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Power-Law Rheology and Flow Behavior of Low-Invasion Coring Fluids

Patrick L. McGuire

Sandia National Laboratories

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Printed in the United States of America

Available from:

National Technical Information Service

U. S. Department of Commerce

5285 Port Royal Road

Springfield, VA 22161

Price: Printed Copy \$4.00; Microfiche \$3.00

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SAND80-0933
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PRINTED AUGUST 1981

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POWER-LAW RHEOLOGY AND FLOW BEHAVIOR
OF LOW-INVASION CORING FLUIDS*

Patrick L. McGuire**

ABSTRACT

An improved pressure coring system has been developed in which an extremely viscous polymer mud is extruded by the core and is used to seal and protect the core from flushing by drilling fluids. The polymer mud must be extremely viscous to minimize invasion, yet must be extruded through a long, narrow annular gap with a minimum of pressure buildup. A highly non-Newtonian shear-thinning polymer is utilized in the low invasion coring fluid. This paper describes the measurement and modeling of non-Newtonian rheology from rotary viscometer data in detail since the simplified equations which are generally used with these instruments can be grossly in error. The development of both an approximate analytical solution and an exact numerical solution of the non-Newtonian extrusion process is presented. These solutions were used to optimize the non-Newtonian rheology of the low-invasion fluid which will be used in actual coring operations.

*This work supported by the U.S. Department of Energy

**Formerly of Div. 4741. Currently with Los Alamos National Scientific Laboratory.

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Acknowledgements

I would like to thank Andrew Kraynik and Jeff Reich, both of the Physical Properties of Polymers Division, for their analytical and experimental support of this project. I would especially like to thank Stan Belak and Bernard Killoran, both of EG&G, for their careful work in preparing the fluids and obtaining filtrate loss, porous-media invasion, and viscosity data for a large number of low-invasion fluids.

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Introduction/Background

In an era of rapidly rising costs and limited resources, energy companies have expended a great deal of time and money to better quantify and exploit existing energy resources. One major area of recent activity has been the development of pressure coring to obtain rock properties and hydrocarbon saturations in old oil and gas fields in order to produce them more effectively. Existing technology was adequate to obtain rock properties, but did not yield reliable saturation data.

In order to overcome this limitation, a new pressure coring system has been developed by Sandia Laboratories to obtain accurate fluid content measurements of oil and gas reservoirs [1,2]. The key element in this new system is the low invasion coring fluid which is used to protect the core from invasion and flushing by the drilling mud (see Figures 1-3). The extremely viscous low invasion fluid is extruded from the core barrel as the core is drilled and serves to seal and protect the core.

One of the major variables controlling invasion and flushing of the core is viscosity. Fluid invasion into the core will decrease if the coring fluid viscosity is increased, but the higher viscosity will also increase the pressure buildup in the core barrel as the core extrudes the low invasion fluid. Since the fluid invasion is directly related to the difference between the coring fluid pressure and the reservoir pressure, this pressure buildup in the core barrel must be minimized.

Thus, we are faced with an optimization problem in which the low invasion fluid must be extremely viscous to minimize invasion and at the same time must not cause a large pressure buildup while being extruded from the core barrel. This paper describes the analytical and experimental procedures which were used to characterize and optimize the rheology of the low invasion coring fluid and also lists some of the pertinent properties of the fluid.

Description of Fluid

The low invasion fluid has the following composition: 10 lb/bbl stabilized hydroxyethyl cellulose polymer (HEC) to develop high viscosity, and 300 lb/bbl carefully sized calcium carbonate particles (CaCO_3) to seal the formation pores and prevent invasion, suspended in calcium bromide eutectic solution (46% by weight CaBr_2) which is non-freezing at dry ice temperatures.

A more detailed explanation of the function of each component can be found in an earlier paper [2]. The HEC is a highly non-Newtonian, shear-thinning, water-based polymer. Although a number of mathematical expressions may be used to represent this non-Newtonian rheology, the high molecular weight grades of HEC polymer, like other water-based polymers used in the oil industry, are best represented by the power-law model [3]. This model is of the form

$$\tau = K\dot{\gamma}^n \quad (1)$$

where τ is the shear stress, γ is the shear rate, K is the consistency index, and n is the power-law index. The power-law index is one for Newtonian fluids, greater than one for dilatant fluids, and less than one for shear-thinning fluids such as HEC polymer solutions.

