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A Mathematical Treatment of Measurement Data for Supplanting the Use of an Optical Comparator in the Inspection of Manufactured Parts

Walter J. Halpin

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A MATHEMATICAL TREATMENT OF MEASUREMENT DATA
FOR SUPPLANTING THE USE OF AN OPTICAL COMPARATOR
IN THE INSPECTION OF MANUFACTURED PARTS

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ABSTRACT

This report explains a mathematical analog to an optical comparator and describes computer program MOCO which employs the analog. It contains results from application of the program to hypothetical data for typical manufactured parts which prove out the effectiveness of the program. It also contains a study of the inspection of a number of actual manufactured parts of the same type using the MOCO program. The study shows the usefulness of MOCO and points out difficulties that may be encountered during inspection if the accuracies of the measurements involved are not consistent with the tolerances placed on the part features and contours. Ways for minimizing the number of computer calculations required are discussed as are means for improving and extending the program, e. g., to three dimensions.

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A MATHEMATICAL TREATMENT OF MEASUREMENT DATA
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Introduction

An optical comparator is an inspection device for determining whether a manufactured part meets its design specifications. The comparator is composed of a projection screen and a magnifying optical system containing some means for adjusting the position and orientation of a part placed in the system. A template of an enlarged drawing of the part (complete with tolerance bands or zones) is placed upon the screen while the part to be inspected is placed in a holder in the optical system and projected over the template with a magnification that produces the same enlargement as that of the template relative to the drawing.* An attempt is then made to adjust the position of the part relative to the template until one is found in which all pertinent part features and contours (hereafter both referred to as features) are within the indicated drawing tolerances. If such a position can be found, the part is considered to have passed inspection; if not, the part is rejected.

Though there are advantages in the use of an optical comparator, especially because it involves a direct comparison of a part with its drawing, there are also two principle difficulties. First, the necessity for having lines of finite width on the drawing and the imperfection of definition for part edges make it uncertain as to just where the positions of features and their tolerances lie on the template used for comparison with the part. Secondly, the movement of the part in attempting to "fit" it with the template involves no systematic method of "search" for the best position. Consequently, a part could be rejected because a position in which it actually meets specifications was overlooked.

The only other types of inspection methods involve making measurements of feature locations on a manufactured part; these measurements are referenced to some xy cartesian coordinate system. In determining whether the part meets specifications, these measurements are compared with their counterparts (so called "nominals") on the design drawing of the part.

* Only aspects of parts that can be described in two dimensions are considered here. Cases involving three dimensions are discussed in Appendix A.

There are also difficulties involved with these methods, however. For example, the measurement coordinate system is not likely to coincide with that of the drawing, either because it would be impractical or simply because there may be present some inherent error in setting up the measurement apparatus relative to the part. The result is a measurement system offset or bias. What is needed in this case, therefore, is a way of transforming the measurement data so that, at the very least, this setup bias is eliminated. It is possible to go even further than this in that the transformation used can be made to provide some sort of "best fitting" of measured and nominal data.^{*} When this is properly done, the use of the measurements can provide more accurate results for part inspection because the relative positions of features are more accurately determined than in the case where the part and its drawing (template) are optically compared. Even when this is done, however, no account is taken during the fitting process of the tolerances involved on the drawing and herein lies another difficulty with these other types of inspection.

This report describes a new method for inspection, based on a "search" scheme suggested by E. W. Kenderdine of Division 2328, which would appear to eliminate most of the difficulties of the others while maintaining their advantages. In effect, this method operates mathematically on part measurement data in a way that simulates the use of an optical comparator, but in addition it includes a systematic approach to the search for the best relative position of a part and its drawing, i. e., one where the part has its best chance for passing inspection.

The report is divided into four sections. In the first section, the principles involved in the use of the new method are explained. In the second section, a computer program MOCO (Mathematical Optical COMparator), which was constructed to implement the method, is described and proven out. In the third section, results from use of the program with actual measurement data from several precision parts of the same type are discussed and analyzed in terms of measurement accuracies required to take proper and full advantage of the program. In the last section, some possible improvements in and extensions of the program are discussed.

Explanation of the Principles

In Figure 1, let the positions of any two feature points on a part drawing be given by $(X_F, Y_F)^{**}$ and (X_E, Y_E) relative to some xy coordinate system in the drawing; and let (X_i, Y_i) represent the position of any one of a number of other feature points. Also, let the zones shown by dashed lines around these points indicate the tolerances allowed in their positions. Furthermore, consider an xy coordinate measurement system used in the inspection of a part made from the drawing to have

^{*} For example, the "least squares" type fitting used with the Coincident Coordinate Method (COCOM) as described in SAND77-1316; W. J. Halpin, Details of the Mathematics Associated with COCOM and COCOMP, March 1978.

^{**} The subscript F was adopted to suggest the fulcrum of a lever and the subscript E its end point.

its axes superimposed on those of the drawing system with the result that, in the latter system, the actual position corresponding to (X_F, Y_F) falls at $(\hat{X}_F, \hat{Y}_F)$, that of (X_E, Y_E) at $(\hat{X}_E, \hat{Y}_E)$, and that of (X_I, Y_I) at $(\hat{X}_I, \hat{Y}_I)$. The question is whether the measured (i. e., the "hatted") points can be moved as a group relative to the drawing points so that each of the former is within its corresponding tolerance zone around each of the latter. This question can only be answered by performing various combinations of translation and rotation (i. e., transformation) of one group of points relative to the other.

Figure 1. Measurement and Drawing Coordinate Systems Superimposed

The transformation may be made in any number of ways, that is, the translation can take place with respect to any set of xy axes and the rotation can occur around any point. A more logical approach to the transformation, however, would be to translate all measured ("hatted") points with respect to the xy axes of the drawing until $(\hat{X}_F, \hat{Y}_F)$ falls within its tolerance zone around (X_F, Y_F) and then to rotate all other measured points around this new position of $(\hat{X}_F, \hat{Y}_F)$ to see whether some angular orientation can be found for which these points are also within their tolerance zones. Such translation and rotation exhausts all the "degrees of freedom" available for rearranging the measured points relative to their nominals. Furthermore, since both $(\hat{X}_F, \hat{Y}_F)$ and $(\hat{X}_I, \hat{Y}_I)$ must each be within tolerance for a part to pass inspection, the following scheme determines all the combinations of translation and rotation that need be tried.

1. Translation of $(\hat{X}_I, \hat{Y}_I)$ to all points within its tolerance zone (obviously all points cannot be tried, but a number sufficient to provide some needed accuracy in the search process can).
2. Rotation about each position of $(\hat{X}_F, \hat{Y}_F)$ such that the "radius" between this point and $(\hat{X}_E, \hat{Y}_E)$ covers all angles from one side of the tolerance zone around (X_E, Y_E) to the other (again, at least a sufficient number of angles to provide the needed accuracy).

For a part to be acceptable, all of the points $(\hat{X}_i, \hat{Y}_i)$ must necessarily be in tolerance for some combination of one of these translations and one of these rotations. An example of a translation and a rotation according to this scheme is indicated in Figures 2a and 2b. Here it is seen that the result has both $(\hat{X}_F, \hat{Y}_F)$ and $(\hat{X}_E, \hat{Y}_E)$ within tolerance but $(\hat{X}_I, \hat{Y}_I)$ out of tolerance.

Figure 2. Example of Transformation of Measured Data

A systematic approach to the use of this scheme and one that has been adopted to perform the necessary search procedure involves the following steps:

1. A translation that places $(\hat{X}_F, \hat{Y}_F)$ in the upper right-hand corner of some search rectangle that envelops the tolerance zone around (X_F, Y_F) .
2. A rotation of $(\hat{X}_E, \hat{Y}_E)$ around this new position of $(\hat{X}_F, \hat{Y}_F)$ until the line between these points lies on or outside the (X_E, Y_E) tolerance zone.

(Steps 1 and 2 merely serve to set up some starting point for the search procedure.)

3. A rotation of $(\hat{X}_I, \hat{Y}_I)$ around $(\hat{X}_F, \hat{Y}_F)$ in successive angular increments (opposite in direction to the rotation in step 2) until the entire tolerance zone of (X_E, Y_E) has been "swept out." The angular increments are small enough so that the tolerance zone of (X_E, Y_E) is adequately covered.
4. Incrementing of the position of $(\hat{X}_F, \hat{Y}_F)$ in either the x or y direction (or both) and a repetition of step 3. The incrementing is such as to cover a sufficient number of points in the (X_I, Y_I) tolerance zone.

For each new translational position of $(\hat{X}_F, \hat{Y}_F)$ and rotational position of $(\hat{X}_I, \hat{Y}_I)$, all the other measured points which are transformed accordingly are checked to see whether they are in tolerance. All of this is discussed in more detail in the next section.

The Computer Program and Its Checkout

The computer program MOCO was constructed by E. L. Frost of Division 1223. In this program, as it now stands, it is assumed that all the two-dimensional features on the drawing of the part can be characterized by

1. Points $P_i; i = 1, 2, \dots, n_P$
2. Straight lines $L_i; i = 1, 2, \dots, n_L$
3. Arcs of circles $A_i; i = 1, 2, \dots, n_A$
4. A special case of 3, i. e., holes $H_i; i = 1, 2, \dots, n_H$.*

This is illustrated in Figures 3a through 3d where it is also indicated what parameters are needed for a geometric description of these features to be used in any mathematical treatment.**

An illustration of tolerance zones corresponding to the above features is given in Figures 4a through 4d, where the symbols adopted to indicate the sizes of these zones are also introduced. In the case of a point as a point, either a rectangular or a circular zone may be used. For the former, there is a tolerance T_{PX1} in x and T_{PY1} in y around the point, while for the latter, the tolerance is T_{PU1} . In the case of a straight line, there is an upper tolerance T_{LU1} away from the origin of the coordinate system and a tolerance T_{LL1} toward the origin. In the case of a circular arc, the tolerances are T_{AU1} on the convex side of the arc and T_{AL1} on the concave side. Finally, in the case of a hole, the tolerances are T_{HU1} outside the hole and T_{HL1} inside.***

For any given position of a point measured on a part, there will be some margin or margins (here indicated by M) between that point and the tolerance zone around its corresponding drawing nominal feature. Mathematically, an expression for a margin of any such point can be constructed such that if the margin is positive, the point is in tolerance, if the margin is negative, the point is out of tolerance. In the case of a point feature and a rectangular tolerance zone, if $(\hat{X}_{P1}, \hat{Y}_{P1})$ are the coordinates of that point in the drawing coordinate system, then x and y margins can be determined from Figure 5a to be given by

$$M_{PX1} = T_{PX1} - |\hat{X}_{P1} - X_{P1}| \quad (1)$$

* Possibilities for the use of other geometrical figures are discussed in Appendix B.

** If a line L_1 should pass through the origin, then D_{L1} is, of course, equal to zero and the following conventions are adopted with regard to determining α_{L1} . If the line coincides with the x-axis, then α_{L1} is given a value of 270° . If the line has a positive slope, α_{L1} is computed as though the line crossed the positive x-axis and the negative y-axis. If the line coincides with the y-axis, then α_{L1} is given a value of zero. If the line has a negative slope, α_{L1} is computed as though the line crossed the positive x-axis and the positive y-axis.

*** These tolerances are given with respect to some hypothetical nominal value of hole radius which is indicated in Figure 3d as R_{H1} and is calculated by dividing the sum of the outer and inner radii allowed for a hole by two.

a. Point

b. Line

c. Circular Arc

d. Hole

Figure 3. Illustration of Geometric Features of a Part Drawing

Figure 4. Illustration of Tolerance Zones

$$M_{\hat{P}Y_i} = T_{PY_i} - |\hat{Y}_{P_i} - Y_{P_i}|^* \quad (2)$$

and for a radial tolerance zone by

$$M_{PU_i} = T_{PU_i} - \sqrt{(\hat{X}_{P_i} - X_{P_i})^2 + (\hat{Y}_{P_i} - Y_{P_i})^2} \quad (3)$$

Figure 5. Illustration of Margins

* In some special cases, the rectangular tolerance zone may be "tilted" with respect to the coordinate axis. The method for handling these cases (not now in the program) is discussed later.

