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Quantitative Petrostructure Analysis

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QUANTITATIVE PETROSTRUCTURE ANALYSIS

by

Nick Warren

ABSTRACT

Petrostructure is defined to be a descriptive (modeling) space spanned by three axes: mineralogical variables, petrographic variables, and microstructural variables. A petrostructure analysis is a study of the correlations among sets of these variables and other experimental quantities (such as moduli-pressure, resistivity-pressure data). This report deals with two aspects of making such a study: parameterizing patterns of topographic features of rock and characterizing variables to allow correlation studies to be carried out. This is part of a basic study being done in application to the Hot Dry Rock Project at the Los Alamos National Laboratory. In this report the principle of quantitative petrostructure analysis is outlined and illustrated using maps from two samples of Conway granite. A Digital Enhancement technique (DIADS), available at Los Alamos, was used in this study and is evaluated here.

I. INTRODUCTION

A. The General Research

This report summarizes some recent results of a research effort conducted at the Los Alamos National Laboratory and the University of California at Los Angeles (UCLA) under the Office of Basic Sciences of the Department of Energy.

The general goal of the research is to develop tools for classifying crack structural patterns in rock in order to associate these patterns with, and to predict, bulk physical properties and in situ crustal properties.

The techniques discussed here, however, are not restricted to this application. They are built on a methodology for decomposing complex

two-dimensional patterns and as such, can be used for a number of topological or morphological pattern analysis tasks. Thus this report may provide notions which will prove useful in other fields as well.

B. General Notions

A large number of interdependent variables relate rock structure to bulk rock properties. With varying success, some physical properties can be predicted by using constitutive relations and a minimum of input parameters. A good example is the prediction of shock Hugoniot of tuff using initial density, water content, and porosity as input parameters (Abey 1981; Butovich 1973). However, the prediction of a large number of other properties depends on knowing explicitly the void structure of the rock. For example, these include all material transport and connectivity properties (such as permeability, resistivity) and the elastic deformation and failure properties.

Following Brace (1969), the void structure may be thought of as composed of quasi-equidimensional pores (and/or vugs) and various hierarchies of cracks and microcrack networks. This total void structure is embedded in, and in part controlled by, rock fabric and mineralogy, and it reflects history and conditions of origin and deposition, etc. Therefore, different classes and subclasses of voids correlate to various mineralogical and petrographic parameters such as grain shape, cleavage planes, the spatial distribution of grain boundaries, crystal habit and preferred orientation, etc. Because of this, a general term petrostructure may be defined to denote the combination of mineralogical variables, petrographic variables and microstructure (microcrack variables). The problem of predicting rock physical properties in terms of basic petrostructure requires a multivariant analysis. In its broadest sense, it is an application in pattern recognition and factor analysis (Joreskog et al. 1976; Davis 1973). The approach discussed in this paper is based on this broad notion.

C. Relation to Other Studies

Partial analysis studies have been done which experimentally correlate observable crack patterns, mineralogy, grain texture (or fabric) and bulk mechanical properties. Among others, Brace (1965), Friedman and Bur (1974), and Hadley (1976) have demonstrated such relationships. However, such work has not been done in a way that generates generalized mathematical tools that allow prediction of the behavior of pressure-dependent mechanical and physical properties over a large range of rock types. Currently, most studies relating

bulk properties to petrographic description mix quantitative and qualitative characterizations. More generalized and moderately economical techniques to quantitatively parameterize rock structure from direct observation are needed.

In other important works, Bennett (1972) and Tilmann and Bennett (1973), have demonstrated that measurements of acoustic properties can be inverted to predict petrostructure. They developed a sonic method for describing the preferred orientation of crystallographic structure of a polycrystal. Their technique, however, is difficult to apply to the problem of separating out the microstructural and crystallographic effects from each other at low pressure.

The development of theories correlating petrostructure to bulk properties are basically limited by the requirement of idealization. Theoretical approaches include calculating the detailed stress-strain models for various geometric shapes of single isolated cracks (Mavko and Nur 1978; Walsh 1965a, b) and the "inverse problem" approach (O'Connell and Budiansky 1974). However, at the very least, such approaches still assume idealized crack properties and there are no easy ways to relate the resultant calculated microstructure spectrum to the sample's observed crack pattern.

Another approach to the problem, the approach taken here, is to treat this as a problem in statistics. Both physical properties and rock structure are then to be taken as statistical data, and the relations between these sets of variables are to be statistical functions. This puts both the phenomenological and the theoretical approaches onto the same footing and allows treatment of large numbers of variables.

The rock itself is taken to be describable as a statistical assemblage of directly observable petrostructure variables. In this approach, actual rock mineralogical and morphological features are required to be measured. The general method of mapping and measurement must be both economical and allow various tests to be made easily, such as similarity tests between different rocks and correlation tests to various bulk physical properties.

In this report I present a prototypal study based on making two thin section maps from two granitic samples. Based on elastic moduli data, the samples have already been grouped together in an earlier study. In the study here, crack patterns observed in the photographs are characterized. The analysis has two goals:

1. to analyze the problem of how best to map and parameterize patterns in rock; and

2. to test for and determine the advantages and disadvantages of two different methods of mapping and digitizing microphotographs. The two methods were: low resolution optical and hand techniques available at UCLA, and second, a technique utilizing DIADS (Digital Image Analysis Display System) at the Los Alamos National Laboratory.