Characterization of Power-Law Rheology

Knowing that the fluid to be characterized follows the power-law model, we can develop experimental methods to measure the viscosity and mathematical methods to express the viscosity/shear rate relationship in terms of the power law. Since the viscosity is a function of shear rate, the viscometer to be used must have a simple geometry such that the shear rate can be calculated and it must have several speeds to develop a number of shear rates over the range of interest. Cone-and-plate viscometers, capillary viscometers, and concentric-cylinder rotary viscometers can be used for this application, while falling ball, vibrating sphere, and disc-type rotary viscometers cannot be used. Three types of viscometers were used to measure the low invasion fluid: Sandia's high pressure/high temperature rotating sleeve viscometer (similar to the Fann VG Meter used routinely in the oil industry [5]), a Mechanical Spectrometer cone-and-plate viscometer, and two different models of the familiar Brookfield viscometer with cylindrical spindles. The Sandia viscometer cannot measure viscosities as high as those of the low invasion

fluid, but was used to determine the performance of HEC polymer at elevated pressures and temperatures as high as 350°F. The Mechanical Spectrometer was used to measure viscosities at low shear rates (0.1 to 1.0 sec⁻¹) and temperatures from -40°F up to 200°F. The Brookfield viscometers were used to measure room temperature viscosities over the shear rates of interest (1 to 40 sec⁻¹).

The Mechanical Spectrometer is a constant shear rate device, with

$$\gamma = \omega / \sin \phi \quad (2)$$

where ω is the angular velocity in radians/sec and ϕ is the angle the cone makes with the plate. This device can be used directly to generate the viscosity/shear rate data from which a power-law model is derived. This is not the case with concentric-cylinder viscometers, however, since the shear rate, and consequently the viscosity, varies throughout the flow field.

The equations commonly used to express shear rate using concentric-cylinder viscometers such as the Brookfield are based upon the assumption that viscosity is constant throughout the flow field. This assumption, and the viscosity/shear rate data it generates, are seriously in error for highly non-Newtonian fluids. The following equations show that the shear rate at the viscometer bob is significantly affected by the power law exponent n .

The shear rate relationships for power law fluid flow in

this geometry have been reported in the literature [5], and a derivation is shown in the Appendix. The applicable equation for a concentric-cylinder viscometer is

$$\gamma \text{ viscosity} = \frac{\pi \cdot \text{RPM}}{15 n \left(1 - (R_1/R_2)^{2/n}\right)} \quad (3)$$

where γ viscosity is the shear rate at which the viscosity measurement is made, R_1 is the inner cylinder (spindle) radius, and R_2 is the outer cylinder (vessel wall) radius. The commonly used shear-rate equation is

$$\gamma \text{ viscosity} = \frac{\pi \cdot \text{RPM}}{15 \left(1 - (R_1/R_2)^2\right)} \quad (4)$$

which is only valid if $n = 1$. For the Brookfield viscometer data in Table 1, R_1 was 0.00625 " and R_2 was 1.64". The shear-rate equation in this case can closely be approximated by

$$\gamma \text{ viscosity} = \frac{0.2097 \cdot \text{RPM}}{n} \quad (5)$$

It is obvious that considerable error will be introduced in the shear rate relationships by assuming a constant viscosity in solving the Navier-Stokes equation if the real fluid behavior is non-Newtonian. For the low invasion fluid study, where n ranged between 0.10 and 0.57, the actual shear rate at the viscometer spindle is two to ten times the shear rate calculated by the commonly used equations.

It is important to note that for concentric-cylinder viscometer data the viscosity/shear rate relationships cannot be developed until the power law index n is known. However, a

log-log plot of viscosity vs. shear rate is required to calculate n . This difficulty is overcome by plotting the measured viscosity vs. the Newtonian shear rate given by Eq. 4 on a log-log scale. The power law index n and a pseudo viscosity index K_p are then calculated from the plot. Eq. 3 can then be used to develop accurate shear rates, and the true viscosity/shear rate curve can then be plotted. An alternative approach is to correct the pseudo viscosity index, K_p , to obtain the true viscosity index, K , by using the following equation

$$K = K_p \times n^{n-1} \quad (6)$$

Eq. 6 can be used where R_1/R_2 , the ratio of the spindle radius to the container radius, is small and Eq. 5 is used to calculate the shear rates. The factor $n^{n-1} - 1$ represents the error involved in the use of Eq. 4 in deriving power law fluid properties. In this study, the error ranged from 27% when n was 0.57 up to 694% when n was 0.10. For a typical low invasion fluid at room temperature, the error involved in using Eq. 4 would be well over 100%. It is therefore imperative that the viscosity/shear rate relationships used to calculate the performance of highly non-Newtonian fluids such as the low invasion coring fluid be based upon Eq. 3 and not on the routinely used Eq. 4.

