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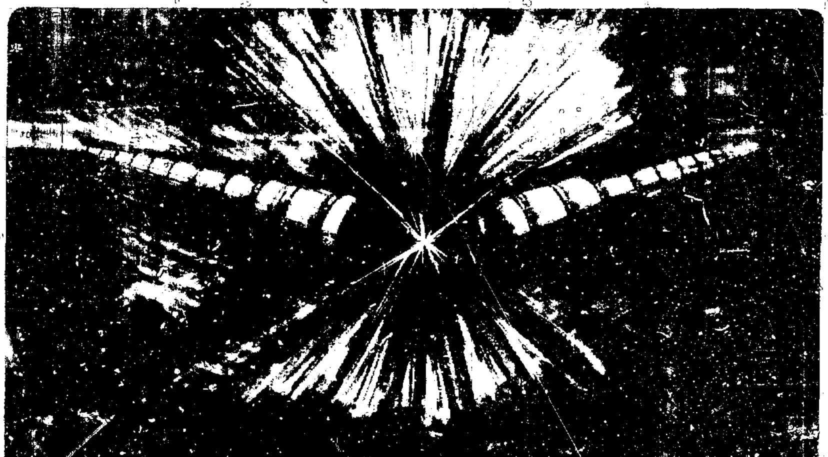

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A Crossed Undulator System for a Variable Polarization

Synchrotron Radiation Source

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Abstract: A crossed undulator system can produce synchrotron radiation whose polarization is arbitrary and adjustable. The polarization can be linear and modulated between two mutually perpendicular directions, or it can be circular and can be modulated between right and left circular polarizations. The system works on low emittance electron storage rings and can cover a wide spectral range. Topics discussed include the basic principle of the system, the design equations and the limitations in performance.

(I) Introduction

Magnetic structures in modern storage rings are intense sources of radiation[1], called synchrotron radiation, which covers a wide region of the photon energy spectrum. However, the applications of synchrotron radiation to polarization sensitive experiments have been limited because the polarization state of the synchrotron radiation could not be changed readily. The crossed undulator system [2]

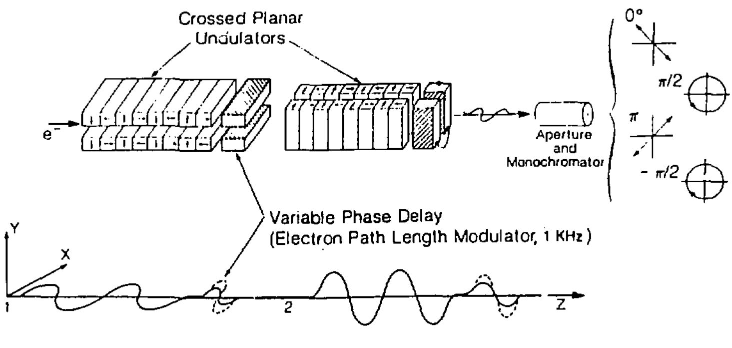

Fig. (1) Operating Principle of the Crossed Undulator System discussed here removes such limitations and offers the possibility of complete polarization control. Thus, the polarization can be rapidly modulated between, for example, left circular and right circular.

After describing the basic principle of the operation in Section (II), various formulas useful in designing the system are derived in Section (III). In Section (IV) the performance of the system is studied, taking into account the electron beam angular spread and also the finite bandpass of the monochromator. Section (V) gives an example system design based on the VUV ring at NSLS. Finally, Section (VI) contains some concluding remarks.

(II) Basic Principle

The operation of the crossed undulator system is illustrated in Fig. (1). An electron oscillates in the x-direction as it passes through the first undulator and radiates photons linearly polarized in the x-direction. In passing through the second undulator, the oscillation is along the y-direction with the resulting radiation polarized along the same direction. The radiations, when observed through a monochromator, combine to give rise to an elliptical polarization vector

$$\underset{\sim}{e} = \frac{1}{\sqrt{2}} (\hat{x} + e^{i\phi}\hat{y}) \quad (1)$$

Here $\hat{x}$ and $\hat{y}$ are unit vectors along the x and y directions, respectively, and ϕ is the relative phase between radiations from the two undulators. The phase ϕ is controlled by a variable field magnet called the modulator (shaded block in Fig. (1), which introduces electron path length modulation. By a proper adjustment of ϕ, it is possible to obtain linear, circular or a general elliptical polarization.