Only one microphotograph of each sample was used. Apparent crack patterns in the microphotographs were mapped as the basic input data for the study. Although limited, these are sufficient to illustrate a petrostructure analysis and to discuss the problem of making a quantitative characterization of microstructure.

An expanded discussion of the basic definitions and notions of a petrostructure analysis are given in Appendix A. Descriptions of the logic of the analysis program itself are given in Appendices B and C.

II. THE TWO GRANITE SAMPLES

The two samples used in this study are Conway granite, sample numbers 704.4 and 381.9. Both of these samples are from a bore hole drilled by the Energy Research and Development Administration in New Hampshire at an exploration hot dry rock site. These two samples are already reported on in Warren and Tiernan (1981) and Warren 1979. These studies describe systematics in the elastic properties of a suite of ten Conway granites. The samples were found to form three groups. Within each group the samples showed qualitative similarities in textures and grain size distributions.

Samples 704.4 and 381.9 are from a single group and are fine grained. Their similarities are demonstrated in Fig. 1. This figure shows "crack spectra" inverted from the velocity data from these two samples. These spectra are not dependent on the exact physics of cracks but are a measure of the microstructure of the samples. The crack spectra are labeled compressional and shear spectra depending on whether they were formed from compressional velocity data or shear velocity data. The strong similarities between the compressional and shear spectra for each sample indicate that the fine structure of the velocity data is a measure of underlying microstructure. The crack spectra are obtained from the derivative of a spline curve fit to the (velocity) data. The technique follows from Simmons et al. (1974) and is given in Eq. 1.

Fig. 1.

"Crack" spectra of Conway samples. Crack spectra are properly derived from strain or compressibility data. Here, however, spectra are derived from velocity data directly to show consistency of their forms. Curves labeled v_p are from compressional wave velocities. Curves labeled v_s are from shear wave velocity data.

$$F_i = P \cdot \frac{d(\rho V^2)^{-1}}{dP} \quad i = p, s \quad (1)$$

Pressure is denoted by P , ρ is density, V_i is either compressional velocity ($i = p$ or shear velocity $i = s$). The quantity F_i is a measure of the derivative of compliance with pressure and is proportional to the zero pressure crack porosity, which is pinned closed at a pressure P to $P+dP$. This result is clear if the acoustic moduli are combined into the volumetric bulk modulus. To illustrate the difference between these two samples and a sample from another group, crack spectra for sample 448.1 is also shown in Fig. 1.

In these studies, a simple statistical measure also was made of a crack density which brings out the similarities between these two samples. Counts were made of the number of cracks intersected by standard circles that were placed on photographs taken at magnifications 32x to 600x. The results for samples 381.9 and 704.4 are shown in Fig. 2. Both of these samples are denoted by a single hierarchy of cracks, which is the only one counted at all magnifications.

These two samples were, therefore, picked for this petrostructure study because they showed strong similarities and provide a basis to illustrate the technique of establishing a measure of correlation between the petrostructure of two distinct but similar rocks.

III. MAPPED DATA

In this study petrostructure was limited to "crack" structure. The term "crack" here is qualified in that no attempt was made to mark open cracks to distinguish them from other features that showed similar visual contrast and morphology. The two reflection microphotographs used are shown in Fig. 3. Portions of the crack patterns seen in the microphotographs were digitized as described in Appendix A. The computer regenerated crack patterns are shown in Fig. 4. The microphotographs were made at a magnification of 63x. Each microphotograph was analyzed separately. One microphotograph was digitized at UCLA, the other one at Los Alamos.

The three goals in this part of the study were:

1. to determine the efficiencies and qualities of different mapping techniques;
2. from this, to develop and select an efficient technique; and

Fig. 2.

Logarithmic plot of cracks intersected by standard circles. The characteristic length (L_{ch}) is the circle circumference corrected for magnification. The number density (N/L) is sensitive to crack length and crack spacing. The vertical height of each box corresponds to the range of spacings that were measured using a number of photographs.

3. to determine operator effects, that is, the effects of the skill and experience of the operator in making subjective judgements on the data.

Digitization at UCLA was done on SAC graph-pen digitizer tied to a Hewlett-Packard 1000 minicomputer.

The second technique involved using a microdensitometer to scan the microphotograph and then digitizing the data using image enhancement techniques (DIADS) (Breedlove, 1978, and personal communication). There were a number of differences in handling the two microphotographs. These included resolution and density of mapping.

3-63-10-18-77

1 millimeter

704.4

6-63-2-16-78

Fig. 3.
Microphotographs used in this study.

381.9

MAPS

3-63-10-18-77

704.4

6-63-2-16-78

Fig. 4.
Computer generated patterns from the microphotographs shown in Fig. 3.

A. Differences in Resolution

At UCLA a pixel grid was overlaid on the microphotographs and features traced onto the grid. The pixel spacing was 1/16 in. The traced map made from the thin section was then enlarged and digitized directly. In this mapping, features smaller than pixel distances were not traced and features that were traced were located to about 1/4 of a pixel spacing unit. The digitizing table was good to ± 0.01 in.

Under DIADS at Los Alamos, no grid was superimposed so the operator was free to decide on the level of resolution. Digitization was made on a television image of the original photograph and resolution on the scan corresponded to ± 0.004 in.

B. Differences in Density of Mapping

At UCLA essentially all of the crack-like features were digitized that were on the scale of resolution at which the maps were made. At Los Alamos not all the features on the microphotographs were mapped. The operator made a prototypical mapping only, which included the major features. The operator also tended to judge whether a crack-like feature constituted a real open crack or a healed or sealed feature. Only those that were judged open were digitized. Neither map was oriented.