Viscosity measurements of two different low invasion fluids are shown in Figures 4 and 5. Figure 4 shows cone-

and-plate viscosity measurements over a temperature range of -41°F up to 177°F. This data clearly shows the power law nature of one fluid in which n increases from 0.10 up to 0.34 and K decreases with increasing temperature. Figure 5 shows Brookfield viscosity data for another low invasion fluid at 150°F over a shear rate of 0.21 to 42 sec^{-1} . A linear least squares analysis can be used to fit the best power law function to the log-log data. In this case, the best fit was $n = 0.50$ and $K = 162,000$. Note that the true dimensions of K are $\text{cp} \cdot \text{sec}^{1-n}$. For the sake of simplicity, however, K can be visualized as the absolute viscosity of the fluid (in centipoise) at a shear rate of 1 sec^{-1} .

Characterization of the Extrusion Process

The extrusion of the low invasion fluid is shown in Figures 2, 3 and 6. Since the annulus is quite small compared to the diameter of the core, the core/core barrel system can be represented by a parallel-plate system with a high degree of accuracy [5]. This conversion from axisymmetric flow to one-dimensional flow greatly simplified the solution of the extrusion problem.

The Appendix contains derivations of both an exact numerical solution and an approximate closed form solution of the parallel plate system. The exact solution results in transcendental functions which must be solved numerically.

The approximate solution is

$$\Delta p' = \left(\frac{D_c (2n + 1) U_c}{8n} \right)^n \left(\frac{2}{h} \right)^{2n + 1} LK \quad (7)$$

where D_c is the core diameter, U_c is the core velocity, h is the annular gap, and L is the instantaneous length of the core along which the pressure buildup is evaluated. This equation yields pressure buildups which are only a few percent in error for pseudo-plastic fluids where $n < 1$ and increasingly larger errors for dilatant fluids where $n > 1$.

Experimental Results

The concentration of HEC polymer is the dominant factor controlling the viscosity of the low invasion fluid. The particle concentration is also important in determining the power law properties of the fluid. These are the two variables which must be optimized to obtain the maximum viscosity and a reasonable pressure buildup to obtain minimum invasion. Optimization in this case refers to obtaining the maximum practical particle content and fluid viscosity while keeping the pressure buildup in the core barrel within design limits. The design criterion for the fluid was to limit the pressure buildup to a maximum of 200 psi through a 9 ft long, 0.125 inch annulus while coring at a rate of 20 ft/hr. Fluid properties used to calculate the pressure buildup were evaluated at 150°F, which was selected as a representative temperature for coring in a petroleum reservoir with very limited cooling of the core.

Table 1 shows the results of viscosity testing for a number of different low invasion fluid candidates.

Fluid 2 shown in Table 1 had a composition of 15 lb/bbl HEC, 5000 ppm formaldehyde, and 100 lb/bbl CaCO_3 in CaBr_2 eutectic brine. At room temperature, this fluid had a power law index of 0.26 and a viscosity index of 2,156,000, resulting in a pressure drop of 1347 psi. At 150°F, the fluid properties were $n = 0.34$, $K = 757,000$, resulting in a pressure drop of 591 psi. In order to reduce the pressure buildup, the concentration of HEC polymer was reduced to 13 lb/bbl, and viscosity measurements were made at room temperature and 150°F. The pressure buildup at 150°F was still too large (see Fluid 3) so the HEC concentration was further reduced. Fluid 5, with a composition of 10 lb/bbl HEC polymer and 300 lb/bbl CaCO_3 particles in CaBr_2 eutectic brine, had power law values of $n = 0.47$ and $K = 198,000$ at 150°F, resulting in a maximum pressure buildup of 220 psi. At room temperature, this fluid had power law values of $n = 0.25$ and $K = 995,000$, resulting in a maximum pressure buildup of 604 psi. Since this fluid was at the upper limit of the pressure buildup criterion and also near the practical limit of particle concentration, this composition was chosen as optimum for the low invasion fluid. The formaldehyde biocide used in Fluid 4 was eliminated from this fluid since it was found to degrade the HEC polymer at elevated temperatures, and the dense eutectic brines appear to adequately protect the HEC from harmful bacteria. Table 2 shows the influence of coring rate on the

pressure buildup for the optimum-viscosity low invasion fluid at room temperature, while Table 3 shows the effects of changing the annular clearance, h , between the core and the core barrel. Changes in the coring rate do not greatly affect the pressure buildup, but the pressure buildup is quite sensitive to changes in the annular clearance. This indicates that great care must be taken to insure that undersized core barrels are not run with the low invasion fluid.

