ARE THERE BACKGROUND FIELDS THAT CAN INDUCE QED PHASE TRANSITIONS AT WEAK COUPLING?*

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ABSTRACT

The existence of a new, non-perturbative phase of QED is indicated by studies of Schwinger-Dyson equations and lattice calculations. The crucial question is whether the phase transition point can be driven down to $\alpha \sim 1/137$ presumably by appropriate background fields. It appears that magnetic fields potentially can induce such a phase transition. Our investigation is related to our original conjecture that the anomalous e^+e^- events at GSI are due to the decay of a new positronium system formed in the new QED phase which is induced by the electromagnetic fields of the heavy-ions.

INTRODUCTION

Several years ago we made a conjecture concerning the GSI anomalous events. We proposed that the e^+e^- peaks observed at GSI [1] are due to the decay of a bound e^+e^- system formed in a new QED strong coupling phase which is induced by the strong as well as rapidly-varying electromagnetic fields present in the heavy-ion collision experiments. [2-4] Now it appears that recent GSI experiments have put the two-body decay scenario of our conjecture in some jeopardy. But we believe the jury is still out on this question (whether our conjecture is right or wrong). In any case, our conjecture, like many other conjectures, once made, tends to take on a life of its own, in the sense that even if it fails to explain the GSI anomalous events, it has motivated us to look into a more general question. Instead of merely asking whether the electromagnetic background field present in the heavy-ion collisions can induce a QED phase transition, we now ask if there are background fields that can induce such a transition and if so, what kind of background

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fields. Of course we want the phase transition to be physically realizable i.e. at a value of the fine structure $\alpha \approx 1/137 \ll 1$, hence the qualification "at weak coupling" in the title.

THEORETICAL INTERPRETATION OF THE ANOMALOUS GSI EVENTS

Our investigation [2] was motivated by the anomalous e^+e^- events found in heavy-ion collision experiments at GSI. The multiple correlated and narrow-peak structures in electron and positron spectra in the range of $\sim 3m_e$ are relatively Z -independent. At least in some experiments the energies of the e^+ and e^- are roughly equal in the center-of-mass frame and they are emitted back-to-back. Most likely these events are not due to nuclear decay or decay of new neutral elementary particles.[5] But they are, to a certain extent, consistent with two-body decays of neutral objects at rest in the center-of-mass frame.

The multiple structure in the e^+e^- spectra is clearly indicative of the formation of bound states. Now there are not too many objects that are (almost) as light as the electron or positron, and since the end products are just electrons and positrons, the simplest guess is that the constituents of the bound states are e^+ and e^- . But the bound state system cannot be the conventional positronium with a Coulomb potential since they have very different mass spectra. Hence it must be a new type of positronium. These considerations led us [2] (and two other groups [3,4]) to conjecture that the observed e^+e^- peaks at GSI are due to the decay of a bound e^+e^- system formed in a new phase of QED, which is induced by the strong and rapidly-varying electromagnetic fields of the large- Z ions. It is quite conceivable that shortly after the scattering of the heavy ions, in the absence of the strong electromagnetic fields, the new positronium system is left in a metastable vacuum and then it decays to the normal phase via quantum tunnelling into the observed e^+e^- pairs. [2,3] This theoretical scenario has the following attractive features: (1) it explains automatically why the peaks appear only in heavy-ion collisions and not elsewhere; (2) it explains the qualitative features of the experimental data; [6] (3) the proposal is a very economical one since there is no need to postulate any new particles (fields) or new types of interactions. Our proposal is also natural in that it makes use of a new, non-perturbative, "strong"-coupling, and chiral-symmetry-breaking phase of QED already known to exist from studies of the Schwinger-Dyson equations [7-11] and lattice gauge theories [11].

