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INFLUENCE OF RESISTIVITY ON ENERGETIC TRAPPED
PARTICLE-INDUCED INTERNAL KINK MODES

By

H. Biglari and L. Chen

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INFLUENCE OF RESISTIVITY ON ENERGETIC TRAPPED PARTICLE-INDUCED INTERNAL KINK MODES

Hamid Biglari and Liu Chen

Department of Astrophysical Sciences

and

Plasma Physics Laboratory

Princeton University

Princeton, New Jersey 08544

ABSTRACT

The influence of resistivity on energetic trapped particle-induced internal kink modes, dubbed "fishbones" in the literature, is explored. A general dispersion relation, which recovers the ideal theory in its appropriate limit, is derived and analyzed. Implications of the theory for present generation fusion devices such as the Joint European Torus are discussed.

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The last few years have witnessed a great deal of theoretical activity focused on the gross magnetohydrodynamic (MHD) stability of plasma systems containing a high-energy component. The great majority of these studies concerned themselves with the hot particle compressional stabilization of plasmas which would otherwise be MHD unstable. More recently, however, it has been observed that energetic particles can contribute in an altogether unfavorable way to the stability of plasma systems. This was registered in a particularly striking way in the Poloidal Divertor Experiment (PDX) heating runs at Princeton¹ where it manifested itself as periodic, sharp X-radiation bursts, as increased MHD activity on the Mirnov coil system as well as in the form of an enhanced flux of charge exchange neutral particles. These oscillations, which came to be dubbed "fishbones" because of their characteristic skeletal signature on the soft X-ray imaging system, rotate toroidally with a frequency comparable to the precessional drift frequency of the energetic trapped beam ions ($\omega \simeq 20$ kHz). A theoretical explanation of these fluctuations in terms of energetic trapped particle-induced ideal internal kink modes² has shown considerable success in qualitatively reproducing experimental observations. It should be emphasized that the latter is a new and separate branch of the internal kink dispersion relation, owing its existence solely to the presence of hot particles trapped in the region of unfavorable curvature (i.e., the outside of the tokamak). In particular, for a sufficiently large population of suprathermal particles, the size of which depends on the MHD stability of the core plasma, the fishbone instability will assert itself even if the ideal MHD branch of the internal kink is stable. The conclusion is that even though energetic particles, at least at high β (where β is the ratio of kinetic to magnetic pressure) and undergoing rapid precessional motion, can have a stabilizing influence on normal MHD modes when the mode frequency is strongly off-resonant with the hot particle precessional drift frequency³ (i.e., $\omega \ll \omega_{dh} = -i\mathbf{v}_{dh} \cdot \nabla$, where $\mathbf{v}_{dh}$ is the magnetic drift velocity), they become destabilizing when the wave structure starts to resonate with the precessional drift motion of the hot particle banana center ($\omega \simeq \omega_{dh}$).

While the theory has been successful in explaining experimental observations, several issues remain. One is the validity of employing an ideal MHD description for the core plasma. A second issue is that as the modes become damped due to a drop in hot particle β below threshold, the ideal eigenfunction becomes singular. Finally, for low enough hot particle precessional drift frequency, the ideal MHD results will cease to be applicable and nonideal effects

will have to enter consideration.

In recent work,⁴ we have sought to resolve these issues by examining the influence of resistivity on these fishbone instabilities. The present letter reports on our findings. To investigate the resistive analogue of the fishbone instability, the only modification that needs to be done to the theory of Chen *et al.*² is to derive the resistive layer Lagrangian and substitute it for the inertial term, δI , in their Eq. (10). First note that since the internal kink mode is radially global in extent, we can legislate hot particle effects inside the localized resistive layer away, since we shall assume that the energetic particle density is three orders of magnitude lower than that of the core plasma; hot particles, it is noted, have a much larger region to tap in the outer rather than the inner layer. With this consideration in mind, the linearized equation of motion and Ohm's law can be written, respectively, as

$$-4\pi\rho\omega^2\xi = \nabla(\mathbf{B} \cdot \delta\mathbf{B} - 4\pi\xi \cdot \nabla p) + \mathbf{B} \cdot \nabla\delta\mathbf{B} + \delta\mathbf{B} \cdot \nabla\mathbf{B} \quad (1)$$

$$\delta\mathbf{B} = \nabla \times (\xi \times \mathbf{B}) - (4\pi\eta/i\omega c^2) \nabla^2\delta\mathbf{B} \quad (2)$$