Summary/Conclusions

Power law viscosity models have been used to represent the rheology of extremely viscous, shear-thinning fluids used in an improved pressure coring system. Rotary viscometer measurements of non-Newtonian power-law fluids have been analyzed previously and expressions have been derived to correctly calculate shear rate and rheological properties for these fluids. The simplified equations which are in general use are grossly in error for fluids with a power law index of 0.5 or less, and should not be used.

The process of extruding the highly non-Newtonian low invasion fluid from the core barrel has been modeled mathematically, and both an approximate analytical solution and an exact numerical solution have been developed. Based upon this model, the rheological properties of the low invasion fluid have been optimized. We found that the rate of coring was relatively unimportant, and that the annular clearance between

the core and the core barrel was the major factor in determining what the pressure buildup in the core barrel would be.

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APPENDIX

Concentric-Cylinder Viscometer Measurements of Power Law Fluids

Many of the drilling fluids used today are highly non-Newtonian, shear thinning fluids, including the low-invasion fluid used in pressure coring. Viscosity measurements and pressure drop calculations based upon Fann or Brookfield viscometer data can be seriously in error unless the proper shear-rate equations are used. The shear-rate equations supplied by the manufacturers and used in the API standards are valid only for constant-viscosity (i.e. Newtonian) fluids.

The applicable Navier-Stokes equation in polar coordinates for an incompressible (see Figure 9), variable viscosity fluid is

$$\begin{aligned} \rho \left[\frac{Dv_{\theta}}{Dt} + \frac{v_r v_{\theta}}{r} \right] = & \frac{-1}{r} \frac{\partial P}{\partial \theta} + F_{\theta} + \frac{2}{r} \frac{\partial}{\partial \theta} \left(\mu r \frac{v_{\theta}}{\partial \theta} + \mu \frac{v_r}{r} \right) + \\ & \frac{\partial}{\partial z} \mu \left[\left(\frac{1}{r} \frac{\partial v_z}{\partial \theta} + \frac{\partial v_{\theta}}{\partial z} \right) \right] + \frac{\partial}{\partial r} \left[\mu \left(\frac{1}{r} \frac{v_{\theta}}{\partial r} + \frac{\partial v_{\theta}}{\partial r} - \frac{v_{\theta}}{r} + \frac{2\mu}{r} \right) \right] \\ & + \frac{2\mu}{r} \left[\frac{1}{r} \frac{\partial v_r}{\partial \theta} + \frac{\partial v_{\theta}}{\partial r} - \frac{v_{\theta}}{r} \right]. \end{aligned} \quad (8)$$

This formidable-looking equation can be greatly simplified since $v_r = v_z = F_{\theta} = 0$ and the flow is axisymmetric (i.e., $\frac{\partial}{\partial \theta} = 0$).

The fluid motion equation is thus reduced to

$$\frac{\partial}{\partial r} \left[\mu \left(\frac{\partial v_{\theta}}{\partial r} - \frac{v_{\theta}}{r} \right) \right] + \frac{2\mu}{r} \left(\frac{\partial v_{\theta}}{\partial r} - \frac{v_{\theta}}{r} \right) = 0 . \quad (9)$$

Eq. 9 can be expanded, combined, and rewritten as

$$\mu \frac{\partial^2 v_{\theta}}{\partial r^2} + r \frac{\partial \mu}{\partial r} \frac{\partial}{\partial r} \left(\frac{v_{\theta}}{r} \right) + \mu \frac{\partial}{\partial r} \left(\frac{v_{\theta}}{r} \right) = 0 . \quad (10)$$

Eq. 10 can then be expressed as a much simpler differential equation;

$$\frac{\partial}{\partial r} \left(r^3 \mu \frac{\partial}{\partial r} \left(\frac{v_{\theta}}{r} \right) \right) = 0 . \quad (11)$$