In the case of a straight line, there may be involved any number of measured points $(\hat{X}_{LiJ}, \hat{Y}_{LiJ})$; $j = 1, 2, \dots, n_{Li}$. For any one of these points, the margins are given by (see Figure 5b)

$$M_{LUiJ} = (T_{LUi} + D_{Li}) - (\hat{X}_{LiJ} \cos \alpha_{Li} + \hat{Y}_{LiJ} \sin \alpha_{Li}) \quad (4)$$

and

$$M_{LLiJ} = (\hat{X}_{LiJ} \cos \alpha_{Li} + \hat{Y}_{LiJ} \sin \alpha_{Li}) - (D_{Li} - T_{LLi}) \quad (5)$$

These latter two equations are obtained by rotating both the drawing line and the measured point about the drawing origin in a clockwise direction through the angle α_{Li} . This makes the line L_1 perpendicular to the x-axis and still a distance D_{Li} from the origin, with the x-component of the measured point being that given by the parenthetical term containing the trigonometric functions. Thus, to find the margin away from the origin, it is only necessary to add the tolerance T_{LUi} to D_{Li} and subtract the x-component. Similarly, to find a margin toward the origin, the difference between D_{Li} and T_{LLi} is subtracted from the x-component. In the case of a circular arc, again there may be involved any number of measured points $(\hat{X}_{AiJ}, \hat{Y}_{AiJ})$; $j = 1, 2, \dots, n_{Ai}$. From Figure 5c it is seen that for any one of these points, the margins are given by

$$M_{AUiJ} = (T_{AUi} + R_{Ai}) - \sqrt{(\hat{X}_{AiJ} - X_{Ai})^2 + (\hat{Y}_{AiJ} - Y_{Ai})^2} \quad (6)$$

and

$$M_{ALiJ} = \sqrt{(\hat{X}_{AiJ} - X_{Ai})^2 + (\hat{Y}_{AiJ} - Y_{Ai})^2} - (R_{Ai} - T_{ALi}) \quad (7)$$

In the case of a hole (Figure 5d), it can be determined that the margins are given by

$$M_{HU1} = (R_{H1} + T_{HU1}) - \left[\hat{R}_{H1} + \sqrt{(\hat{X}_{H1} - X_{H1})^2 + (\hat{Y}_{H1} - Y_{H1})^2} \right] \quad (8)$$

and

$$M_{HL1} = \hat{R}_{H1} - \left[(R_{H1} - T_{HL1}) + \sqrt{(\hat{X}_{H1} - X_{H1})^2 + (\hat{Y}_{H1} - Y_{H1})^2} \right] \quad (9)$$

where $\hat{R}_{H1}$ is the measurement of the radius of the hole on the part (made with a gage, for example).

In setting up the systematic approach to the use of the search procedure as discussed in the last section, the translational parameters, which place $(\hat{X}_F, \hat{Y}_F)$ in the upper right-hand corner of some search rectangle around (X_F, Y_F) , can first be determined. This rectangle need only be

large enough to envelop the tolerance zones. The parameter for this translation in the x-direction will be denoted by h and for the y-direction by k , where

$$h = X_F - \hat{X}_F + a_F \quad (10)$$

$$k = Y_F - \hat{Y}_F + b_F \quad (11)$$

with a_F and b_F being the half-widths of the search rectangle in the x- and y-directions, respectively. The next step in the initial setup requires calculating an angle such that when the point $(\hat{X}_E, \hat{Y}_E)$ is rotated in a clockwise direction around $(\hat{X}_F, \hat{Y}_F)$, the line between these points becomes parallel to the line between (X_E, Y_E) and (X_F, Y_F) . From Figure 6a this is seen to be given by

$$\theta = \tan^{-1} \left[\frac{\hat{Y}_E - \hat{Y}_F}{\hat{X}_E - \hat{X}_F} \right] - \tan^{-1} \left[\frac{Y_E - Y_F}{X_E - X_F} \right] \quad (12)$$

Figure 6. Illustration of the Angles θ and ρ Used in Reaching the Initial Setup of the Search Procedure

Finally, the last step in reaching the initial setup requires calculation of an angle ρ so that when, after initial transformation by h , k and θ , $(\hat{X}_I, \hat{Y}_I)$ is rotated by ρ around $(\hat{X}_I, \hat{Y}_I)$ in a clockwise direction, the line between these points will lie on or outside of the tolerance zone of (X_E, Y_E) . The intention here is to sweep in the opposite (counterclockwise) direction through twice the value of this angle during the search procedure. To assure that all of the tolerance zone around (X_E, Y_E) is swept regardless of the position of $(\hat{X}_I, \hat{Y}_I)$ in the search rectangle,

the angle ρ is conservatively calculated on the large side.* From Figure 6b it can be determined that a value of this angle, which will cover all cases during the search, is given by

$$\rho = \tan^{-1} \left\{ \frac{\sqrt{(a_E^2 + b_E^2)} + \sqrt{(a_F^2 + b_F^2)}}{\sqrt{(X_E - X_F)^2 + (Y_E - Y_F)^2} - \left[\sqrt{(a_E^2 + b_E^2)} + \sqrt{(a_F^2 + b_F^2)} \right]} \right\} \quad (13)$$

If the tolerance zone around (X_E, Y_E) is circular, then $\sqrt{(a_E^2 + b_E^2)}$ can be replaced by the radius of that zone. Thus to reach the initial setup for the search procedure, there is a translation in x by h and in y by k and a clockwise rotation of the new position of $(\hat{X}_E, \hat{Y}_E)$ around the new position of $(\hat{X}_F, \hat{Y}_F)$ by an angle ϕ where

$$\phi = \theta + \rho \quad (14)$$

From this point on, the position of $(\hat{X}_F, \hat{Y}_F)$ in its search zone can be changed by increments Δh in x and Δk in y, while the angle of rotation of $(\hat{X}_E, \hat{Y}_E)$ around $(\hat{X}_F, \hat{Y}_F)$ can be incremented in the angular search zone by amounts $\Delta\phi$ where

$$\Delta h = \frac{2a_F}{d_h} \quad (15)$$

$$\Delta k = \frac{2b_F}{d_k} \quad (16)$$

$$\Delta\phi = \frac{2\rho}{d_\phi} \quad (17)$$

and where d_h , d_k , and d_ϕ are integer divisors that determine the "fineness" of the search. Because it is desirable to make the linear increments of travel of all points (whether due to translation or rotation) roughly equal while performing the search, the following calculational approach is used to determine values for the divisors: choose, say, d_h for adequate "fineness"; then let

$$d_k = d_h \frac{b_F}{a_F} \quad (\text{to nearest integer}) \quad (18)$$

* Calculating ρ in this way roughly doubles or triples the number of calculations that must be made since some of them will involve points outside of the (X_E, Y_E) tolerance zone. A method of programming to eliminate this shortcoming is discussed in Appendix C.

$$d_{\phi} = d_h \rho \sqrt{\frac{(\hat{X}_E - \hat{X}_F)^2 + (\hat{Y}_E - \hat{Y}_F)^2}{a_F}} * \quad \text{(to nearest integer)} \quad . \quad (19)$$

From all of this, if m_h , m_k , and m_{ϕ} are integer multipliers for their respective increments, the transformations of a set of coordinates for the various initial measurements of the features of a part to a new set of coordinates (indicated by primes) during the search procedure are given by the following equations.

1. Points $\hat{P}_i$, $i = 1, 2, \dots, n_P$

$$\begin{aligned} \hat{X}'_{Pi} &= (\hat{X}_{Pi} - \hat{X}_F) \cos (\phi - m_{\phi} \Delta\phi) + (\hat{Y}_{Pi} - \hat{Y}_F) \sin (\phi - m_{\phi} \Delta\phi) + \\ &\quad (h - m_h \Delta h) + \hat{X}_F \end{aligned} \quad (20)$$

$$\begin{aligned} \hat{Y}'_{Pi} &= (\hat{Y}_{Pi} - \hat{Y}_F) \cos (\phi - m_{\phi} \Delta\phi) - (\hat{X}_{Pi} - \hat{X}_F) \sin (\phi - m_{\phi} \Delta\phi) + \\ &\quad (k - m_k \Delta k) + \hat{Y}_F \quad . \end{aligned} \quad (21)$$

2. Points $(\hat{X}_{Lij}, \hat{Y}_{Lij})$ along lines, $j = 1, 2, \dots, n_{Li}$, $i = 1, 2, \dots, n_L$

$$\begin{aligned} \hat{X}'_{Lij} &= (\hat{X}_{Lij} - \hat{X}_F) \cos (\phi - m_{\phi} \Delta\phi) + (\hat{Y}_{Lij} - \hat{Y}_F) \sin (\phi - m_{\phi} \Delta\phi) + \\ &\quad (h - m_h \Delta h) + \hat{X}_F \end{aligned} \quad (22)$$

$$\begin{aligned} \hat{Y}'_{Lij} &= (\hat{Y}_{Lij} - \hat{Y}_F) \cos (\phi - m_{\phi} \Delta\phi) - (\hat{X}_{Lij} - \hat{X}_F) \sin (\phi - m_{\phi} \Delta\phi) + \\ &\quad (k - m_k \Delta k) + \hat{Y}_F \quad . \end{aligned} \quad (23)$$

* In order to make the linear search increments due to angle as small as that in the tolerance region of the end point for all points on the part, in the equation for d_{ϕ} the square root quantity would be replaced by a quantity representing the distance from the fulcrum point to the most remote point on the part.

3. Points $(\hat{X}_{Aij}, \hat{Y}_{Aij})$ along arcs, $j = 1, 2, \dots, n_{Ai}$, $i = 1, 2, \dots, n_A$

$$\begin{aligned} \hat{X}'_{Aij} = & (\hat{X}_{Aij} - \hat{X}_F) \cos (\phi - m_\phi \Delta\phi) + (\hat{Y}_{Aij} - \hat{Y}_F) \sin (\phi - m_\phi \Delta\phi) + \\ & (h - m_h \Delta h) + \hat{X}_F \end{aligned} \quad (24)$$

$$\begin{aligned} \hat{Y}'_{Aij} = & (\hat{Y}_{Aij} - \hat{Y}_F) \cos (\phi - m_\phi \Delta\phi) - (\hat{X}_{Aij} - \hat{X}_F) \sin (\phi - m_\phi \Delta\phi) + \\ & (k - m_k \Delta k) + \hat{Y}_F \end{aligned} \quad (25)$$

4. Holes $\hat{H}_i$, $i = 1, 2, \dots, n_H$

$$\hat{X}'_{Hi} = (\hat{X}_{Hi} - \hat{X}_F) \cos (\phi - m_\phi \Delta\phi) + (\hat{Y}_{Hi} - \hat{Y}_F) \sin (\phi - m_\phi \Delta\phi) + (h - m_h \Delta h) + \hat{X}_F \quad (26)$$

$$\begin{aligned} \hat{Y}'_{Hi} = & (\hat{Y}_{Hi} - \hat{Y}_F) \cos (\phi - m_\phi \Delta\phi) - (\hat{X}_{Hi} - \hat{X}_F) \sin (\phi - m_\phi \Delta\phi) + \\ & (k - m_k \Delta k) + \hat{Y}_F \end{aligned} \quad (27)$$

The actual search procedure that is used, given some initial drawing (nominal) and measurement data on the various points, lines, etc., involves the following steps.

1. With m_h , m_k set at zero, m_ϕ is allowed to range from zero to d_ϕ . For each of these values of m_ϕ , the $\hat{X}'$'s and the $\hat{Y}'$'s are calculated by using Equations (20) through (27). Then by using Equations (1) through (9) with the $\hat{X}'$'s and the $\hat{Y}'$'s replacing the $\hat{X}$'s and the $\hat{Y}$'s, values for the margins M are calculated. All of the M 's so obtained are normalized by dividing them by their corresponding tolerances, e.g., M_{Pui} would be divided by T_{Pui} in Equation (3) (see the discussion below concerning the reason for this). At each stage (i.e., set of values for m_h , m_k and m_ϕ), the smallest value of the normalized M 's is noted, and the largest of these smallest values (over all sets of the integers) is kept along with the set of integers that produced it.

2. Step 1 is repeated but with m_h set first at 1, then 2, ... at d_h . Again, the largest of the smallest values of normalized margins over all sets of integers tried is kept.
3. Steps 1 and 2 are repeated but with m_k set first at 1, then 2, ... then d_k with the maximum of the minimum normalized margins and its corresponding set of integers always being kept over all sets of the integers.

The reason for normalizing the margins of the various features is to prevent any one feature with a large tolerance from "overpowering" features with smaller tolerances; that is, in the procedure outlined, if the maximum least margin (nonnormalized) over all sets of the integers were sought, a given value for this maximum could be a relatively small fraction of a large tolerance zone and a large fraction of a small tolerance zone. The result might then be that the "best fit" obtained during the search procedure is not truly the one sought when relative magnitudes of the tolerance zones are considered.