We will express all quantities in terms of the distance away from the location of the resistive layer, i.e., $r = r_s$. Thus, $x = r - r_s$ is our independent radial variable. $\mathbf{B} \cdot \nabla \equiv iF \sim F'x$ and in what follows, we assume $r_s\partial/\partial x$ acting on perturbed quantities is large. The relevant ordering is obtained from two physical considerations: that there exist a balance i) among all three terms in the induction equation and ii) between the inertial term and the restoring force arising from the field-line bending term in the equation of motion. This yields $x \sim |\omega| \sim \eta^{1/2}$. With this ordering, the equation of motion, to leading order, becomes

$$\delta B_{\parallel}/B \simeq \beta'\xi_r/2 \quad ,$$

which is just the mathematical statement that perturbed fluid and magnetic pressure balance perpendicular to the field lines. To next order, we have

$$-4\pi\rho\omega^2\xi = iF\delta\mathbf{B}$$

which after some manipulation yields

$$\tilde{\Omega}^2\xi'' = x\psi'' \quad , \quad (3)$$

where $\tilde{\Omega} = -i\omega/\omega_A$, $\omega_A = (1 + 2q_s^2)^{-1/2} v_{A\theta}/r_s$, $v_{A\theta}^2|_{q=1} = (B_{\theta}q'r)^2/(4\pi\rho)$, $\psi = i\delta B_r/(B_{\theta}q')r_s$, and all primes refer to x -derivatives. Note that we have incorporated the toroidal Pfirsch-Schlüter factor, $(1 + 2q_s^2) = 3$, into the definition

of the Alfvén frequency. Ohm's law reads

$$\psi = -x\xi + \psi''/(\tilde{\Omega}S_M) \quad , \quad (4)$$

where $S_M = \tau_R/\tau_A \gg 1$, $\tau_R = 4\pi r_s^2/\eta c^2$, and $\tau_A = \omega_A^{-1}$. The Lagrangian is obtained upon integrating these equations against ξ and ψ

$$\begin{aligned} L_R &= -(\mathcal{B}q'\tau)_s \pi R \cdot 2\pi r_s \tilde{L}_R \\ \tilde{L}_R &= \int_{\tau_s-\Delta}^{\tau_s+\Delta} d\tau (\tilde{\Omega}^2 |\xi'|^2 + x\xi' \psi'') \quad . \end{aligned} \quad (5)$$

Following Coppi *et al.*,⁵ define a new variable χ such that $\xi' = (\chi - \chi_\infty)/\tilde{\Omega}^2$, where χ_∞ is a constant. From Eq. (3), it then follows that $\chi' = x\psi''$ and $\psi = -x \int_x^\infty d\zeta \chi/\zeta^2$. In terms of χ , $\tilde{L}_R$ becomes

$$\tilde{L}_R = \int_{-\infty}^{\infty} \frac{dx}{\tilde{\Omega}^2} \left\{ \left[\left(1 + \frac{\tilde{\Omega}^2}{x^2}\right) \chi - \chi_\infty \right] (\chi - \chi_\infty) + \frac{\tilde{\Omega}}{S_M} \frac{|\chi'|^2}{x^2} \right\} \quad . \quad (6)$$

Varying with respect to ξ , we obtain the Euler-Lagrange equation

$$\frac{\tilde{\Omega}}{S_M} (\chi'' - 2 \frac{\chi'}{x}) - (\tilde{\Omega}^2 + x^2) \chi = -(\frac{\tilde{\Omega}^2}{2} + x^2) \chi_\infty \quad . \quad (7)$$