The crossed undulator system produces two wave trains, one from the first undulator polarized along the x-direction, the other from the second undulator polarized along the y-direction. These two wave trains are separated in space. For our purpose, these waves need to interfere to produce an elliptically polarized wave. The superposition of the two waves can be achieved by the action of a monochromator[3], which is to stretch a short pulse of radiation into a long wave train of $n = \lambda/\delta\lambda$ periods, where $\delta\lambda$ is the monochromator bandpass. This is illustrated in Fig. (2.a). When the bandpass is small so that $n \gg N$, where N is the number of undulator magnet periods, the waves from the two undulators overlap each other after passing through the monochromator and become a single wave of an elliptical polarization. This is illustrated in Fig. (2.b). In this way, one sees that the degree of polarization obtainable in this scheme is limited by the monochromator bandpass employed. A more precise discussion of this and other limitations on the performance of the system will be discussed later in this paper.

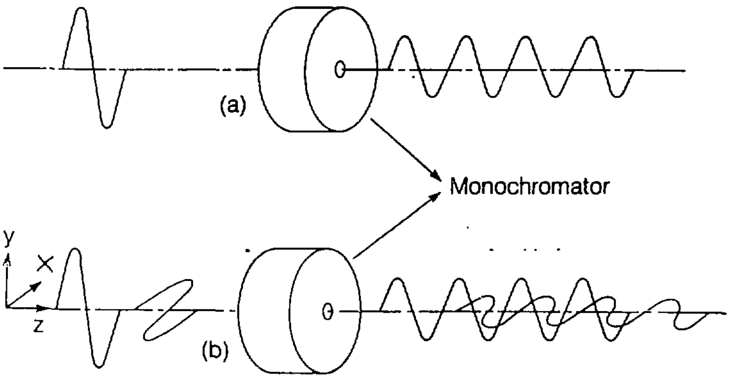

Fig. (2) Monochromater Action

(III) Analysis and Design of a Crossed Undulator System

A. General Analysis

From standard electromagnetic theory[4], the number of photons radiated into the forward direction per unit solid angle by an electron during its motion through a crossed undulator system is given by

$$\frac{dN}{d\Omega} = \alpha\frac{\delta\lambda}{\lambda}\left|\underset{\sim}{\epsilon}\right|^2 , \tag{2}$$

$$\underset{\sim}{\epsilon} = \frac{1}{\lambda}\int_{z_1}^{z_3} dz\ \underset{\sim}{\beta}(z)e^{ik\int_{z'}^{z}(1-\beta_z(z'))dz'} \tag{3}$$

Here α is the fine structure constant (= 1/137), $\delta\lambda$ is the bandpass, λ is the photon wavelength, $k = 2\pi/\lambda$, and

$$\underset{\sim}{\beta} = (\underset{\sim}{\beta}_{\perp}, \beta_z) = (\beta_x, \beta_y, \beta_z)$$

is the electron's velocity divided by the velocity of light. The coordinate system here is the one shown in Fig. (1). The limits of integration z_1 and z_3 in Eq (3) are the z coordinates of the entrance to the first undulator and the exit of the second undulator, respectively.

The second undulator is assumed to be identical to the first except that its orientation is rotated by 90° relative to the first one. The radiation vector ϵ can then be expressed in terms of those quantities involving the first undulator only as follows:

$$\underset{\sim}{\epsilon} = (\hat{x} + e^{i\phi}\,\hat{y})\ \epsilon_0 . \tag{4}$$

$$\phi = \frac{2\pi}{\lambda} \int_{z_1}^{z_2} dz \, (1 - \beta_z(z)), \tag{5}$$

$$\varepsilon_o = \frac{1}{\lambda} \int_{z_1}^{z_2} dz \, \beta_x(z) e^{ik\int_{z_1}^{z} (1-\beta_z(z'))dz'}, \tag{6}$$

Here z_2 is the z-coordinate of the entrance of the second undulator. ϕ in Eq (4) specifies the polarization ellipse and was introduced in Eq (1).

B. Design Objectives

For a satisfactory polarization control, the crossed undulator system needs to satisfy the following requirements:

i) The radiation spectrum has a peak at the desired photon energy.

ii) The polarization can be modulated rapidly between two arbitrary states, e.g., the left circular and the right circular polarizations.

iii) The radiation intensity remains unchanged as the polarization modulates.

