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THE COUPLING OF MECHANICAL DYNAMICS AND INDUCED CURRENTS
IN A CANTILEVER BEAM

By

J.M. Bialek and D.W. Weissenburger

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The Coupling of Mechanical Dynamics and Induced Currents
in a Cantilever Beam

by J. M. Bialek and D. W. Weissenburger,
Plasma Physics Laboratory, Princeton University, Princeton, NJ 08544

Abstract

Electrical eddy currents induced in a conducting structure subjected to a background magnetic field produce forces which may result in significant mechanical reactions and deflections. The dynamics of the conductive structure are modified by additional eddy currents which are induced by the structural motion. Frequently, the observed effects of these secondary eddy currents are referred to as magnetic damping and magnetic stiffness.

Simple rigid body models of coupled magneto/mechanical systems have been previously investigated by the authors. This paper addresses the coupled structural deformation and eddy currents in a simple cantilever beam. A coupled system of equations was formulated using finite element techniques for the mechanical aspects and a mesh network method for the electrical aspects of the problem. The eigenvalues of the governing equations are examined using the background magnetic field as a parameter, and the solution of the equations is presented for a sample problem. The expected effects of magnetic damping and magnetic stiffness are observed in the solutions of the coupled equations.

Introduction

When a mechanical structure built of conducting material is placed in a strong magnetic field, it is found that the dynamic behavior of that structure depends on the coupled behavior of its mechanical and electrical characteristics. This is because the motion of the structure in a magnetic field produces additional eddy currents and forces which alter the dynamic behavior of the structure. This effect, basically Lenz's law applied to structural dynamics, is important when designing mechanical structures used in a magnetic fusion reactor/experiment.

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Our previous work ^{1,2} has been centered on the rigid body behavior of a pivoting loop or plate that is restrained by a rotational spring. In that system two main effects were observed: 1) a change in mechanical frequency due to the presence of the magnetic field and 2) a large effective damping. These effects were described as magnetic stiffness and magnetic damping.

In this paper we have formulated the complete equations for the problem shown in Fig. 1. The changing magnetic field $B_y(t)$ induces an eddy current pattern in the beam. These eddy currents interact with the constant magnetic field B_x to produce forces which cause lateral deflections (y motion) in the cantilever beam. Motion of the beam then induces changes in the eddy current patterns, and this produces a significant change in the dynamics of the beam. In standard analyses, this coupling of mechanical motion and electrical performance has not been included. It turns out to be an important effect.

This paper presents the derivation of the equations, the reformulation of the problem into a form more suitable to computer simulation, an examination of the eigenvalues of the problem and finally the results for a sample problem where the importance of this effect is contrasted by a comparison to the same physical problem with the motional emf effect "turned off".

Derivation of Governing Equations

The physical problem shown in Fig. 1 has both electrical and mechanical characteristics. Figure 2 illustrates an electrical mesh (top), an electrical branch (middle), and a mechanical panel (bottom) characterization of the problem. Electrically, the cantilever is considered to be a series of overlapping mesh elements similar to those used by the eddy current code SPARK³. Meshes are characterized by self and mutual resistances and inductances. The overlaps between meshes are electrical branches and also mechanical boundaries. Note in Fig. 2 that the mechanical boundaries are offset at the ends of the beam from the electrical branches. We used the (n) equal sized mesh spacing as shown in the figure for all our analyses.

Mechanically, the problem may be characterized as a series of Euler beams. We may idealize the mechanical problem as shown in the bottom of Fig. 2. The stiffness, mass, and dimensions of the short beams are characterized by the lateral motion and rotation of nodes that are located at the apparent boundaries of the panels. References 4, 5, and 6 document this type of modeling in more detail.

The variables used in this analysis are the lateral motion (U_y) and rotation about the z-axis (θ_z) at each node, and the current in each electrical mesh (I). For n meshes, we have n mesh currents and $2(n+1)$ structural variables. The structural boundary condition of being built-in at a beam end fixes the lateral motion and nodal rotation to be identically zero at the built-in end. Hence we end up with $2n$ structural variables and n mesh currents.