If after the search procedure is completed it is thought that the results are too rough, i.e., the divisions d_h , d_k and d_ϕ were too small, then it may be necessary to increase the "fineness" of the search. Rather than simply increasing the values of the d's for the original sizes of the search rectangle and angular segment, it should be much more economical, as far as the number of calculations are concerned, to use the following method:

1. Note the set (m_h, m_k, m_ϕ) for which the least maximum, normalized margin is obtained for the first choice of the d's.
2. Choose a search rectangle that is centered around a point corresponding to (m_h, m_k) and has sides equal to $2\Delta h$ in the x-direction and $2\Delta k$ in the y-direction.
3. Choose an angular segment that is centered around the line between $(\hat{X}_E, \hat{Y}_E)$ and $(\hat{X}_F, \hat{Y}_F)$ corresponding to (m_h, m_k, m_ϕ) and extends an amount somewhat greater than $\Delta\phi$ on either side, say $C\Delta\phi$ where $C > 1$.*
4. To determine the "fineness" of the search with the new rectangle and angular segment, choose some appropriate value for the x direction divisor d'_h and calculate the other two divisors from

$$d'_k = d'_h \frac{\Delta k}{\Delta h} \quad (28)$$

*The reason for increasing the angle somewhat is the same as that given for calculating ρ conservatively on the high side in Equation (13). In the program MOCO, the value of C has been set rather arbitrarily at 3.

$$d'_\phi = C\Delta\phi \frac{\sqrt{(\hat{X}_E - \hat{X}_F)^2 + (\hat{Y}_E - \hat{Y}_F)^2}}{\Delta h} d'_h . \quad (29)$$

The new search increments are then given by:

$$\Delta h' = \frac{2\Delta h}{d'_h} \quad (30)$$

$$\Delta k' = \frac{2\Delta k}{d'_k} \quad (31)$$

$$\Delta\phi' = \frac{2C\Delta\phi}{d'_\phi} . \quad (32)$$

5. In Equations (20) through (27), while using the m_h , m_k , and m_ϕ of 1 above, add the terms $(\Delta h - m'_h \Delta h')$, $(\Delta k - m'_k \Delta k')$, and $(\Delta\phi - m'_\phi \Delta\phi')$ in their respective, proper parentheses. Here, $m'_h = 0, 1, 2, \dots, d'_h$, $m'_k = 0, 1, 2, \dots, d'_k$, and $m'_\phi = 0, 1, 2, \dots, d'_\phi$ are integer multipliers for their respective increments.
6. Follow the search procedure as outlined earlier for the first and larger rectangle and angular segment with the new initial setup indicated.

The method indicated in steps 1 through 6 above merely narrows the search region after the initial run with the larger rectangle and angular segment, excluding regions that have already been tried but that did not produce the maximum least margin. As an example of the amount of calculations that can be saved with this method, suppose it were required that the large rectangle and angular segment had to have divisors (i.e., values of the d's) of 100 in order to obtain adequate "fineness" of the search. This would mean that a total of about 10^6 calculations of all the transformation coordinates and their margins would have to be made. With the method under discussion, however, the same "fineness" can be obtained with initial divisors of, say, 10 followed by divisors for the smaller rectangle and angular segment of 20. This would require a total of less than 11,000 calculations for coordinates and margins.* Since there is some chance that the "best answer" for the initial subdivision might lie somewhere outside the smaller rectangle as defined,

*Analyses having to do with choosing d_h and d'_h so as to minimize the number of calculations for a given fineness of search are discussed in Appendix D.

it may be wise to increase its size somewhat. In any event, the savings in calculations should still be considerable. Obviously, the process could be continued with a third and still smaller rectangle and angular segment being chosen and subdivided, and then a fourth, etc., and this is what is done in MOCO until some predetermined precision of search is obtained.* However, for most practical cases of part inspection that may be encountered, the process need only include the first two steps, and the first example that follows will show results in which this is the case.

A drawing of a hypothetical part to be manufactured is shown in Figure 7. The nominal data for various part features indicated and corresponding tolerances chosen follow.

Figure 7. Drawing for a Hypothetical Part Used in an Example of the Use of MOCO

1. Points P_i

$$n_P = 2, (X_{P1}, Y_{P1}) = (3, 2), (X_{P2}, Y_{P2}) = (5, 0), T_{PU1} = 0.001,$$

$$T_{PX2} = 0.002, T_{PY2} = 0.002 .$$

* The method of subdividing search regions so as to reduce the amount of calculations, especially for cases where great search precision is required, is discussed in Appendix D.

2. Straight Lines L_i

$$n_L = 2, \alpha_{L1} = \pi/2, D_{L1} = 2, \alpha_{L2} = \pi/4, D_{L2} = 3.53553,$$

$$T_{LU1} = 0.002, T_{LL1} = 0.002, T_{LU2} = 0.002, T_{LL2} = 0.002 \quad .$$

3. Circular Arcs A_i

$$n_A = 2, (X_{A1}, Y_{A1}) = (0.4, 1.6), (X_{A2}, Y_{A2}) = (0.6, 0.6),$$

$$R_{A1} = 0.4, R_{A2} = 0.6, T_{AU1} = 0.001, T_{AL1} = 0.001, T_{AU2} = 0.001,$$

$$T_{AL2} = 0.001 \quad .$$

4. Holes H_i

$$n_H = 2, (X_{H1}, Y_{H1}) = (1.8, 1.4), (X_{H2}, Y_{H2}) = (3.8, 0.4),$$

$$R_{H1} = 0.2, R_{H2} = 0.15, T_{HU1} = 0.001, T_{HL1} = 0.001, T_{HU2} = 0.002,$$

$$T_{HL2} = 0.002 \quad .$$

The fulcrum and end points were chosen to be

$$(X_F, Y_F) = (X_{H1}, Y_{H1}) = (1.8, 1.4), (X_E, Y_E) = (X_{P2}, Y_{P2}) = (5, 0)$$

from which, because of the choice of these points and their tolerances, the sizes of the search rectangle and angular segment were fixed by the dimensions

$$a_F = 0.001, b_F = 0.001$$

$$a_E = 0.002, b_E = 0.002 \quad .$$

In order to "synthesize" measurement data, points were taken directly from the drawing features and then translated and rotated in some arbitrary way with respect to the drawing coordinate system. This simulated the measurements that might be obtained on a part with respect to an offset measurement coordinate system if there were no errors made by the machines that manufactured the part. The advantage of using these synthesized data is not only that they fully test the procedure of searching for a "best fit," but also because the latter is known in advance.

That is, the simulated measurements should return to precisely their positions on the drawing, at least to the extent that the search procedure is precise.

The points taken from the drawing were, in this example, rotated by an amount of 10 degrees (clockwise) around the origin of the drawing system and then translated by an amount of 0.1 in the new x-direction and 0.1 in the new y-direction; that is, given a point (X, Y) on the drawing, the corresponding synthesized measured point ($\hat{X}$, $\hat{Y}$) was given by

$$\hat{X} = X \cos (10^{\circ}) + Y \sin (10^{\circ}) + 0.1 \quad (33)$$

$$\hat{Y} = Y \cos (10^{\circ}) - X \sin (10^{\circ}) + 0.1 \quad (34)$$

The points taken from the drawing in this example and their corresponding "measurements" are listed in Table I.* Also shown in the table are the measured radii assumed in the case of the holes.

TABLE I
Nominal and Measured Data Used in an Example of the Use of MOCO

<u>Feature</u>	<u>No.</u>	<u>X</u>	<u>$\hat{X}$</u>	<u>Y</u>	<u>$\hat{Y}$</u>	<u>R</u>	<u>$\hat{R}$</u>
P ₁	1	3.0	3.401720	2.0	1.548671		
P ₂	2	5.0	5.024039	0.0	-0.768241		
L ₁	3	1.0	1.432104	2.0	1.895967		
L ₁	4	2.6	3.007797	2.0	1.618130		
L ₂	5	3.4	3.726183	1.6	1.085289		
L ₂	6	4.6	4.699575	0.4	-0.304859		
A ₁	7	0.0	0.377837	1.6	1.675692		
A ₁	8	0.117157	0.542329	1.882843	1.933894		
A ₁	9	0.4	0.841219	2.0	2.000156		
A ₂	10	0.0	0.204189	0.6	0.690885		
A ₂	11	0.175736	0.303582	0.175736	0.242550		
A ₂	12	0.6	0.690885	0.0	-0.004189		
H ₁	13	1.8	2.115761	1.4	1.166164	0.2	0.2
H ₂	14	3.8	3.911729	0.4	-0.165940	0.15	0.15

*These points are indicated by circles on Figure 7 and identified by the number on the table.

The nominal and measured data from Table I were entered into the program MOCO. The "fineness" of the search procedure was determined by a choice of $d_h = d_k = d_\phi = 10$. The formula of Equation (19) for d_ϕ was not used in this case so as to assure that all divisors would be even numbers. The reason for this was that an "exact" answer could then be found by the search procedure; that is, the search procedure for an exact answer must result in $(\hat{X}_F, \hat{Y}_F)$ transforming to the center of the search rectangle while the line between $(\hat{X}_F, \hat{Y}_F)$ and $(\hat{X}_E, \hat{Y}_E)$ must become coincident with the line between (X_F, Y_F) and (X_E, Y_E) . This can only occur for even values of the divisors, combined in this case with the integers $m_h = m_k = m_\phi = 5$. These values of the integers provide the maximum least normalized margin, which should in fact be equal to 1 for all the measured points considered. Similarly, as stated, the measured points should all return to the values they had on the drawing before they were translated and rotated to provide synthetic measurement data. The program provided these results within the accuracy of the numbers provided as input.

As another check of the program, the same points from Figure 7 were used but were slightly offset from their drawing positions so as to simulate measurements of a part that was not made with complete accuracy. At the same time, the normalized margins for these offset points were determined. These data and the pertinent nominals were entered into the program so that a "best fit" could be found which maximized the minimum margin. The original coordinates of the "measured" points and their margins are compared in Table II with their counterparts which were obtained after a search through two stages of sizes of rectangles and angular segments. The same fulcrum and end point and values for $\hat{R}$ were used as in the previous check. The results in Table II show clearly that the maximum least margin (indicated by an asterisk) increases after application of the program, being larger after the second stage of the search as might be expected.

Still another check of the program involved changing the fulcrum and end points from the center of H_1 and the point P_2 to the point P_1 and the center of H_2 , respectively. The results were not significantly different.

For a final check of the program, a drawing for a typical manufactured part (clock plate) was used. A sketch of this part, which shows the labeling of the many features it contains, is given in Figure 8. (Actually, features P_1 through P_4 are hole centers but were used as points so that all feature types could be included.) Hypothetical measurement data for a part made from this drawing were synthesized by using points somewhat offset from their nominal positions for the various features. These data along with others required for input to the MOCO program are listed in Table III with their normalized margins. It will be noted that two of the points are on their tolerance boundaries (margin of 0) while one is even outside its boundary (i. e., negative margin). In this check MOCO was adjusted to proceed through enough stages (i. e., subdivisions to smaller and smaller rectangles) to obtain a precision of at least 10^{-7} . The results from application of MOCO are summarized in Table IV where average normalized (lesser) margins are compared before and after the application for points, lines, arcs, and holes separately, and finally for the

combination of all of these features. Also shown are the least normalized margins over all feature points both before and after. It is to be expected that the average normalized margins would decrease with application of MOCO because features with ample margins will have these decreased in order to maximize the least margin. The benefit of MOCO in this respect is evident here where it is seen that the least normalized margin increased from -0.0768 (out of tolerance) to 0.1436 (in tolerance). In this example, the fulcrum and end points were chosen to be the centers of holes H_2 and H_4 , respectively. Nearly identical results were obtained when other sets of points were used in separate runs of the data from Table III.

TABLE II
Measured Data and Their Margins Before and After Application of MOCO

Feature	No.	Before MOCO			After MOCO					
		$\hat{X}$	$\hat{Y}$	Margin	1st Stage			2nd Stage		
		$\hat{X}$	$\hat{Y}$	Margin	$\hat{X}$	$\hat{Y}$	Margin	$\hat{X}$	$\hat{Y}$	Margin
P_1	1	3.0	2.000700	0.3	3.000363	2.000375	0.4786	3.000484	2.000371	0.3897
P_2	2	5.0	0.001500	0.25	5.000487	0.0012989	0.3505*	5.000536	0.00122318	0.3884
L_1	3	1.0	2.0	1.0	1.000363	1.999550	0.7751	1.000484	1.999619	0.8096
L_1	4	2.6	2.000500	0.75	2.600363	2.000150	0.9251	2.600484	2.000161	0.9196
L_2	5	3.400707	1.600707	0.5	3.401095	1.600407	0.4674	3.401202	1.600389	0.4358
L_2	6	4.599010	0.399010	0.3	4.599472	0.3987840	0.3854	4.599536	0.3987228	0.3863
A_1	7	0.0	1.6	1.0	0.00038757	1.599488	0.6128	0.00049480	1.599593	0.5054
A_1	8	0.116450	1.883550	0.0*	0.1168199	1.883045	0.6182	0.1169374	1.883146	0.6299
A_1	9	0.4	2.0	1.0	0.4003627	1.999513	0.5131	0.4004844	1.999604	0.6039
A_2	10	0.0	0.6	1.0	0.00044973	0.5994881	0.5505	0.000520798	0.5995932	0.4793
A_2	11	0.175382	0.175382	0.5	0.1758581	0.1748810	0.4815	0.1759138	0.1749798	0.5907
A_2	12	0.6	0.0	1.0	0.6004870	-0.00047459	0.5252	0.6005364	-0.0003912	0.6086
H_1	13	1.8	1.40	1.0	1.800400	1.399600	0.4343	1.800500	1.399640	0.3839*
H_2	14	3.8	0.4	1.0	3.800462	0.3997243	0.7309	3.800526	0.3996920	0.6952

*The divisors used in the first stage of the search were $d_h = 10$, $d_k = 10$, and $d_\phi = 48$. In the second stage the divisors were $d_h' = 20$, $d_k' = 20$, and $d_\phi' = 28$. First stage maximized least margin of 0.3505 found for $m_h = 8$, $m_k = 4$, $m_\phi = 34$. Second stage maximized least margin of 0.3839 found for $m_h' = 9$, $m_k' = 6$, $m_\phi' = 5$.

