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DECAY CONSTANTS and WAVE FUNCTIONS of HEAVY-LIGHT PSEUDOSCALARS *

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Calculations of the pseudoscalar decay constant for mesons containing one heavy quark (i.e., f_D , f_B) are reported. Results at $\beta=6.0$ using the static-quark approximation for the heavy-quark propagator are compared to results using a propagating heavy quark with Wilson fermions at ($\beta=6.0$, $\beta=6.1$ and $\beta=6.3$). Further investigation into the dynamics governing these systems is made by measuring the Bethe-Salpeter wave function for each case.

1. INTRODUCTION

Matrix elements of pseudoscalar mesons P , containing one heavy quark ($Q = c, b$) and one light quark ($q = u, d, s$), are of great phenomenological interest and have recently been the subject of a variety of calculational efforts on the lattice.¹⁻⁴ Here, the major obstacle to a successful treatment of such “heavy-light” physics is clear: the existence of a heavy quark (mass m_Q) as a dynamical degree of freedom restricts the grid spacing a to a value $a \ll m_Q^{-1}$ so that maintaining a physical dimension Na comparable to typical hadronic scales quickly exhausts current computing capabilities.

There have been two approaches to circumvent this problem. The first, which we will refer to as the “conventional” technique,¹ is to simulate both heavy and light quarks as standard lattice (Wilson) fermions and to measure some physical quantity, such as f_P , as a function of M_P while increasing the mass of the *heavy* quark. Clearly at some point in this process, the error induced by large aM effects becomes intolerable, and one must introduce some extrapolation procedure in order to reach physics at higher mass scales. At what value of aM_P direct simulation becomes unreliable and extrapolation is necessary is in general a numerical question to which one must address by repeating the calculation at a weaker coupling

and checking for scaling behavior.

The second approach to the heavy-light problem on the lattice was initially proposed by Eichten,³ and by Lepage and Thacker,⁵ and is commonly referred to as the static-quark technique. Here, one removes the heavy-quark mass as a dynamical scale in the problem by retaining only the leading term in an expansion of the heavy-quark Lagrangian in powers of $1/m_Q$.

We report here recent results for calculations of f_P , using both methods outlined above. The expected behavior at very large mass scales,

$$f_P \sqrt{M_P} \sim \text{constant (up to logs)}, \quad (1)$$

is implicit in the static approximation and may be checked for in conventional calculations. As the estimate of f_B made previously using the conventional technique is in poor agreement with the results in the static limit, we report, in Section 4, measurements of the Bethe-Salpeter wave functions as a way to study the difference between the “static-light” and the conventional heavy-light mesons. The lattices used in the calculations are listed in Table 1.

2. D MESONS AND BEYOND

We discuss now the heavy-light results from the conventional method where “light” is defined to be in the vicinity of the strange-quark mass.

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name	β	$\sim a^{-1}(\text{GeV})$	size	$L(\text{fm})$	confs
A_{16}	5.7	1.0	$16^3 \times 25$	3.2	16
B_{16}	6.0	1.75	$16^3 \times 39$	1.8	19
B_{24}	6.0	1.75	$24^3 \times 39$	2.7	8
C_{12}	6.1	2.0	$12^3 \times 33$	1.2	30
D_{24}	6.3	2.75	$24^3 \times 49$	1.7	4

Table 1: The lattices and nominal parameters used.

Using the strange quark eliminates the complication of an extrapolation in κ_q to the chiral limit and allows us to examine directly the properties of f_P as a function of M_P at several different couplings.

In Fig. 1 and Table 2 we re-examine a subset of the results at $\beta=6.1$ (originally presented in Ref. 1) and compare them to recent results at $\beta=6.0$ and preliminary results (from only four configurations) at $\beta=6.3$. Only statistical errors are shown. We draw the following conclusions:

- 1) As noted in Ref. 1, the data beyond the mass of the D meson appear consistent with the expected large-mass scaling law given by (1).
- 2) The suspicion that such behavior could be a lattice artifact due to the largeness of aM is so far unsubstantiated, as the general shape in Fig. 1 appears to be scaling over the range of couplings tested.