Basically, the problem may be written as

$$\{M\} \ddot{\{X\}} + \{K\} \{X\} = \{F(t)\} \quad (1a)$$

$$\{L\} \frac{d}{dt} \{I\} + \{R\} \{I\} = - \frac{d}{dt} \{\phi\} \quad (1b)$$

where

$\{F(t)\}$ is a vector of applied forces

$- \frac{d}{dt} \{\phi\}$ is a vector of time change in magnetic flux

Here $\{M\}$, $\{K\}$, $\{L\}$, and $\{R\}$ are, respectively, the mass, stiffness, inductance, and resistance matrices of the problem, while $\{X\}$ and $\{I\}$ are the vectors of structural variables and mesh currents. The right-hand side of Eq. (1a) is basically a Lorentz force calculation ($I dx \times B$) and hence is linear in mesh current. The right-hand side of Eq. (1b) is the change in flux. This may be broken down into two parts; a flux change due to the changing field, $A \frac{dB_y}{dt}$, and a flux change due to motion of the structure. If we write these equations in combined form with our vector of unknowns defined by:

$$\langle Z \rangle = \begin{pmatrix} \langle X \rangle \\ \langle I \rangle \end{pmatrix} = (u_{y1}, a_{z1}, u_{y2}, a_{z2}, \dots, I_1, I_2, I_3, \dots)^t \quad (2)$$

we may write our equations as,

$$\begin{bmatrix} \{M\} & 0 \\ 0 & 0 \end{bmatrix} \langle \ddot{Z} \rangle + \begin{bmatrix} 0 & 0 \\ \{C_2\} & \{L\} \end{bmatrix} \langle \dot{Z} \rangle + \begin{bmatrix} \{K\} & \{C_1\} \\ 0 & \{R\} \end{bmatrix} \langle Z \rangle = \begin{pmatrix} 0 \\ -AB_y \end{pmatrix} \quad (3)$$

In Eq. (3) matrices $\{M\}$, $\{K\}$, $\{L\}$, and $\{R\}$ are as before while $\{C_1\}\langle I \rangle$ represents the Lorentz force calculation and $\{C_2\}\langle \dot{X} \rangle$ represents the flux change induced by motion of the structure. The right-hand side of Eq. (3) has zero for the first $2n$ unknowns since the Lorentz forces now appear as $\{C_1\}\langle I \rangle$ on the left-hand side of the problem. The last n terms on the right-hand side of Eq. (3) are flux changes due to time changes in the B_y field. We will now derive in detail each of these submatrices.

As described in Ref. 1, the self-inductance of a rectangular plate was empirically derived using the SPARK code to be

$$L_s = (1.665) \frac{2\mu_0(a+b)}{9}, \quad (4)$$

$$\text{where } \mu_0 = 4\pi \times 10^{-7}.$$

Using the effective area formula which was similarly derived in Ref. 1, let us consider each mesh to be an ideal magnetic dipole with a magnetic moment equal to the product of its mesh current and effective area. Then we can approximate the mutual inductance between mesh i and mesh j as

$$M_{ij} = \frac{\mu_{ij}}{I_j} = \frac{B_{ij} A_i}{I_j} = \frac{-\mu_0 I_j A_j}{4\pi (|i-j|b/2)^3} \frac{A_i}{I_j}. \quad (5)$$

Upon canceling common terms we obtain

$$M_{ij} = \frac{-10^{-7} A_i A_j}{(|i-j| b/2)^3}, \quad (6)$$

where

- M_{ij} = mutual inductance between mesh i and mesh j
 Φ_{ij} = flux in panel i due to current in mesh j
 B_{ij} = magnetic induction at the centroid of i due to current in mesh j
 A_i = effective area of mesh i = $(0.695)^2 ab$
 $b/2 |i-j|$ = distance between centroids of panel i and panel j.

