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Jet Angular Distribution From Quantum Chromodynamics*

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Abstract

We calculate the quantum chromodynamic correction to jet (defined à la Sterman and Weinberg) angular distribution in energetic e^+e^- annihilation to order α_s keeping exact dependence on ϵ and δ . We find deviations from the zeroth order distribution, $1 + \cos^2 \theta$, for relatively large values of ϵ and δ . This effect could be tested at the existing e^+e^- colliding beam facilities.

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1. Introduction

It was first shown by Sterman and Weinberg¹ that suitably defined jet cross sections are free of infrared singularities, and, therefore, can be computed in QCD perturbation theory. Their results demonstrated for the first time the dominance of the 2-jet final states in energetic e^+e^- annihilation. In the above work, neglecting the contributions which vanishes in the limit $\epsilon \rightarrow 0$, $\delta \rightarrow 0$, the angular distribution of the axis of the jet cone was found to be the same as in the zeroth order, i.e., $1 + \cos^2 \theta$. The purpose of this work is to calculate the angular distribution of the jet axis to order α_s keeping exact dependence on ϵ and δ .² [α_s is the square of the gauge coupling constant divided by 4π defined at a renormalization point with four-momenta square of the order s .] We find that it is given by $1 + \beta(\alpha_s, \epsilon, \delta) \cos^2 \theta$ where the deviation of $\beta(\alpha_s, \epsilon, \delta)$ from unity can be as large as 10% for relatively large allowed values of ϵ and δ . Using the appropriate jet prescriptions of Ref. 1, this could be tested in the existing e^+e^- colliding beam facilities.

2. Differential cross sections

Following Ref. 1, we consider the 2-jet cross section $\sigma(E, \theta, \epsilon, \delta)$ defined as the cross section for all e^+e^- annihilation events in which $(1 - \epsilon)$ fraction of the total e^+e^- energy E is emitted in a pair of oppositely directed cones of opening angle 2δ . θ is the angle between the axis of the cone and the e^+e^- beam line. Axis of the cone is defined to be the thrust axis.³ Let us call these cones the $(\epsilon - \delta)$ jets. To order α_s , the contributions to the cross section arising from a quark, an antiquark, and a gluon in the final state are given by the diagrams (A), in Fig. 1, while those with a quark and an antiquark alone in the final state by the diagrams (B). As in Ref. 1, for simplicity

We have set the quark masses equal to zero from the beginning, but have used a finite gluon mass $\mu \ll \epsilon E$ as an infrared cutoff.

For the purpose of defining the jet axis precisely, it is necessary that the three-particle phase space of the final state be divided into non-overlapping regions. These are designated as I, II, and III in Fig.2. Let x_1 , x_2 , and x_3 be the fractions of the energy $E/2$ carried by the quark, anti-quark, and gluon, respectively. The axis of the jet cone then coincides with the direction of the particle carrying the largest momentum. We have

$$x_1 + x_2 + x_3 = 2, \quad (1)$$

and the three distinct regions are:

$$\text{I: } x_1 > x_2, x_3$$

$$\text{II: } x_2 > x_1, x_3$$

$$\text{III: } x_3 > x_1, x_2. \quad (2)$$

In region I, the quark carries the largest momentum, therefore its direction is the axis of the jet, with one cone containing the quark and the other the antiquark and the gluon. Similarly for the regions II and III. The three particles have the equal energy, i.e., $x_1 = x_2 = x_3 = \frac{2}{3}$, at the point of confluence of the three regions I, II, and III.

The differential cross sections in the three regions corresponding to the diagrams (A) are found to be⁴

$$\frac{d\sigma_A^I}{d(\cos\theta)dx_1dx_2} = \sigma_0 \frac{\alpha_s}{4\pi} \left[\frac{(x_1^2 + x_2^2)}{(1-x_1)(1-x_2)} (1 + \cos^2\theta) + \frac{2(x_1 + x_2 - 1)}{x_1^2} (1 - 3\cos^2\theta) \right], \quad (3)$$

$$\frac{d\sigma_A^{II}}{d(\cos\theta)dx_1dx_2} = \text{same as above with } x_1 \leftrightarrow x_2, \quad (4)$$

$$\frac{d\sigma_A^{III}}{d(\cos\theta)dx_1dx_2} = \sigma_0 \frac{\alpha_s}{4\pi} \left[\frac{x_1^2 + x_2^2}{(1-x_1)(1-x_2)} (1 + \cos^2\theta) + \frac{4(x_1 + x_2 - 1)}{(2 - x_1 - x_2)^2} (1 - 3\cos^2\theta) \right], \quad (5)$$

where

$$\sigma_0 \equiv \frac{4\pi\alpha^2}{3s} \sum_{\text{flavor}} 3Q^2, \quad s \equiv E^2. \quad (6)$$