Equation (7) can be solved for χ to yield⁵

$$\frac{\chi}{\chi_\infty} = 1 - \frac{\Omega^{3/2}}{8\sqrt{2}} \int_0^1 dy y^{\Lambda-1} (1+y)^{1/2} \exp\left(-\frac{1-y}{1+y} S_M^{2/3} \frac{x^2}{2\Omega^{1/2}}\right) \quad , \quad (8)$$

where $\Omega = S_M^{1/3} \tilde{\Omega}$ and $\Lambda = (\Omega^{3/2} - 1)/4$.

L_R can now be obtained upon substituting Eq. (8) into Eq. (6). However, it is much simpler to integrate the Euler-Lagrange equation by parts and obtain the boundary equivalent of Eq. (6). After some manipulation, we obtain

$$\delta L_R = (\mathcal{B}q'\tau)_s \chi_\infty \xi_{-\infty}/16 \quad , \quad (9)$$

where $\xi_{-\infty}$ is the constant value of ξ in the outer region (cf. Fig. 1) and is given by

$$\frac{\xi_{-\infty}}{\chi_\infty} = 2 \int_0^\infty dx \frac{\chi'/\chi_\infty}{x} = \frac{\pi}{8} S_M^{1/3} \Omega^{5/4} \frac{\Gamma[(\Omega^{3/2} - 1)/4]}{\Gamma[(\Omega^{3/2} + 5)/4]} \quad . \quad (10)$$

Finally, the dispersion relation can be schematically written as

$$\delta \tilde{W}_c + \left(\frac{\omega}{\tilde{\omega}_{dh} - \omega} \beta_{h,t} \right) = -S_M^{-1/3} \Omega^{-5/4} \frac{\Gamma[(\Omega^{3/2} + 5)/4]}{\Gamma[(\Omega^{3/2} - 1)/4]} \quad . \quad (11)$$

In Eq. (11), $\delta\tilde{W}_c$ is a normalized version of the usual core plasma ideal MHD potential energy, while the second term is a measure of the energy content of the energetic trapped particles. The reader is referred to Ref. 2 for exact details.

We now proceed to analyze this dispersion relation in various limiting cases of interest. Our classification is premised upon the character of the resonant mode. As an aid in this task, it is convenient to define two resistive temporal scales: a resistive interchange time scale defined by $\omega_R = S_M^{-1/3} \omega_A$ and a resistive tearing time scale defined by $\omega_T = S_M^{-3/5} \omega_A$.

A. IDEAL LIMIT ($|\tilde{\omega}_{dh}| \gg \omega_R$)

1. Resonance ($|\omega| \sim |\tilde{\omega}_{dh}|$)

In the ideal limit where $\Omega \gg 1$, the large argument limit of the gamma functions can be taken to recover the relation of Chen *et al.*²

$$\delta\tilde{W}_c + \frac{5}{4} \frac{\omega_A^2}{S_M \omega^2} + \left(\frac{\omega}{\tilde{\omega}_{dh} - \omega} \beta_{h,i} \right) = i \frac{\omega}{\omega_A}, \quad (12)$$

where we have kept the leading order resistive contribution to the dispersion relation. Recalling that $\delta\tilde{W}_c \sim O(\epsilon^3) \sim 10^{-3}$, we see that the resistive contribution becomes comparable in magnitude to it for $S_M \lesssim 10^7$. Thus, the resistive correction to the ideal dispersion relation affects the growth rate only as an effective $\delta\tilde{W}$ and this contribution is a stabilizing one.