Thus we have the $n \times n$ submatrix

$$[L] = \begin{bmatrix} (0.37) \mu_0 (a+b) & \frac{-10^{-7} A_1 A_2}{(b/2)^3} & \frac{-10^{-7} A_1 A_3}{(2 b/2)^3} & \dots \\ \frac{(-10^{-7}) A_2 A_1}{(b/2)^3} & (0.37) \mu_0 (a+b) & \frac{(-10^{-7}) A_2 A_3}{(b/2)^3} & \dots \\ \frac{(-10^{-7}) A_3 A_1}{(2 b/2)^3} & \frac{(-10^{-7}) A_3 A_2}{(b/2)^3} & (0.37) \mu_0 (a+b) & \dots \\ \vdots & \vdots & \vdots & \ddots \end{bmatrix} \quad (7)$$

Again referring to Ref. 1, the resistance of a mesh may be written as

$$R_s = (1.665) \frac{2 \rho_e}{h} \left(\frac{a^2 + b^2}{ab} \right), \quad (8)$$

where ρ_e is the electrical resistivity of the material in the panel.

This may be thought of as four subsections of a mesh in series, i.e.,

$$R_s = (1.665) \frac{\rho_e}{h} \left(\frac{a}{b} + \frac{b}{a} + \frac{a}{b} + \frac{b}{a} \right). \quad (9)$$

We emphasize this characteristic in order to aid us in deriving the off-diagonal terms in the resistance matrix. Figure 3 illustrates the implicit numbering scheme we use in order to go from a branch resistance matrix to the mesh resistance matrix needed in this problem. As shown in Refs. 4 and 5 we may transform a branch resistance matrix $\{R_B\}$ into the mesh resistance matrix $\{R\}$ used in Eq. (3) via the relationship

$$\{R\} = \{T\} \{R_B\} \{T\}^t, \quad (10)$$

where $\{T\}$ is a mesh-branch incidence matrix.

The matrix $\{T\}$ is best explained by example: consider the two-mesh, seven-branch configuration illustrated in Fig. 3. We write +1 for branches whose assumed direction coincides with the assumed direction of mesh current, -1 for those that are opposite, and 0 if the branch in question is not part of the mesh. For this sample problem we have two meshes (rows) and seven branches (columns) giving:

$$\{T\} = \begin{bmatrix} 1 & 1 & -1 & -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 1 & 1 & -1 \end{bmatrix}. \quad (11)$$

If we write the branch resistance matrix as a diagonal matrix with the j th branch resistance in the (j, j) position and carry out the transformation in Eq. (10), we obtain for our problem

$$\{R\} = \begin{bmatrix} R_s & -R_a & 0 & 0 & 0 & 0 & \cdot & \cdot & \cdot \\ -R_s & R_a & -R_s & 0 & 0 & 0 & \cdot & \cdot & \cdot \\ 0 & -R_a & R_s & -R_a & 0 & 0 & \cdot & \cdot & \cdot \\ 0 & 0 & -R_s & R_a & -R_a & 0 & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \end{bmatrix} \quad (12)$$

where

R_s is given in Eq. (8)

and

$$R_a = (1.665) \frac{\rho_e a^3}{h b}.$$

The matrices $\{M\}$ and $\{K\}$ are the standard consistent mass and stiffness matrices. The derivation of the matrices for each element may be found in Ref. 6. For each beam element between two adjacent nodes separated by distance ℓ we use the following standard element matrices

$$\{K_{ele}\} = \frac{2E}{\ell^3} \frac{ah^3}{12} \begin{bmatrix} 6 & 3\ell & -6 & 3\ell \\ & 2\ell^2 & -3\ell & \ell^2 \\ & & 6 & -3\ell \\ \text{(symmetric)} & & & 2\ell^2 \end{bmatrix} \quad (13)$$

and

$$\{M_{ele}\} = \rho_m ah \begin{bmatrix} \frac{11}{35} \ell & \frac{11}{210} \ell & \frac{18}{140} \ell & \frac{-26}{840} \ell^2 \\ & \frac{2}{210} \ell^3 & \frac{26}{840} \ell^2 & \frac{-2}{280} \ell^3 \\ & & \frac{13}{35} \ell & \frac{-11}{210} \ell^2 \\ \text{(symmetric)} & & & \frac{2}{210} \ell^3 \end{bmatrix} \quad (14)$$

where ρ_m is the mass density of the material and E is Young's modulus of the material.