Figure 8. Sketch for a Clock Plate Part

TABLE III

Data Used for Input to MOCO Associated with Figure 8

Points (P_i)	λ_{Pi}	Y_{Pi}	T_{Pui}	$\hat{X}_{Pi}$	$\hat{Y}_{Pi}$	M_{Pui}
P_1	-0.1255	0.2871	0.0005	-0.1255	0.2868	0.400 (U)*
P_2	0.1255	0.2871	0.0005	0.1255	0.2873	0.800 (U)
P_3	0.2780	0.0878	0.0005	0.2780	0.0877	0.800 (U)
P_4	0.2900	-0.1078	0.0005	0.2900	-0.1080	0.600 (U)

Lines (L_i)	α_{Li} (deg)	D_{Li}	T_{LUi}	T_{LLi}	Point (j)	$\hat{X}_{Lij}$	$\hat{Y}_{Lij}$	Lesser (M_{LUi} or M_{LLi})
L_1	180	0.5	0.0025	0.0025	1	-0.5025	-0.250	0.000 (U)
					2	-0.498	0.250	0.200 (L)
L_2	270	0.5	0.0025	0.0025	1	-0.3	-0.5005	0.800 (U)
					2	-0.1	-0.5005	0.800 (U)
L_3	180	0.068	0.0025	0.0025	1	-0.0685	-0.4	0.800 (U)
					2	-0.067	-0.45	0.600 (L)
L_4	0	0.140	0.0025	0.0025	1	0.140	-0.45	1.000
					2	---	---	---
L_5	270	0.5	0.0025	0.0025	1	0.25	-0.4998	0.920 (L)
					2	0.4	-0.4998	0.920 (L)
L_6	0	0.5	0.0025	0.0025	1	0.4995	0.25	0.800 (L)
					2	0.5005	-0.25	0.800 (U)
L_7	0	0.5	0.0025	0.0025	1	0.5010	0.388	0.600 (U)
					2	---	---	---
L_8	90	0.5	0.0025	0.0025	1	-0.25	0.5	1.000
					2	0.25	0.4975	0.000 (L)
L_9	99.63647	0.2273755	0.0025	0.0025	1	-0.000244045	0.2319255	0.473 (U)
					2	-0.2699187	0.185432	0.751 (U)
L_{10}	188.1301	0.3282965	0.0025	0.0025	1	-0.345049	0.086859	0.600 (U)
					2	---	---	---
L_{11}	95.39393	0.0552998	0.0025	0.0025	1	0.15	0.069704	1.000
					2	-0.270047	0.030550	0.800 (U)

Arcs (λ_i)	$\lambda_{\lambda i}$	$Y_{\lambda i}$	R_{Ai}	$T_{AU i}$	$T_{AL i}$	Point (j)	$\hat{X}_{Aij}$	$\hat{Y}_{Aij}$	Lesser ($M_{AU i}$ or $M_{AL i}$)
λ_1	0	0	0.645	0.0025	0.0025	1	-0.486184	-0.422633	0.680 (L)
						2	-0.422699	-0.486259	0.720 (L)
λ_2	0	0	0.645	0.0025	0.0025	1	0.485656	-0.422174	0.400 (L)
						2	0.422305	-0.485807	0.480 (L)
λ_3	0.510	0.304	0.050	0.0025	0.0025	1	0.479380	0.340062	-0.0768 (L)
						2	0.474291	0.268291	0.800 (U)
λ_4	0	0	0.645	0.0025	0.0025	1	0.422305	0.485807	0.480 (L)
						2	0.485882	0.422371	0.520 (L)
λ_5	0	0	0.645	0.0025	0.0025	1	-0.422764	0.486335	0.760 (L)
						2	-0.486335	0.422764	0.760 (L)
λ_6	-0.294	0.076	0.048	0.0025	0.0025	1	-0.328648	0.041352	0.600 (L)
						2	---	---	---
λ_7	0.172	0.120	0.048	0.0025	0.0025	1	0.205234	0.086766	0.600 (L)
						2	---	---	---
λ_8	0.003244	-0.001596	0.256	0.0025	0.0025	1	0.180312	0.180312	0.143 (L)
						2	---	---	---
λ_9	0.074	0.194	0.048	0.0025	0.0025	1	0.090588	0.239575	0.800 (U)
						2	---	---	---

TABLE III (cont)

λ_{i1}	λ_{Ai}	Y_{Ai}	R_{Ai}	T_{AU1}	T_{AL1}	Point (j)	$\hat{X}_{Ai}$	$\hat{Y}_{Ai}$	Lesser (M_{AU1} or M_{AL1})
λ_{10}	-0.302	0.132	0.048	0.0025	0.0025	1 2	-0.336295 ---	0.166295 ---	0.800 (U) ---
λ_{11}	0.1511	-0.131	0.040	0.001	0.001	1 2	0.121857 ---	-0.158143 ---	0.899 (L) ---
λ_{12}	-0.000016	0.000012	0.240	0.0025	0.0025	1 2	0.207413 ---	-0.119750 ---	0.808 (L) ---
λ_{13}	0.1959	-0.0402	0.040	0.001	0.002	1 2	0.223426 ---	-0.011574 ---	0.857 (L) ---
λ_{14}	-0.000016	0.000012	0.160	0.0025	0.0025	1 2	0.138131 ---	-0.079750 ---	0.808 (L) ---
λ_{15}	0.24	-0.414817	0.100	0.0025	0.0025	1 2	0.189500 0.154263	-0.327348 -0.365317	0.600 (U) 0.600 (L)
λ_{16}	0.182	-0.270	0.056	0.0025	0.0025	1 2	0.221244 0.238	-0.309244 -0.270	0.800 (L) 1.000
λ_{17}	-0.000967	-0.000186	0.270	0.0025	0.0025	1 2	0.07011 0.174195	-0.261283 -0.207598	0.761 (U) 0.408 (U)
λ_{18}	-0.012	-0.326	0.056	0.0025	0.0025	1 2	-0.066575 -0.026753	-0.311377 -0.270942	0.800 (U) 0.600 (U)

Holes (H_i)	X_{Hi}	Y_{Hi}	R_{Hi}	T_{HU1}^{**}	T_{HL1}^{**}	$\hat{R}_{Hi}^{***}$	$\hat{X}_{Hi}$	$\hat{Y}_{Hi}$	Lesser (M_{HU1} or M_{HL1})
H_1	0	0	0.02360	1.0	0.00065	0.0236432	0	0.0003	0.605 (L)
H_2	-0.388	-0.388	0.047025	1.0	0.000925	0.0471120	-0.388	-0.3882	0.878 (L)
H_3	0.388	-0.388	0.047025	1.0	0.000925	0.0471120	0.388	-0.3882	0.878 (L)
H_4	0.388	0.388	0.047025	1.0	0.000925	0.0471120	0.388	0.3883	0.770 (L)
H_5	-0.388	0.388	0.047025	1.0	0.000925	0.0471120	-0.388	0.3878	0.878 (L)
H_6	0.250	-0.417	0.03145	1.0	0.00120	0.0315245	0.25	-0.4175	0.645 (L)
H_7	-0.412	0.087	0.03140	1.0	0.00265	0.0315245	-0.412	0.0885	0.481 (L)
H_8	-0.212	-0.212	0.02015	1.0	0.00065	0.0202427	-0.212	-0.2122	0.835 (L)
H_9	-0.3346	-0.1507	0.01800	1.0	0.0055	0.0182780	-0.3346	-0.1517	0.869 (L)
H_{10}	0.423	-0.048	0.03140	1.0	0.00285	0.03140	0.423	-0.047	0.623 (L)

* The letter U refers to the lesser margin at the upper tolerance, the letter L refers to the lesser margin at the lower tolerance.

** The calculations for T_{HU1} and T_{HL1} are normally done as follows: Let r_{11} and r_{21} be, respectively, the inner and outer radii allowed for a hole, and let d_1 be the deviation allowed for the hole center (either x- or y-direction). Then

$$T_{HL1} = d_1 + r_{21} - R_{H1}, \quad T_{HU1} = R_{H1} - (r_{11} - d_1).$$

These tolerances thus include the "leeway" associated with the hole size allowance, i.e., maximum or minimum material conditions or MMC. For this clock plate part, the upper tolerance is of no concern and so was arbitrarily set at a conservative value of 1.

*** These numbers were averages of radii measurements taken over a number of actual manufactured parts.

TABLE IV
Margins for the Data of Table III Compared Before
and After Application of MOCO

Feature	Average Lesser Margins		
	Before MOCO	After MOCO	
		1st Stage	Last (9th) Stage
Points	0.600	0.418	0.235
Lines	0.677	0.657	0.641
Arcs	0.645	0.649	0.645
Holes	0.746	0.538	0.402
Combined	0.667	0.566	0.481

Least margin before MOCO = -0.0768

Least margins after MOCO

1st Stage = 0.0970

$$d_h = 10, d_k = 10, d_\phi = 33^*$$

$$m_h = 3, m_k = 7, m_\phi = 23$$

Last (9th) Stage** = 0.1436

$$d'_h = 8, d'_k = 8, d'_\phi = 24$$

Fulcrum point was center of $H_2 = (-0.388, -0.388)$

End point was center of $H_4 = (0.388, 0.388)$

* The divisors used in the search stages were rather arbitrarily chosen. However, methods for making choices that would use less calculations and yet not sacrifice precision or accuracy are discussed in Appendix D.

** For all stages from 2 through 9.

Analyses of Actual Measurement Data for a Number of Manufactured Precision Parts

It was decided to use MOCO to inspect a number of manufactured parts. Consequently, six parts of the type shown in Figure 8 (i.e., clock plates) were obtained from E. W. Kenderdine of Division 2328. These parts were stamped from a die tool which was made in accordance with the nominal data and tolerances indicated in Table III. In addition as previously mentioned, the points P_1 through P_4 were actually holes which were relabeled H_{11} through H_{14} . This is shown in the sketch of Figure 9. The nominal centers of these holes were identical with the x- and y-values given for the corresponding points in Table III, while their nominal radii were all 0.01385 with a lower tolerance T_{HLi} of 0.00065. The set of measurements taken for each part included:

1. Positions of all hole centers (14 in all)
2. The diameters (thus the radii $\hat{R}_{Hi}$) of all holes
3. Positions of points along feature contours located approximately as shown by the solid circles in Figure 9 (16 along lines and 20 along arcs).

Figure 9. Sketch for Clock Plate Part Showing Points of Inspection Measurements on Contours

The first set of measurements for the six clock plates were taken by Mechanical Inspection Section 1485-3 (H. V. Riley, Supervisor) with a Zeiss toolmaker's microscope to obtain positions and with gage pins to obtain diameters. Because the parts had been heat treated (and therefore shrunk) after manufacture, the measurements were treated in two ways: (1) just as they were taken and (2) with a shrinkage factor applied to "stretch" the data back to their values that would be obtained without the heat treatment. The factor to use had previously been established by experiment to be 1.00115. The results for the first set of measurements both without and with the use of the factor and after application of MOCO are given in the first part of Table V. It is seen that the results vary widely from part to part, both as to the number of features out of tolerance and the least margin of tolerance. The fact that the parts might be found to be out of tolerance would not in itself be surprising; however, because the parts were stamped with the same die it is to be expected that they would all be out of tolerance in much the same way. Since this was not the case, the accuracy and repeatability of the measurements become suspect. Therefore, a second set of measurements was taken on a more accurate Moore measuring machine by the Inspection Section.* The results after application of MOCO are given in the second part of Table V. Here it is seen that vastly improved margins were obtained. For the first five parts, in fact, all of the features were in tolerance for the case where the shrinkage factor was used. As for Part 6, the results were similar to those obtained with the first set of measurements. It is suspected that this part simply was generally poorly measured with the Moore machine because no gross errors on one or a few features could be detected.