With the inclusion of resistivity, we can resolve another outstanding issue left over from the ideal calculation. The integration of the Euler-Lagrange equation resulting from the variation of the ideal inertial layer Lagrangian gives

$$\left. \frac{d\xi}{dx} \right|_{(1,1)} = \frac{C_0}{x^2 + \tilde{\Omega}^2}, \quad (13)$$

where C_0 is a constant of integration. Note the singularity in ξ' when $x = \pm i\tilde{\Omega}$. The singularity in the eigenfunction gives rise to a continuous spectrum on the real axis of the complex frequency plane for $|\tilde{\Omega}| \leq x$. In fact, the reason that the frequency in the kinetic energy term on the right-hand side of Eq. (12) is of first rather than of second power (as is usually the case in the calculation of the energy principle) is a direct consequence of this singular behavior. In order to be able to relate the unstable solutions to the stable ones across this continuum, we need to

invoke causality. Mathematically, causality shows us how to continue the solution analytically from the unstable to the stable half of the complex frequency plane across the branch cut singularity. To see how the inclusion of resistivity lays these mathematical issues to rest, we asymptotically evaluate the integral in Eq. (8) to obtain

$$\lim_{\eta \rightarrow 0} \xi' = -\frac{\chi_\infty}{2} \left\{ P\left(\frac{1}{x^2 + \tilde{\Omega}^2}\right) + i\pi \frac{1}{2|\tilde{\Omega}|} [\delta(x + i\tilde{\Omega}) + \delta(x - i\tilde{\Omega})] \right\}, \quad (14)$$

where $P(\dots)$ denotes the Cauchy principal value and $\delta(\dots)$ is the Dirac delta function. Equation (14) recovers the ideal result, Eq. (13), and shows how the mode picks up simple poles as it approaches the continuous spectrum from the unstable side. More careful stationary phase analysis of the integral reveals that finite resistivity regularizes the ideal delta function singularity by giving it a characteristic width of $S_M^{-1/3}$. The ideal dispersion relation has been discussed in Ref. 2 and will not be further discussed here.

2. Adiabatic Limit ($|\omega| \sim \omega_R$)

In the limit where the hot particles precess much faster than time scales of interest, they act as a rigid ring and do not interact with the core plasma. Then the energetic particle branch of the dispersion relation is ignorable and the relation of Coppi *et al.*⁵ for the usual resistive internal kink modes is obtained.

B. RESISTIVE INTERCHANGE LIMIT ($\omega_R \gg |\bar{\omega}_{dh}| \gg \omega_T$)

1. Hot Particle-Modified Resistive Internal Kink Mode ($|\omega| \sim \omega_R$)

In this limit, $\delta\hat{W}_k$ reduces to the guiding center energy integral of Kruskal and Oberman⁶ and we again recover the dispersion relation of Coppi *et al.*,⁵ but now with $\delta\hat{W}_c$ replaced by $\delta\hat{W}_c - \langle\beta_{h,t}\rangle$. The hot particles now interact with the core plasma and act as an effective δW , making a (de)stabilizing contribution when trapped in the region of (un)favorable curvature. Noting that $\Delta' \propto (\delta\hat{W}_c - \langle\beta_{h,t}\rangle)^{-1}$, where Δ' is the usual measure of resistive stability,^{7,8} the present limit is valid only close to ideal marginal stability, i.e., $\Delta' \gtrsim S_M^{1/3}$.

2. Resonance ($|\omega| \sim |\bar{\omega}_{dh}|$)

The new regime of interest, as discussed before, is the resistive regime,

where $\Omega \ll 1$. The dispersion relation reduces to

$$\delta\hat{W}_c + \left\langle \frac{\omega}{\bar{\omega}_{dh} - \omega} \beta_{h,t} \right\rangle = A_S e^{i5\pi/8} S_M^{-1/3} \bar{\Omega}^{-5/4}, \quad (15)$$

where $A_S = -(|\bar{\omega}_{dm}|/\omega_R)^{-5/4} \Gamma(5/4)/\Gamma(-1/4) > 0$, $\bar{\Omega} = \omega/|\bar{\omega}_{dm}|$, and $\bar{\omega}_{dm}$ is a measure of the hot particle precessional drift frequency. Writing $\bar{\Omega} = 1 + \Delta\bar{\Omega}$, $\Delta\bar{\Omega} \ll 1$, and using a slowing-down model hot particle distribution function of the form