One may entertain the additional assumption that the new QED phase is confining so that to a fair approximation one may calculate the mass spectra of the new confined positronium system by using $V(\vec{r}) = \lambda r$ [2], the linear potential between two charged particles obtained in the (static) strong-coupling regime of lattice QED. To minimize the number of parameters in the problem one may use, for the electron mass, $m_e \approx 0.5$ MeV (although in the new chiral-symmetry breaking phase it is conceivable that the electron has an additional contribution to its mass.) From the observed e^+e^- spectra the mass splitting between the various states is typically ~ 100 keV, hence one gets $\sqrt{\lambda} \sim 200$ keV. The mean size of a typical state is ~ 1000 Fermi.[12] It is to be noted that such a large size appears to be necessary in order not to run into conflict with the existing experimental

bounds on light neutral particles.[5]

Two predictions follow from our interpretation of the GSI anomalous events:

(1) According to us, the strong and rapidly-varying electromagnetic fields in the heavy-ion experiments are responsible for the QED phase transition. So the anomalous e^+e^- peaks in the mass range of $\sim 1.5 - 1.8$ MeV are not expected in e^+e^- elastic scattering.[2] (Note that there is no violation of time-reversal invariance here.) (2) Since the new QED phase is characterized by spontaneous chiral symmetry breaking, we expect that the lightest new positronium state (the analogue of the pion) would be considerably lighter than the other states.[2,3] We encourage the experimentalists to search for it.

STRONG-COUPLING QED

The existence of a new, non-perturbative and strong-coupling phase of QED, the theoretical underpinning of our interpretation of the GSI events (and our on-going investigation), has been shown in two approaches using either (1) the (analytical) Schwinger-Dyson equations [7-11] or (2) the (numerical) lattice gauge calculations. Here we will go over briefly some of the ingredients in the SD equation approach.

The Schwinger-Dyson equations for QED are a set of complicated coupled integral equations. In order to make the problem tractable we will consider the SD equation for the fermion self-energy in the ladder-approximation. In this approximation the photon propagator and the vertices are replaced by the bare ones. With the choice of the Landau gauge the SD equation takes the form

$$\Sigma(p) = m_0 + \frac{3e^2}{(2\pi)^4} \int d^4q \frac{1}{(p-q)^2} \frac{\Sigma(q)}{q^2 + \Sigma^2(q)}$$

where m_0 is the bare fermion mass and we have made a Wick rotation into the Euclidean momentum space. To study the ultra-violet behaviour of the theory let us introduce an ultra-violet cut-off Λ . The integral SD equation can be converted into a differential equation ($x \equiv p^2$, $\alpha \equiv \frac{e^2}{4\pi}$)

$$(x \Sigma(x))' + \frac{3\alpha}{4\pi} \frac{\Sigma(x)}{x + \Sigma^2(x)} = 0$$

satisfying the boundary conditions

$$(x \Sigma(x))' \big|_{x=\Lambda^2} = m_0(\Lambda) \quad (\text{Ultra-violet b.c.})$$

$$x^2 \Sigma'(x) \big|_{x=0} = 0 \quad (\text{Infrared b.c.})$$

By considering large x one can linearize the differential equation. The solutions take on two different forms depending on whether α is smaller or larger than a critical value

$$\alpha_c \equiv \pi/3.$$

For weak coupling ($\alpha < \alpha_c$) one has

$$\Sigma(p) \sim p^{-1 + (1 - \frac{\alpha}{\alpha_c})^{\frac{1}{2}}}$$

with the UV boundary condition given by

$$m_0(\Lambda) \sim \Lambda^{-1 + (1 - \frac{\alpha}{\alpha_c})^{\frac{1}{2}}}$$

so that there is no spontaneous symmetry breaking solution.

For strong coupling ($\alpha > \alpha_c$) one gets

$$\Sigma(p) \sim \frac{1}{p} \frac{1}{(\frac{\alpha}{\alpha_c} - 1)^{\frac{1}{2}}} \sin[(\frac{\alpha}{\alpha_c} - 1)^{\frac{1}{2}} (\log p + \delta)]$$

with the UV boundary condition given by

$$m_0(\Lambda) \sim \frac{1}{\Lambda} \sin[\theta + \tan^{-1}(\frac{\alpha}{\alpha_c} - 1)^{\frac{1}{2}}]$$

where δ is a function of α ($\delta(\alpha) = 0.715$ for $\alpha \approx \alpha_c$) and

$$\theta = \left| \frac{\alpha}{\alpha_c} - 1 \right|^{\frac{1}{2}} (\log \frac{\Lambda}{\Sigma(0)} + \delta)$$