Another procedural difference was that no standard instruction was given for digitizing along crack-like features. Only general guidelines were given:

1. That enough points be digitized to resolve crack trends with fair accuracy and with the minimum number of points necessary to do this.
2. That if a crack-like segment seemed to be straight and then kink, the kinked point should be digitized if that segment was on the order of or greater than that of the resolving grid unit.

C. Comparison

In summary, the differences in the two approaches are the following. At Los Alamos the photo image was worked with directly. The display was much more readable and the image quality was much higher. However, the operator tended to digitize just a fraction or portion of the features which included mainly, but not all, apparently open cracks. At UCLA the digitized map was already of lower resolution but all mapped features were digitized.

These differences showed themselves in comparison of the statistics of the two maps. Relative to the Los Alamos map, the UCLA map contained a larger crack density. That is, a greater number of data points were taken at UCLA.

In the Los Alamos data, very few cracks were judged to join other cracks. That is, the number of actual forks or intersections was smaller than the number mapped in the UCLA section. Also, fewer crack grains (that is regions closed by a continuous crack) were digitized on the Los Alamos microphotograph.

The digitized data from both microphotographs were carefully checked for errors (repeated points, etc.). In the UCLA data the crack-grains mapped were augmented by closing almost closed or grain-like crack features. This was done in order to emphasize a small but definite population of grain-like features for the demonstration in this report.

D. Relative Advantages of DIADS vs Other Digitization Techniques

The much higher resolving power at the same magnification and the ability to digitize the resolved differences indicate the importance of using DIADS. The ability to scan and judge the photo directly during digitization greatly enhances objectivity. Neither technique resolves the problem of marking cracks for positive identification of open features or crack networks. It can be suggested here that for some studies positive identification of cracks is not necessary. Statistical correlation of the patterns proceeds much more efficiently than the actual finding of open cracks. It is expected that actual cracks will be directly reflected in the statistical pattern. In this approach we also emphasize mapping these features at relatively low magnifications (that is, 32 power to 600 power) well below usual (Scanning Electron Microscopy) SEM magnifications used in crack studies.

IV. ANALYSIS

A number of variables were chosen that characterize the mapped crack patterns and have physical significance. The analysis was done using a computer program developed explicitly for this type of study. The program is named SOCIOLOGY, in order to emphasize that the analysis performed is one of pattern decomposition and parameterization devoid of modeling theory (see the Appendices for program description). In brief, through SOCIOLOGY, the mapped pattern is decomposed into tables of segment length, angle orientations, and connectivity, and then from these tables, distributions and characteristic parameters are generated which quantify the description of the mapped pattern. The variables chosen, and their significances are given in Table I. The mathematical definitions of key variables are given in Appendix B.

V. RESULTS

A. Crack Lengths

The first sets of histograms (Fig. 5) show a strong similarity between the two samples. The similarity is demonstrated to be real; that is, to be independent of techniques of digitization and of the persons doing the digitization. Three distributions are shown. These are the distributions of nearest-neighbor distances between crack tips $D(J = 1, K = 1)$, the equivalent distributions for crack branch and fork points $D(J = 3^+, K = 1)$, and the directly digitized crack segment lengths (CLS). The nearest neighbor distributions give the spacing between nodes of degrees 1 and 3^+ , independent of their connectivity. The CLS distribution gives the distances between adjacent connected nodes independent of degree. The physical significance of the $(J = 1, K = 1)$ and the $J = 3^+, K = 1)$ distributions is indicated in Table I.

Three implications are directly apparent from the histograms. First, for either sample, the spatial probabilities of crack stopping or branching are equal (i.e., the distributions $D(J = 1, K = 1)$ and $D(J = 3^+, K = 1)$ are commensurate). This would not be expected for a highly anisotropic or schistose rock. [The problem with $(J = 3^+, K = 1)$ population for 381.9 may arise from the low count of intersections and forks as discussed in the first part of this section.]

Second, for either sample, the segment length distributions are commensurate with the nearest-neighbor distributions. This has a two-fold significance. First, it implies that crack segments are straight features between kink points. There is no a priori reason for a segment length distribution to have a physical significance, since $J = 2$ nodes can be picked subjectively to give sufficient resolution of a curved line. Only the crack tip ($J = 1$) and branch point ($J = 3$) distributions may be expected to be a priori significant. However, if the crack between its ends and branch points is formed of straight segments between kink points then the tendency of the person doing the digitizing would be to only digitize kink points. The segment length distribution would then be a measure of the shortest distances along a crack between kink points. In Fig. 5 the fact that for each sample all three distributions are commensurate demonstrates that the spatial probability distributions for a crack to stop, to kink, or to branch are equal. The samples can, therefore, be modeled as having, to first order, uniform distributions of

TABLE I
SOCIOLOGICAL VARIABLES
(See Appendices A and B for nomenclature)