Each of these 4×4 matrices are submatrices in the global stiffness and mass matrices. Once the sequence of node numbers has been fixed, each elemental matrix may be written as a $(2n+1) \times (2n+1)$ matrix with the nonzero terms appropriately placed in the larger matrix. The global stiffness and mass matrices are then assembled by adding together the elemental matrices that have been written in the basis for the global problem. Once the global mass and stiffness equations are assembled, the physical boundary conditions of no motion or rotation at the cantilever support are imposed by setting the motion and rotation identically equal to zero at the appropriate node.

The matrix $\{C_1\}$ defines the Lorentz forces acting on the physical structure: it is an $2n \times n$ matrix. If we consider the torque on a panel, we obtain

$$\text{torque} = I \vec{A} \times \vec{B} = I (ab) (0.695)^2 B_x e_z. \quad (15)$$

Writing this torque as a force couple, we obtain the equivalent force on the nodes defining the panel ends as

$$F_y = \pm \frac{Iab(0.695)^2 B_x}{(b/2)} \quad (\text{for central meshes}) \quad (16a)$$

$$F_y = \pm \frac{Iab(0.695)^2 B_x}{(3b/4)} \quad (\text{for end meshes}). \quad (16b)$$

$$\dot{\Phi} = \frac{d}{dt} (\text{flux}) = -A B_x \cos \alpha \dot{\alpha} - AB_y \sin \alpha \dot{\alpha} + A \dot{B}_y \cos \alpha. \quad (19)$$

Now using the small angle approximation, and assuming $|B_x| > |B_y|$, we approximate this expression by

$$\dot{\Phi} = \frac{d}{dt} (\text{flux}) = -A B_x \dot{\alpha} + A \dot{B}_y. \quad (20)$$

The second term in Eq. (20) is the flux change produced by the change in the B_y component of the field, and the first term is what we seek to represent by matrix $\{C_2\}$. For the mesh between nodes i and $i+1$, we may write this as

$$\dot{\Phi}_{i \ i+1} = -AB_x \frac{\dot{y}_{i+1} - \dot{y}_i}{\ell_{i \ i+1}}, \quad (21)$$

where $\dot{y}_i$ is the time derivative of the y motion of point i , and

$\ell_{i \ i+1}$ is the original length between nodes i and $i+1$.

When we specialize this expression to the geometry of Fig. 2, we obtain

$$\dot{\Phi}_{i \ i+1} = -ab(0.695)^2 B_x \frac{\dot{y}_{i+1} - \dot{y}_i}{b/2} \quad \text{for central meshes,} \quad (22)$$

or

$$\dot{\Phi}_{i \ i+1} = -ab(0.695)^2 B_x \frac{\dot{y}_{i+1} - \dot{y}_i}{3b/4} \quad \text{for end meshes,}$$

and define a new extended column vector by

$$\{Q\} = \begin{pmatrix} \{P\} \\ \{X\} \\ \{I\} \end{pmatrix} \quad (25)$$

we may reformulate Eq. (3) as the system of first order differential equations,

$$\begin{bmatrix} 0 & \{I\} & 0 \\ \{M\} & 0 & 0 \\ 0 & \{C_2\} & \{L\} \end{bmatrix} \{\dot{Q}\} + \begin{bmatrix} \{-I\} & 0 & 0 \\ 0 & \{K\} & \{C_1\} \\ 0 & 0 & \{R\} \end{bmatrix} \{Q\} = \begin{pmatrix} 0 \\ 0 \\ \{-A \quad \dot{B}_y\} \end{pmatrix} \quad (26)$$

where $\{I\}$ is the identity matrix.