The UV boundary condition with $m_0(\Lambda) = 0$ admits infinitely many spontaneous chiral symmetry breaking solutions given by, for $\alpha \approx \alpha_c$,

$$\theta \approx n\pi - (\frac{\alpha}{\alpha_c} - 1)^{\frac{1}{2}}$$

where $n - 1$ counts the number of nodes in $\Sigma(p)$. For the ground state, i.e., the lowest vacuum energy, [13] $n = 1$ so that $0 < \theta < \pi$ yielding

$$\Sigma(0) = e^{\delta + 1} \Lambda e^{-\pi(\frac{\alpha}{\alpha_c} - 1)^{\frac{1}{2}}}$$

which, for fixed α , is proportional to Λ . But, for a physically meaningful solution the fermion mass $\Sigma(0)$ must remain finite in the large Λ (i.e. continuum) limit. This is possible if α_c is the (Miransky) ultra-violet fixed point[8]:

$$\frac{\alpha(\Lambda)}{\alpha_c} = 1 + \frac{\pi^2}{\log^2(\Lambda/\kappa)}$$

where κ is an infrared mass scale proportional to $\Sigma(0)$.

How good is the ladder approximation? The replacement of the full vertex Γ_μ by the bare vertex γ_μ turns out not to affect our linearized differential equation since we are interested in $\Sigma^2(p^2) \ll p^2$. [11] This can be seen by applying the Ward identity $q^\mu \Gamma_\mu(p, p+q) = S^{-1}(p+q) - S^{-1}(p)$. For large p , $iS^{-1}(p) \rightarrow \gamma^\mu p_\mu + \text{constant}$, so that the Ward identity implies $\Gamma_\mu = \gamma_\mu + O(\Sigma(p)/p)$. How about the quenched approximation of replacing the full photon propagator by the bare propagator? It is harder to estimate its reliability, but numerical works [14] and lattice calculations [11] have shown that the approximation is qualitatively admissible.

The solutions to the SD equation found above give some unexpected "renormalization group" interpretations to non-perturbative QED. The anomalous

dimension of the fermion mass operator defined by

$$\gamma(\alpha) = - \frac{\partial \log m_0(\Lambda)}{\partial \log \Lambda}$$

for $\alpha \approx \alpha_c$ is particularly interesting:

$$\gamma(\alpha) = 1 - \left(1 - \frac{\alpha}{\alpha_c}\right)^{\frac{1}{2}} \quad \alpha < \alpha_c$$

$$\gamma(\alpha) = 1 \quad \alpha > \alpha_c$$

Due to this large value of the anomalous dimension the well-known Appelquist-Carazzone decoupling theorem [15] is violated with the consequence that ultra-violet physics affects infra-red dynamics (e.g. $\Sigma(0)$). In the ladder approximation, the four-fermion operators have twice the dimension of the fermion mass operator, so that $d_{(\bar{\psi}\psi)^2} = 2(3 - \gamma)$ which approaches 4 as $\alpha \rightarrow \alpha_c$. Thus the $(\bar{\psi}\psi)^2$ interactions are relevant operators and can mix with the electromagnetic interactions.[9]

For a consistent analysis of the critical behaviors one is thus led to enlarge the pure QED Lagrangian to include all chiral invariant four-fermion interactions [9] such as those in the Nambu-Jona-Lasinio model (NJL)[16]:

$$\mathcal{L} = \bar{\psi}(i\gamma^\mu D_\mu - \mu_0) \psi + \frac{G_0}{2}[(\bar{\psi}\psi)^2 + (\bar{\psi}i\gamma_5 \psi)^2]$$

where we have now denoted the bare fermion mass by μ_0 . (Note that the vector- and axial-vector four-fermion interactions are also chiral invariant. But a combination of them is, via Fierz transformation, equivalent to the scalar and pseudoscalar combination we have introduced. Also note that the pure vector $(\bar{\psi}\gamma_\mu \psi)^2$ term is irrelevant due to current conservation leading to zero anomalous dimension. On the other hand, terms like $\bar{\psi} \sigma_{\mu\nu} \psi F^{\mu\nu}$ may have to be included.[17]) In the ladder approximation the four-fermion interactions are easy to incorporate into the SD equation since the only extra contribution comes from the fermion tadpole which is independent of the external momentum. Its only effect is to change the bare fermion mass μ_0 to

$$m_0 = \mu_0 - G_0 \langle \bar{\psi}\psi \rangle_0$$

without changing the differential equation. The closed fermion loop $\langle \bar{\psi}\psi \rangle_0$ in the tadpole can be calculated by using the previously found solutions.