The second conclusion is clearly highly preliminary, as only a fraction of the available data set at $\beta=6.3$ has been analyzed so far.

3. THE STATIC-QUARK APPROXIMATION

We assume that the reader is familiar with the static-quark approximation on the lattice^{3,5} and provide only those details necessary for the presentation of our numerical results.

3.1. Calculational Methods

For the static-quark propagator, we use³

$$S_Q^{latt}(x, y) = \delta^3(\vec{x} - \vec{y}) e^{-am_Q|x_0 - y_0|} \times \left\{ \begin{aligned} &\theta(x_0 - y_0) \left(\frac{1+\gamma_0}{2} \right) \prod_{t=x_0-1}^{y_0} U_0^\dagger(\vec{y}, t) \\ &+ \theta(y_0 - x_0) \left(\frac{1-\gamma_0}{2} \right) \prod_{t=x_0}^{y_0-1} U_0(\vec{y}, t) \end{aligned} \right\}. \quad (2)$$

Given, then, the standard definitions

Figure 1: The decay constant of “heavy-strange” mesons from the conventional method. See also Table 2.

name	β	κ_q	κ_Q	aM_P	$a^{3/2} f \sqrt{M}$
B_{24}	6.0	.154	.154	0.373(70)	.059(09)
			.135	0.852(11)	.095(12)
			.088	1.805(17)	.110(13)
C_{12}	6.1	“.152”	“.152”	0.353(28)	.0371(42)
			.130	0.887(15)	.0752(43)
			.100	1.504(14)	.0820(45)
D_{24}	6.3	.150	.150	0.268(07)	.0402(64)
			.140	0.547(05)	.0529(25)
			.110	1.211(12)	.0636(68)

Table 2: A subset of the data plotted in Fig. 1. Note that $\kappa_q = \text{“.152”}$ is a linear interpolation between $\kappa_q = .153$ and $\kappa_q = .151$.

$$A_\mu = \bar{\psi}_Q \gamma_\mu \gamma_5 \psi_q, \quad \text{and} \quad (3)$$

$$\langle 0 | A_\mu(x) | P(\vec{p}) \rangle = -i f_P p_\mu e^{-ip \cdot x}, \quad (4)$$

the calculation of the decay constant in the static limit follows from a conventional computation of the axial-axial Green function, with the substitution of (2) for the heavy-quark propagator. In practice, however, such a computation is highly contaminated with contributions from excited states,⁶ as illustrated in Fig. 2.

The calculation is made feasible through the use of “smeared” operators to enhance the overlap with the ground state (see Fig. 2) and gauge fixing to extract gauge-invariant physics from these gauge-non-invariant quantities. Define the smeared axial current as

$$A_\mu^s(x) \equiv \frac{1}{n^3} \int_{V_n} d^3y \bar{\psi}_q(x) \gamma_\mu \gamma_5 \psi_Q(x + \vec{y}). \quad (5)$$

The “source-smeared” correlator then reduces to

$$\langle A_0 A_0^s \rangle^{latt} = \left\langle \frac{\text{Tr}}{n^3} \sum_i^{(n)} \left[\frac{1 \pm \gamma_0}{2} P^\pm S_q(\vec{i}, t_x; \vec{o}, t_o) \right] \right\rangle \quad (6)$$

$$\longrightarrow \zeta_s e^{-a(M_P - m_Q)|t_x - t_o|}, \quad (7)$$

where $P^\pm$ stands for the appropriate product of links in (2). The source/sink-smeared correlator $\langle A_0^s A_0^s \rangle^{latt}$ is similarly defined;⁷ its residue is denoted ζ_{ss} . In terms of these quantities, the decay constant is given by

$$f_P \sqrt{M_P} = Z_A^{stat} \sqrt{2\kappa_q} \sqrt{\frac{2\zeta_s^2}{\zeta_{ss}}} \times a^{-3/2}, \quad (8)$$

where Z_A^{stat} is the perturbatively-calculated renormalization constant for the axial-vector current.⁸

Fits to a single exponential were done on the quantities $\langle A_0 A_0^s \rangle^{latt}$ and $\langle A_0^s A_0^s \rangle^{latt}$ using single-elimination subsets of configurations; the best fits were then used with (8) and the standard jack-knifing procedure to give our estimates of the statistical error in the computation.