When it was learned, after the fact, that both the probe and optics options were used in the second measurement set, it was decided that further checks should be made because it was suspected that those features measured with the optics were not done as accurately as those for which the probe was used. Furthermore, it was decided to have the measurements done by an independent group. Consequently, measurements were done with a Moore machine (probe only) by Mechanical Calibration and Electronic Inspection Section 1485-4 (H. M. Meidal, Supervisor) on Parts 5 and 6 only, with Part 5 receiving two separate inspections and Part 6 one. The results after application of MOCO are given in the third part of Table V. It is seen that both parts were found to be completely in tolerance, by substantial margins regardless of whether the shrinkage factor was used. As for the two measurements of Part 5, their results are as nearly identical as could be expected given the measuring machine's accuracy; that is, considering the tolerance of 0.002 for the feature with the least margin, the actual difference is between 0.0002 and 0.00004.

* All except for the hole diameters, which were taken once and then checked later by another source and found to be the same to at least 0.0001.

TABLE V

Comparisons of Results from Various Inspections
of Six Clock Plate Parts by MOCO

Measurement Source		Part No.	Margins Without Shrinkage Factor				Margins With Shrinkage Factor				Comments
Place	Machine		# Neg.	Least Margin	Feature	Tolerance	# Neg.	Least Margin	Feature	Tolerance	
Insp. Section 1485-3 ↓	Zeiss TMM ↓	1	14	-1.780 (L)	H10	0.00265	14	-1.750 (L)	H14	0.00065	1 bad measurement MOCO not run
		2	9	-0.978 (L)	H2	0.000925	10	-0.803 (L)	H1	0.00065	
		3	13	-1.400 (L)	H3	0.000925	13	-1.200 (L)	L9	0.0025	
		4	5	-0.261 (L)	A13	0.0020	3	-0.025 (L)	A11	0.0010	
		5	-	---	---	---	-	---	---	---	
		6	4	-0.109 (L)	A13	0.0020	4	-0.028 (L)	A11	0.0010	
Insp. Section 1485-3 ↓	Moore, both probe & optics ↓	1	2	-0.051 (L)	H11	0.00065	0	0.201 (U)	L9	0.0025	
		2	3	-0.136 (L)	H4	0.000925	0	0.194 (U)	L9	0.0025	
		3	6	-0.105 (L)	H3	0.000925	0	0.213 (L)	H2	0.000925	
		4	4	-0.095 (L)	H3	0.000925	0	0.209 (L)	A13	0.0020	
		5	3	-0.007 (L)	H13	0.00065	0	0.216 (U)	L9	0.0025	
		6	8	-0.627 (L)	A15	0.0025	8	-0.467 (L)	H8	0.00065	
Std. Section 1485-4 ↓	Moore, probe only ↓	5 (1st)	0	0.105 (L)	A13	0.0020	0	0.263 (L)	A13	0.0020	
		5 (2nd)	0	0.114 (L)	A13	0.0020	0	0.243 (L)	A13	0.0020	
		6	0	0.148 (L)	H2	0.000925	0	0.251 (L)	L9	0.0025	

* The letter L refers to lower margin and U to upper.

From this brief analyses of actual measurement data, two points become rather clear. The first is that even measurements which are supposedly taken in the same way by the same machine can differ greatly, so much care must be taken to check on their correctness. The second is that a sufficient accuracy of measurement, which must be predicated on the sizes of the tolerances of the part measured, is required before a meaningful inspection of the part can be made with MOCO, or by any other method for that matter. In the case at hand, after all the measurements are considered it seems correct to conclude that the measurement taken by the Standards Section are more nearly representative of the six parts and, therefore, that these parts are all within tolerance, with or without the use of the shrinkage factor.

Conclusions - Program Improvements and Extensions

The mathematical analog to an optical comparator and its program MOCO with its particular search scheme appears to work very well, provided sufficiently accurate measurement data are available. Other search schemes could have been employed in the program, but they might have introduced certain difficulties. For example, the well-established computer subroutine SIMIN might have been used, but in many instances it seems to require that at least a fair approximation to the values of the variables of search be known in advance. These are labeled in the subroutine as "starter values". In cases involving measurements where the measurement coordinate system is offset from that of the part by some significant yet unknown amount, problems could arise in establishing starter values accurately enough so that SIMIN would converge to a correct solution. No such problems exist for the MOCO search scheme, i. e., no starter values are needed regardless of the amount of measurement system offset.

As already discussed earlier, it may be desirable to find ways of reducing the number of calculations involved in using the MOCO search scheme. The purpose of this would be to minimize the amount of computer time involved. Several methods of obtaining such an improvement are discussed fully in Appendix D. Other improvements that might be made would involve making the program flexible enough to handle situations that may occur rather infrequently; for example, more general types of curves than circular arcs (such as spirals) may need to be handled occasionally. All that would be needed to make such an accommodation would be the geometrical description of the curve and a way of determining deviations of points in its vicinity. Analyses for the incorporation of such curves are given in Appendix B. As another example, the program might be set up to allow for a "tilt" of rectangular tolerance zones. As it now stands, the sides of the rectangle are required to be parallel to either the x- or y-axis of the drawing. In order to allow for this, Equations (1) and (2) for margins would be replaced by

$$M_{PXi} = T_{PXi} - |(\hat{X}_{Pi} - X_{Pi}) \cos \psi + (\hat{Y}_{Pi} - Y_{Pi}) \sin \psi| \quad (35)$$

$$M_{PYi} = T_{PYi} - |(\hat{Y}_{Pi} - Y_{Pi}) \cos \psi - (\hat{X}_{Pi} - X_{Pi}) \sin \psi| \quad (36)$$

where ψ is the tilt angle with respect to the drawing x-axis.

As for extensions to the program MOCO, some that immediately come to mind have to do with statistics. At present, MOCO will accept a number of sets of measurement data from a number of parts. If these parts are of the same type, it would be advantageous to have the program compute such things as means and standard deviations of quantities such as the main one of interest so far; i. e., the maximized least margin over the number of parts measured. Results from this could provide valuable information about the accuracy of the processes involved in making the parts. It should also be helpful for MOCO to search not only for the maximum least margin but also for the maximum average margin. Results from the latter would tend to point out much more clearly any gross errors that may occur in the measurements, from whatever sources. Another extension of the program that could prove useful in some instances is one to make it handle three-dimensional cases. This can be done by using only those principles used so far for two dimensions, but many complications would be introduced. The analyses involved are discussed in Appendix A.

APPENDIX A

Development of Equations to be Used for the Counterpart of MOCO in Cases of Three-Dimensional Measurements

A three-dimensional version of MOCO can be developed by direct analogy with the procedures used in the two-dimensional case. Fulcrum and end points (X_F, Y_F, Z_F) and (X_E, Y_E, Z_E) , respectively, are chosen on the three-dimensional "drawing", and these will have corresponding measurements $(\hat{X}_F, \hat{Y}_F, \hat{Z}_F)$ and $(\hat{X}_E, \hat{Y}_E, \hat{Z}_E)$. Likewise, there will in general be other drawing points (X_1, Y_1, Z_1) with their measurements $(\hat{X}_1, \hat{Y}_1, \hat{Z}_1)$. The search zone around the fulcrum is either a sphere or, more generally, a rectangular parallelepiped centered on that point and having half widths of a_F, b_F , and c_F in the x-, y-, and z-directions, respectively. Similarly, the search region in the vicinity of the end point is defined by an angle ρ which in turn defines a solid angle that is large enough to encompass the tolerance zone around the end point. The latter is defined at its largest by a rectangular parallelepiped having half widths of a_E, b_E , and c_E in the x-, y-, and z-directions, respectively.

Calculations of preliminary quantities required are made without comment (their need will become apparent) and with reference to Figure A-1 according to the following equations:

$$\left. \begin{aligned}
 R &= \sqrt{(X_E - X_F)^2 + (Y_E - Y_F)^2 + (Z_E - Z_F)^2}, \quad \hat{R} = \sqrt{(\hat{X}_E - \hat{X}_F)^2 + (\hat{Y}_E - \hat{Y}_F)^2 + (\hat{Z}_E - \hat{Z}_F)^2} \\
 R' &= \sqrt{(X_E - X_F)^2 + (Y_E - Y_F)^2}, \quad \hat{R}' = \sqrt{(\hat{X}_E - \hat{X}_F)^2 + (\hat{Y}_E - \hat{Y}_F)^2} \\
 \alpha_o &= \tan^{-1} \left(\frac{(Y_E - Y_F)}{(X_E - X_F)} \right), \quad \hat{\alpha}_o = \tan^{-1} \left(\frac{(\hat{Y}_E - \hat{Y}_F)}{(\hat{X}_E - \hat{X}_F)} \right) \\
 \beta_o &= \tan^{-1} \left(\frac{(Z_E - Z_F)}{R'} \right), \quad \hat{\beta}_o = \tan^{-1} \left(\frac{(\hat{Z}_E - \hat{Z}_F)}{\hat{R}'} \right) \\
 \rho &= \tan^{-1} \left[\frac{\sqrt{a_F^2 + b_F^2 + c_F^2} + \sqrt{a_E^2 + b_E^2 + c_E^2}}{R - \left(\sqrt{a_F^2 + b_F^2 + c_F^2} + \sqrt{a_E^2 + b_E^2 + c_E^2} \right)} \right]^*
 \end{aligned} \right\} \quad (A-1)$$

* The angle ρ is, as in the two-dimensional case, calculated conservatively large.

Figure A-1

Again, a divisor d_h for the x-direction of the search zone around the fulcrum is specified, and the other divisors with corresponding search increments are obtained (to the nearest integer) as follows:

$$\left. \begin{aligned} d_h &= d_h, \quad d_k = \left(\frac{b_F}{a_F}\right) d_h, \quad d_l = \left(\frac{c_F}{a_F}\right) d_h, \quad d_\alpha = \left(\frac{\hat{R}\rho}{a_F}\right) d_h = d_\beta \\ \Delta_h &= \frac{2a_F}{d_h}, \quad \Delta_k = \frac{2b_F}{d_k}, \quad \Delta_l = \frac{2c_F}{d_l}, \quad \Delta_\alpha = \frac{2\hat{R}\rho}{d_\alpha} = \Delta_\beta \end{aligned} \right\} \quad (A-2)$$

Here k, again, refers to the y-direction and l to the z-direction.

As in the two-dimensional case, the idea is first to translate all measured points until $(\hat{X}_F, \hat{Y}_F, \hat{Z}_F)$ is superimposed on a point which corresponds to an intersection of a three-dimensional grid into which the tolerance zone around the fulcrum has been divided. Then all measured points are rotated around $(\hat{X}_F, \hat{Y}_F, \hat{Z}_F)$ in a way that will adequately cover the tolerance zone around the end point. This rotation must be accomplished so as to stop at each point defined by some solid angle grid. For each combination of a point in the first (translational) grid with one in the second (rotational) grid, all measured points are checked to see whether they are in tolerance. Again, the precision of the results of the search will depend on the "fineness" of the grid divisions.

The equations of transformation to be used during the search are best worked out by first rotating all measured points around $(\hat{X}_F, \hat{Y}_F, \hat{Z}_F)$ until the line between this point and $(\hat{X}_E, \hat{Y}_E, \hat{Z}_E)$ is parallel to, say, the x-axis of the "drawing" and then working backwards. This can be done in three steps which also include setting up for the initial and subsequent positions on the angular grid.

1. Rotation of the line (represented by $\hat{R}$ in Figure A-1b) into a plane parallel to the xz-plane and then past it by an angle $(\rho - m_\alpha \Delta\alpha)$, $m_\alpha = 0, 1, \dots, (d_\alpha + 1)$. When this is done, a measured point $(\hat{X}, \hat{Y}, \hat{Z})$ transforms to the point $(\hat{X}_1, \hat{Y}_1, \hat{Z}_1)$ according to the equations:

$$\left. \begin{aligned} \hat{X}_1 &= (\hat{X} - \hat{X}_F) \cos(\hat{\alpha} - m_\alpha \Delta\alpha) + (\hat{Y} - \hat{Y}_F) \sin(\hat{\alpha} - m_\alpha \Delta\alpha) \\ \hat{Y}_1 &= (\hat{Y} - \hat{Y}_F) \cos(\hat{\alpha} - m_\alpha \Delta\alpha) - (\hat{X} - \hat{X}_F) \sin(\hat{\alpha} - m_\alpha \Delta\alpha) \\ \hat{Z}_1 &= \hat{Z} \end{aligned} \right\} \quad (A-3)$$

where $\hat{\alpha} = \hat{\alpha}_0 + \rho$.