A little algebra with the component matrices then yields

$$\begin{bmatrix} \{I\} & & \\ & \{I\} & \\ & & \{I\} \end{bmatrix} \{\dot{Q}\} + \begin{bmatrix} 0 & \{M^{-1}K\} & \{M^{-1}C_1\} \\ \{-I\} & 0 & 0 \\ \{L^{-1}C_2\} & 0 & \{L^{-1}R\} \end{bmatrix} \{Q\} = \begin{pmatrix} 0 \\ 0 \\ \{-L^{-1}A \quad \dot{B}_y\} \end{pmatrix} \quad (27)$$

which is ideally suited to numerical solution.

Eigenvalues of the Problem

Equation (27) is the basic formulation of the coupled mechanical and electrical model of a cantilever beam. It is worthwhile to examine the eigenvalues of this problem before proceeding further.

Typically, the eigenvalues of the mechanical problem alone are formulated as

$$\{M\} (\phi) + \{K\} (\phi) = 0, \quad (28)$$

where the eigenvalues (real numbers) represent the squares of the natural frequencies of the problem. The eigenvectors here represent a set of "mode shapes" which may be used to express the structural displacement. The mechanical part of Eq. (27) has been formulated as a first order problem which is slightly nonstandard from a mechanical engineering viewpoint. The mechanical equations for the eigenvalue problem are

$$\begin{bmatrix} \{M\} & 0 \\ 0 & \{I\} \end{bmatrix} \begin{bmatrix} \dot{P} \\ \dot{X} \end{bmatrix} + \begin{bmatrix} 0 & \{K\} \\ \{-I\} & 0 \end{bmatrix} \begin{bmatrix} P \\ X \end{bmatrix} = 0. \quad (29)$$

This first order formulation is twice the size of Eq. (28) and the eigenvalues are pure imaginary numbers in complex conjugate pairs. The eigenvector for a particular eigenvalue ($i\omega_j$) is simply the mode shape from Eq. (28) for the X variables, and $\pm i\omega_j$ multiplied by the mode shape for the P variables.

The eigenvalues of the electrical problem alone already are formulated as the first order system

$$\{L\} \frac{d}{dt} (I) + \{R\} (I) = 0, \quad (30)$$

and the eigenvalues are real negative numbers, while the eigenvectors are real vectors corresponding to "current mode shapes." By setting the right-hand side of Eq. (27) equal to zero, we may now examine the eigenvalues of the coupled problem.

When the background field (B_x) is set equal to zero, matrices $\{C_1\}$ and $\{C_2\}$ are identically zero and the separate eigenvalues and eigenvectors described above are recovered. For the full problem each eigenvalue now has an eigenvector associated with mechanical and electrical variables and for B_x

$= 0$ the eigenvectors decouple nicely. As B_x is made nonzero, the mixing of mechanical and electrical aspects begins. Those eigenvectors associated with the purely mechanical aspects of the problem at $B_x = 0$ now have nonzero mesh currents, and similar mixing occurs in the electrical eigenvectors. It is no longer possible to make a distinction between the electrical and mechanical aspects of the problem, we can however, follow the eigenvalues as they move in the complex plane. The eigenvalues associated with the mechanical problem at $B_x = 0$ now develop negative real parts and an effective damping in the transient solution is expected because of this. In fact these eigenvalues may become negative real numbers in certain exceptional instances. The eigenvalues associated with the electrical part of the problem at $B_x = 0$ are observed to remain pure real negative numbers as the coupling background field is turned on. The eigenvectors associated with these eigenvalues are real.

The behavior of the eigenvalues as the background coupling field is changed is presented in Figs. 5-9. The particular physical problem examined in these plots is a cantilever made of copper with dimensions and properties as listed in Table 1. Since Eq. (27) has $(5n)$ variables for a cantilever made of (n) mesh/panels, we have chosen to model the cantilever with 15 mesh/panels.

