Depleted Uranium Valuation

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April 1979

Prepared for the U.S. Department of Energy
under Contract EY-76-C-06-1830

Pacific Northwest Laboratory
Operated for the U.S. Department of Energy
by Battelle Memorial Institute

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PACIFIC NORTHWEST LABORATORY
operated by
BATTELLE
for the
UNITED STATES DEPARTMENT OF ENERGY
Under Contract EY-76-C-06-1830

Printed in the United States of America
Available from
National Technical Information Service
United States Department of Commerce
5285 Port Royal Road
Springfield, Virginia 22151

Price: Printed Copy \$____*; Microfiche \$3.00

*Pages	NTIS
	Selling Price
001-025	\$4.00
026-050	\$4.50
051-075	\$5.25
076-100	\$6.00
101-125	\$6.50
126-150	\$7.25
151-175	\$8.00
176-200	\$9.00
201-225	\$9.25
226-250	\$9.50
251-275	\$10.75
276-300	\$11.00

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1.0 INTRODUCTION

In order to use natural uranium to fuel existing commercial Light Water Reactor (LWR) power plants, the uranium must be processed to enrich its ^{235}U content. In the enrichment process, natural uranium is fed to the plant, enriched uranium is produced and the residual depleted uranium (called the tails) is recovered and stored in anticipation of future use. The future uses include: production of plutonium to fuel new power plants (e.g., fast breeder reactors), feed for advanced process enrichment plants, and material for radiation shields.

Current markets for depleted uranium are limited. The future markets are subject to large technological and political uncertainties. Thus, accurately computing a value for depleted uranium is a difficult exercise. A complete analysis of the value of depleted uranium should include: 1) computing a value for depleted uranium in each of its potential uses; 2) estimating the probability of occurrence and period of these future applications; and 3) appropriately weighting the computed values with the estimated probabilities and periods of occurrence. This analysis will focus on the valuation step (as in one above) for several postulated scenarios where depleted uranium could have value.

The implicit value of depleted uranium is derived from the following characteristics:

- The large density of uranium (especially for shielding or ammunition applications).
- The residual quantities of ^{235}U contained in depleted uranium.
- The fertile material, ^{238}U , present.

This analysis postulates employing these attributes in various ways to compute the value of depleted uranium.

2.0 SUMMARY

The following uses for depleted uranium were examined to determine its value:

- A substitute for lead in shielding applications.
- Feed material in gaseous diffusion enrichment facilities.
- Feed material for an advanced enrichment concept.
- Mixed Oxide (MOx) diluent and blanket material in liquid metal fast breeder reactors (LMFBR).
- Fertile material in LMFBR systems.

A range of depleted uranium values was calculated for each of these applications. The sensitivity of these values to analysis assumptions will be discussed in the remainder of the report. The calculated range of depleted uranium values is listed in Table 2.1.

TABLE 2.1. Estimated Depleted Uranium Value, \$/kg U
Chemical Form: UF_6 , Assay: 0.30 wt% ^{235}U

Scenario	Tails Value Range, \$/kg U		Comments
	Lower Bound	Upper Bound	
Shielding Material	(Present Value Storage Costs)	~1	Substitute for lead
Gaseous Diffusion Feed Material	(Present Value Storage Costs)	18	For variable cost recovery at optimum tails
Advanced Concept Feed Material	--	41	Maximum value
MOx Diluent in LMFBR	--	78	Threshold ^{238}U plus linear ^{235}U contribution
Fertile Material in LMFBR Systems	38	--	Corresponds to estimate of thorium price

These values are reported in constant 1978 dollars. The first two scenarios represent currently achievable applications for depleted uranium. For these two cases the lower bound for tails value is the negative of the present value of all future storage costs. This represents a dollar value outlay in 1978 that equals the sum of all discounted future storage charges.

The final three price ranges reported in Table 2.1 represent potential applications for depleted uranium. To meaningfully compare the depleted uranium value for these three applications to that for the first two would require estimating their probabilities and time of their occurrence.

3.0 VALUATION AS SHIELDING MATERIAL

Depleted uranium has occasionally been used as a shielding material. In certain applications, where weight considerations are important, depleted uranium has been used as a replacement for lead to maintain adequate shielding properties while concurrently reducing weight. Oak Ridge National Laboratory has designed and built casks with depleted uranium. More recently General Electric used depleted uranium in the construction of their IF-300 spent fuel shipping cask. For these applications depleted uranium is superior to lead from a weight-shielding standpoint but more difficult to fabricate. Because of this trade-off, it can be postulated that the approximate value of depleted uranium is that of lead. Since lead is currently priced at \$0.39/lb or \$0.86/kg,⁽¹⁾ the approximate value of depleted uranium for shielding applications is \$1/kg.