2. Rotation of $\hat{R}$ into a plane parallel to the xy-plane and then past it by an angle $(\rho - m_\beta \Delta\beta)$, $m_\beta = 0, 1, \dots, (d_\beta + 1)$. The measured point $(\hat{X}_1, \hat{Y}_1, \hat{Z}_1)$ then transforms to the point $(\hat{X}_2, \hat{Y}_2, \hat{Z}_2)$ according to the equations:

$$\left. \begin{aligned} \hat{X}_2 &= \hat{X}_1 \cos (\hat{\beta} - m_\beta \Delta\beta) + \hat{Z}_1 \sin (\hat{\beta} - m_\beta \Delta\beta) \\ \hat{Y}_2 &= \hat{Y}_1 \\ \hat{Z}_2 &= \hat{Z}_1 \cos (\hat{\beta} - m_\beta \Delta\beta) - \hat{X}_1 \sin (\hat{\beta} - m_\beta \Delta\beta) \end{aligned} \right\} \quad (A-4)$$

where

$$\hat{\beta} = \hat{\beta}_0 + \rho .$$

3. Rotation of the line back through the angle β_0 around the y-axis and then through α_0 around the z-axis. The point $(\hat{X}_2, \hat{Y}_2, \hat{Z}_2)$ then becomes .

$$\left. \begin{aligned} \hat{X}_3 &= (\hat{X}_2 \cos \beta_0 - \hat{Z}_2 \sin \beta_0) \cos \alpha_0 - \hat{Y}_2 \sin \alpha_0 \\ \hat{Y}_3 &= \hat{Y}_2 \cos \alpha_0 + (\hat{X}_2 \cos \beta_0 - \hat{Z}_2 \sin \beta_0) \sin \alpha_0 \\ \hat{Z}_3 &= \hat{Z}_2 \cos \beta_0 + \hat{X}_2 \sin \beta_0 \end{aligned} \right\} \quad (A-5)$$

These equations pertain only to the angular part of the search and in effect set up a grid on a region of the surface of a sphere having radius $\hat{R}$ where the region is subtended by a solid angle as indicated in Figure A-1d. (This grid is shown in the three-dimensional drawing as being made up of straight lines for the sake of simplifying the drawing. Actually, of course, the lines would be curved since the whole grid represents a region of a spherical surface.)

In order to provide for the translational part of the search, Equations (A-5) are further modified so that $(\hat{X}_3, \hat{Y}_3, \hat{Z}_3)$ becomes

$$\left. \begin{aligned} \hat{X}_4 &= \hat{X}_3 + h - m_h \Delta h, \quad m_h = 0, 1, \dots, (d_h + 1) \\ \hat{Y}_4 &= \hat{Y}_3 + k - m_k \Delta k, \quad m_k = 0, 1, \dots, (d_k + 1) \\ \hat{Z}_4 &= \hat{Z}_3 + \ell - m_\ell \Delta \ell, \quad m_\ell = 0, 1, \dots, (d_\ell + 1) \end{aligned} \right\} \quad (A-6)$$

where

$$h = X_F + a_F, \quad k = Y_F + b_F, \quad \ell = Z_F + c_F . \quad (A-7)$$

The complete equations of transformation to be used during the search can now be obtained by successive substitutions of the set of the Equations (A-1) into (A-4), (A-4) into (A-5), and finally (A-5) into (A-6). If a measured point $(\hat{X}, \hat{Y}, \hat{Z})$ transforms into $(\hat{X}', \hat{Y}', \hat{Z}')$, then the equations are:

$$\begin{aligned} \hat{X}' = & \left\langle \left\{ \left[(\hat{X} - \hat{X}_F) \cos (\hat{\alpha} - m_{\alpha} \Delta \alpha) + (\hat{Y} - \hat{Y}_F) \sin (\hat{\alpha} - m_{\alpha} \Delta \alpha) \right] \cos (\hat{\beta} - m_{\beta} \Delta \beta) \right. \right. \\ & + (\hat{Z} - \hat{Z}_F) \sin (\hat{\beta} - m_{\beta} \Delta \beta) \left. \right\} \cos \beta_o - \left\{ (\hat{Z} - \hat{Z}_F) \cos (\hat{\beta} - m_{\beta} \Delta \beta) - [(\hat{X} - \hat{X}_F) \right. \\ & \cos (\hat{\alpha} - m_{\alpha} \Delta \alpha) + (\hat{Y} - \hat{Y}_F) \sin (\hat{\alpha} - m_{\alpha} \Delta \alpha)] \sin (\hat{\beta} - m_{\beta} \Delta \beta) \left. \right\} \sin \beta_o \left. \right\rangle \cos \alpha_o \\ & - \left[(\hat{Y} - \hat{Y}_F) \cos (\hat{\alpha} - m_{\alpha} \Delta \alpha) - (\hat{X} - \hat{X}_F) \sin (\hat{\alpha} - m_{\alpha} \Delta \alpha) \right] \sin \alpha_o + h - m_h \Delta h \end{aligned} \quad (A-8)$$

$$\begin{aligned} \hat{Y}' = & \left[(\hat{Y} - \hat{Y}_F) \cos (\hat{\alpha} - m_{\alpha} \Delta \alpha) - (\hat{X} - \hat{X}_F) \sin (\hat{\alpha} - m_{\alpha} \Delta \alpha) \right] \cos \alpha_o + \left\langle \left\{ \left[(\hat{X} - \hat{X}_F) \cos (\hat{\alpha} - m_{\alpha} \Delta \alpha) \right. \right. \right. \\ & + (\hat{Y} - \hat{Y}_F) \sin (\hat{\alpha} - m_{\alpha} \Delta \alpha) \left. \right\} \cos (\hat{\beta} - m_{\beta} \Delta \beta) + (\hat{Z} - \hat{Z}_F) \sin (\hat{\beta} - m_{\beta} \Delta \beta) \left. \right\} \cos \beta_o \\ & - \left\{ (\hat{Z} - \hat{Z}_F) \cos (\hat{\beta} - m_{\beta} \Delta \beta) - [(\hat{X} - \hat{X}_F) \cos (\hat{\alpha} - m_{\alpha} \Delta \alpha) + (\hat{Y} - \hat{Y}_F) \sin (\hat{\alpha} - m_{\alpha} \Delta \alpha)] \right. \\ & \left. \sin (\hat{\beta} - m_{\beta} \Delta \beta) \right\} \sin \beta_o \left. \right\rangle \sin \alpha_o + k - m_k \Delta k \end{aligned} \quad (A-9)$$

$$\begin{aligned} \hat{Z}' = & \left\{ (\hat{Z} - \hat{Z}_F) \cos (\hat{\beta} - m_{\beta} \Delta \beta) - [(\hat{X} - \hat{X}_F) \cos (\hat{\alpha} - m_{\alpha} \Delta \alpha) + (\hat{Y} - \hat{Y}_F) \sin (\hat{\alpha} - m_{\alpha} \Delta \alpha)] \right. \\ & \left. \sin (\hat{\beta} - m_{\beta} \Delta \beta) \right\} \cos \beta_o + \left\{ [(\hat{X} - \hat{X}_F) \cos (\hat{\alpha} - m_{\alpha} \Delta \alpha) + (\hat{Y} - \hat{Y}_F) \sin (\hat{\alpha} - m_{\alpha} \Delta \alpha)] \right. \\ & \left. \cos (\hat{\beta} - m_{\beta} \Delta \beta) + (\hat{Z} - \hat{Z}_F) \sin (\hat{\beta} - m_{\beta} \Delta \beta) \right\} \sin \beta_o + \ell - m_{\ell} \Delta \ell \end{aligned} \quad (A-10)$$

For an initial subdivision of the translational and rotational grids, the actual search procedure is completely analogous with the case for two dimensions; that is, for every possible combination of the integers m_h , m_k , m_{ℓ} , m_{α} , and m_{β} , the transformations of all measured points are made according to Equations (A-8) through (A-10); for each of these combinations, a set of normalized margins for the transformed measured points are computed. The combination of integers that produces the set with the maximum least margin is the one that is sought. This combination may then be used in a second stage of search in which the search regions can be reduced. The process of going to smaller and smaller regions of search can be continued to obtain any degree of "finess" desired.

The margins computed will depend on the types of features (again, meant to include contours) that are considered in the measurements of a three-dimensional object. Some of the possibilities for features are worked out below with respect to calculations of margins. This is done without a step by step explanation of the results. The reader may verify these results for himself.

1. Points P_i , $i = 1, 2, \dots, n_p$

Parameters: Position (X_{Pi}, Y_{Pi}, Z_{Pi})

Measurement of position $(\hat{X}_{Pi}, \hat{Y}_{Pi}, \hat{Z}_{Pi})$

Margins

Spherical tolerance around fulcrum

$$M_{Pi} = T_{Pui} - \sqrt{(\hat{X}_{Pi}' - \hat{X}_{Pi})^2 + (\hat{Y}_{Pi}' - \hat{Y}_{Pi})^2 + (\hat{Z}_{Pi}' - \hat{Z}_{Pi})^2} \quad (A-11)$$

Rectangular tolerance zone around fulcrum

$$\left. \begin{aligned} M_{P\hat{X}i} &= T_{PXi} - |\hat{X}_{Pi} - X_{Pi}|, \quad M_{P\hat{Y}i} = T_{PYi} - |\hat{Y}_{Pi} - Y_{Pi}|, \\ M_{P\hat{Z}i} &= T_{PZi} - |\hat{Z}_{Pi} - Z_{Pi}| \end{aligned} \right\} \quad (A-12)$$

2. Lines L_i , $i = 1, 2, \dots, n_L$

Parameters: Two points on the line, $(X_{Li1}, Y_{Li1}, Z_{Li1})$ and $(X_{Li2}, Y_{Li2}, Z_{Li2})$

Measurements of point along the line $(\hat{X}_{Lij}, \hat{Y}_{Lij}, \hat{Z}_{Lij})$ (Figure A-2)

Figure A-2

From Figure A-2

$$\left. \begin{aligned} r_2^2 &= r_1^2 + r^2 - 2r_1 r \cos \theta \\ \theta &= \cos^{-1} \left(\frac{r^2 + r_1^2 - r_2^2}{2r_1 r} \right) \end{aligned} \right\} \quad (\text{A-13})$$

$$\left. \begin{aligned} r &= \sqrt{(X_{Li2} - X_{Li1})^2 + (Y_{Li2} - Y_{Li1})^2 + (Z_{Li2} - Z_{Li1})^2} \\ r_1 &= \sqrt{(\hat{X}_{Lij} - X_{Li1})^2 + (\hat{Y}_{Lij} - Y_{Li1})^2 + (\hat{Z}_{Lij} - Z_{Li1})^2} \\ r_2 &= \sqrt{(\hat{X}_{Lij} - X_{Li2})^2 + (\hat{Y}_{Lij} - Y_{Li2})^2 + (\hat{Z}_{Lij} - Z_{Li2})^2} \end{aligned} \right\} \quad (\text{A-14})$$

$$\Delta = r_1 \sin \theta \quad (\text{A-15})$$

Margins

$$M_{LUij} = T_{LUi} - \Delta, \quad j = 1, 2, \dots, n_{Li} \quad (\text{A-16})$$

3. Planes PL_i , $i = 1, 2, \dots, n_{PL}$

Parameters: Distance from origin to plane D_{PLi} ; angles that this distance (line) makes with the x-, y-, and z-axis, respectively, ϕ_{PLi} , θ_{PLi} , ψ_{PLi}

Measurements of points near the plane $(\hat{X}_{PLij}, \hat{Y}_{PLij}, \hat{Z}_{PLij})$

Projections along the axes are

$$\left. \begin{aligned} X_{PLDi} &= D_{PLi} \cos \phi_{PLi} \\ Y_{PLDi} &= D_{PLi} \cos \theta_{PLi} \\ Z_{PLDi} &= D_{PLi} \cos \psi_{PLi} \end{aligned} \right\} \quad (\text{A-17})$$

The angle that the projection of D_{PLi} in the xy-plane makes with the x-axis is

$$\alpha_{PLDi} = \tan^{-1} \left(\frac{Y_{PLDi}}{X_{PLDi}} \right) \quad (A-18)$$

The angle that D_{PLi} makes with the xy-plane is

$$\beta_{PLDi} = \sin^{-1} \left(\frac{Z_{PLDi}}{D_{PLi}} \right) \text{ or } = 90^\circ - \psi_{PLi} \quad (A-19)$$

Then rotation of D_{PLi} into the xy-plane results in

$$\left. \begin{aligned} \hat{X}_{PLij} &= \hat{X}_{PLij} \cos \alpha_{PLDi} + \hat{Y}_{PLij} \sin \alpha_{PLDi} \\ \hat{Y}_{PLij} &= \hat{Y}_{PLij} \cos \alpha_{PLDi} - \hat{X}_{PLij} \sin \alpha_{PLDi} \\ \hat{Z}_{PLij} &= \hat{Z}_{PLij} \end{aligned} \right\}^* \quad (A-20)$$

and subsequent rotation of D_{PLi} into alignment with the x-axis gives

$$\left. \begin{aligned} \hat{X}_{PLij} &= \hat{X}_{PLij} \cos \beta_{PLDi} + \hat{Z}_{PLij} \sin \beta_{PLDi} \\ \hat{Y}_{PLij} &= \hat{Y}_{PLij} \\ \hat{Z}_{PLij} &= \hat{Z}_{PLij} \cos \beta_{PLDi} - \hat{X}_{PLij} \sin \beta_{PLDi} \end{aligned} \right\} \quad (A-21)$$

By combining Equations (A-20) with (A-21), the point $(\hat{X}, \hat{Y}, \hat{Z})$ transforms into one with an x-component of

$$\hat{X}'_{PLij} = (\hat{X}_{PLij} \cos \alpha_{PLDi} + \hat{Y}_{PLij} \sin \alpha_{PLDi}) \cos \beta_{PLDi} + \hat{Z}_{PLij} \sin \beta_{PLDi} \quad (A-22)$$

* Here rather than introducing more subscripts, the license of repeating variables used by programmers is adopted (e.g., $A = A + c$).