3.2. Numerical Results

The decay constant was computed using the static approximation on lattices A_{16} and B_{24} , fixed to Landau gauge. The results from lattice B_{24} , quoted as our best values for this method, are shown in Fig. 3; lattice A_{16} gives a consistent lower bound, but the result is unstable to the choice of fitting interval.

A comparison of Figs. 1 and 3 reveals the magnitude of the discrepancy between the static and conventional results. Note that the quantity in Fig. 3 is a bare lattice quantity; its perturbative renormalization contains an uncertainty stemming from the choice of coupling but has been estimated to be^{8,9} $Z_A^{stat} \approx 0.6 - 0.8$. The inclusion of this factor improves the situation but falls short of a resolution. There are then several possible explanations for the discrepancy:

- First, we may still be at values of aM that are too large to produce reliable results in the conventional method. Completion of the calculation at $\beta=6.3$ should shed light on this question.
- Secondly, $\mathcal{O}(a)$ corrections may be significant. This issue we again defer to “computations in progress,” which will use an improved action.¹⁰

Figure 2: The “binding energy” from local ($\square$) and source-smeared ($\diamond$) correlators in the static approximation, from lattice B_{24} , with $\kappa_q = .154$.

Figure 3: The decay constant in the static limit at $\beta=6.0$ (lattice B_{24}), before renormalization by Z_A^{stat} .

- Thirdly, the fact that Z_A^{stat} is calculated to be such a large “correction” may be an indication that some non-perturbative method of renormalization is required.
- Finally, there exists the possibility that the static-quark results may be altered in the mass region below and including the B meson by the inclusion of $(1/m_Q)$ terms. However, this, by itself, cannot explain the discrepancy, since the conventional results seem to show asymptotic behavior (no large $(1/m_Q)$ corrections) at a value of $f\sqrt{M}$ that is inconsistent with the static approximation.

Thus, we proceed to investigate the normalization of the static-quark measurements in the following section.

4. WAVE FUNCTIONS

4.1. Motivation

The Bethe-Salpeter wave functions (BSWF) of SU(3) hadronic states were first computed on the lattice by Gottlieb.¹¹ They are of interest in the

present context of the heavy-light system primarily for two reasons. First, as was pointed out to us by Eichten,¹² the wave function may provide a quantitative check on the decay constant calculations: for highly non-relativistic bound states, the BSWF reduces to the ordinary quantum-mechanical wave function, whose value at the origin is then determined by properly normalizing the measurement. For such states, the decay constant is given by (again, up to logs)

$$f_P \sqrt{M_P} = 2\sqrt{3} |\phi_P(0)|. \quad (9)$$

The validity of the approximation here (one is requiring the *light* quark to be non-relativistic) must, however, be checked empirically. Secondly, wave functions may provide useful qualitative information—e.g., the approximate physical size of the states, how this size compares to the parameters of the computation, and how the states differ under the two methods.

4.2. The Calculation

We define, in addition to the local pseudoscalar current $J_5(x)$, the non-local current

$$J_5(x, \vec{r}) \equiv \bar{\psi}_Q(x + \vec{r}) \gamma_5 \psi_q(x). \quad (10)$$

The BSWF, defined in a fixed gauge¹¹ as

$$\Phi_P(\vec{r}) \equiv \langle 0 | J_5(x, \vec{r}) | P \rangle, \quad (11)$$

is extracted from a lattice calculation of the two-point correlator, defined by

$$\Gamma(\vec{r}, t) \equiv \sum_{\vec{x}} \langle 0 | \hat{T} J_5(x, \vec{r}) J_5^\dagger(0) | 0 \rangle, \quad (12)$$

via the large-time relation

$$\frac{\Gamma(\vec{r}, t)}{\Gamma(\vec{0}, t)} \longrightarrow \frac{\Phi_P(\vec{r})}{\Phi_P(\vec{0})} \equiv \Psi_P(\vec{r}). \quad (13)$$

We average the quantity $\Gamma(\vec{r}, t)$ over all points symmetric under cubic rotations. In the static approximation, Ψ_P is computed using a smeared current, J_5^s (analogous to (5)), in place of the local current at the origin of the correlation function (12). Our discussion here is limited to Landau gauge.¹³ Errors are again computed using the jackknife technique and are purely statistical.