The power law viscosity model stated that

$$\tau = \mu \dot{\gamma} = K \dot{\gamma}^n \quad (12)$$

which leads to

$$\mu = K \dot{\gamma}^{n-1} = K \left(r \frac{\partial}{\partial r} \left(\frac{v_{\theta}}{r} \right) \right)^{n-1} \quad (13)$$

By substituting the power law viscosity into Eq. 11 the governing differential equation becomes

$$\frac{\partial}{\partial r} \left[r^{2+n} \left(\frac{\partial}{\partial r} \left(\frac{v_{\theta}}{r} \right) \right)^n \right] = 0 . \quad (14)$$

This equation can be integrated twice, which yields

$$v_{\theta} = c_1 r^{1-2/n} + c_2 r \quad (15)$$

For Newtonian fluids, where $n = 1$, this results in the familiar equation

$$v_{\theta} = \frac{c_1}{r} + c_2 r \quad (16)$$

which leads to

$$\gamma \text{ viscosity} = \frac{\pi \times \text{RPM}}{15 \left(1 - (R_1/R_2)^2 \right)} \quad (17)$$

By applying the boundary conditions at R_2 , the vessel wall where the velocity is zero, and at R_1 , the rotating spindle where the velocity is known, Eq. 15 becomes

$$v_{\theta} = \frac{\pi \times \text{RPM}}{30 \left(R_1^{-2/n} - R_2^{-2/n} \right)} \left(r^{1-2/n} - r R_2^{-2/n} \right) \quad (18)$$

and the shear rate at the spindle becomes

$$\gamma \text{ viscosity} = \frac{\pi \times \text{RPM}}{15n \left(1 - (R_1/R_2)^{2/n} \right)} \quad (19)$$

Exact Numerical Solution

The extrusion of the low-invasion fluid is shown in Figures 2, 3 and 6. Since the annulus is quite small compared to the diameter of the core, the core/core barrel system can be represented by a parallel-plate system with a high degree of accuracy [5]. This conversion from axisymmetric flow to one-dimensional flow will simplify the solution of the extrusion problem. The basic Navier-Stokes equation for this problem then becomes

$$\begin{aligned} -\rho \frac{Du}{Dt} = & -\frac{\partial P}{\partial x} + F_x + 2 \frac{\partial}{\partial x} \left(\mu \frac{\partial u}{\partial x} \right) \\ & + \frac{\partial}{\partial y} \left[\mu \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right) \right] + \frac{\partial}{\partial z} \left[\mu \left(\frac{\partial w}{\partial x} + \frac{\partial u}{\partial z} \right) \right]. \end{aligned} \quad (20)$$

The equation can be greatly simplified by applying

$$\frac{\partial u}{\partial x} = \frac{\partial u}{\partial z} = \frac{\partial u}{\partial t} = v = w = F_x = 0.$$

The Navier-Stokes equation thus reduces to

$$\frac{dP}{dx} = \frac{d}{dy} \left(\mu \frac{du}{dy} \right) = \frac{d\tau}{dy} \quad (21)$$

By inserting the power law equation, Eq. 1, into Eq. 21, we obtain

$$\frac{dP}{dx} = K \frac{d}{dy} \left[\left(\frac{du}{dy} \right)^n \right]. \quad (22)$$

Since K , n , and u are all independent of x , we can rewrite Eq. 22 as

$$\frac{d}{dy} \left[\left(\frac{du}{dy} \right)^n \right] = -\alpha \quad (23)$$

where $\alpha = -\frac{1}{K} \frac{dP}{dx}$, which is a positive constant. Eq. 23 can be integrated and rewritten as

$$\frac{du}{dy} = (-\alpha y + A)^{1/n}.$$

A second integration yields

$$u(y) = \frac{-1}{\alpha\beta} (-\alpha y + A)^\beta + B \quad (24)$$

where A and B are constants of integration and $\beta = 1 + \frac{1}{n}$.