4.0 VALUATION BASED ON ^{235}U CONTENT

Depleted uranium contains varying amounts of ^{235}U . The Department of Energy is currently storing approximately 250 thousand metric tons of depleted uranium. Of this, 14% is at an assay 0.30 wt% ^{235}U , 31% at 0.25 wt% ^{235}U , and 55% at 0.20 wt% ^{235}U . At present the enrichment cascade is operating at a tails assay of 0.20 wt% ^{235}U and will produce about 20 thousand metric tons of tails annually.

4.1 ENRICHMENT FEED MATERIAL - GASEOUS DIFFUSION FACILITY

The value of depleted uranium can be estimated by assuming its use as feed material to produce natural uranium. A price for the depleted uranium can be estimated for which a utility would be indifferent to using this process to obtain natural uranium or purchasing the natural uranium directly. This relationship can be formulated as:

$$P = W_Q \cdot W_P + U_Q \cdot U_P \quad (1)$$

where

P is the price of natural uranium as UF_6 , \$/kg U (product)

W_Q is the quantity of depleted uranium fed to cascade per kg of product, kg U (feed)/kg U (product)

W_P is the price of depleted uranium as UF_6 , \$/kg U (feed)

U_Q is the quantity of separative work required per kg of product, SWU/kg U (product)

U_P is the price of separative work, \$/SWU

Solving for W_P , the tails indifference price, yields:

$$W_P = \frac{P - U_Q \cdot U_P}{W_Q} \quad (2)$$

Table 4.1 shows resultant prices of tails for various combinations of P , U_Q , U_P , and W_Q . Several tails assays as well as two different SWU prices were considered. These analyses resulted in a range of indifference values for depleted uranium of \$2.97 to \$17.65/kg uranium.

TABLE 4.1. Depleted Uranium Valued as Feed Material - Gaseous Diffusion

Feed Assay wt% ²³⁵ U	Tails ^{a)} Assay, wt% ²³⁵ U	Natural Uranium Value, \$/kg U	Separative Work Price \$/SWU	Separative Work Requirements, SWU	Feed Requirements kg U	Comments	Depleted Uranium Value, \$/kg
0.30	0.20	118	86	0.806	5.10	Current market conditions	9.55
0.30	0.14	118	50 ^{b)}	1.10	3.57	SWU price at estimated marginal cost	17.65
0.25	0.20	118	86	1.02	10.2	Current market con- ditions	2.97
0.25	0.14	118	50 ^{b)}	1.39	5.18	SWU price at estimated marginal cost	9.36
0.20	0.14	118	50 ^{b)}	1.78	9.52	SWU price at estimated marginal cost	3.05

a) The value of this tails stream is assumed to be zero.

b) This is an estimate of the marginal cost of a SWU. This value represents the variable operating cost for an incremental unit of enrichment service.

4.2 ENRICHMENT FEED MATERIAL - ADVANCED CONCEPT FACILITY

The value of depleted uranium as feed material to an advanced enrichment concept could differ from its value as feed to current enrichment processes. Production of typical light water reactor (LWR) fuel is represented as follows:

The cost of this product as UF₆ can be expressed as:

$$V_P = F_Q \cdot F_P + C_Q \cdot C_P + U_Q \cdot U_P \quad (3)$$

where the variables and representative values are defined as follows:

V_P is the cost of 3.0 wt% ²³⁵U as UF₆, \$(kg U product)

F_Q is the quantity of natural uranium required per kg of product, 5.48 kg U (feed)/kg U (product)

F_p is the price of natural uranium as U_3O_8 , \$113/kg U (feed)

C_Q is the quantity of converted natural uranium,
5.48 kg U (feed)/kg U (product)

C_p is the price of conversion operation, \$5.00/kg U (feed)

U_Q is the separative work per kg U of product,
4.306 SWU/kg U (product)

U_p is the separative work price, \$86/SWU.

Substituting into (3) yields:

$$V_p = 5.48 \cdot 113 + 5.48 \cdot 5 + 4.306 \cdot 86$$

$$V_p = \$1017 \text{ per kg uranium (product)}$$

An advanced enrichment concept, if it were to be competitive, would require providing UF_6 at 3.0 wt% ^{235}U for no more than the above cost. A hypothetical flow diagram for such a concept is shown below.