Corresponding to these three capabilities, each undulator of the crossed undulator system will be assumed to consist of three distinct magnetic parts. The first part serves the function (i) in the above,

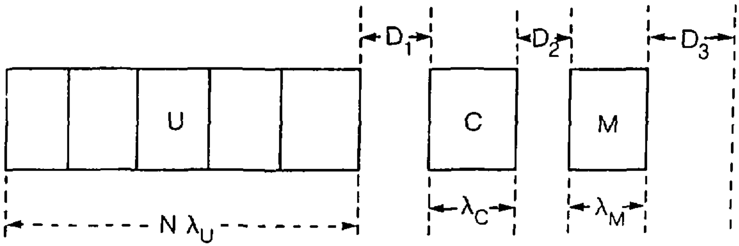

Fig. (3) Three Parts of Undulator

and controls the overall radiation intensity. It will be characterized by a subscript u, and has N periods with period length λ_u. The second part, called the corrector, serves the function (iii) and is a single period magnet of length λ_c. The third part, called the modulator, serves the function (ii) and is a single period magnet with period length λ_m. These are shown in Fig. (3), in which the distances D_1, D_2, and D_3 are also specified. Here D_3 is the distance between the exit of the modulator to the entrance of the second undulator.

The magnetic field within each part will be assumed to be sinusoidal with peak field B_i ($i = u, c, m$). One also defines the magnet strength parameter K_i via

$$K_i = .934\ \lambda_i\ (\text{cm})\ B_i\ (\text{Tesla}). \tag{7}$$

The parameter K is useful in characterizing the electron trajectory in undulator fields.(5)

C. Derivation

With the above assumptions, electron motion in the undulator is completely specified and one obtains

$$\phi = \frac{\pi}{\lambda\gamma^2}\left(N\lambda_u(1 + K_u^2/2) + \lambda_c\ (1 + K_c^2/2) + \lambda_m(1 + K_m^2/2) + D\right) \tag{8}$$

Here γ is electron energy in units of its rest energy and $D = D_1 + D_2 + D_3$. We assume $N >> 1$, in which case the undulator spectrum is peaked at the wave length

$$\lambda_1 = \frac{\lambda_u(1 + K_u^2/2)}{2\gamma^2} \tag{9}$$

At this wave length, Eq (8) becomes

$$\phi = 2\pi\ \left(N + \frac{\lambda_c(1 + K_c^2/2) + \lambda_m(1 + K_m^2/2) + D}{\lambda_u\ (1 + K_u^2/2)}\right) \tag{10}$$

To modulate the polarization between the left circular and the right circular one, ϕ needs to modulate between values

$$\phi_1 = 2\pi(n + 1/4) \text{ and } \phi_2 = 2\pi(m - 1/4). \tag{11.a}$$

Here n and m are arbitrary integers. For a modulation between two linear polarizations, on the other hand, the limiting values are

$$\phi_1 = 2\pi(n + 1/2) \text{ and } \phi_2 = 2\pi m. \tag{11.b}$$

From Eq (8), one finds that the corresponding modulation in K_m is, in the circular case (11.a), between the values

$$K_{m1} = \sqrt{\bar{K}^2 + \Delta^2} \quad , \quad K_{m2} = \sqrt{\bar{K}^2 - \Delta^2} \quad , \tag{12}$$

where

$$\begin{aligned} \bar{K}^2 &= \frac{1}{\lambda_m} (\lambda_u(1 + K_0^2/2)(p + 2m) - 2 (\lambda_c + \lambda_m + D) - K_c^2 \lambda_c), \\ \Delta^2 &= \frac{\lambda_u}{\lambda_m} (1 + K_0^2/2) (p + 1/2). \end{aligned} \tag{13}$$

Here p is another arbitrary integer. The values for the linear case (11.b) are obtained by replacing m by m + 1/4 in Eq (13).

