THE REAL OPTICAL- AND SHELL-MODEL POTENTIALS⁺

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ABSTRACT

From fits to neutron scattering data over a wide range of nuclei it is shown that r_v , the reduced radius of the real optical-model potential, decreases with increasing A . The value of the isovector part of the real potential is discussed and a simple argument is given for its magnitude. The dispersion relationship and the method of moments are used to extrapolate the scattering potential to the bound-state regime. The possibility of deducing the spin-orbit strength from the observed single-particle binding energies is discussed.

Over the past few years the elastic scattering of neutrons with incident energies of 1.5 - 10 MeV has been studied extensively in this laboratory using ^{51}V , ^{58}Ni , ^{59}Co , ^{89}Y , Zr , ^{93}Nb , In and ^{209}Bi targets. (Where the mass numbers are not shown, elemental samples were used.) The data obtained were analyzed using the spherical optical model (SOM) assuming Woods-Saxon and derivative Woods-Saxon shapes for the real and imaginary potentials, respectively, and a Thomas term for the spin-orbit interaction. The parameters describing the real Woods-Saxon well for 8 MeV incident neutrons¹ are shown in Table I. From this table it is clear that the reduced radius, r_v , ($R = r_v \cdot A^{1/3}$) is decreasing as A increases, and if we parameterize this decrease by assuming that

$$r_v = r_0 + r_1/A^{1/3}, \quad (1)$$

the values shown in column 5 of Table I are obtained when

$$\begin{aligned} r_0 &= 1.148 \text{ fm} \\ r_1 &= 0.442 \text{ fm.} \end{aligned} \quad (2)$$

These values are similar to those found by Moldauer² from considerations of lower-energy neutron processes.

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Table I. Values of r_v , a_v and J_v for the real SOM Woods Saxon potential used in describing scattering of 8 MeV neutrons from the targets of column 1. Columns 5 and 6 (Systematics) show the values obtained using Eqs. 1, 2 and 5.

Target	SOM Fits			Systematics	
	r_v^b	a_v^b	J_v^c	r_v^b	J_v^c
^{51}V	1.268	0.615	440.4	1.267	451.3
^{58}Ni	1.254	0.646	462.9	1.262	461.7
^{59}Co	1.262	0.636	454.8	1.262	448.6
^{89}Y	1.240	0.703	424.5	1.247	423.9
^{90}Zr	1.259	0.667	431.9	1.247	426.6
^{93}Nb	1.250	0.700	426.3	1.246	423.7
^{115}In	1.234	0.640	400.1	1.239	410.1
^{209}Bi	1.220	0.700	385.5	1.222	380.9

^b Radii and diffusenesses in fm.

^c Volume-integrals-per-nucleon in MeV fm^3 .

Turning to the diffusenesses, given in column 3 of Table I, there is some slight evidence that this quantity increases with increasing A . However, the improvement in χ^2 over that obtained when the average value, $a_v = 0.663$ fm, is used is not significant.

Finally, the values of J_v , the volume integral per nucleon of the real potential, show a very distinct decrease with increasing A . Since the SOM potential is expected to have an isovector component³, one might try fitting these values assuming

$$J_v = J_0(1 - \xi \cdot (N-Z)/A), \quad (3)$$

where J_0 is a constant and ξ is the relative strength of the isovector potential.

An excellent fit to the data is obtained with $J_0 = 486.8 \text{ MeV fm}^3$ and $\xi = 1.04$. These parameters are quite similar to the 8 MeV values obtained by Holmqvist and Wiedling⁴ ($J_0 = 480 \text{ MeV fm}^3$, $\xi = 0.98$), and to the 11.1 MeV

conclusions of Ferrer et al.⁵ ($J_0 = 495 \text{ MeV fm}^3$, $\xi = 0.95$). On the other hand, the nucleon–nucleon scattering data⁶ indicates that ξ should be ≈ 0.48 , and the (p,n) results are consistent⁷ with a value of 0.4. The global SOM fits of Walter and Guss⁸ and of Rapaport⁹ give $\xi = 0.32$ and $= 0.42$, respectively, at 8 MeV. Thus, in order to fit the rapid A–variation of J_v one needs a value of ξ much larger than deduced from other considerations.