Figure 5 illustrates the magnitude of the real eigenvalues associated with the circuit equations (at $B_x = 0$). At $B_x = 0$ we have the 15 eigenvalues clustered very closely together. As the coupling field is increased, the eigenvalues spread out as shown in Fig. 5.

The eigenvalues that correspond to the structural part of the problem at $B_x = 0$ are, in general, complex numbers. Figure 6 illustrates how the magnitude of these eigenvalues changes as B_x is increased. In general, a monotonic increase in magnitude is observed as B_x is increased. The lower magnitude complex eigenvalues are more heavily influenced by the coupling field than the higher frequency eigenvalues. The rapid change in magnitude of the smallest complex eigenvalue, near B_x equal to 2, 3.4, and 4. Tesla occurs as this eigenvalue collides with one of the real eigenvalues. The curve is actually continuous.

Figures 7-9 clarify what is happening at these regions of rapid change in eigenvalue magnitude. Figure 7 illustrates the phase angle of the complex eigenvalues whose magnitudes are less than 10^4 . A very rapid change in phase angle occurs for the smallest complex eigenvalue at B_x equal to 2, 3.4, and 4 Tesla. In fact, one of the complex eigenvalues becomes real near 2 Tesla. The curves are truncated when their magnitude exceeds 10^4 for purposes of clarity. Figure 8 shows the imaginary part of the eigenvalues as B_x is varied. Since the imaginary part of the eigenvalues corresponds to the observed frequency of structural and electrical vibrations, we see that the magnetic field may increase or decrease the natural frequencies of the physical problem. Finally, Fig. 9 illustrates the motion of the complex eigenvalues that originate in the mechanical aspect of the problem plotted in the complex plane as B_x is varied. Figure 9 illustrates one quadrant of the complex plane. The complex conjugates of these eigenvalues would appear in the third quadrant of the complex plane.

Sample Problem, Comparison to Uncoupled Calculation

We now present typical results for the solution of Eq. (27). Table 1 defines the problem we have examined. A copper cantilever 12-cm long and 2-cm wide is subjected to a linear field change of 0.24 Tesla in 0.003 seconds. In order to provide a standard to which the reader may compare these results, we also provide a solution to what we call the "uncoupled problem."

If we ignore the extra eddy currents produced by the motion of the structure, as most previous analyses have done, there is no coupling of the mechanical and electrical eigenvalues of the physical problem. We refer to this formulation as the "uncoupled problem." If matrix $\{C_2\}$ is set equal to zero in Eq. (27), we recover the uncoupled problem.

Figures 10 and 11 illustrate the solution obtained for the dimensions, materials, and forcing function summarized in Table 1. Figures 10 and 11 correspond to the solution of the coupled equations and should be compared to Figs. 12 and 13, which correspond to the solution of the uncoupled problem. The reader should note that the maximum deflection observed is about five times smaller for the coupled equations (see Fig. 14). It is also of interest

to compare the mesh currents at the start of the transient in Figs. 11 and 12. The first mesh current contour is almost identical in the two cases. However, once mechanical motion develops and becomes nonzero, the pattern of mesh currents becomes quite different. Figures 15-17 allow the reader to compare mesh currents in the coupled and uncoupled problems for the first, middle, and last mesh in the cantilever beam.

Conclusions

The coupling of mechanical dynamics and induced currents in a cantilever beam is an important effect. We feel that efforts should be made to include these effects in the analysis and design of future magnetic fusion reactors and experiments. The basic method presented here can be extended to more complex magnetic field patterns and arbitrarily shaped structures.

If experimental verification of this type of analysis is attempted, appropriate modification of the governing equations would be required to account for the finite extent of the built-in end of the cantilever.