$$F_{0h} = C_0(r) E^{-3/2} \delta(\alpha - \alpha_0),$$

we obtain after some manipulations

$$\Delta\bar{\Omega} = \exp(i\sigma - \alpha + \delta\hat{W}_c/\langle\beta I_0\rangle) \ll 1,$$

where

$$\begin{pmatrix} \alpha \\ \sigma \end{pmatrix} = \frac{S_M^{-1/3}}{\langle\beta I_0\rangle} \left(\frac{|\bar{\omega}_{dm}|}{\omega_R} \right)^{-5/4} \begin{pmatrix} \cos 3\pi/8 \\ \sin 3\pi/8 \end{pmatrix} \gg 1,$$

i.e., we get a weakly growing instability.

3. Tearing modes ($|\omega| \sim \omega_T$)

In this case, the energetic particle branch is again negligible and we recover the tearing limit of Coppi *et al.*,⁵ i.e.,

$$-i\omega \simeq \delta\hat{W}_c^{-4/5} \omega_T. \quad (16)$$

C. RESISTIVE TEARING LIMIT ($\omega_T \gg |\bar{\omega}_{dh}|$)

1. Hot Particle-Modified Resistive Tearing Mode ($|\omega| \gg |\bar{\omega}_{dh}|$)

This limit is the analogue of case (B.1), where the substitution $\delta\hat{W}_c \rightarrow \delta\hat{W}_c - \langle\beta_{h,t}\rangle$ into Eq. (16) recovers energetic trapped particle-modified tearing modes. The opposite limit of case (B.1), this case is valid only away from marginal stability ($\Delta' \ll S_M^{1/3}$).

2. Resonance ($|\omega| \simeq |\bar{\omega}_{dh}|$)

Same as in case (B.2).

Finally, we investigate the threshold condition in more detail. Adopting a slowing-down model as before, the dispersion relation corresponding to case

(B.2), i.e., Eq. (15), can be written as

$$D(\bar{\Omega}) = \delta\hat{W}_c + \langle\beta_{h,t}I_0\rangle_{cr}\bar{\Omega}\ln(1 - \frac{1}{\bar{\Omega}}) - A_S e^{i5\pi/8} S_M^{-1/3} \bar{\Omega}^{-5/4} = 0 \quad (17)$$

The qualitative features of Eq. (17) can be analyzed with the aid of Nyquist diagrammatic techniques. For the case of no hot particles then, depending on whether $\delta\hat{W}_c$ is greater or lesser than $A_S S_M^{-1/3} \sin(3\pi/8)$, one unstable root corresponding to the usual resistive internal kink mode will assert itself. If we now include hot deeply trapped particles, then an instability will only assert itself if two conditions are met: first, $\text{Im}D(\bar{\Omega})$ has to change sign for some $\bar{\Omega}_c$ such that $0 < \bar{\Omega}_c < 1$ and second, $\text{Re}D(\bar{\Omega}_c) > 0$. These two conditions impose upper and lower bounds on $\beta_{h,t}$. Moreover, the Nyquist analysis reveals that resonant destabilization will be promoted only if the hot particles are deeply trapped in a region of adverse curvature, so that they undergo unfavorable drifts, i.e., $\hat{\omega}_h/\hat{\omega}_{dh} > 0$. Quantitatively, we can calculate threshold conditions near marginal stability, i.e., $\bar{\Omega} = \bar{\Omega}_{cr} + i0^+$

$$\begin{aligned} \langle\beta_{h,t}I_0\rangle_{cr} &= S_M^{-1/3} A_S \frac{\sin 3\pi/8}{\pi} \bar{\Omega}_{cr}^{-9/4} \\ -\delta\hat{W}_c &= S_M^{-1/3} A_S \frac{\sin 3\pi/8}{\pi} \left[\pi \ln\left(\frac{1}{\bar{\Omega}_{cr}} - 1\right) + \cot \frac{3\pi}{8} \right] \end{aligned} \quad (18)$$