In order to solve for the constants A and B, it is necessary to apply the principle of symmetry and then apply the appropriate boundary conditions in the region where Eq. 22 is mathematically defined. Figure 6 illustrates the principle of symmetry and the use of the symmetry distance, s, which are used in applying the boundary conditions. The symmetry distance, s, is the distance in the y-direction from the core such that the fluid velocity is zero. By making use of the symmetry distance, we can write three boundary conditions:

$$\frac{du(0)}{dy} = 0 \quad (25)$$

$$u\left(-\frac{(h-s)}{2}\right) = u(-C) = 0 \quad (26)$$

$$u\left(-\frac{(h+s)}{2}\right) = u(-D) = -U_c \quad (27)$$

where U_c is the coring rate. The velocity equation then becomes

$$u(y) = \frac{1}{\alpha\beta} \left[(\alpha C)^\beta - (-\alpha y)^\beta \right] \quad (28)$$

Evaluation of Eq. 27 leads to

$$F(s, \alpha) = U_c + \frac{1}{\alpha\beta} \left[(\alpha C)^\beta - (\alpha D)^\beta \right] = 0 \quad (29)$$

where both s and α are unknown variables.

The final equation which must be used to solve the extrusion problem is the continuity equation, which in this case states that the integral of the velocity across the annulus must be equal to the rate of fluid displacement by the core. The velocity integral can be evaluated by parts and combined to form

$$Q_c(s, \alpha) = \bar{R} \int_{-D}^C u(y) dy = \frac{\bar{R}}{\alpha\beta} \left[\frac{-1}{\alpha^\nu} \left((\alpha D)^\nu + (\alpha C)^\nu \right) + C(\alpha C)^\beta + D(\alpha D)^\beta \right] = Q_a \quad (30)$$

where $\bar{R}$ is π times the mean annular diameter, $\nu = 2 + \frac{1}{n}$, and Q_c is the calculated flow rate which is a function of the symmetry distance s and the pressure term α . The actual flow rate, Q_a , is

$$Q_a = \pi (D_c/2)^2 U_c \quad (31)$$

where D_c is the diameter of the core.

By combining Eqs. 30 and 31 to form the continuity equation and considering along with it Eq. 29, a system of two equations containing

two unknown variables is formed. Unfortunately, these equations are complex transcendental functions of both s and α and cannot be solved in closed form. A simple Newton's-method numerical solution may be evaluated, however, by treating F as a function of s only and Q_c as a function of α only. By successively improving the value of an s by use of Eq. 29 with an initial value of α , then using the new s and Eq. 30 to improve the value of α , a substantially improved solution can be calculated. After a few iterations of this process, which is diagrammed in Figure 7, the numerical solution will converge upon the exact solution.

Approximate Solution

Numerical approaches such as Newton's method require an initial estimate, or a guess, of the unknown variable. If the guess is close to the true value, the method converges on the exact solution quickly. If the initial guess is poor, the method may converge slowly or not at all. In order to make a good initial estimate (and to verify the exact solution), an approximate solution was used. By utilizing the parallel plate model as before and making the assumption that the fluid velocity at the core is zero (i.e., Poiseuille flow which ignores the Couette boundary conditions) the velocity field becomes symmetric and can be solved analytically [5]. By applying the boundary conditions $\frac{du'}{dy}(0) = 0$ and $u'(-\frac{h}{2}) = 0$ to Eq. 24, we find that

$$u'(y) = \frac{\alpha^{1/n}}{\beta} \left[(h/2)^\beta - (-y)^\beta \right] . \quad (32)$$

Integration of Eq. 32 over the annulus yields

$$Q_c = 2\pi D_c \alpha^{1/n} \left(\frac{h}{2}\right)^{n+1} \frac{n}{2n+1} = \pi (D_c/2)^2 U_c \quad (33)$$

which, when solved for α becomes

$$\alpha = \left(\frac{D_c}{2} U_c \frac{2n+1}{4n} \right)^n \left(\frac{h}{2} \right)^{-2n-1} = \frac{1}{K} \frac{dp'}{dx} \quad (34)$$

The approximate pressure buildup is thus

$$\Delta p' = \left(D_c U_c \frac{2n+1}{8n} \right)^n \left(\frac{h}{2} \right)^{-2n-1} LK \quad (35)$$

where L is the instantaneous length of the core along which the pressure drop is evaluated. This equation yields pressure buildups which are only a few percent in error for pseudo-plastic fluids where $n < 1$ and increasingly larger errors for dilatant fluids where $n > 1$.