The product cost for this process (as UF_6) can be expressed as:

$$V_p = F_Q \cdot F_p + C_Q \cdot C_p + E$$

where

E is the enrichment charge, \$/kg U (product)

F_Q is the quantity of natural uranium required per kg of product,
4.63 kg U (feed)/kg U (product)

F_p is the price of natural uranium as U_3O_8 , \$113/kg U (feed)

C_Q is the quantity of converted natural uranium,
4.63 kg U (feed)/kg U (product)

C_p is the price of conversion operation, \$5/kg U (feed)

V_p is the value of 3.0 wt% ^{235}U as UF_6 , \$1017 per kg U (product)

Thus solving for E and substituting yields:

$$E = V_p - F_Q \cdot F_p - C_Q \cdot C_p = \$471 \text{ per kg uranium product.}$$

If it is assumed that for the hypothetical advanced concept, the enrichment charge is relatively indifferent to the quantity of feed material processed^(a), a value for tails can be calculated. The system flow diagram for this calculation is shown below:

Reformulating equation (2) for this process yields:

$$V_p = W_Q \cdot W_p + E \quad (4)$$

(a) This is equivalent to assuming the cost of the process is more dependent on the amount of ^{235}U removed than the amount of material processed.

where

W_Q is the quantity of depleted uranium fed to enrichment process,
kg U (feed)/kg U (product)

W_P is the price of depleted uranium as UF_6 , \$/kg U (feed)

E is the Enrichment Change, \$471/kg U (product)

W_Q for this case (assuming 0.30 wt% ^{235}U) is 13.3 kg tails per kg product, and
 U_P is \$1017/kg as previously calculated. Substituting into (4) and solving for
 W_P yields:

$$W_P = \frac{V_P - E}{W_Q} = \$41 \text{ per kg uranium (feed)}$$

This is the maximum that 0.3 wt% $^{235}UF_6$ material could be valued at for this
hypothetical advanced enrichment concept. Tails with lower ^{235}U assay would
have less value.

5.0 VALUATION AS MOx DILUENT AND BLANKET MATERIAL

Depleted uranium may be used as the diluent in mixed oxide cores and as the fertile material in the blanket regions of LMFBRs. The residual quantities of ^{235}U in depleted uranium represent fissile material, which reduces plutonium charge requirements. As the ^{235}U assay increases the plutonium requirement decreases. The ^{238}U in depleted uranium is a fertile material which absorbs a neutron to produce additional plutonium.

Both of these properties impart value to depleted uranium. Calculations to determine the value of depleted uranium for this application were made based on a conceptual LMFBR system.^(2,3,4) These calculations compare the required plutonium charge enrichment for several different assays of ^{235}U in depleted uranium diluent. The resultant mass flows are shown in Table 5.1. Figure 5.1 shows the relationship between assay and charge enrichment.

TABLE 5.1. Annual Mass Flows for MOx LMFBR Core + Axial Blanket and Radial Blanket, kg

		0.0 wt%	^{235}U	0.2 wt%	^{235}U	0.3 wt%	^{235}U	0.5 wt%	^{235}U	Natural U (.711 wt% ^{235}U)	
		Core	Radial	Core	Radial	Core	Radial	Core	Radial	Core	Radial
Charge	U-235	---	9256	52	19	78	28	130	46	185	66
	U-238	25848	9256	25831	9237	25822	9228	25807	9210	25789	9189
	Total U	25848	9256	25883	9256	25900	9256	25937	9256	25974	9255
	Pu-239	1558	--	1536	--	1526	--	1504	--	1482	--
	Pu-240	679	--	670	--	666	--	656	--	646	--
	Pu-241	323	--	318	--	316	--	312	--	307	--
	Pu-242	85	--	84	--	83	--	82	--	80	--
	Total Pu	2645	--	2608	--	2591	--	2554	--	2515	--
	Total	28493	9256	28491	9256	28491	9256	28491	9256	28489	9255
Discharge	U-235	--	--	33	15	50	23	84	38	119	54
	U-238	24374	9060	24365	9041	24361	9032	24354	9015	24345	8994
	Total U	24374	9060	24398	9056	24411	9055	24438	9053	24464	9048
	Pu-239	1884	171	1871	170	1864	170	1851	169	1837	169
	Pu-240	751	5	741	5	736	5	726	5	715	5
	Pu-241	204	0	202	0	200	0	198	0	187	0
	Pu-242	100	0	99	0	98	0	97	0	95	0
	Total Pu	2939	176	2913	175	2898	175	2872	174	2834	174
	Total	27313	9236	27311	9231	27309	9230	27310	9227	27298	9222