The volume–integral–per–nucleon of the Woods–Saxon potential is given by the expression¹⁰

$$J_v = \frac{4\pi}{3} \cdot r_v^3 \cdot V_0 \cdot \left[1 + \left[\frac{\pi \cdot a_v}{R} \right]^2 \right], \quad (4)$$

where $R = r_v \cdot A^{1/3}$. However, according to Eq. 1, r_v decreases with increasing A. Thus part of the rapid decrease in J_v with mass number observed experimentally can be attributed to the decrease in r_v . To take this into account we parameterize J_v by the expression

$$J_v = K_0 \cdot (1 - \xi \cdot (N-Z)/A) \cdot (r_0 + r_1/A^{1/3})^3. \quad (5)$$

When r_0 and r_1 have the values given by Eq. 2, one finds $K_0 = 234.2 \text{ MeV}$ and $\xi = 0.53$. This parameterization yields a value of ξ similar to those obtained by other considerations and, moreover, the predicted J_v , shown in the last column of Table I, are in good agreement with experiment.

That a value of ξ in the neighborhood of 0.3 – 0.5 should be observed can be deduced from very simple considerations. Let us assume that the SOM potential arises from the interaction of the incident nucleon with the constituent nucleons of the target. From experiment, we know that the range of nucleon–nucleon interaction is small compared to nuclear dimensions. Therefore, in first approximation, we assume that this interaction is a delta function, and from this we estimate the isospin dependence of the SOM. Since the exclusion principle does not get in the way when one of the Z protons in the nucleus interacts with an incident neutron, there are Z possible pairs. On the other hand, the number of target neutrons with which the projectile can interact is only N/2; i.e., the Pauli Principle demands that if the two neutrons sit on top of each other spatially, their spins must be oppositely oriented. Thus the SOM for the neutron will be given by

$$\begin{aligned} V &= U_0 \cdot Z + U_0 \cdot N/2 \\ &= \frac{3}{4} \cdot U_0 \cdot (N+Z) - \frac{1}{4} \cdot U_0 (N-Z) \\ &= \frac{3}{4} \cdot U_0 \cdot A \cdot \left(1 - \frac{1}{3} \cdot (N-Z)/A \right), \end{aligned} \quad (6)$$

where $A = N + Z$ is the total number of target nucleons and U_0 is the product of the potential strength and the probability that two nucleons sit on top of each other. A similar argument can be made for an incident proton, the only difference being that $N \rightarrow Z$ so that the coefficient $-1/3$ becomes $+1/3$. Furthermore, if the nucleon wave function in the nucleus is uniformly spread over the nuclear volume, the normalization factor will be proportional to $A^{-1/2}$. Thus the product $U_0 A$ in Eq. 6 is expected, and indeed found, to have a very weak A dependence. Therefore, the coefficient of the isovector part of the SOM, that is ξ , should be about $1/3$. Thus the observed magnitude of ξ is a direct consequence of the short-range nature and nearly equal strengths of (n,p) and (n,n) free-nucleon interactions.

We now examine what can be said about the shell-model potential by the use of the scattering data. There is a well known dispersion relationship linking real and imaginary optical potentials¹¹

$$V(r,E) = V_{HF}(r,E) + \frac{P}{\pi} \cdot \int_{-\infty}^{+\infty} \frac{W(r,E')}{E-E'} \cdot dE', \quad (7)$$

where $V(r,E)$ is the total real OM interaction, $V_{HF}(r,E)$ is its Hartree-Fock component, P is the principal-value integral, and $W(r,E)$ is the absorptive potential. The same dispersion relationship holds for the radial moments of the potentials, so that

$$\langle r(E)^q \rangle_v = \langle r(E)^q \rangle_{HF} + \frac{P}{\pi} \cdot \int_{-\infty}^{+\infty} \frac{\langle r(E')^q \rangle_w}{E-E'} \cdot dE', \quad (8)$$

where, for example,

$$\langle r(E)^q \rangle_w = \frac{4\pi}{A} \cdot \int W(r,E) \cdot r^q \cdot dr. \quad (9)$$