Of the two terms in each of (3), (4), and (5), the second ($\sim 1 - 3\cos^2\theta$) gives rise to contributions involving only positive powers of ϵ and δ upon integration over x_1 and x_2 . In the approximation in which such contributions are disregarded, as was done in Ref. 1, these terms could have been dropped in the differential cross sections. However, we find that they give contributions to the differential cross sections over a wide (ϵ, δ) domain, and lead to the quantum chromodynamic correction to the zeroth order angular distribution given by

$$\left(\frac{d\sigma}{d\cos\theta}\right)_0 = \sigma_0 \frac{3}{8}(1 + \cos^2\theta). \quad (7)$$

The contributions to the 2-jet cross section from the diagrams (A) can be obtained by carrying out the integrations in (3)-(5) over the appropriate (x_1, x_2) phase space shown in Fig.3. In anticipation of infrared divergences in (3) and (4) arising from $[(1-x_1)(1-x_2)]^{-1}$ in each integral proportional to $1 + \cos^2\theta$, a finite gluon mass $\mu < eE$ has been introduced as an infrared cut-off. We note, however, that no infrared divergence occurs in the integration of (5) over the region III. The upper-right-half triangle is, of course, the allowed phase space for the three particles in the limit $\mu \rightarrow 0$.

The criteria for 2-jet configurations given in Ref. 1 restrict these events to the peripheral regions A, B, C, D, E, and F in Fig.3. In region I the antiquark and the gluon lie within a cone of half-angle δ whose axis coincides with the momentum of the opposing quark. This gives rise to the restriction that defines the peripheral region A:

$$\frac{1-x_2}{1-x_2 \sin^2 \delta} \geq 2-x_1-x_2. \quad (8)$$

Exchanging the roles of the quark and the antiquark, we obtain the peripheral region B of II defined by

$$\frac{1-x_1}{1-x_1 \sin^2 \delta} \geq 2-x_1-x_2. \quad (9)$$

In region III, the condition that the quark and the antiquark lie within a cone of half-angle δ whose axis coincides with the momentum of the opposing gluon gives rise to the restriction that defines region C:

$$\frac{1-x_1}{1-x_1 \sin^2 \delta} \geq x_2 \geq 1-x_1. \quad (10)$$

For an arbitrary value of δ regions A, B, and C could overlap. Such an overlapping must be avoided. This is because if, for example, the regions B and C overlapped, a point in the overlapping region would correspond to both the antiquark and the gluon simultaneously having the largest momentum. It can easily be shown from (8)-(10) that this imposes the restriction $\delta \leq 45^\circ$.

The other criterion for 2-jet event of Ref. 1 is that the two cones together contains all but fraction ϵ of the total c.m. energy E. This leads to the restrictions

$$x_1 \leq 2\epsilon, \quad x_2 \leq 2\epsilon, \quad x_3 \leq 2\epsilon \quad (11)$$

in the phase space excluded by the regions A, B, and C, and give rise to the additional regions D, E, and F for the 2-jet configuration. The remaining region of the phase space, designated by R in Fig.3, then becomes that for the 3-jet configuration.

One important restriction on ϵ is the following. The line $x_1 = 2\epsilon$ must not include any area of the region I. Otherwise, a point in the region to the left of the line $x_1 = 2\epsilon$ will have the antiquark carrying the largest energy, in contradiction to the definition of the region I. Using (11), this leads to the restriction $2\epsilon < \frac{2}{3}$, or

$$\epsilon < \frac{1}{3}. \quad (12)$$

There is an additional restriction on ϵ and δ which is explained in Ref. 5. This arises from the fact that in any three-particle final state (a quark, an antiquark, and a gluon) in which the angle between the soft particle (with energy $< \epsilon E$) and another is larger than 2δ , ϵ has to be chosen in such a way that the condition

$$\sin \delta \geq \frac{\epsilon}{1 - \epsilon} \quad (13)$$

is satisfied.