Variable Type	Variable Description/Name	Significance
Node	<u>D(J=1, K=1)</u> Nearest neighbor (K=1) distances for nodes of degree 1(J=1).	This is a spatial distribution of crack tips in a cross section. This may be taken to be a measure of the distribution of a crack-stopping variable; namely, either low stress regions or barriers (high K_C).
Node	<u>D(J=3⁺, K=1)</u> Nearest neighbor (K=1) distances for nodes of degree equal to or greater than 3 (J=3 ⁺).	<p>This is the spatial distribution of a crack forking variable (either low stress regions into which cracks "sink," or higher K_C regions at which cracks may fork if they are growing in region II. The super-script (+) simply denotes that the degree of the node is equal to or greater than 3.</p> <p>The similarity or difference of these two distributions is a measure of whether the (spatial) probability for a crack to stop is the same as for a crack to fork, and hence, it gives an inference as to whether the same material properties control both probabilities.</p>
Linear	<u>CLS</u> Segment lengths between connected neighbors.	This variable may be taken as a measure of the mean-free-path of crack growth. Kink points can be mapped either where an apparently straight crack segment kinks (small change in angle), or can be mapped as a point needed to resolve the curvature of a crack. That is kink-points (J=2) are not as uniquely specifiable between operators as are nodes J=1 or J=3 ⁺ . However, IN A WELL PLANNED MAPPING STUDY this distribution can become a measure of the probable distance a crack may grow without meeting any barrier. THIS BECOMES A MEASURE OF THE SCALE OF HOMOGENEITY.
Linear	<u>CL(1, 1)</u> crack lengths of non-forking cracks.	These are crack lengths for cracks defined by strings of connected nodes (J=1, 1) and (J=1, 2... 2, 1) only. These distances can be taken as crack-diffusion-distances. The diffusion distance is the probable

TABLE I (cont.)

Variable Type	Variable Description/Name	Significance
		distance a crack will grow without branching.
Linear	<u>CL(1, 3⁺)</u>	These are crack lengths for cracks CL(3 ⁺ , 3 ⁺) defined by strings of connected nodes ending at branch points (at least at one end). CL(1, 1), CL(1, 3 ⁺), and CL(3 ⁺ , 3 ⁺) are all possible crack length subpopulations. Combined with angle distributions for the principle directions of these crack subpopulations, measures can be obtained of THE SCALE OF ISOTROPY for crack diffusion in different directions (e.g. all 3 - 3 cracks may be short and have preferred orientation relative to the 1 - 1 population).
Angle	<u>Segment angle orientation of CLS.</u>	This is a direct measure of preferred orientation. The orientation distribution is determined by stress field, crystallographic, and mineralogical variables.
Linear/ Angle	<u>CLS length versus angle.</u>	Characterizes the degree of isotropy of the crack growth mean-free-path.
Angle	<u>Max., min. branch angles that is the largest and smallest inter-angles at a branch point.</u>	This gives a measure of interaction of stress fields and the relations to crystallographic and mineralogical control.
Closure	<u>Crack-grain area and shape.</u>	Area and shape (flatness and aspect ratio) are defined in Appendix B. A cross-plot of these two variables shows size-shape clustering of crack grains, indicating for example, mosaic fitting of cracks due to crystallographic control.
Angle	<u>Grain principal direction.</u>	This is the direction of the principal axis of a grain, and characterizes preferred orientation of crack grains (stress freed regions).
Node	<u>Nearest-neighbor distances between crack grains.</u>	A crack grain indicates a stress-freed region. The spatial distribution of such regions is a measure of the distribution of stress concentrations.

Fig. 5.
 Histograms of nearest-neighbor distances between crack tips [$D(J=1, K=1)$], between crack forks [$D(J=3, K=1)$], and the segment lengths of continuous crack segments (CLS).

crack-growth-barriers; the spatial probabilities of a crack stopping, kinking, or branching being equal.

Third, this equivalence in spatial probabilities for the two samples is consistent with the predicted similarities in microstructure based on the velocity-pressure data.

Although these distributions are not direct measures of the actual microstructure or fine cracks which control elastic moduli, a very striking agreement is found for these results and those of other studies designed to determine actual crack length distributions. (The problem of measuring true crack length distributions is discussed in Appendix D.) An important effort was made by Hadley (1976). She used 400x SEM microphotographs of Westerly granite from which she measured crack lengths and widths. She defined a single crack to be "any open flaw of aspect ratio less than 1 which did not change orientation by more than 20° over any significant portion of its length and which was either continuously open or bridged by material of exposed thickness no more than three crack width or one tenth of the total crack length, whichever was less."

Figure 6 is a superposition of her crack length distributions for two samples of westerly granite along with the CLS distribution for 704.4. The lowest curve is for unstressed granite, and the uppermost curve is for prestressed granite W.5, (which was cycled to 0.68 GP differential stress at 0.08 GP confining pressure). None of the sets of data are normalized to each other. However, Hadley's micrograph covered "about 1 mm²," and the areas covered in our mapping of 704.4 were almost exactly 1 mm². The crack length ranges plotted for the three samples are strikingly similar. Hadley's definition of crack length agrees well with the natural CLS determined for the Conway granite sample. In this case, the CLS can be taken as a measure of the probable distance a crack may grow without meeting a significant barrier. That is, the mean CLS can be interpreted as a measure of the mean-free-path of crack growth.

Although samples 381.9 and 704.4 can be said to have first order uniform distributions of crack-growth-barriers, there are anisotropic components in the distribution of the CLS, or more appropriately the crack growth-free-paths. For the mapped area of sample 704.4, the mean lengths of the free-paths and thus the probability of crack growth are not independent of direction, as shown in Figs. 7a and b, which contours the crack-segment frequency as functions of segment length and angle. The 90° direction is arbitrarily chosen as the vertical in the image plane. Peaks appear in the 0°-180° line, and in the 90°-270° direction. The effect of this orthogonal gridding shows in the control of crack-grain formation which is discussed

Fig. 6.