It remains to determine the value of the corrector field K_c by requiring that the radiation intensity remains invariant as K_m varies between the values (12). When the undulator field is sinusoidal as discussed in the above, the radiation intensity becomes

$$\frac{dN}{d\Omega} = \alpha \frac{\Delta \lambda}{\lambda} \frac{4\gamma^2 K_0^2 N^2}{(1 + K_0^2/2)} G(K_c, K_m) J^2(1, q_u), \tag{14}$$

where

$$G(K_c, K_m) = \left| 1 + r_c e^{i\theta_c} + r_m e^{i\theta_m} \right|^2, \tag{15}$$

$$r_i = \frac{1}{N} \frac{K_i \lambda_i}{K_u \lambda_u} \frac{J(\nu_i, q_i)}{J(1, q_u)}, \qquad (i = c,\ m)$$

$$J(\nu, q) = \frac{1}{\pi} \int_0^{\pi} \sin \phi \sin(\nu\phi - q \sin 2\phi) d\phi,$$

$$q_i = \frac{K_i^2 \nu_i}{4(1 + K_i^2/2)} \quad (i = u,\ c,\ m)$$

$$\nu_i = \frac{1 + K_i^2/2}{1 + K_0^2/2} \quad (i = u,\ c,\ m)$$

$$\theta_c = \pi \frac{2D_1 + \lambda_c(1 + K_c^2/2)}{\lambda_u(1 + K_0^2/2)}$$

$$\theta_m = \pi \frac{2(D_1 + D_2) + \lambda_m(1 + K_m^2/2) + 2\lambda_c(1 + K_c^2/2)}{\lambda_u(1 + K_0^2/2)}$$

The equation that determines K_c is, therefore,

$$G(K_c, K_{m1}) - G(K_c, K_{m2}) = 0 \tag{16}$$

Eq (16) can be solved numerically.

The design of a crossed undulator system proceeds as follows: First, the optimum values of K_u and λ_u for a given magnet gap are selected to cover the desired spectral range. Then the set of equations, Eq (12) and Eq (16), are solved to obtain K_{m1}, K_{m2} and K_c. Given the magnetic period lengths λ_m and λ_c, which could be chosen to be equal to λ_u, the peak magnetic field strengths B_i in each magnetic part are determined. The process may need to be iterated if the resulting B_i's are not consistent with the gap requirement and λ_i.

The time dependence of the modulator field is given by

$$B_m(t) = B_m + \Delta B_m\, f(t) \tag{17}$$

where $B_m = (B_{m1} + B_{m2})/2$, $\Delta B_m = (B_{m1} - B_{m2})/2$ and $f(t)$ is a periodic function which varies between -1 and +1. Ideally, $f(t)$ is a sequence of step functions. In practice it will vary smoothly between -1 and 1, and it may be necessary to discriminate experimental data taken when $f(t) \neq \pm 1$ by a suitable method.

IV Depolarizing Effects

In section (V), electrons are assumed to travel on an ideal orbit. In general, the finite angular spread of electron beam as well as the finite monochromator bandpass implies that the observed radiation is not in a pure polarization state. To study this effect, consider an electron which enters the undulator with a slope $\hat{a}$ with respect to the z-axis, with an energy deviation $\delta\gamma$ relative to the ideal energy. Let the wave length of the observed photon be $\lambda_1 + \delta\lambda$, where $\delta\lambda$ lies within the monochromator bandpass. The polarization phase ϕ in this case is

$$\phi = \langle\phi\rangle - \phi_o \frac{\Delta\lambda}{\lambda} + \frac{\pi L}{\lambda}\left(\theta^2 - \langle\theta^2\rangle\right). \tag{18}$$

Here the angular bracket < > indicates the average value, ϕ_o is the phase of an ideal electron given by Eq (10), $L = 2(N\lambda_u + \lambda_c + \lambda_m + D)$ is the full length of the crossed undulator system, and

$$\frac{\Delta\lambda}{\lambda} = \frac{\delta\lambda}{\lambda} + \frac{\delta\gamma}{\gamma}. \tag{19}$$

Eq (18) implies that ϕ has a probability distribution around its average. The polarization property in this case can be studied using the formalism of the coherency matrix[6], which is defined as follows:

$$J = \begin{pmatrix} \langle E_x E_x^* \rangle & \langle E_x E_y^* \rangle \\ \langle E_y E_x^* \rangle & \langle E_y E_y^* \rangle \end{pmatrix}$$

$$= \left|E_o\right|^2 \begin{pmatrix} 1 & \langle e^{-i\phi}\rangle \\ \langle e^{i\phi}\rangle & 1 \end{pmatrix} . \tag{20}$$