The differential cross section for the 2-jet event corresponding to the diagrams (A) can conveniently be calculated as the difference between the sum of (3), (4), and (5) integrated over the entire allowed region (designated by Δ) and that over the region R. We obtain

$$\left. \frac{d\sigma_A}{d(\cos\theta)} \right|_{\Delta} = \sigma_0 \frac{\alpha_s}{2\pi} \left[\left(2 \ln^2 \frac{E}{\mu} - 3 \ln \frac{E}{\mu} + 3 - \frac{\pi^2}{6} \right) (1 + \cos^2 \theta) + (8 \ln(\frac{3}{2}) - 3) (1 - 3 \cos^2 \theta) \right], \quad (14)$$

$$\left. \frac{d\sigma_A}{d(\cos\theta)} \right|_R = \sigma_0 \frac{\alpha_s}{4\pi} \left[F_1(\epsilon, \delta) (1 + \cos^2 \theta) + F_2(\epsilon, \delta) (1 - 3 \cos^2 \theta) \right], \quad (15)$$

where

$$F_1(\epsilon, \delta) \equiv \int_R dx_1 dx_2 \frac{(x_1^2 + x_2^2)}{(1 - x_1)(1 - x_2)}, \quad (16)$$

$$\begin{aligned} F_2(\epsilon, \delta) = & \int_{R_1} dx_1 dx_2 \frac{2(x_1 + x_2 - 1)}{x_1^2} \\ & + \int_{R_2} dx_1 dx_2 \frac{2(x_1 + x_2 - 1)}{x_2^2} \\ & + \int_{R_3} dx_1 dx_2 \frac{4(x_1 + x_2 - 1)}{(2 - x_1 - x_2)^2}. \end{aligned} \quad (17)$$

The regions of integrations R_1 , R_2 , and R_3 in (17) can be seen in Fig.3: R_1 is the portion of the region I excluded by the peripheral regions A, E, and F; R_2 that of II excluded by B, D, and F; and R_3 that of III excluded by C, D, and E. The boundaries of the various regions can be determined by the restrictions (8)-(13).

The contribution to the differential cross section from the diagrams (A) for the 2-jet event is now given by

$$\left. \frac{d\sigma_A}{d(\cos\theta)} \right|_{2\text{-jet}} = \left. \frac{d\sigma_A}{d(\cos\theta)} \right|_{\Delta} - \left. \frac{d\sigma_A}{d(\cos\theta)} \right|_R. \quad (18)$$

3. The total cross sections for quark-antiquark-gluon production

The contribution to the 2-jet cross section from the diagrams (B) has been calculated by the standard method⁶ and is found to be⁷

$$\frac{d\sigma_B}{d(\cos\theta)} = \sigma_0 \frac{\alpha_s}{2\pi} \left[-2 \ln^2 \frac{E}{\mu} + 3 \ln \frac{E}{\mu} - \frac{9}{4} + \frac{\pi^2}{6} \right] (1 + \cos^2 \theta). \quad (19)$$

The complete differential cross section to order α_s is then obtained by adding (7), (14), and (19):

$$\frac{d\sigma}{d(\cos\theta)} = \sigma_0 \frac{3}{8} \left[\left(1 + \frac{\alpha_s}{\pi} \right) (1 + \cos^2 \theta) + \frac{4\alpha_s}{3\pi} (8 \ln \frac{3}{2} - 3) (1 - 3\cos^2 \theta) \right]. \quad (20)$$

The infrared divergence has been eliminated. Integrating (20), one finds the total cross sections to be $\sigma_0 \left(1 + \frac{\alpha_s}{\pi} \right)$. It is interesting to note that this result obtained from QCD perturbation theory agrees with that obtained from the solution of the renormalization group equation in QCD for the time-like photon.⁸