A rough estimate of the instability window alluded to above gives

$$2^{9/4} \gtrsim \langle\beta_{h,t}I_0\rangle_{cr}/S_M^{-1/3} A_S \gtrsim 1 \quad ,$$

and the growth rate just above threshold can be calculated to yield

$$\gamma \simeq |\bar{\omega}_{dm}| \left(\langle\beta_{h,t}I_0\rangle / \langle\beta_{h,t}I_0\rangle_{cr} - 1 \right) \quad .$$

Finally, since [as is clear from Eq. (18)] $\bar{\Omega}_{cr}$ is bounded between 0.5 and 1 for all positive values of $\delta\hat{W}_c$, we can compare the order of magnitude of the ideal and resistive thresholds

$$\langle\beta_{h,t}I_0\rangle_{cr}^{\text{resistive}} \sim (\omega_R/|\bar{\omega}_{dm}|)^{9/4} \langle\beta_{h,t}I_0\rangle_{cr}^{\text{ideal}} \quad (19)$$

Note that the ideal and resistive theories match up for $|\bar{\omega}_{dm}| \sim \omega_R$. The instability mechanism has the character of a negative energy/dissipation trapped particle precession mode in the ideal region which becomes positively dissipated at the inertial layer. The inclusion of resistivity acts to enhance this sink mechanism so that it becomes harder to drive the instability.

We remark that the preceding analysis has ignored average toroidal curvature effects. From the classic work of Glasser, Greene, and Johnson on resistive instabilities,⁸ it is known that the validity of this approximation hinges on the condition $D_{Rc} < \Omega$, where $D_{Rc} \sim O(\epsilon\beta_{pc})^2$ represents average curvature effects and is the usual measure of resistive stability. This condition is not satisfied in the deeply resistive regime of our calculation, i.e., $\Omega \ll 1$. Straightforward generalization of our dispersion relation to include finite- D_{Rc} effects will result in $\Omega^{5/4} \rightarrow \Omega^{5/4}(1 - D_{Rc}/\Omega^{3/2})$ on the right-hand side of Eq. (15). Using the same arguments as before, it can then be shown that favorable average curvature effects, i.e., $D_{Rc} < 0$, such as exist in tokamaks, have a stabilizing effect on the mode and further enhance the threshold condition.

To wit, we have derived a general dispersion relation, i.e., Eq. (11), which governs energetic trapped particle-induced internal kink modes in low- β plasmas valid for all relative orderings of $|\bar{\omega}_{dm}|$ and ω_R . Note that even for devices such as PDX, where $\omega_A \sim 10$ MHz and $|\bar{\omega}_{dm}| \simeq 100$ kHz, the validity of taking the ideal limit of Eq. (11), which corresponds to the theory of Ref. 2 is, at best, only marginal. For present generation tokamaks of the size of JET, due to a larger size and stronger magnetic field strength, the precessional drift frequency (~ 1 kHz) actually drops below what we have defined as the resistive interchange frequency (~ 100 kHz). Premature extrapolation from the ideal theory, which predicts $(\beta_{h,t} I_0)_{cr} \sim |\bar{\omega}_{dm}|/\omega_A$, would have led us to the pessimistic conclusion that the fishbone is virulently unstable in such present generation devices. However, the resistive limit of Eq. (11), namely Eq. (15), shows that quite to the contrary, the threshold is actually raised by dissipation to such an extent that it becomes prohibitive for these "resistive fishbones" to assert themselves (cf. Fig. 2).

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FIGURE CAPTIONS

Fig. 1 The form of the minimizing displacement eigenfunction.

Fig. 2 Schematic plot of the threshold hot particle beta as a function of $|\bar{\omega}_{dm}|/\omega_R$.

Fig. 1

Fig. 2

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