Nomenclature

- C - Distance from line of symmetry to inner core barrel
- D - Distance from line of symmetry to core
- D_c - Diameter of core
- F_x, F_θ - Body forces in the x and θ directions
- h - Annular gap between the core and the core barrel
- K - Consistency index, cp - sec¹⁻ⁿ
- L - Length of core
- n - Power-law index, dimensionless
- P - Coring fluid pressure
- Δp' - Approximate annular pressure buildup
- Q_a, Q_c - Actual and calculated coring fluid flow rates
- r, θ, z - Cylindrical coordinates
- $\bar{R}$ - Mean circumference of annulus
- R₁ - Radius of the inner cylinder of a viscometer
- R₂ - Radius of the outer cylinder of a viscometer
- s - Symmetry distance such that $u(\pm(h - s)/2) = 0$
- u, v, w - Axial, radial, and tangential velocities of extruded coring fluid
- U_c - Coring rate
- v_r, v_θ, v_z - Viscometer fluid velocities in the radial, tangential, and axial directions
- α - A positive constant, $-\frac{1}{k} \frac{dP}{dx}$
- β, ν - Positive constants, $1 + 1/n$ and $2 + 2/n$, respectively
- γ - Shear rate, sec⁻¹

(Nomenclature, continued)

$\dot{\gamma}$ viscosity - The shear rate at which the viscosity is measured

μ - Viscosity, centipoise

τ - Shear stress

Fig. 1 - Schematic Drawing of Improved Pressure Coring System

Fig. 2 - End View of the Core and the Inner Core Barrel

Fig. 3 - Schematic of the Incoming Core Extruding the Low-Invasion Fluid

Fig. 4 - Cone-and-Plate Viscometer Data for a Typical Low-Invasion Fluid

Fig. 5 - Brookfield Viscometer Data for Fluid 4 at 150°F

Fig. 6 - Mathematical Detail Drawing of the Extrusion Process

Fig. 7 - Logic Diagram for Exact Numerical Solution of Extrusion Process

EXTRUSION PROFILE OF LOW-INVASION FLUID

Fig. 8 - Extrusion Velocity Profile as a Function of the Power Law Index

Fig. 9 - COMPRESSIBILITY OF LOW - INVASION
CORING FLUID, 11.6 lb/gal

Table 1. Summary of Viscosity Testing for Various Low-Invasion Fluid Candidates

		<u>Fluid 1</u>	<u>Fluid 2</u>	<u>Fluid 3</u>	<u>Fluid 4</u>	<u>Fluid 5</u>	<u>Fluid 6</u>
Fluid Properties @ 70°F	n	0.27	0.26	0.31	0.33	0.25	1.0
	K,cp	1,837,000	2,156,000	1,251,000	638,000	995,000	61,500
Pressure Buildup	psi	1181	1347	900	485	604	265
Fluid Properties @ 150°F	n	0.44	0.34	0.48	0.50	0.47	1.0
	K,cp	376,000	757,000	289,000	162,000	198,000	30,000
Pressure Buildup	psi	385	591	329	194	220	130

Fluid 1 -- 15 lb/bbl HEC and 100 lb/bbl CaCO_3 in CaCl_2 eutectic brine

Fluid 2 -- 15 lb/bbl HEC, 300 lb/bbl CaCO_3 , and 1.75 lb/bbl CH_2O in CaBr_2 eutectic brine

Fluid 3 -- 13 lb/bbl HEC, 300 lb/bbl CaCO_3 , and 1.75 lb/bbl CH_2O in CaBr_2 eutectic brine

Fluid 4 -- 10 lb/bbl HEC, 300 lb/bbl CaCO_3 , and 1.75 lb/bbl CH_2O in CaBr_2 eutectic brine

Fluid 5 -- 10 lb/bbl HEC and 300 lb/bbl CaCO_3 in CaBr_2 eutectic brine

Fluid 6 -- Dimethyl Siloxane polymer, Dow Corning 200, 60,000 centistokes

Table 2. Pressure buildup vs annular clearance for low-invasion fluid at room temperature ($n = 0.25$, $K = 995,000$, $L = 9$ ft, $U_c = 20$ ft/hr)

<u>Annular Clearance, Inches</u>	<u>Pressure Buildup, psi</u>
0.500	79
0.250	217
0.125	604
0.100	842
0.075	1291
0.050	2364
0.025	6662

Table 3. Pressure buildup vs coring rate for low-invasion fluid at room temperature ($n = 0.25$, $K = 995,000$, $L = 9$ ft, $h = 0.125$ in)

<u>Coring Rate, ft/hr</u>	<u>Pressure Buildup, psi</u>
100	904
50	760
20	604
10	508
5	427
1	286
0.1	161

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