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TABLE 1

Parameters of Sample Problem

material	copper
number of mesh/panels= no. of nodes	15
width, a	0.02 m
total length	0.12 m
mesh length, b	0.015 m
thickness, h	0.001 m
mass density, ρ_m	8000 kgm/m ³
electrical resistivity, ρ_e	1.7E-8 ohm m
Youngs modulus, E	1.1E+11 (N/m ²)
background field, B_x	4 T
driving field, $B_y(t)$	0.24 T spatially uniform field ramped to zero in 0.003 s
initial conditions	all variables set to zero identically
duration	0 to 0.032 s

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Fig. 1

#84E0142

Basic cantilever beam model.

Fig. 2

#84E0145

Illustrative problem characterizations: overlapping electrical meshes (top), electrical branch network (middle), and mechanical beam series (bottom).

Fig. 3

#84E0144

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Example electrical network with two meshes and seven branches.

Fig. 4

Example deflected beam with detail of typical mesh/panel element .

Fig. 5

Magnitudes of eigenvalues originating in the electrical aspects of the problem versus B_x . All values are pure real and negative.

Fig. 6

Magnitudes of complex conjugate eigenvalues originating in the mechanical aspects of the problem versus B_x .

Fig. 7

Phase angles of originally mechanical, complex conjugate, eigenvalues versus B_x .

Fig. 8

Imaginary components of originally mechanical, complex conjugate, eigenvalues versus B_x .

Fig. 9

One quadrant display of originally mechanical, complex conjugate, eigenvalues for $B_x = 0-5$ T. Pure real values not shown.

Fig. 10

Coupled equation solution of sample problem for lateral displacement versus time along cantilever. Total beam length is 12 cm.

Fig. 11

Coupled equation solution of sample problem for negative mesh current
versus time along cantilever. Total beam length is 12 cm.

Fig. 12

$n1 = 15$
 $a1 = 0.0150$
 $b = 0.0200$
 $h = 0.0010$
 $\rho_{ho} = 1.70e-08$
 $\rho_{ho} = 8.00e+03$
 $E = 1.10e+11$
 $B_x = 4.0000$

Uncoupled equation solution of sample problem for negative mesh current versus time along cantilever. Total beam length is 12 cm.

Fig. 13

$n1 = 15$
 $a1 = 0.0150$
 $b = 0.0200$
 $h = 0.0010$
 $\rho_{oe} = 1.70e-08$
 $\rho_{om} = 8.00e+03$
 $E = 1.10e+11$
 $Bx = 4.0000$

Uncoupled equation solution of sample problem for lateral displacement versus time along cantilever. Total beam length is 12 cm.

Fig. 14

variable 59
 n1 = 15
 a1 = 0.0150
 b = 0.0200
 h = 0.0010
 rhoe = 1.70e-08
 rhom = 8.00e+03
 E = 1.10e+11
 Bx = 4.0000

Coupled and uncoupled equation solutions of sample problem
 for displacement versus time at node 15.

Fig. 15

variable 6i
 n! = 15
 a1 = 0.0150
 b = 0.0200
 h = 0.0010
 rhoe = 1.70e-08
 rhom = 8.00e+03
 E = 1.10e+11
 Bx = 4.0000

Coupled and uncoupled equation solutions of sample problem
 for mesh current versus time in mesh 1.

Fig. 16

current in mesh 8 coupled

current in mesh 8 uncoupled

variable 68
 n1 = 15
 a1 = 0.0150
 b = 0.0200
 h = 0.0010
 rhoe = 1.70e-08
 rhom = 8.00e+03
 E = 1.10e+11
 Bx = 4.0000

Coupled and uncoupled equation solutions of sample problem
 for mesh current versus time in mesh 8.

Fig. 17

variable 75
n1 = 15
a1 = 0.0150
b = 0.0200
h = 0.0010
rhoe = 1.70e-08
rhom = 8.00e+03
E = 1.10e+11
Bx = 4.0000

Coupled and uncoupled equation solutions of sample problem
for mesh current versus time in mesh 15.

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