Figure 4: The Landau-gauge wave functions, as given by eq. 15, from lattice B_{24} , with $\kappa_q = .154$.

4.3. Results

In Fig. 4 are shown the results of the wave function measurements at $\beta=6.0$ using both the conventional and static-quark techniques, with a “light” quark of $\kappa_q = .154$. The qualitative behavior of the two states is quite different: the static-light wave functions in general fall more quickly than the conventional ones, and *empirically* fit well to a simple exponential (such as that produced by a Coulombic potential).

However, independent of such observations, let us momentarily *assume* non-relativistic behavior, integrate the radial distribution $r^2 |\Psi(r)|^2$, and compute the decay constant via (9). For clarity, we enumerate the expanded range of possible techniques:

- 1a. conventional, direct (as in Fig. 1),
- 1b. conventional, wave function (from eq. 9),
- 2a. static, direct (as in Fig. 3), and
- 2b. static, wave function.

For the static case (using “light” quarks of $\kappa_q = .155$, $.154$ and $.152$), we find close agreement between the bare lattice results. At $\kappa_q = .154$, for example, method 2a. gives $a^{3/2} f \sqrt{M} = .43(2)$, whereas method 2b. gives $a^{3/2} f \sqrt{M} = .41(2)$. Note, however, that such precise agreement is certainly fortuitous—the (large and uncertain) renormalization constants have not been included.

Moreover, any numerical agreement in the check described above may in fact be very misleading. In Fig. 5 we plot the ratio of *bare* decay

Figure 5: The ratio of degenerate-meson, bare decay-constants, $f(\text{direct})/f(\text{wave function})$, from lattice B_{24} .

constants from methods 1a. and 1b. for *degenerate* (instead of heavy-light) mesons. In the low-mass region of the plot, we interpret the smallness of this ratio as evidence of a relativistic bound state—the wave-function technique gives absurdly large results.^{14,15} Despite the difference in reduced mass between these degenerate systems and the corresponding static-light systems, it thus seems unlikely that the light quarks of the latter are non-relativistic. Further, one can calculate the mean velocity of the light quark directly from the wave function under the non-relativistic assumption. For the static-light wave function in Fig. 4, for example, we find $v/c \approx 1$, in violation of the assumption. Thus, for the light-quark masses used so far, the static method cannot be realistically tested by the wave-function approach.

5. CONCLUSIONS

In general, our results indicate that the static-quark and conventional methods of computing heavy-light physics are producing highly inconsistent answers for the decay constant calculation. Thus we feel that it is still premature to base an estimate of f_B on the static results. However, we are proceeding to clarify the discrepancies by 1) completing the computation at $\beta=6.3$ with at least fifteen configurations, 2) further studying the wave functions for a wide variety of cases, and 3) including the $1/m_Q$ and $\mathcal{O}(a)$ corrections into the existing calculations.

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13. At $\beta=5.7$ we have made some preliminary measurements of Coulomb-gauge, static-light wave functions ($\kappa_q = .155, .160$). We find $|\phi(0)|^{Coul} \approx 1.3|\phi(0)|^{Lan}$.
14. A qualitatively similar result for $f\sqrt{M}$ via method 1b. is obtained from staggered-fermion wave functions computed by Gupta *et al.* See, for example, S. Sharpe, University of Washington preprint, 40423-10 P90 (1990).
15. We note that even in the moderate mass region ($\kappa_q = .148 - .135$), the ratio in Fig.5 does not approach 1. Possible causes for this behavior—such as the absence here of renormalization factors, or the onset of large aM effects—are under investigation.

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