FIGURE 5.1. Plutonium Core Enrichment and ^{235}U Assay

For each of the mass flow sets in Table 5.1 the fuel cycle cost can be evaluated as a function of depleted uranium feed and credit price. This relationship is expressed as:

$$\text{Annual Fuel Cycle Cost} = F(T_F, T_C) \quad (5)$$

where

T_F is the Tails feed price, \$/kg U.

T_C is the Tails credit price, \$/kg U.

Since the value of tails is derived from both the ^{235}U and ^{238}U content, a relationship for value as a function of assay can be postulated as:

$$T (\$/\text{kg U}) = a + bA \quad (6)$$

where

A is the ^{235}U assay wt%, and a and b are constants to be determined.

A boundary condition for equation (6) is the market price of natural uranium (as UF_6), which is currently \$118/kg U. Substituting this into equation (6) yields:

$$118 = a + 0.711 b \quad (7)$$

To determine a and b the fuel cycle costs for the mass flows in Table 5.1 were evaluated. In these evaluations, the tails prices were expressed in form of equation (6) to arrive at a set of equations for fuel cycle costs expressed as functions of a and b. Equating fuel cycle costs for any two sets of mass flows derives an indifference expression such that the power costs are equal utilizing either tails assay. These indifference equations, now expressed in terms of a and b, together with boundary condition equation (7), allow a unique solution for a and b. The intermediate steps for this process are given in Tables A.7 through A.9 in the Appendix. The results of all combinations of tails assay considered are shown in Table 5.2.

TABLE 5.2. Calculated Coefficients for a and b
in the Equation " $T(\$/\text{kg}) = a + bA$ "

<u>Data Combination</u>	<u>a</u>	<u>b</u>
0.0 wt% and Natural	47.95	98.52
0.0 wt% and 0.5 wt%	48.66	97.52
0.0 wt% and 0.3 wt%	51.92	92.93
0.0 wt% and 0.2 wt%	48.61	97.59
0.2 wt% and Natural	47.70	98.87
0.2 wt% and 0.5 wt%	48.69	97.48
0.2 wt% and 0.3 wt%	58.59	83.56
0.3 wt% and Natural	45.07	102.58
0.3 wt% and 0.5 wt%	43.78	104.39
0.5 wt% and Natural	46.29	100.86
Mean	48.72	97.43
Standard Deviation	4.12	5.80

Averaging the calculated values for a and b, gives a depleted uranium price equation of:

$$\text{Tails Price (\$/kg U)} = 49 + 97 A \quad (8)$$

where

A is ^{235}U assay in wt% ^{235}U .

Therefore, this analysis results in a range of \$49/kg to 118 \$/kg U for the value of depleted uranium with ^{235}U assay ranging from 0.0 wt% ^{235}U to 0.711 wt% ^{235}U .

6.0 VALUATION COMPARED TO ALTERNATIVE FERTILE MATERIAL-THORIUM

In the previous section the use of depleted uranium as fertile material in LMFBRs was described. Other designs have also been proposed using thorium as the fertile blanket material. In these systems neutrons absorbed in the thorium produce ^{233}U .

The comparison of fuel cycle costs using depleted uranium and thorium for fertile material allows the relative values of thorium and depleted uranium to be determined. Tables 6.1 and 6.2 display the steady state mass flows for two representative mixed oxide LMFBR designs. Both systems use mixed oxide cores and depleted uranium for the axial blanket. One system uses thorium for the radial blanket and the other uses depleted uranium. The Appendix contains the reactor characteristics and data used for this analysis.

Note that the core and axial blanket mass flows are almost identical. From this data it can be inferred that the majority of fuel cycle cost variation in these two systems will be in radial blanket performance. For each of these systems the annual fuel cycle costs can be expressed as the following functional relationship:

$$\text{Annual Fuel Cycle Costs (\$/year)} = F(T, P, Th, U) \quad (9)$$

where

T is the depleted uranium price in \$/kg U

P is the plutonium (core charge) price in \$/kg Pu

Th is the thorium price in \$/kg Th

U is the ^{233}U price in \$/kg U.