4. The 2-jet cross section and the angular distribution

The total $(\epsilon - \delta)$ 2-jet cross section, to order α_s , is now obtained by adding (7), (18), and (19):

$$\begin{aligned} \frac{d\sigma^{2\text{-jet}}}{d(\cos\theta)} = \sigma_0 \frac{3}{8} \left[\left\{ 1 + \frac{\alpha_s}{\pi} - \frac{2\alpha_s}{3\pi} F_1(\epsilon, \delta) \right\} (1 + \cos^2 \theta) \right. \\ \left. + \frac{2\alpha_s}{3\pi} \{ 16 \ln(\frac{3}{2}) - 6 - F_2(\epsilon, \delta) \} (1 - 3\cos^2 \theta) \right]. \end{aligned} \quad (21)$$

We note that, as in Ref. 1, the infrared singularities, present separately in diagrams (A) and (B), have canceled in the resulting observable cross sections given in (20) and (21). The functions $F_1(\epsilon, \delta)$ and $F_2(\epsilon, \delta)$, as defined by (16)-(17), are free of any infrared singularity.

In the limit $\epsilon \rightarrow 0$ and $\delta \rightarrow 0$, the terms involving positive powers of ϵ and δ in the expansion of $F_1(\epsilon, \delta)$ and $F_2(\epsilon, \delta)$ drop out, and we obtain

$$\begin{aligned} F_1(\epsilon, \delta) &= 2 \left[(3 + 4 \ln 2\epsilon) \ln \delta + \frac{\pi^2}{3} - \frac{7}{4} \right], \\ F_2(\epsilon, \delta) &= 16 \ln\left(\frac{3}{2}\right) - 6, \end{aligned} \quad (22)$$

so that (21) becomes

$$\frac{d\sigma^{2\text{-jet}}}{d(\cos\theta)} = \sigma_0^{\frac{3}{8}} \left[1 - \frac{4\alpha_s}{3\pi} \left\{ (3 + 4 \ln 2\epsilon) \ln \delta + \frac{\pi^2}{3} - \frac{5}{2} \right\} \right] (1 + \cos^2\theta). \quad (23)$$

We note that this is in agreement with the expression (6) in Ref. 1.

The $(\epsilon - \delta)$ jet angular distribution in (21) can be expressed as

$$\begin{aligned} \frac{d\sigma^{2\text{-jet}}}{d(\cos\theta)} &= \sigma_0^{\frac{3}{8}} \left[1 + \frac{\alpha_s}{3\pi} \left\{ -9 + 32 \ln\left(\frac{3}{2}\right) - 2F_1(\epsilon, \delta) - 2F_2(\epsilon, \delta) \right\} \right] \\ &\quad \cdot \left[1 + \beta(\alpha_s, \epsilon, \delta) \cos^2\theta \right], \end{aligned} \quad (24)$$

where

$$\beta(\alpha_s, \epsilon, \delta) \equiv \frac{1 + \frac{\alpha_s}{3\pi} \{ 39 - 96 \ln\left(\frac{3}{2}\right) - 2F_1(\epsilon, \delta) + 6F_2(\epsilon, \delta) \}}{1 + \frac{\alpha_s}{3\pi} \{ -9 + 32 \ln\left(\frac{3}{2}\right) - 2F_1(\epsilon, \delta) - 2F_2(\epsilon, \delta) \}}. \quad (25)$$

Keeping terms to order α_s , we obtain

$$\beta(\alpha_s, \epsilon, \delta) = 1 - \frac{8\alpha_s}{3\pi} [16 \ln(\frac{3}{2}) - 6 - F_2(\epsilon, \delta)]. \quad (26)$$

We have evaluated $\beta(\alpha_s, \epsilon, \delta)$ numerically for the domain $(\alpha_s, \epsilon, \delta) = (0.1 - 0.3, 0.01 - 0.33, 2^\circ - 45^\circ)$ taking the appropriate restrictions on the phase space into account (see Fig.3). In Fig.4, using $\alpha_s = 0.25$,⁹ $\beta(\alpha_s, \epsilon, \delta)$ is plotted against ϵ for various values of δ . That some curves terminate at right for certain values of ϵ is due to the condition (13). We see that there are deviations in $\beta(\alpha_s, \epsilon, \delta)$ from unity by as much as $\approx 10\%$ for relatively large δ and ϵ .