Margins

$$\left. \begin{aligned} M_{PLUi} &= D_{PLi} + T_{PLUi} - \hat{X}'_{PLij} \\ M_{PLLi} &= \hat{X}'_{PLij} - (D_{PLi} - T_{PLLi}) , j = 1, 2, \dots, n_{PLi} \end{aligned} \right\} \quad (A-23)$$

4. Spherical Segments S_i , $i = 1, 2, \dots, n_S$

Parameters: Center (X_{Si}, Y_{Si}, Z_{Si}) and radius R_{Si}

Measurements along the segment $(\hat{X}_{Sij}, \hat{Y}_{Sij}, \hat{Z}_{Sij})$

Margins

$$\left. \begin{aligned} M_{SUi} &= R_{Si} + T_{SUi} - \sqrt{(\hat{X}_{Sij} - X_{Si})^2 + (\hat{Y}_{Sij} - Y_{Si})^2 + (\hat{Z}_{Sij} - Z_{Si})^2} \\ M_{SLi} &= \sqrt{(\hat{X}_{Sij} - X_{Si})^2 + (\hat{Y}_{Sij} - Y_{Si})^2 + (\hat{Z}_{Sij} - Z_{Si})^2} - (R_{Si} - T_{SLi}) \end{aligned} \right\} \quad (A-24)$$

APPENDIX B

Incorporation of Other Contours (Features) That May Occur Rather Infrequently Into MOCO

Elliptical Segments E_i , $i = 1, 2, \dots, n_E$

An ellipse having a general position and orientation with respect to the xy -axis of the drawing system is shown in Figure B-1a. Also shown are the parameters needed to identify it properly and a point $(\hat{X}_{Eij}, \hat{Y}_{Eij})$ corresponding to a measurement along the surface of the ellipse in a manufactured part. In Figure B-1b the same ellipse and measured point are shown after they have been rotated by a clockwise angle α_{Ei} around the point (X_{Ei}, Y_{Ei}) and then translated so that the "center" of the ellipse coincides with the origin of the drawing system. The transformation equations to accomplish this for the measured point are

$$\hat{X}_{Eij} = (\hat{X}_{Eij} - X_{Ei}) \cos \alpha_{Ei} + (\hat{Y}_{Eij} - Y_{Ei}) \sin \alpha_{Ei} \quad (B-1)$$

$$\hat{Y}_{Eij} = (\hat{Y}_{Eij} - Y_{Ei}) \cos \alpha_{Ei} - (\hat{X}_{Eij} - X_{Ei}) \sin \alpha_{Ei} \quad (B-2)$$

Figure B-1

In order to begin working toward finding the margin of the measured point from the ellipse, it is necessary to consider the line from this point perpendicular to the ellipse at the nearest point. This line will intersect the ellipse at (X_{Eij1}, Y_{Eij1}) and at (X_{Eij2}, Y_{Eij2}) . For temporary brevity $(\hat{X}_{Eij}, \hat{Y}_{Eij})$ will be denoted by $(\hat{X}, \hat{Y})$, (X_{Eij1}, Y_{Eij1}) by (X_1, Y_1) , and (X_{Eij2}, Y_{Eij2}) by (X_2, Y_2) . Since the slope of the ellipse at any point (x, y) can be shown to be given by $-b^2x/a^2y$ (where b and a are the distances from the center of the ellipse to the ellipse along the x - and y -axes, respectively), then the slope of the perpendicular line through the measured point is given by a_i^2y/b_i^2x . In order to find the points of intersection of the line perpendicular to the ellipse with the ellipse, it is necessary to solve the following equations simultaneously:

$$\frac{\hat{Y} - y}{\hat{X} - x} = \frac{a_i^2 y}{b_i^2 x} \quad (\text{for the line}) \quad (B-3)$$

$$\frac{x^2}{a_i^2} + \frac{y^2}{b_i^2} = 1 \quad (\text{for the ellipse}) \quad (B-4)$$

By solving Equation (B-1) for y and substituting the result in Equation (B-2), it is found, after some manipulation, that

$$Ax^4 + Bx^3 + Cx^2 + Dx + E = 0 \quad (B-5)$$

where

$$\left. \begin{aligned} A &= (a_i^2 - b_i^2)^2 \\ B &= 2a_i^2 \hat{X} (b_i^2 - a_i^2) \\ C &= a_i^2 \left[a_i^2 \hat{X}^2 + b_i^2 \hat{Y}^2 - (a_i^2 - b_i^2)^2 \right] \\ D &= 2a_i^4 \hat{X} (a_i^2 - b_i^2) \\ E &= -a_i^6 \hat{X}^2 \end{aligned} \right\} \quad (B-6)$$

Solution of the fourth-degree equation (B-3) will give four values for x . Two of these are the X_1 and X_2 sought; the other two will be the intersections of the line from $(-\hat{X}, -\hat{Y})$ perpendicular to the ellipse with the ellipse. To find the proper values for X_1 and X_2 and their counterparts Y_1 and Y_2 , the following procedure should be observed.

For Certain Special Cases Regardless of the Roots of Equation (B-3)

If $\hat{X} > 0$ and $\hat{Y} = 0$, then $(X_1, Y_1) = (a_i, 0)$ and $(X_2, Y_2) = (-a_i, 0)$

If $\hat{X} < 0$ and $\hat{Y} = 0$, then $(X_1, Y_1) = (-a_i, 0)$ and $(X_2, Y_2) = (a_i, 0)$

If $\hat{Y} > 0$ and $\hat{X} = 0$, then $(X_1, Y_1) = (0, b_i)$ and $(X_2, Y_2) = (0, -b_i)$

If $\hat{Y} < 0$ and $\hat{X} = 0$, then $(X_1, Y_1) = (0, -b_i)$ and $(X_2, Y_2) = (0, b_i)$

All Other Cases

Solve for the roots of Equation (B-3). Suppose these are given by r_1, r_2, r_3, r_4 . If each of these roots is substituted into Equation (B-2) and the result solved for y , there is obtained

$$y = \pm b_i \sqrt{1 - \left(\frac{r_\ell}{a_i}\right)^2}, \quad \ell = 1, 2, 3, \text{ and } 4. \quad (\text{B-7})$$

These are the possible values of Y_1 and Y_2 while the r_ℓ are the possible values for X_1 and X_2 . Each possible value of X_1 is combined with each possible value of Y_1 to give a distance Δ according to the formula

$$\Delta = \sqrt{(\hat{X} - X_1)^2 + (\hat{Y} - Y_1)^2}, \quad (\text{B-8})$$

and the proper choice for (X_1, Y_1) is that which gives the least value for Δ . Similarly, each possible value of X_2 is combined with each possible value of Y_2 to give a slope m according to the formula

$$m = \frac{Y_1 - Y_2}{X_1 - X_2}, \quad (\text{B-9})$$

and the proper choice for (X_2, Y_2) is that which gives an

$$m = \frac{a_i^2 Y_1}{b_i^2 X_1}$$

where X_1 and Y_1 are those values chosen from the Δ test.

With all of the above, if T_{EUi} and T_{ELi} are the respective upper and lower tolerances around the ellipse, the margins for the point $(\hat{X}, \hat{Y})$, i. e., $(\hat{X}_{Eij}, \hat{Y}_{Eij})$, are given by

$$M_{EUij} = T_{EUi} + \sqrt{(X_1 - X_2)^2 + (Y_1 - Y_2)^2} - \sqrt{(\hat{X} - X_2)^2 + (\hat{Y} - Y_2)^2} \quad (B-10)$$

$$M_{ELij} = \sqrt{(\hat{X} - X_2)^2 + (\hat{Y} - Y_2)^2} - \left[\sqrt{(X_1 - X_2)^2 + (Y_1 - Y_2)^2} - T_{ELi} \right] \quad (B-11)$$

Spiral Segments S_i , $i = 1, 2, \dots, n_S$

In Figure B-2 below is drawn a portion of an Archimedes spiral (solid line) which is given by the equation, $R = k\theta$, where R is the distance of the line from the origin of some coordinate system to the spiral and θ is the angle that the line makes with the x axis of the system. From the figure, $(\hat{X}, \hat{Y})$ is a measured point whose distance Δ from the spiral is required. Let (x, y) be any point along the spiral; then its distance D is given by

$$D = \left[\hat{R}^2 + R^2 - 2\hat{R}R \cos(\theta - \alpha) \right]^{1/2} \quad (B-12)$$

where R and θ are the spiral parameters corresponding to (x, y) , $\hat{R}$ is the distance of the line from the origin to $(\hat{X}, \hat{Y})$, and α is the angle that this line makes with the x axis with α thus given by $\tan^{-1}\left(\frac{\hat{Y}}{\hat{X}}\right)$. Equation (B-12) can be rewritten by substituting $k\theta$ for R to give

$$D = \left[\hat{R}^2 + k^2\theta^2 - 2\hat{R}k\theta \cos(\theta - \alpha) \right]^{1/2} \quad (B-13)$$

Figure B-2

The value for Δ will correspond to a value of θ for which D is a minimum. A necessary condition for this is

$$\frac{dD}{d\theta} = \frac{1}{2} \left[\hat{R}^2 + k^2 \theta^2 - 2\hat{R}k\theta \cos(\theta - \alpha) \right]^{1/2} \left[2k^2\theta + 2k\hat{R} \sin(\theta - \alpha) - 2k\hat{R} \cos(\theta - \alpha) \right] = 0 \quad (B-14)$$

which reduces to

$$\left[k\theta + \hat{R}\theta \sin(\theta - \alpha) - \hat{R} \cos(\theta - \alpha) \right] = 0 \quad (B-15)$$

This equation can be solved for θ by a variety of fairly simple numerical methods for given values of k , $\hat{R}$, and α . The value of θ so found can be substituted into Equation (B-13) to give a value of $D = \Delta$. To determine whether $(\hat{X}, \hat{Y})$ is "inside" or "outside" the spiral (i.e., whether Δ is on the lower or upper tolerance side of the spiral), it is only necessary to compute the quantity $k\alpha$. If $k\alpha$ is larger than $\hat{R}$, then Δ is on the lower side, while if $k\alpha$ is smaller than $\hat{R}$, it is on the upper side.

Let T_{SUi} and T_{SLi} be the respective upper and lower tolerances around the spiral. Then the margins for the j th particular measured point $(\hat{X}_{Sij}, \hat{Y}_{Sij})$ along the i th spiral are given by

$$M_{SUi} = T_{SUi} - \Delta_{ij}, \Delta_{ij} \text{ on upper side} \quad (B-16)$$

$$M_{SLi} = T_{SLi} - \Delta_{ij}, \Delta_{ij} \text{ on lower side} \quad (B-17)$$

where the i and j subscripts on Δ now refer to the j th point along the i th spiral.

The axes of the coordinate system of Figure B-2 need not be mutually parallel to the axes of the drawing system. If they are not, then the system related to the spiral (including any measured point for which Δ is to be found) can first be rotated and translated to coincide with the drawing system. After this, the equations would follow as given above in a manner similar to that of the previous case for the ellipse.