In general the radial moments of the imaginary potential can be fairly well parameterized by an expression of the form

$$\langle r(E)^q \rangle_w = \frac{C_q (E - E_F)^2}{(E - E_F)^2 + D_q^2}, \quad (10)$$

where C_q and D_q are constants to be fitted to the various moments and E_F is the Fermi energy. In addition, one expects $V_{HF}(r,E)$ to be a smooth function of the energy, and it is reasonable to approximate its moments by

$$\langle r(E)^q \rangle_{HF} = A_q + B_q \cdot E. \quad (11)$$

When Eqs. 10 and 11 are substituted into Eq. 8, an analytic expression can be obtained for the various moments, $\langle r(E)^q \rangle_v$, of the total real potential,

$$\langle r(E)^q \rangle_v = A_q + B_q \cdot E + \frac{C_q \cdot D_q \cdot (E - E_F)}{(E - E_F)^2 + D_q^2}. \quad (12)$$

To be specific, we shall apply the foregoing considerations to the case of ^{51}V . For this nucleus high quality data are available¹² over the energy range 1.8 – 11.1 MeV. These data were fitted using the SOM defined earlier. The radii, strengths and diffusenesses found from this fitting were used to calculate the radial moments of the real and imaginary SOM potentials. Mahaux and Sartor¹³ have studied which moments are most stable and have concluded that the best results are obtained when q , in Eq. 8, takes the values 0.8, 2 and 4. For these values, the calculated $\langle r(E)^q \rangle_w$ were parameterized by Eq. 10, and in this way values of C_q and D_q were obtained. Finally, the parameters A_q and B_q were deduced using Eq. 12 to fit the moments $\langle r(E)^q \rangle_v$ obtained from the real potential.

If one now assumes that Eq. 12 holds for all values of E and , further, takes $V(r,E)$ to be a Woods–Saxon potential, the three moments of the interaction can be used to determine the strength and geometry, V_0 , r_v and a_v , for energies outside the 1.8 – 11.1 MeV region. This provides a way of estimating the bound–state potential to be used in shell–model calculations.

The variations in the strength and geometry of the real Woods–Saxon well for ^{51}V are shown in Fig. 1. For this nucleus there are six shell–model single particle states for which reasonable estimates of the binding energy can be made, and these are listed in Table II. Using these experimental values, one can obtain from Eq. 12 the predicted V_0 , r_v and a_v which should reproduce the binding energies. In column 3 of Table II we list the values actually obtained when these Woods–Saxon parameters are used. These binding energies, which are based upon the assumption that the spin–orbit interaction has the Thomas form, and the same strength as used in the analysis of the scattering data¹²,

$$\begin{aligned} V_{so} &= 8.36 \text{ MeV} \\ r_{so} &= 1.00 \text{ fm} \\ a_{so} &= 0.65 \text{ fm}, \end{aligned} \quad (13)$$

are somewhat less than experiment for the particle states, whereas two of the three hole states are predicted to be overbound. The rms deviation between predicted and observed values is 650 keV.

Fig. 1. The solid lines are the predicted variation with energy of V_0 , r_v and a_v for a Woods-Saxon potential. These result when the $q = 0.8, 2$ and 4 moments of the real potential, obtained from the best four-parameter fit to the experimental data, are approximated by Eq. 12. The "O" symbols at positive energies are the values obtained from the fitting of the experimental scattering data. The Fermi energy, $E_F = -9.68$ MeV, is also shown.