Superposition of crack length distribution from Hadley (1976) and the CLS distribution for 704.4 shown in Fig. 5.

later. The map for 381.9 (Figs. 7c and d) shows a main horizontal girdle. There may be secondary (nonorthogonal) griddings at 40° and 120°.

The second set of histograms (Fig. 8) gives distributions of cumulated crack-segment lengths, or more simple, the 'crack-length' distributions $CL(1, 1)$, $CL(1, 3^+)$, and $CL(3^+, 3^+)$. Obviously, here cracks are now defined to go from a tip to another tip, or to a branch point, or between two branch points. A crack defined this way is not necessarily straight but as defined in the table, may be called a "diffusion distance."

The data here are biased by two effects. First, for 381.9 the number of free ends may be skewed with respect to the number of branch points since fewer branch points were mapped in comparison to the data for 704.4. Second, the data include artificial augmentations of the $CL(1, 1)$ and $CL(1, 3^+)$ populations. Any crack that cuts the boundary of the map area generates a $J = 1$ node on the map boundary. This subpopulation was not removed for the demonstration study here. The result, nevertheless, is that for either rock, crack

Fig. 7.

The crack segment length-angle distributions. Figures 7a and c show contour plots. Figures b and d show the three-dimensional display of the same data.

diffusion distances are about the same for all three types of crack populations (although the distributions may be flatter for sample 381.9). This is consistent with the results in Fig. 5.

Figure 9 shows the dominant crack branching angles for the two samples. If at a branch point, all angles are equal, then they are equal to 120° . If a crack branch grows off (or into) a straight crack, then the maximum interangle is 180° and the minimum intercrack angle is equal to or less than 90° . Both samples show tendencies to form this latter form of crack pattern. Peaks are

Fig. 7. (cont)

observable in the data for preferred interangles. These are especially clear in the distribution for 704.4.

B. Crack-Grains

As discussed earlier, the crack pattern for 704.4 tended to form closed areas or grain-like regions bounded by cracks. These closed areas were characterized using the subroutine of SOCIOLOGY named BORDER. The crack-grain size distribution tends to be bimodal. There are a number of small crack-grains and fewer larger grains. These two populations are also separated by differences in shape and preferred orientation.

Fig. 8.
Cumulated crack-segment lengths. The three distributions are for cracks which do not fork [CL(1,1)], for cracks which start from a crack tip and run to a crack fork [CL(1, 3⁺)], and for cracks which run from a crack fork to a crack fork [CL(3⁺, 3⁺)].

Fig. 9.
 Distributions of maximum and minimum interangles at crack branch points.

Figure 10 is a plot of grain area versus flatness. The larger grains tend to be equidimensional; that is, to have flatness near 1.0 to 1.5. The smaller crack-grains have a greater range of flatnesses with the grain shapes varying from almost round to very flat.

Fig. 10.

Grain area vs flatness. Area units are in map units (not converted to mm^2).

The major axes of small crack-grains tend to form an orthogonal grid that is rotated 45° about the dominant orientation of the large crack grains (Fig. 11). Therefore, the two sets of crack grains each generate their own sets of cracks with preferred orientation. Large grains align with the dominant cracks in the mapped sample, and form the major mosaic of the fracture pattern, whereas the small grains occupy corners of this dominant grid.

Figure 12 shows the spatial distributions of these two populations. In the figure are plotted the nearest neighbor and second nearest neighbor distances for the grains marked by their centers-of-mass. Large grains, being aligned with the dominant crack pattern, can form contiguous groups. On the other hand, there are seldom more than two small grains contiguous with each other. Therefore, as seen in the figure, the nearest neighbor spacings of the grains are almost equal to the mean grain dimensions (given by the square-roots of the averaged grain area) but the spacings to the second nearest neighbors are very different for the two populations.

Fig. 11.
Grain area vs grain orientation.

VI. DISCUSSION

A large volume of data has been generated from the two maps. The basic data reduction was done very efficiently on the UCLA 3033 IBM computer. The running times for SOCIOLOGY (PRUNE and BORDER) were less than 6 s for either the 452 node map of 704.4 or the 303 node map of 381.9. Therefore, multimap large studies are economically feasible.

At present the software system for making a full study is not completed. We do not yet have interactive capacity and direct integration of output from SOCIOLOGY with various statistical factor analysis packages. However, such a fully interactive system can be developed using, for example, a Hewlett-Packard IMAGE Data Base Management System Package.

What Can Be Learned From Such Studies? Even in the extremely limited study here, enough consistency appears in the reduced data to suggest that

Fig. 12.

Nearest neighbor and second nearest neighbor spacings for the grains plotted in Figs. 10 and 11. The upper plot shows the data for the small grains. The lower plot shows the data for the large grains. Average grain dimensions are simply the square root of the average grain areas (in mm^2).

some of the results are significant, in particular the match between the distributions of CLS and Hadley's crack measurements. The similarity of the distributions for both Conway granite samples is satisfying in light of the similarity of the velocity spectra obtained earlier. The mean-free-path and diffusion distances are interesting physical quantities.

The ability to readily establish such parameters has several implications. First, the established measures are statistical parameters. Therefore, one can phrase problems of crack growth in the framework of statistics.