Thus it is only necessary to evaluate the average value $e^{i\phi}$ to completely characterize the polarization state. If one defines the complex number u by

$$u = \langle e^{-i(\phi - \langle\phi\rangle)}\rangle \quad = u\, e^{i\alpha} , \tag{21}$$

one easily obtains

$$J = |E_0^2| \left[(1 - |u|) \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} 1 + |u| \begin{pmatrix} 1 & e^{-i\psi} \\ e^{i\psi} & 1 \end{pmatrix} \right] \tag{22}$$

Where $\psi = \alpha + \langle\phi\rangle$ Eq (22) is a decomposition of the radiation into its unpolarized part and completely polarized part u . The latter part has the polarization vector

$$\underset{\sim}{e} = \frac{1}{2}(\hat{x} + e^{i\psi}\hat{y}) \tag{23}$$

Let us assume that the distributions in $\Delta\lambda$ and θ are Gaussian with rms values σ_λ and σ_θ, respectively. The averaging operation in Eq (21) is then easy to perform and one obtains

$$u = \frac{e^{-1/2\left(\frac{\sigma_\lambda \phi}{\lambda}\right)^2} e^{i \frac{\pi L}{\lambda} \sigma_\theta^2}}{\left(1 + i \frac{\pi L}{\lambda} \sigma_\theta{}^2\right)^{1/2}} \tag{24}$$

As noted in the above, this number completely specifies the polarization state. In particular, the degree of polarization is (7)

$$P = |u| = \frac{e^{-1/2\left(\frac{\sigma_\lambda}{\lambda}\phi_o\right)^2}}{\left(1 + \frac{\pi L}{\lambda} \sigma_\theta^2\right)^{1/4}} . \tag{25}$$

In order to obtain a significant polarization, the following conditions must be satisfied:

$$\frac{\sigma_\lambda}{\lambda} = \left(\left(\frac{\delta\lambda}{\lambda}\right)^2 + \left(\frac{2\delta\gamma}{\gamma}\right)^2\right)^{1/2} << \frac{1}{\phi_o} \sim \frac{1}{2\pi N} , \tag{26}$$

$$\sigma_\theta > \sqrt{\frac{\lambda}{L}} \tag{27}$$

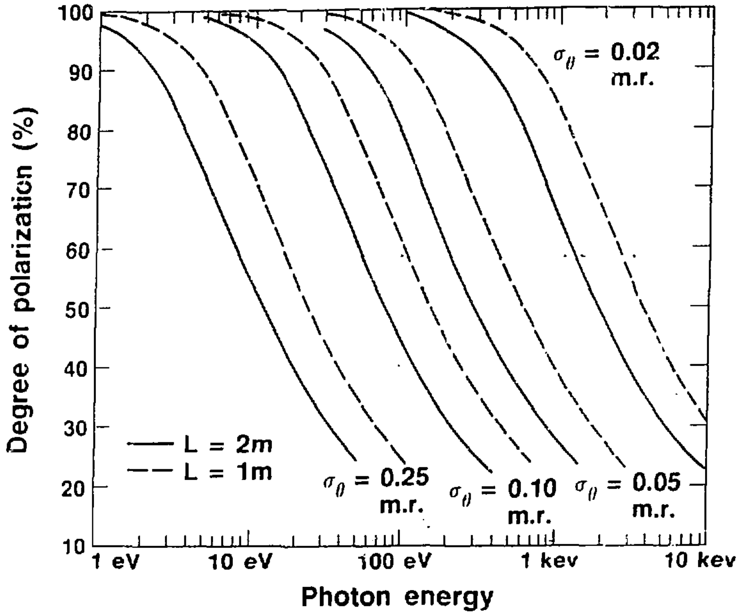

Fig. (4) Polarization From Crossed Undulator

Neglecting the electron beam energy spread, the condition (26) says that the relative monochromator bandpass must be smaller than N^{-1}, in agreement with the discussion in Sec. (II).

In general the inequality (26) is easy to satisfy. More serious is the effect of electron beam angular divergence σ_θ. Fig. (4) shows the degree of polarization as a function of photon energy for different values of σ_θ, assuming $\sigma_\lambda/\lambda = 0$. Two sets of graphs are shown, one for L = 2 meters (solid lines) the other for L = 1 meter (dashed lines). The necessity of a low emittance electron storage ring to operate a crossed undulator system, especially at photon energies higher than 100 eV, is clear from this figure.