The fit to the elastic-scattering and total cross section data is not very sensitive to the spin-orbit potential. Thus, in the absence of low-energy polarization data for ^{51}V , the strength of this interaction, Eq. 13, could be, and was, somewhat arbitrarily chosen. On the other hand, the binding energies of all but the $s_{1/2}$ level are quite sensitive to this force. Therefore, for nuclei with none or only sketchy polarization data, the spin-orbit strength can be estimated by optimizing the single-particle binding energies. If this is done the optimum value of V_{so} , assuming that r_{so} and a_{so} are fixed at the values of Eq. 13, is 6.6 MeV, which is quite close to the global values.^{8,9} The

rms deviation between theory (shown in column 4 of Table II) and experiment is reduced to 340 keV.

Table II. Single-particle and single-hole binding energies in ^{51}V . The second column gives the experimental estimate, and the third and fourth the predicted results when V_0 , r_v and a_v have the values given by Eq. 12 at the experimental binding energies. The spin-orbit interaction used in column 3 is given by Eq. 13, whereas in column 4 $V_{so} = 6.6$ MeV. In column 5 results are listed when r_v and a_v are given by Eq. 14, and V_0 is adjusted so as to reproduce the J_v values (Eq. 12 with $q = 2$) obtained from the analysis of the scattering data.

State	Binding Energy (MeV)			
	Exp.	$V_{so}=8.36$	$V_{so}=6.6$	$V_{so}= 6.6$
$p_{1/2}$	-5.5	-5.38	-5.65	-5.94
$f_{5/2}$	-6.0	-5.26	-6.14	-5.98
$p_{3/2}$	-7.3	-7.18	-6.99	-7.23
$f_{7/2}^{-1}$	-11.1	-12.26	-11.48	-10.23
$d_{3/2}^{-1}$	-15.1	-14.55	-15.35	-14.16
$s_{1/2}^{-1}$	-15.2	-15.79	-15.79	-14.89

The possibility of determining the spin-orbit interaction from the single-particle binding energies depends on the sensitivity of these quantities to the parameters characterizing the Woods-Saxon well. To investigate this sensitivity let us assume that r_v and a_v are constant for negative energies and

have the values given by the scattering analysis when $E = 0$. For ^{51}V this implies that

$$\begin{aligned} r_v &= 1.34 \text{ fm} \\ a_v &= 0.574 \text{ fm.} \end{aligned} \tag{14}$$

Generally, the scattering data lead to a rather smooth energy variation of J_v and J_w , the volume integrals per nucleon of the real and imaginary SOM potentials. Therefore, we will use the negative energy values predicted by Eq. 12 with $q = 2$, together with Eq. 14, to determine the depth of the bound-state potential. Under these conditions the binding energies shown in the last column of Table II are obtained when $V_{so} = 6.6$ MeV. As can be seen

from Fig. 1, the dispersion relation approach using $q = 0.8, 2$ and 4 moments of the SOM potential gives $V_0 \approx 43.3$ MeV and $r_v \approx 1.37$ fm for the $p_{1/2}, f_{5/2}$ and $p_{3/2}$ particle states, whereas when Eq. 14 is used the constraint that J_v be the same for the two approaches implies that V_0 be about 2 MeV larger. Despite these differences in V_0 and r_v , the calculated binding energies for the particle states are quite similar for the two approaches. On the other hand, when Eq. 14 is used, the $f_{7/2}, d_{3/2}$ and $s_{1/2}$ hole states are all less tightly bound by about 1 MeV. From Fig. 1 it is clear that for these states the radii used in the two different approaches are almost identical. However, a_v given by Eq. 14 is more than 10% larger than the values shown in Fig. 1, and as a consequence V_0 will be smaller than shown in the figure. Thus binding energies are quite sensitive to details of the Woods-Saxon potential. Consequently, before determination of the spin-orbit strength can be made using the bound-state data, one must check whether or not the parameters describing the Woods Saxon well depend significantly on which moments of the potential are used in the extrapolation to negative energies.

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