In the $\mu_0 = 0$ chiral limit, the gap equation reads [9]

$$\alpha > \alpha_c : \tan \theta = \frac{G+1}{G-1} \left(\frac{\alpha}{\alpha_c} - 1\right)^{\frac{1}{2}}$$

$$\alpha < \alpha_c : \tanh \theta = \frac{G+1}{G-1} \left(1 - \frac{\alpha}{\alpha_c}\right)^{\frac{1}{2}}$$

where $G \equiv G_0 \Lambda^2 \alpha_c / (\pi^2 \alpha)$ is the renormalized four-fermion coupling. For strong coupling ($\alpha > \alpha_c$), the vacuum solution again requires $0 < \theta < \pi$ so that the continuum limit

demands $\alpha \rightarrow \alpha_c$, $G \rightarrow 1$. For weak coupling ($\alpha < \alpha_c$), (surprisingly) nontrivial solutions exist for $G > 1$: in the continuum limit θ becomes infinitely large, yielding the critical line given by [10]

$$\frac{G_0 \Lambda^2}{\pi^2} = [1 + (1 - \frac{\alpha}{\alpha_c})^{\frac{1}{2}}]^2$$

which goes smoothly to the pure NJL limit (more about this later) as $\alpha \rightarrow 0$. It may not be totally crazy to speculate that the critical behaviour for weak electromagnetic coupling at $\alpha \sim 1/137$ (and $G_0 \Lambda^2/\pi^2 \approx 4$) is reasonably approximated by that for the pure NJL model ($\alpha = 0$ and $G_0 \Lambda^2/\pi^2 = 4$, to be shown later). The pure NJL model has the distinct advantage of being amenable to simple analytical analyses involving only algebraic equations. In the next section we will take this speculation seriously and our investigation will concentrate on the NJL model.

The importance of the four-fermion interactions for physical consistency can also be glimpsed by an examination of the renormalization-group β -function. The Miransky solution for pure QED demands a dynamical running of α for $\alpha > \alpha_c$. But in the quenched approximation, the origin of the running of α is quite unclear. On the other hand, the four-fermion coupling in the QED-NJL theory is renormalized even in the ladder approximation. It is the dynamical running of the four-fermion coupling that leads to a finite dynamical fermion mass and hence an infrared meaningful theory when $G \rightarrow 1$ and $\alpha \rightarrow \alpha_c$ in the continuum limit. [9]

Needless to say, in our discussion, the dynamical degree of freedom of the photon is utterly crucial for phase transitions in QED (with or without the NJL terms.) We believe that the dynamical photon also plays an indispensable role in the critical behaviour of QED in background fields. [18]

INDUCING QED PHASE TRANSITIONS BY BACKGROUND FIELDS?

Our theoretical interpretation of the GSI events described above hinges on the speculation that there is a connection between the strong (as well as rapidly-varying) electromagnetic fields and strong-coupling QED. This connection has not been proven so far. To prove it, one will have to show that the QED critical coupling separating the two phases, which is ~ 1 in the absence of background fields, can be driven down to the normal weak value $\sim 1/137$ by the appropriate electromagnetic background fields. At present it is not known what attributes of the background fields are relevant for QED phase transitions.

The more general question we are asking now is whether there are background fields, not necessarily pure electromagnetic ones, which can induce the phase transition point to move towards *weaker* coupling.[19] So far we have examined only QED and the NJL model with some background fields that are easily amenable to analytical analysis.