5. Discussion

We emphasize that in deriving the jet angular distribution, we have followed the perturbative QCD approach of Ref. 1. We have made no attempt to deal with any non-perturbative effect. At low energies, non-perturbative effect is expected to have a large contribution so far as the width of the jet is concerned. However, we believe it to have a very little effect in changing the direction of the axis of the jet.¹⁰

To conclude, we have presented the quantum chromodynamic correction to the angular distribution of the axis of the $(\epsilon - \delta)$ jet cone (defined to be the thrust axis) in energetic e^+e^- annihilation to order α_s keeping exact dependence on ϵ and δ . Over a wide domain of (ϵ, δ) , we find deviations which could be as large as 10% from the zeroth order angular distribution. If the predicted pattern of deviation is verified experimentally, it would naturally point out that the non-perturbative effects are small, and that the perturbative QCD jet calculation leads to reliable results. In view of the importance of this information, experimental search for the possible deviations of the jet angular distribution from $1 + \cos^2 \theta$ as a function of ϵ and δ , as discussed in this work, is highly desirable. We believe that the QCD correction thus predicted for the

angular distribution of the cones of the jet constitutes a detailed test of the theory as compared with the corresponding correction for the e^+e^- annihilation cross section in $\sigma_0 \left(1 + \frac{\alpha_s}{\pi}\right)$.

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References

1. G. Sterman and S. Weinberg, Phys. Rev. Lett 39, 1438 (1977).
2. A number of authors have calculated the corrections arising from finite ϵ and δ to the total 2-jet fraction of the Sterman-Weinberg jet formula, Eq.(7) in Ref. 1. However, the problem of the correction to the angular dependence, Eq.(6) in Ref. 1, has not been addressed. See P. M. Stevenson, Phys. Lett. 78B, 451 (1978); B. G. Weeks, Phys. Lett. 81B, 377 (1979); P. Binétruy and G. Girardi, CERN Report, TH.2617.
3. E. Fahren, Phys. Rev. Lett. 39, 1587 (1977); H. Georgi and M. Machacek, Phys. Rev. Lett. 39, 1237 (1977).
4. Somewhat similar expressions for the differential cross sections including the derivatives with respect to an additional azimuthal angle χ have been given in G. Kramer, G. Schierholz, and J. Willrodt, Phys. Lett. 79B, 249 (1978).
5. See P. M. Stevenson in Ref. 2.
6. See, for example, J. D. Bjorken and S. D. Drell, Relativistic Quantum Mechanics, McGraw-Hill, New York (1964).
7. Note that there are discrepancies in the constant terms in (14) and (19) when compared with the corresponding expressions in Ref. 1, namely $\sigma_A + \sigma_B$ vs. (14) and σ_C vs. (19). These discrepancies are presumably due to the different methods of regularizations used (private communication with S. Weinberg). However, as mentioned in the text, our final 2-jet cross section (23) agrees with the corresponding expression in Ref. 1.
8. T. Appelquist and H. Georgi, Phys. Rev. D 8, 4000 (1973); A. Zee, Phys. Rev. D 8, 4038 (1973).

9. Using the asymptotic QCD formula $\alpha_s = 12\pi / (25 \ln S / \Lambda^2)$, one obtains $\alpha_s = 0.23(0.28)$ for $\Lambda = 0.275(0.500)$ GeV at $\sqrt{s} = 7.4$ GeV. For any other value of α_s , $\beta(\alpha_s, \epsilon, \delta)$ can be obtained by scaling α_s according to (26).
10. Another possible source of deviation of $\beta(\alpha_s, \epsilon, \delta)$ from unity is the effect of finite quark mass, which above threshold gives $1 - \beta(\alpha_s, \epsilon, \delta) = 4 \langle m_q^2 \rangle / E^2$ in the zeroth order. At energy $E = 8$ GeV, the charmed quark contribution to the deviation amounts to $\sim 2\%$.

Figure Captions

- Fig. 1 Diagrams that contribute to the 2-jet cross section to order α_s .
- Fig. 2 The division of the phase space depending upon the largest momentum carried by one of the three particles (see text).
- Fig. 3 The phase space for the quark, antiquark, and gluon (see Ref. 5).
For the various regions indicated, see text.
- Fig. 4 $\beta(\alpha_s, \epsilon, \delta)$ plotted against $\epsilon (\leq \frac{1}{3})$ for various values of $\delta (\leq 45^\circ)$ at $\alpha_s = 0.25$.

Fig. 1

Diagrams (A)

Diagrams (B)

Fig. 2

Fig. 4

Fig. 3