Probability distributions of crack lengths can be established without needing to make a complete identification of all cracks. Such probability distributions of length can be, for example, convolved with the preferred crack spectra to yield most probable distributions of crack-aspect ratios or widths, etc. In this way probability distributions can be used to model the medium in which cracks may grow. That is, they may constrain models of materials in crack growth studies. Data generated by SOCIOLOGY can be used to provide quantitative definitions of other material properties. Although not done here, a measure of tortuosity can be defined as well as measures of crack connectivity, which enter directly into modeling, permeability, and resistivity.

Secondly the problem arises of how such data as generated here can be correlated to, or can be summarized to describe rock structure, or generate predictions of other kinds of rock properties. A formal treatment of this task has not been carried out here, but a simple demonstration follows to show how the data presented in the last section can be synthesized in a form suitable for this task.

In doing this, two analytic questions are apparent and of immediate interest. The first concerns similarity, the second concerns predication. First, what is the measure of similarity between two samples? This is a problem in measuring the grouping or closeness of objects within a multivariant space. Second, can the elastic moduli and other physical properties be predicted from factor analysis of petrostructural variables?

Prediction can be made at two levels. First, what is the measure of similarity between the two samples? This is a problem in measuring the grouping or closeness of objects within a multivariant space. Figure 13a shows the data for CLS for 381.9 plotted against those for 704.4. Figure 13b shows a similar cross plot for the crack spectra. The fairly well-developed diagonal trends indicate the measure of similarity for these two samples over these two variables.

Second, can the elastic moduli and other physical properties be predicted from factor analysis of petrostructure variables? This prediction can be made at two levels. First, one can ask whether the physical properties of any one group of rocks should be similar based on the similarity in their petrostructure. This would be given by a measure of cross-correlation between such plots as 13a and 13b. Figure 14 is illustrative. It is a plot of the

Fig. 13.
Correlation plots of segment lengths and of crack spectra.

Fig. 14.
Correlation plot for four variables.

correlation over three petrostructure variables plus the crack spectra. The basic similarity of the two samples is established.

At the second level, the quantitative behavior of the physical properties are to be predicted. This is not considered in this study, but could be accomplished with the development of a sufficiently large data base.

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APPENDIX A

PETROSTRUCTURE ANALYSIS PHILOSOPHY AND LOGIC

I. GENERAL COMMENTS

In in this study, both physical properties and petrostructure are considered as statistical data. This approach necessitates the demonstration that statistical systematics exist in some class of rock properties. The systematics must occur at a sufficient level of resolution to infer that underlying variables can be found. The systematics in one class of properties must be of a type to reasonably imply correlated systematics and predictable systematics among other variables.

A. Moduli Systematics

In a study which addressed this problem, Warren and Trice (1975, 1977) showed that over a large spectrum of rock types, from competent granites to porous sandstones and breccias, the functional forms of velocity-pressure data exhibit predictable systematics that are characteristic of the texture and fabric of that specific rock type.

Continuing this work, Warren (1979) showed that such systematics can be represented by a theoretical model of crack and pore behavior. Warren and Tiernan (1981) applied the concept to rocks of similar fabric and texture. The study demonstrated that texture and fabric differences between rocks with essentially the same mineralogy and geologic history are reflected in the pressure-dependence of the mechanical properties. We therefore, established that correlations between bulk properties and observable rock structure do exist at a level of significance. The functional forms of static strains (Simmons et al. 1974) and velocity-pressure give measures of microstructure in rocks.

B. Structure Systematics

A second task is to devise a method to economically characterize and parameterize rock structure, so that similar tests can be made and correlated between different rocks.

There are three parts to this second task. These are

1. obtaining input data from rock thin sections,
2. sorting and binning the values of the variables generated from the input data, and
3. developing statistically meaningful tests.

There are four requirements on such an analysis. The parameterization must

1. allow one to group rocks together based on their microstructure and textural patterns;
2. provide a characterization of the crack networks and fabric sufficient to qualitatively associate network features with various mechanical properties; and
3. establish quantitative relations between measurable petrographic features and details of bulk properties.

The methodology of petrostructure analysis is being developed to meet these tasks.

II. BASIC DEFINITIONS AND NOTIONS OF A PETROSTRUCTURE ANALYSIS

A. Definition of Petrostructure

Petrostructure is a modeling space spanned by three variables: the mineralogical, the petrographic, and the microstructural. Into this space the bulk physical and mechanical properties are correlated into classes.

Petrostructure variables fall broadly into two groups:

- (1) compositional
- (2) topographic

Compositional variables (such as, chemistry and mineralogy) are generally quantifiable.

Topographic variables are variables of patterns which are capable of being mapped. Every topographic pattern is composed of some combination of line element networks: open networks with at least one free end, and/or closed networks. Closed networks are line patterns having no free ends. These two classes of patterns are separated by a mapping technique which can preserve the statistics of the pattern geometry.

Mapping variables fall into four classes.

- Nodal variables: point relations such as crack end-points and crack branch-points.
- Linear variables: line features such as lengths of crack segments.
- Angle variables: orientation of cracks and crack-related features, such as the major axes of crack-grains.
- Closure variables: features of crack-grains such as areas and shapes.

B. Mapping and Analysis

For mapping of thin sections, three sets of map parameters have been chosen.

1. Sets of coordinate points (pattern nodes) (x, y).
2. Listings of connected points ($n_1 n_2 n_3 n_4 \dots n_j$).
3. Flag values (F_j) which specify the nature of connectivity between points, or identify, or distinguish a group of points.