APPENDIX C

An Alternative to the Programming of the Rotational Parameters to Avoid the Increase in Calculations Due to the Conservatism Used in Calculating ρ as Done in Equation (13) of the Text

The calculation of the rotational parameters used to transform the measured data relative to their nominals as indicated in the text can be improved upon if some complications in the programming are acceptable. Suppose, for example, for some combination of m_h and m_k the measured fulcrum point $(\hat{X}_F, \hat{Y}_F)$ should fall in the search rectangle at the point indicated in Figure C-1 below. The idea here is to calculate θ as in the text, but then to add to that a $\Delta\theta$ (which may be positive or negative) and a ρ (which is always positive). Clockwise rotation of $(\hat{X}_E, \hat{Y}_E)$ around its fulcrum position point $(\hat{X}_F, \hat{Y}_F)$ through the sum of these angles will place the line between these points just at the edge of the tolerance zone around $(\hat{X}_E, \hat{Y}_E)$ as shown in the figure. The expressions for $\Delta\theta$ and ρ will be seen to be given by

$$\Delta\theta(m_h, m_k) = \tan^{-1} \left[\frac{Y_E - (Y_F + b_F - m_k \Delta k)}{X_E - (X_F + a_F - m_h \Delta h)} \right] - \theta \quad (C-1)$$

and

$$\rho(m_h, m_k) = \cos^{-1} \left[\frac{A^2 + B^2 - C}{2AB} \right] \quad (C-2)$$

where

$$\left. \begin{aligned} A &= \sqrt{[X_E - (X_F + a_F - m_h \Delta h)]^2 + [Y_E - (Y_F + b_F - m_k \Delta k)]^2} \\ B &= \sqrt{[(X_E + a_E) - (X_F + a_F - m_h \Delta h)]^2 + [(Y_E - b_E) - (Y_F + b_F - m_k \Delta k)]^2} \end{aligned} \right\} \quad (C-3)$$

In changing the programming, it is only necessary to substitute $[\theta + \Delta\theta(m_h, m_k) + \rho(m_h, m_k)]$ for ϕ wherever it occurs in the equations of the text.

The fact that ρ becomes a function of m_h and m_k makes d_ϕ (and thus $\Delta\phi$) a function of these variables as well; that is, following Equation (19) of the text,

$$d_\phi = \frac{\rho(m_h, m_k) \sqrt{(\hat{X}_E - \hat{X}_F)^2 + (\hat{Y}_E - \hat{Y}_F)^2}}{a_F} d_h \quad (C-4)$$

This does further complicate the programming.

Figure C-1

APPENDIX D

Regarding the Minimization of the Amount of Calculations by Proper Choice of the Divisors for the Translational and Rotational Grids

With all other symbols defined as in the main text, let T_s be the smallest tolerance for any of the part features being considered. Furthermore, let $1/k$ be that part of T_s to which accuracy is desired in the search procedure. Then if the two-stage search explained in the main text is used,

$$\frac{T_s}{k} = \frac{4a_F}{d_h d_h'} \quad \text{or} \quad d_h' = \frac{4a_F k}{T_s d_h} \quad . \quad (D-1)$$

If N is the number of times that the calculations of transformed coordinates and their margins must be made, it should be clear from the description of the search scheme given that

$$\begin{aligned} N &= \left[(d_h + 1) (d_k + 1) (d_\phi + 1) + (d_h' + 1) (d_k' + 1) (d_\phi' + 1) \right] \\ &= (d_h d_k d_\phi + d_h d_k + d_h d_\phi + d_k d_\phi + d_h + d_k + d_\phi + 1) \\ &\quad + (d_h' d_k' d_\phi' + d_h' d_k' + d_h' d_\phi' + d_k' d_\phi' + d_h' + d_k' + d_\phi' + 1) \quad . \end{aligned} \quad (D-2)$$

The following relationships were defined in the text:

$$d_k = \frac{b_F}{a_F} d_h \quad , \quad d_\phi = \frac{\hat{R}_{FE} \rho}{a_F} d_h \quad , \quad \Delta h = \frac{2a_F}{d_h} \quad , \quad \Delta k = \frac{2b_F}{d_k} \quad , \quad \Delta \phi = \frac{2\rho}{d_\phi} \quad (D-3)$$

and

$$d_k' = \frac{\Delta k}{\Delta h} d_h' \quad , \quad d_\phi' = \frac{\hat{R}_{FE} \Delta \phi}{\Delta h} d_h' \quad (D-4)$$

where

$$\hat{R}_{FE} = \sqrt{(\hat{X}_E - \hat{X}_F)^2 + (\hat{Y}_E - \hat{Y}_F)^2} \quad .$$

i. e., the distance between the measured points corresponding to the fulcrum and end point. Substitution of Equations (D-1) and (D-3) into (D-4) show that

$$d_k' \approx d_\phi' = d_h' = \frac{4a_F^k}{T_s} d_h^{-1} . \quad (D-5)$$

Further substitution in Equation (D-2) for the expressions for d_k and d_ϕ from Equation (D-3) and for d_h' , d_k' , and d_ϕ' from Equations (D-1) and (D-4) yield

$$N = \left[\left(\frac{b_F R_{FE} \rho}{a_F^2} \right) d_h^3 + \left(\frac{b_F}{a_F} + \frac{b_F R_{FE} \rho}{a_F^2} + \frac{R_{FE} \rho}{a_F} \right) d_h^2 + \left(1 + \frac{b_F}{a_F} + \frac{R_{FE} \rho}{a_F} \right) d_h + 1 \right] \\ + \left[\left(\frac{4a_F^k}{T_s} \right)^3 d_h^{-3} + 3 \left(\frac{4a_F^k}{T_s} \right)^2 d_h^{-2} + 3 \left(\frac{4a_F^k}{T_s} \right) d_h^{-1} + 1 \right] . \quad (D-6)$$

A necessary condition for a minimum for N is that its derivative with respect to d_h be equal to zero; that is,

$$\frac{dN}{d(d_h)} = 3 \left(\frac{b_F R_{FE} \rho}{a_F^2} \right) d_h^2 + 2 \left(\frac{b_F}{a_F} + \frac{b_F R_{FE} \rho}{a_F^2} + \frac{R_{FE} \rho}{a_F} \right) d_h \\ + \left(1 + \frac{b_F}{a_F} + \frac{R_{FE} \rho}{a_F} \right) - 3 \left(\frac{4a_F^k}{T_s} \right)^3 d_h^{-4} - 6 \left(\frac{4a_F^k}{T_s} \right)^2 d_h^{-3} \\ - 3 \left(\frac{4a_F^k}{T_s} \right) d_h^{-2} = 0 . \quad (D-7)$$

If the latter is multiplied through by $a_F^2 d_h^4$, the result is

$$A d_h^6 + B d_h^5 + C d_h^4 + D d_h^3 + E d_h^2 + F d_h + G = 0 \quad (D-8)$$

where

$$\left. \begin{aligned} A &= 3(b_F R_{FE} \rho), \quad B = 2(a_F R_{FE} \rho + b_F R_{FE} \rho + a_F b_F) , \\ C &= (a_F R_{FE} \rho + a_F b_F) , \quad D = 0, \quad E = \frac{-12a_F^3 k}{T_S} , \quad F = \frac{-96a_F^4 k^2}{T_S^2} , \\ G &= \frac{-192a_F^5 k^3}{T_S^3} . \end{aligned} \right\} \quad (D-9)$$

The value for d_h that would afford a minimum to N (if it exists) would have to be found among the zeros of the sixth degree polynomial of Equation (D-8). However, an analytical solution for these zeros would be a formidable if not impossible task, so some other approaches must be used.

One approach would involve simply plotting Equation (D-6) for N over a sufficient range of the variable d_h for each particular set of values A through G of interest. Because, in any practical situation, d_h should be only of the order of 10, it would not add unduly to the amount of computations in MOCO to include a subroutine which would perform these calculations over a range of integers from, say, 1 to 50 or 100 and at the same time determine the value of d_h (nearest integer) which corresponds to the least value for N . As an example, a typical set of data might be

$$a_F = 0.001, \quad k = 100, \quad T_S = 0.001, \quad b_F = 0.001, \quad R_{FE} = 1, \quad \rho = 0.001 \text{ (radians)}.$$

Equation (D-6) then becomes

$$N = d_h^3 + 3d_h^2 + 3d_h + 6.4 \times 10^7 d_h^{-3} + 4.8 \times 10^5 d_h^{-2} + 1200d_h^{-1} + 2 . \quad (D-10)$$

A number of calculations were made for N with various values substituted for d_h in Equation (D-10). The results are tabulated and graphed in Figure D-1. It is seen that the minimum for N appears to occur at a value for d_h of about 20 (nearest integer). With this, d_h' would be calculated from Equation (D-1) to be also 20 (nearest integer). Note that if instead d_h were arbitrarily chosen to be 10, the number of calculations jumps from about 18,000 to about 70,000. The fact that d_h equals d_h' in this example is coincidental. In order to show this, another typical but different set of data were used with the equations. These are

$$a_F = 0.001, \quad k = 100, \quad T_S = 0.0005, \quad b_F = 0.002, \quad R_{FE} = 1, \quad \rho = 0.001 \text{ (radians)}.$$

With these data the minimum value for N turned out to be 70,413 which was afforded by a value for d_h of 25 and for d_h' of 32.

Figure D-1

Apparently a close approximation to a solution for the d_h that minimizes N can be had by ignoring the 1's in Equation (D-2). In this case

$$\begin{aligned}
 N &= d_h d_k d_\phi + d_h' d_k' d_\phi' \\
 &= \left(\frac{b_F R_{FE} \rho}{a_F^2} \right) d_h^3 + \left(\frac{4a_F^k}{T_s} \right)^3 d_h^{-3} ;
 \end{aligned}
 \tag{D-11}$$

from this

$$\frac{dN}{d(d_h)} = 3 \left(\frac{b_F R_{FE} \rho}{a_F^2} \right) d_h^2 - 3 \left(\frac{4a_F^k}{T_s} \right)^3 d_h^{-4} = 0
 \tag{D-12}$$

so that

$$d_h = \left[\frac{64a_F^{5k3}}{T_s^3 b_F R_{FE} \rho} \right]^{1/6}
 \tag{D-13}$$

With this formula the same values for d_h are calculated for the data of the two examples above, at least to the nearest integer.

If for some reason, it is required that a very fine subdivision of the search regions be used for high accuracy (e.g., k equals 1000 or higher), it would be advisable to continue past the two-stage division of search regions, as discussed so far, to a third or fourth, etc. If the divisors for the successive rectangles and angles are given by $d_{h1}, d_{h2}, \dots, d_{hn}; d_{k1}, d_{k2}, \dots, d_{kn}; d_{\phi 1}, d_{\phi 2}, \dots, d_{\phi n}$ for n such subdivisions, then by analogy to the two-stage case the following equations are obtained:

$$\frac{T_s}{k} = \frac{2^n a_F}{\prod_{i=1}^n d_{hi}} \quad (D-14)$$

and

$$N = (d_{h1} + 1) \left(\frac{b_F}{a_F} d_{h1} + 1 \right) \left(\frac{\rho \hat{R}_{FE}}{a_F} d_{h1} + 1 \right) + \sum_{i=2}^n (2d_{hi} + 1)^3 \quad (D-15)$$

Obviously, the task of finding the values of n and all the d_h 's which would minimize the number of calculations, N , would be a formidable one. If certain simplifying assumptions are made, some idea can be obtained about the size of N . For example, if all the d_h 's were assumed to be equal to some d_h for each stage of subdivision, the above equations become

$$\frac{T_s}{k} = \left(\frac{2}{d_h} \right)^n a_F \quad (D-16)$$

and

$$N = (d_h + 1) \left(\frac{b_F}{a_F} d_h + 1 \right) \left(\frac{\rho \hat{R}_{FE}}{a_F} d_h + 1 \right) + (n - 1) (2d_h + 1)^3 \quad (D-17)$$

For a numerical case, let

$$T_s = 0.001, \quad k = 10,000, \quad a_F = 0.001, \quad \rho = 0.001, \quad \hat{R}_{FE} = 1, \quad b_F = 0.001,$$

then

$$d_h = 2(10,000)^{1/n}, \quad N = (d_h + 1)^3 + (n - 1) (2d_h + 1)^3$$

The following table shows values of N corresponding to various pairs of values for n and d_h

n	d_h	N
1	20,000	8×10^{12}
2	200	7.3×10^7
3	43	1.4×10^6
6	9	35,295
8	6	15,722
10	5	12,195
20	3	6,581

By comparison, the two-stage method discussed above for use with smaller values of k would, for this case, yield a $d_h = 200$ and an $N = 2 \times 10^7$. Thus, it is seen how continued iteration of the process of going to smaller and smaller search regions can save considerable amounts of calculations when greater precision of the search should be required. The program MOCO has already been modified to take advantage of this approach along with two-stage approach.

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