As the first example let us consider QED with a (small) chemical potential μ , a non-zero value of which signifies the presence of some background e^- (or e^+). This example may have bearing on the interpretation of the GSI events since the heavy ions in the

collisions are presumably surrounded by clouds of electrons. Note that the addition of a chemical potential does not break chiral symmetry by itself. For $\mu \neq 0$, the electron propagator is given by [20]

$$G_\mu(p) = \frac{\gamma p + m}{[p_0 + i\epsilon \text{sign}(p_0 + \mu)]^2 - (\vec{p}^2 + m^2)}$$

where m is the electron mass and ϵ is infinitesimally small but positive. Accordingly the Schwinger-Dyson equation for the electron self energy in the ladder approximation with the choice of Landau gauge is [21]

$$\Sigma_\mu(p) = m_0 - 3ie^2 \int \frac{d^4 q}{(2\pi)^4} \frac{1}{(p-q)^2} \frac{\Sigma_\mu(q)}{[q_0 + i\epsilon \text{sign}(q_0 + \mu)]^2 - \vec{q}^2 - \Sigma_\mu^2(q)}$$

In going to the Euclidean momentum space by a Wick rotation we have to examine more closely the pole structure of the electron propagator in the complex q_0 plane. Poles are located at $q_0 = \pm \sqrt{\vec{q}^2 + \Sigma_\mu^2(q)} - i\epsilon \text{sign}(q_0 + \mu)$. For $0 < |\mu| < \sqrt{\vec{q}^2 + \Sigma_\mu^2(q)}$, i.e., $\vec{q}^2 > \mu^2 - \Sigma_\mu^2(q)$, the usual two poles appear with one pole each in the upper and lower half planes. But for $0 < \vec{q}^2 < \mu^2 - \Sigma_\mu^2(q)$ both poles are found in either the upper (for $\mu < 0$) or lower (for $\mu > 0$) half planes. Thus in performing the Wick rotation ($q_0 \rightarrow iq_0$) one picks up an additional contribution to $\Sigma_\mu(p)$ from the momentum interval $0 < \vec{q}^2 < \mu^2 - \Sigma_\mu^2(q)$ (if such an interval exists). Assuming that this momentum interval is small we can replace $\Sigma_\mu(q)$ in the integrand by $\Sigma_\mu(0)$. For large p , the extra piece of $\Sigma_\mu(p)$ is thus proportional to p^{-2} . One can solve the SD equation for large p following the same procedure as in the $\mu = 0$ case. The differential equation is not affected by the extra piece of $\Sigma_\mu(p)$. Neither is the ultra-violet boundary condition. The only effect is in the infrared region (replacing $\Sigma(0)$ by $\Sigma_\mu(0)$). Hence for this case the critical coupling is not changed.[21]

To gain further insights let us turn to the pure NJL model which, as shown above, is relevant to our study of critical behavior of QED. In the absence of background fields the model, [16]

$$\mathcal{L}_{NJL} = \frac{G_0}{2} [(\bar{\psi}\psi)^2 + (\bar{\psi}i\gamma_5\psi)^2],$$

has nontrivial chiral-symmetry breaking solutions ($m \neq 0$) with the gap equation given by

$$1 = -\frac{G_0 i}{4\pi^4} \int \frac{d^4 p}{p^2 + m^2 - i\epsilon}$$

At this point, we have a choice of two different cut-off regularizations. We will make use of both of them later. If one uses a non-covariant cut-off at $|\vec{p}| = \Lambda$ to regularize the integral, the critical coupling is given by

$$\frac{2\pi^2}{G_o \Lambda^2} = 1$$

One can also adopt a covariant cut-off at $p_{\text{Euclidean}}^2 = \Lambda^2$, then the critical coupling is given by

$$\frac{4\pi^2}{G_o \Lambda^2} = 1$$

which as mentioned before, is nothing other than the $\alpha = 0$ limit of the critical line for the QED-NJL model.