C. The Network Form

In the digitized pattern every node has some flag value and connectiveness to its neighbor(s). The degree (n) of a node is equal to its number of immediate connected neighbors, which is denoted by the number $J = n$. The evaluated map parameters are arranged (by computer program) into a table of points, distances, and angles between any one point and up to six of its connected neighbors. The tabulated network is operated on by a number of statistical analysis routines using three specially written major subroutines. These subroutines are based on the intrinsic nature of any network mapped in input format.

D. Major Subroutines

SOCIOLOGY consists of three major subroutines, which operate on the network table. The subroutines are PRUNE, BORDER, AND PSUEDOCLOSURE.

1. PRUNE: This subroutine decomposes any open line network in a step-wise process, and in the process, characterizes the open network by resultant length and angle distributions. Output from PRUNE includes distributions of the node distances, the line segment lengths, cumulative line length per unit area, the angle distributions of the segments, and a distribution of segment length versus angle.
2. BORDER: This subroutine decomposes any remaining network of closed paths. The output consists of grain areas, roundness, aspect ratio, principal axes direction.
3. PSUEDOCLOSURE: This subroutine generates a measure of crack pattern closure. Line segments are extended from the free ends ($J = 1$) to test for intersection with other local line segments. The output of this subroutine is a distribution (length and angle) of new (virtual) line segments.

Additional subroutines, used in conjunction with the three above are:

- TORT: A measure of "tortuosity." This is the distribution of the summed line segments between specified node-types (e.g., $J = 1$, $J = 3$, etc.) divided by the distance between the endpoints of the summed line segments.
- KNEIGH: The calculated distributions of distances between nodes. The nodes need not be connected. The node population can be freely specified and requested ($K = 1$ implies nearest neighbor, etc.).
- JLENG: Gives the distributions of the cumulative line lengths of specifiable subpopulations of the network pattern (e.g., lengths of line segments terminated by two free ends, or by one free end and a forkpoint, etc.).
- INANG: Calculates the interangles of line segments radiating from branch points.

E. Statistical Analysis

Output from SOCIOLOGY is applicable to various levels of statistical analysis. On the first level, the output is an analysis itself in terms of mean values, first variations, and distributions or histograms.

On the next level the data is applicable to Abstract Factor Analysis programs.

APPENDIX B

SUBROUTINE ALGORITHMS

The following summarizes the major algorithms and functional definitions used in the various subroutines of SOCIOLOGY.

A. Definitions:

1. distance: $d_{ij} = \sqrt{(x_i - x_j)^2 + (y_i - y_j)^2}$

2. angle: $\phi = \tan^{-1} (x_i - x_j) / (y_i - y_j)$

where (x_i, y_i) and (x_j, y_j) are two node points in the mapped pattern.

3. crack density: $CD = \text{cumulated line length} / \text{mapped area}.$

$$CD = \sum_{ij} d_{ij} / 2 \cdot \text{area}$$

4. grain area: $GAREA = \oint x dy$ where the integration is done using a trapezoidal rule in which the steps are given by $\Delta x_i = x_i - x_{i-1}$, and $\Delta y_i = y_i - y_{i-1}$.

5. grain shape (roundness):

$$R = 4\pi \cdot \frac{GAREA}{(GP)^2}$$

where GP is the grain perimeter which for an n node grain is given by

$$GP = \sum_i^n \sqrt{\Delta x_i^2 + \Delta y_i^2} + \sqrt{(x_n - x_1)^2 + (y_n - y_1)^2}$$

5a. flatness (inverse roundness):

$$F = (GP)^2 / 4\pi GAREA$$

5b. aspect ratio: This is an alternate definition for flatness, not illustrated in this report. To define aspect ratio, first define DTEST

$$DTEST = \sqrt{(XCG - X_f)^2 + (YCG - Y_f)^2}$$

where XCG, YCG, X_f , Y_f are defined under 6 and 7.

then ASPECT RATIO $\equiv$ AR

$$AR = \pi DTEST^2 / GAREA$$

Comparison of Flatness to Aspect Ratio

Square

$$F = 1.273$$

$$AR = 1.571$$

Circle

$$F = 1.0$$

$$AR = 1.0$$

Ellipse

$$F \approx \frac{1}{2} \frac{a}{b} (1 + b^2/a^2)$$

$$AR \approx a/b$$

where a and b are the major and minor semiaxes respectively.

6. grain center of gravity: $XCG = \frac{1}{GAREA} \oint xydx$

$$YCG = \frac{1}{GAREA} \oint xydy$$

7. principle grain direction:

$$\theta = \tan^{-1} \frac{XCG - x_f}{YCG - y_f}$$

where (X_f, Y_f) is the coordinate of the node furthest from the center of gravity.

APPENDIX C

SUBROUTINE FLOW DIAGRAMS

1. Typical SOCIOLOGICAL; Flow Path

II. PRUNE

III. BORDER

APPENDIX D

THE PROBLEM OF CHARACTERIZING MICROSTRUCTURE

Typically a minimum number of parameters are used to specify the microstructure of a rock in theoretical modeling of rock properties. A crack is specified by its shape, its length, and its width, or an equivalent set of variables. A crack may also be replaced by a model distribution of asperities. In a petrostructure analysis the statistics of the patterns formed by the crack-like features are found. It is of interest to consider the relation of such a characterization to these model variables.

Crack shape: There is no direct measurement of individual crack morphology. Observable classes of crack pattern morphology in combination with velocity-pressure data can in theory be used to infer average closure laws for comparison to theoretical closure laws given by the specification of crack shapes.