(V) An Example Design for VUV Ring at NSLS

The VUV ring [8] at NSLS runs at an electron energy of 750 MeV. The beam angular divergence σ_θ is 0.1 m.r. at straight section. Thus one sees from Fig. (4) that the degree of polarization lies in the range between 80 to 90% for photon energies between 10 eV and 30 eV for a 2 m device. Thus a crossed undulator system placed at the VUV ring at NSLS would be a good source of VUV radiation with arbitrarily adjustable polarization.

In order to implement various control features discussed in the previous section, the design is based on an electromagnetic structure. The magnet gap is set at 7 cm in accordance with the current vacuum chamber specification of the ring. The magnet dimensions are

$$\lambda_u = \lambda_c = \lambda_m = 15 \text{ cm}, \ D_1 = D_2 = 0, \ D_3 = 10 \text{ cm}.$$

These values ensure adequate field strength at desired photon energies. The main undulator has 3 periods and has an adjustable dc peak field up to 3 KG. The corrector has an adjustable dc peak field of 3 KG. The modulator magnet has an adjustable dc peak field of 3 KG plus a modulating field with amplitudes up to .5 KG at a frequency of 50 H_z. The poles of the modulator magnet are laminated. Fig. (5) gives a layout of the magnet design[9].

Calculation shows that the crossed undulator design here generates about 10^{14} photons per sec per $(\text{m.r.})^2$ per (.1% B.W.) in the photon energy range 10 to 30 eV. The polarization can be modulated either between two mutually perpendicular linear polarizations, or between left circular and right circular.

The design example considered here was constrained by the 7 cm vacuum chamber gap. In the future, the gap dimension could be reduced to 4 cm, and a more compact and powerful device covering a wider range of photon energies could be designed. A similar design should also work for the Aladdin storage ring at Wisconsin[10].

(VI) Discussions

The crossed undulator system studied here is a versatile source of polarized radiation for photon energies higher than 10 eV. The device works only in conjunction with a low emittance electron storage ring.

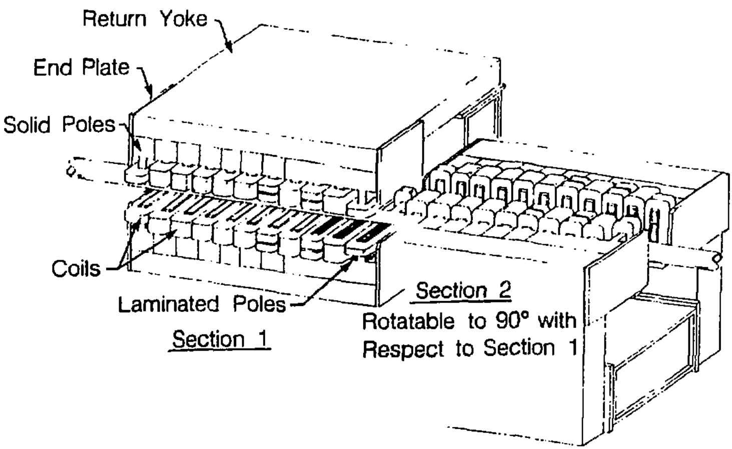

Fig. (5) A Crossed Undulator Design for VUV Ring

In the VUV energy range, (photon energies below 100eV), the current generation of VUV rings, such as Aladdin and the 750 MeV ring at NSLS, can serve as sites for the crossed undulator device. For photons of higher energy, a next generation storage ring such as the ALS[11] is required.

In general, the shape of the polarization ellipse will change due to reflections from optical elements, such as mirrors and gratings. In the case of mirrors, the effect should be small both for the low energy photons ($\epsilon \leq 30$eV) with normal incidence optics, and also for the high energy cases ($\epsilon \geq 200$eV) for sufficiently grazing angles of incidence (Θ_g). In any case, it should be possible to correct for the anticipated change of the polarization due to reflections, since the source is capable of producing an arbitrary state of polarization. The case of gratings is more complicated. It is known that if

$$\frac{\lambda}{d \sin \Theta_g} < .2 \qquad (28)$$

then gratings behave similarly to mirrors as far as the polarization effects are concerned[12]. In the above, d is the groove spacing and Θ_g is the angle of incidence. The inequality (28) is usually satisfied for VUV and soft x-ray grating systems.

The crossed undulator system works as a free electron laser if placed in an optical cavity. The operational principle here is similar to the case of the optical klystron[13], and is discussed elsewhere.[14]

Acknowledgments

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References and Footnotes

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