THE SPHEROMAK AS A PROTOTYPE FOR
ULTRA-HIGH-FIELD SUPERCONDUCTING MAGNETS

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The Spheromak as a Prototype for
Ultra-High-Field Superconducting Magnets

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Abstract

In view of current progress in the development of superconductor materials, the ultimate high-field limit of superconducting magnets is likely to be set by mechanical stress problems. Maximum field strength should be attainable by means of approximately force-free magnet windings having favorable "MHD" stability properties (so that small winding errors will not grow). Since a low-beta finite-flux-hole spheromak configuration qualifies as a suitable prototype, the theoretical and experimental spheromak research effort of the past decade has served to create a substantial technical basis for the design of ultra-high-field superconducting coils.

In view of current progress in the development of superconductor materials,¹ the ultimate high-field limit of superconducting magnets is likely to be set by mechanical stress problems. Maximum field strength should be attainable by means of force-free magnet windings.

The theory of force-free plasma equilibria was first investigated by R. Lüst and A. Schlüter,² and the possibility of applying these ideas to magnet designs was suggested by L. Spitzer, Jr. In the context of the high-field superconductor application, meeting the force-free condition $\vec{J} = \lambda \vec{B}$ has the additional advantage³ of maximizing the critical current density (since $J \equiv J_{\parallel}$).

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In the present paper, we explore the possibility of utilizing a finite-flux-hole spheromak as a prototype to provide the winding law for a superconducting high-field magnet.⁴⁻⁶ The configuration under consideration is illustrated in Fig. 1a. A conventional solenoidal magnet of constant current density per unit height, inside major radius x_c , and half-height z_c provides the bias equilibrium field for a force-free superconducting insert with midplane endpoints x_L and x_R . The area inside the radius $x < x_L$ is the high-field experimental area. The magnetic field in this region is reversed in sign relative to the field of the external solenoid, and is typically amplified severalfold.

To make quantitative calculations, we approximate the discrete finite-size superconducting windings with a continuum limit. The local current density vector in the windings $\vec{J}$ must be related to the local magnetic field $\vec{B}$ by the familiar equations

$$\vec{J} \times \vec{B} = 0, \quad (1)$$

$$\nabla \times \vec{B} = \mu_0 \vec{J}, \quad (2)$$

$$\nabla \cdot \vec{B} = 0, \quad (3)$$

or equivalently,

$$\vec{J} = \lambda \vec{B}, \quad (4)$$

$$\vec{B} \cdot \nabla \lambda = 0. \quad (5)$$

Force-free solutions with $\lambda = \text{constant}$ everywhere were studied in Ref. 2. We begin by examining a model where $\lambda = \text{constant}$ inside the coil volume, but $\lambda = 0$ outside. Using the methods described in Ref. 7, we have computed a two-parameter family of solutions where we let x_L and x_R vary separately while keeping z_c and x_c fixed, both equal to 1.0. The results are presented in Figs. 1b and 1c where we plot the field amplification factor R^* and the field-gradient parameter B_z^* for variations in x_L and x_R . Here, we define

$$R^* \equiv B_z(0, 0)/B_z^e(0, 0) \times B_z^e(x_c, 0)/B_z(x_c, 0),$$

$$B_z^{**} \equiv B_z''(0, 0)/B_z(0, 0),$$

where $B_z(x, z)$ is the field due to the solenoid plus the force-free magnet, and $B_z^e(x, z)$ is the field due to the solenoid alone. In Fig. 2 we plot the flux surfaces of three of the configurations included in Fig. 1.

The trade-offs for this class of solutions are easily recognized from Figs. 1b and 1c. In Fig. 1b, the amplification factor R^* increases and the field gradient parameter B_z^{**} decreases as x_L goes to zero -- but these improvements are made at the expense of the size of the useful experimental area. In Fig. 1c, we see that R^* increases slightly as x_R is decreased from the coil radius x_c , at the expense of increased field gradient B_z^{**} .

To improve the overall usefulness of the configuration, we now drop the requirement that $\lambda = \text{const}$. Figure 3 gives an example of one such configuration, with $\lambda = \Psi^{-1/2}$, where Ψ is the normalized poloidal flux, which is equal to unity on the magnetic axis and zero at the coil boundary. We also allow the current density in the solenoid to vary as a function of z , having minimum current at the midplane and maximum about half way up. This configuration lends itself to elongated solutions with extremely high field uniformity, while also maintaining a high field-ratio R^* and a reasonably sized experimental area. For the configuration shown in Fig. 3, we have $x_L = 0.25$, $x_R = 0.95$, $R^* = 2.7$, and $B_z^{**} \leq 0.1$ -- more than an order-of-magnitude improvement in field uniformity relative to the cases presented in Fig. 1.

Actual coil windings will tend to depart from the ideal equilibrium prescription of Eqs. (4) and (5), because of finite conductor size, as well as imperfect manufacture. Accordingly, the "MHD stability" properties^{7,8} of the coil winding are of interest. Coils with negative Δ' should be intrinsically

stable.⁹ (The existence of low- δ spheromaks in cold-plasma experiments provides a physical demonstration that configurations of this type can maintain themselves in the absence of mechanical rigidity.) More generally, one should be able to count on some help from the strength of materials in resisting the growth of winding perturbations. This is important, because configurations like that in Fig. 3 tend to have less favorable stability against tearing modes than those of Fig. 2. As pointed out in Ref. 10, a spheromak-type configuration in an external magnetic field will always be unstable against "flipping over." This type of gross mode which has been troublesome for plasma spheromaks, could be restrained quite effectively by mechanical means in the case of force-free coils.

We note, finally, that force-free coils other than those of the spheromak type may find useful applications. In the limit of very large aspect ratio R/a , the configuration of Fig. 1a goes over into cylindrical geometry and could serve to provide an external poloidal guide field -- or simply to carry electric power.³ Another option is a hollow spheromak-type coil, to provide an internal toroidal field¹¹ for magnetic confinement of plasma.

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Figure Captions

Fig. 1. (a) magnet geometry, (b) variation of field amplification factor R^* and field gradient parameter B_z^{**} with x_L and (c) x_R .

Fig. 2. Magnetic flux surfaces for (a) $x_L = 0.5$, $x_R = 0.95$, (b) $x_L = 0.15$, $x_R = 0.75$, and (c) $x_L = 0.1$, $x_R = 0.95$. The x -dependence of the magnetic field strength $B_z(x,0)$ is shown with and without the force-free coil insert.

Fig. 3. Elongated configuration with $x = \psi^{-1/2}$. (a) flux surfaces, (b) toroidal current surfaces, (c) toroidal field contours.

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Fig. 1

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Fig. 2

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Fig. 3

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