Length: Distributions of crack segment lengths and cumulative crack lengths are measured directly. Segment lengths can be determined as a function of resolution for comparison to theoretical crack length distributions.

Width: No direct measurement of crack width is made. However, planar area can be mapped, so, in practice, any resolvable open crack or pore feature can be mapped and analyzed as a "grainlike" feature. For very narrow cracks this is neither practical nor economical. Inferred width distributions can be calculated to be self consistent with the best average closure laws and length distributions. The self-consistent set of variables are testable against observed bulk rock properties.

Porosity: There is no direct measurement of crack-porosity in map parameterization. Any finite density of cracks can be mapped and parameterized. Averages of pore-pore interaction effects are theoretically derivable from the statistics of the crack patterns and the generated best model distributions.

Crack structure is a function both of scale and of resolution. That is, hierarchy is a fundamental aspect of crack patterns which must be accounted for in petrostructure studies. Fractals (Mandelbrot 1979) offer a way of defining hierarchy and characterizing similarity and changes in patterns with scale.

REFERENCES

- Abey, A. E., "A Computer Model for Stress-Volume Relationships of Porous Rocks with Various Saturations," *Rock Mechanics* 13, no. 4, 235-244, March (1981).
- Bennett, H. F., "A Simple Seismic Model for Determining Principal Anisotropic Direction," *J. Geophys. Res.* 77, 3078-3080 (1972).
- Brace, W. F., "Relation of Elastic Properties of Rocks to Fabric," *J. Geophys. Res.* 70, 5657-5668 (1965).
- Brace, W. F., "Micromechanics in Rock Systems" in Structure, Solid Mechanics and Engineering Design; The Proceedings of the Southampton 1969 Civil Engineering Materials Conference, Ed. M. Te'eni, Wiley-Interscience, 187-204 (1969).
- Breedlove, J., "Mini Computer Based Interactive Image Processing" in Proceedings of the First Micro and Mini Computer Conference, November 1978, Houston (1978).
- Butkovich, T. R., "A Technique for Generating Pressure-Volume Relationships and Failure Envelopes for Rocks," Manuscript No. UCRL-51411, Livermore, California: Lawrence Livermore Laboratory (1973).
- Davis, J. C., Statistics and Data Analysis in Geology, John Wiley and Sons, Inc., New York, 550 pp. (1973).
- Friedman, M., and T. R. Bur, "Investigations of the Relations Among Residual Strain, Fabric, Fracture, and Ultrasonic Attenuation and Velocity in Rocks," *International Journal of Rock Mechanics and Mining Sciences* 11, 221-234 (1974).
- Hadley, K., "Comparison of Calculated and Observed Crack Densities and Seismic Velocities in Westerly Granite," *J. Geophys. Res.* 81, no. 20, 3484-3493, July (1976).
- Joreskog, K. G., J. E. Klován, and R. A. Reymont, "Geological Factor Analysis (Methods in Geomathematics I)", Elsevier Scientific Publishing Company, Amsterdam, 174 pp. (1976).
- Mandelbrot, B. B., Fractals: Form, Chance, and Dimension, W. H. Freeman, revised translation, pp 365 (1977).
- Mavko, G. M. and A. Nur, "The Effect of Nonelliptical Cracks on the Compressibility of Rocks," *J. Geophys. Res.* 83, 4459-4468 (1978).

- O'Connell, R. J. and B. Budiansky, "Seismic Velocities in Dry and Saturated Cracked Solids," J. Geophys. Res. 79, no. 35, 5412-5426 (1974).
- Simmons, B., R. W. Siegfried, and M. Feves, "A New Method for Examining Cracks in Rocks," J. Geophys. Res. 79, 4383-4385 (1974).
- Tilman, S. E. and Hugh F. Bennett, "A Sonic Method for Petrographic Analysis," J. Geophys. Res. 78, no. 35, 8463-8469 (1973).
- Trice, R. and N. Warren, "Preliminary Study of the Correlation of Acoustic Velocity and Permeability in Two Granodiorites from the LASL Fenton Hill Deep Borehole, GT-2, Near the Valles Caldera, New Mexico," Los Alamos Scientific Laboratory report LA-6851-MS (July 1977).
- Walsh, J. B., "The Effect of Cracks on the Compressibility of Rock," J. Geophys. Res., 70 381-389 (1965a).
- Walsh, J. B., "The Effect of Cracks on the Uniaxial Elastic Compression of Rocks," J. Geophys. Res. 70, 399-411 (1965b).
- Warren, N., "Rock Physics Characterization of Conway Granite from a DOE Borehole, Conway, New Hampshire," Los Alamos Scientific Laboratory report LA-8102-MS (November 1979).
- Warren, N., "Characterization of Modulus-Pressure Systematics of Rocks: Dependence on Microstructure," Geophysical Monograph 20, Am. Geophys. Union, The Earth's Crust, 119-148 (1977).
- Warren, N., and M. Tiernan, "Systematics of Crack Controlled Mechanical Properties for a Suite of Conway Granites from the White Mountains, New Hampshire," Tectonophysics 73, 295-322 (1981).
- Warren, N. and R. Trice, "Correlation of Elastic Moduli Systematics with Texture in Lunar Material," Proc. 6th Lunar Sci. Conference, Suppl. 6, Geochimica et Cosmochimica Acta 3, 3255-3268 (1975).