For the NJL model with a chemical potential μ , the gap equation

$$1 = -\frac{G_o i}{4\pi^4} \int d^3 p \int d p_o \frac{1}{- [p_o + i\epsilon \text{sign}(p_o - \mu)]^2 + \vec{p}^2 + m^2}$$

yields the critical coupling

$$\frac{2\pi^2}{G_o \Lambda^2} = 1 - \frac{\mu^2}{\Lambda^2}$$

if we use the non-covariant cut-off. The critical coupling is thus larger than for the $\mu = 0$ case, i.e., with a chemical potential the phase transition point moves towards stronger coupling, consistent with the physical picture that a nonvanishing e^- (e^+) background density repels the e^- (e^+) but attracts the e^+ (e^-) in the e^+e^- condensate making it harder to have e^+e^- condensates the formation of which is necessary for spontaneous chiral symmetry breaking.[21]

For the NJL model in a heat bath ($T \neq 0$), the gap equation is [22]

$$1 = \frac{G_o}{\pi^2} \int dp \frac{p^2}{(p^2 + m^2)^{\frac{3}{2}}} \left[1 - \frac{2}{e^{(p^2 + m^2)/(kT)} + 1} \right]$$

where the non-covariant cut-off has been adopted. A nonvanishing temperature reduces the positive definite integrand, leading to a larger critical coupling. This result is physically reasonable since thermal excitations tend to dissociate the e^+e^- pairs in the condensate.

Finally let us consider the NJL model with constant electromagnetic background field. One can employ Schwinger's proper-time method [23] to cast the gap equation in the form [24]

$$1 = \frac{G_o}{2\pi^2} \int_{\Lambda^{-2}}^{\infty} \frac{ds}{s} e^{-ms} \left\{ (es)^2 \mathcal{G} \frac{\text{Re} [\cosh(esX)]}{\text{Im} [\cosh(esX)]} \right\}$$

where $\mathcal{G} = \vec{E} \cdot \vec{H}$ and $X^2 = (\vec{H} + i\vec{E})^2$ for constant electric field $\vec{E}$ and constant magnetic field $\vec{H}$; here the covariant cut-off has been used to regularize the integral. Four limiting cases can be easily worked out. The quantity inside the braces is 1, $e\text{Escot}(eEs)$, 1, $eHs \coth(eHs)$ respectively for the cases $\vec{E} = \vec{H} = 0$, $\vec{H} = 0$ but $\vec{E} \neq 0$, $\vec{E} \perp \vec{H}$ with $E = H$,

and $\vec{E} = 0$ but $\vec{H} \neq 0$. (Here, e , E and H denote their respective magnitudes.) The critical coupling for the respective cases is given by [24]

$$\frac{4\pi^2}{G_0 \Lambda^2} = 1, \quad 1 - \frac{\pi e E}{2\Lambda^2}, \quad 1, \quad 1 + \frac{eH}{\Lambda^2} \ln \frac{2eH}{n^2}$$

Thus a constant electric field drives the transition point towards stronger coupling while a constant magnetic field has the opposite effect. The electric field effect is consistent with our intuition since an electric field acts on e^+ and e^- in the condensates with opposite forces. It is harder to understand the magnetic field effect. A possible explanation lies in the formation of the mixed condensate $\langle \bar{\psi} \sigma^{\mu\nu} \psi F_{\mu\nu} \rangle$. While chiral symmetry guarantees that it is zero in the perturbative weak-coupling phase, a non-zero value would indicate chiral symmetry breaking. A fermion moving through this mixed condensate acquires an anomalous magnetic moment to which an existing magnetic background field can couple. So it is not unlikely that a magnetic field helps to lower the critical coupling.[25]

SUMMARY

We are not ready to give a definite answer to the question posed in the title. But if one accepts the premise that the critical behavior of QED (supplemented by appropriate relevant operators like those in the NJL model) at small electromagnetic coupling ($\alpha \sim 1/137 \ll 1$) is not too different from that of the pure NJL model, then the example considered above with a pure magnetic background field does give one a glimmer of hope that one can perhaps answer the question affirmatively. Judging from the form of the critical coupling for the pure electric and pure magnetic cases it seems even possible that in some electromagnetic field configurations a relatively small magnetic component can overcome the effect of a larger electric field. For our conjecture in connection with the GSI anomalous events to be correct, this must be the case since in heavy-ion collisions the dominant electromagnetic field is electric. A detailed study is warranted.

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