric concentrations to within 20%, which is outstanding agreement. However, as we have discussed previously, the effective energy model is theoretically unsound, and is not a good approximation. The accurate predictions may be fortuitous, or they may indicate that the effective energy model is accurate in some sense as an average when applied to an oscillating shock front. Extensive investigation would be required before this speculation could be taken to the point of using the effective energy model for predictive purposes.

Figure 8 demonstrates the effect of reducing the induction time by a factor of 0.382. This brings the prediction into exact agreement with the data at low equivalence ratios. Note that the minimum charge was reduced by exactly the cube of this factor. It is not credible that the induction time data of Burcat et al [21] could be a factor of 2.6 too high, so the problem lies elsewhere.

Another possible source of error is the use of a correlation for ignition delay times instead of the time to half the energy release. The predicted minimum charge will vary with the cube of the relative induction time, so a 25 percent error in τ would result in a factor of 2 error in E_0 . Note however that the energy release time must be larger than the measured ignition delay, not smaller. Use of a longer time span would increase the discrepancy.

Figure 9 demonstrates the effect of using Taylor's strong blast model for the initiating charge shock speed, instead of the correlation in Equation (11). For this case, the Taylor model roughly doubles the minimum charge. Since Equation (11) is much more accurate than the Taylor model, the error it causes is much too small to explain the discrepancy between prediction and experiment.

Summary and Conclusions

An exact solution has been obtained for the minimum charge to detonate a gas explosive, using a square wave approximation for the ZND zone, and assuming a stable, non-oscillatory induction zone. Comparison of theoretical predictions to experimental data on the minimum charge to detonate propane in air, results in a factor of ten discrepancy. There appears to be no obvious resolution to this contradiction. The discrepancy can not be explained by errors in the experimental data on minimum charge, or induction time. The correlation for the combined shock speed is too accurate to be the problem, as well as the correlation for the shock speed from a point charge.

The use of the effective energy model for the shock speed yields excellent agreement between theory and experiment. However, even though this shock speed model has been widely applied in the past, it is theoretically unsound and inaccurate. The good agreement is most likely fortuitous.

One may speculate about a number of other possible resolutions to the discrepancy, but the most likely possibility is that the basic theoretical approach is unsound. A dynamic theory based on an oscillating induction zone may be necessary to accurately predict minimum charge. This is supported by the experimental data indicating strong oscillations in the shock and fire velocities, for charges near the minimum. Further work along the line of Ul'yanitskii's [15] model, and with numerical simulation of the initiation process, may provide a better answer.

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Figure 1. Shock speed trajectories for initiation of a gas detonation.

Figure 2. The locus of minimum shock speeds.

Figure 3. Comparison of the Taylor strong blast solution and the fitted correlation to the Baker data.

Figure 4. Correlations for the combined shock speed compared to numerical data.

Figure 5. Correlations for the combined shock speed compared to numerical data.

Figure 6. Minimum charge to trigger a detonation in propane/air mixtures: comparison of theoretical prediction and experiment

Figure 7. Minimum charge to trigger a detonation in propane/air mixtures: effective energy model vs experiment

Figure 8. Minimum charge to trigger a detonation in propane/air mixtures: effect of reducing the induction time

Figure 9. Minimum charge to trigger a detonation in propane/air mixtures: effect of using the Taylor strong shock model

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