By choosing representative values for fuel cycle lead times, fabrication and reprocessing costs, and discount factor, equation (9) can be evaluated for both the depleted uranium and thorium radial blanket systems. Details of these calculations appear in the Appendix.

By equating these two functions, which represent the uranium and thorium fuel cycle power costs, and solving for T, the depleted uranium price, an

TABLE 6.1. Steady State Mass Flows for Depleted Uranium Radial Blanket Design, kg HM/Year

Component	Isotope	Core		Axial Blanket	Radial Blanket
		Zone 1	Zone 2		
Annual Charge	Th-232	--	--	--	--
	U-233	--	--	--	--
	U-235	23	22	31	22
	U-238	7965	7611	10240	7376
	Pu-239	661	866	--	--
	Pu-240	289	378	--	--
	Pu-241	138	179	--	--
	Pu-242	37	48	--	--
	Totals	9113	9104	10277	7408
Annual Discharge	Th-232	--	--	--	--
	U-233	--	--	--	--
	U-235	11	13	26	16
	U-238	7217	7109	10026	7146
	Pu-239	806	882	188	199
	Pu-240	329	401	5	11
	Pu-241	84	119	--	--
	Pu-242	44	55	--	--
	Totals	8491	8579	10245	7372

TABLE 6.2. Steady State Mass Flows Thorium Blanket Design, \$/kg-HM Design, kg HM/Year

Component	Isotope	Core		Axial Blanket	Radial Blanket
		Zone 1	Zone 2		
Annual Charge	Th-232	--	--	--	6637
	U-233	--	--	--	--
	U-235	24	23	31	--
	U-238	7963	7607	10244	--
	Pu-239	663	867	--	--
	Pu-240	289	379	--	--
	Pu-241	138	179	--	--
	Pu-242	36	47	--	--
	Totals	9077	9102	10275	6637
Annual Discharge	Th-232	--	--	--	6440
	U-233	--	--	--	180
	U-235	11	14	26	--
	U-238	7193	7117	10024	--
	Pu-239	809	882	190	--
	Pu-240	330	401	5	--
	Pu-241	83	120	--	--
	Pu-242	44	54	--	--
	Totals	8470	8588	10245	6620

expression is obtained for T as a function of other material prices. Using the mass flows on Tables 6.1 and 6.2 and cost data from the Appendix yields the expression:

$$T (\$/\text{kg}) = -23 + 0.935 \text{ Th} + 0.0510 \text{ P} - 0.026 \text{ U} \quad (10)$$

Equation (10) yields the price for depleted uranium that will give the two LMFBR systems analyzed the same fuel cycle costs.

Conclusions can be drawn from Equation (10) without obtaining precise values for Th, P and U. Equation (10) can be reformulated to isolate the tails and thorium price variables. This is expressed by:

$$T - 0.935 \text{ Th} = -23 + 0.051 \text{ P} - 0.026 \text{ U} \quad (11)$$

If the right-hand side of Equation (11) is much greater than zero when a maximum value of P and a minimum value of U are assumed, it may be concluded that:

$$T \gg 0.935 \text{ Th, or}$$

$$T \gg \text{Th}$$

If the right-hand side of Equation (11) is much greater than zero for the maximum value of P and the minimum value of U, then it will be much greater than zero for all possible values of these variables, and the conclusion that depleted uranium is always more valuable than thorium will be justified.

In a synergistic LWR-LMFBR economy, plutonium is more valuable to the LMFBR operator than to the operators of LWRs. The minimum value for plutonium in such an economy is calculated by comparing its use to low-enriched uranium in a LWR. The minimum value for plutonium is the value such that the LWR operator is indifferent to operating a LWR on the plutonium (MOx) fuel cycle or on the low-enriched uranium cycle. Tables A.3 through A.6 in the Appendix summarize this fuel cycle cost comparison. For the assumed scenario the minimum value of plutonium (as fed to the LMFBR or LWR) is \$16.80/g.

In a similar manner, the maximum value of ^{233}U can be calculated. Its value is a maximum when it is used in a LWR. Again, Tables A.3 through A.6 in the Appendix report the details of this analysis. For the scenario considered, the maximum value of ^{233}U is \$24.40/g.

Substituting these values into the right-hand side of Equation (11) yields

$$-23 + 0.0510 (16800) - 0.26 (24400) \gg 0, \text{ or}$$

$$245 \gg 0$$

Since the relationship is satisfied with the minimum value P and the maximum value of U , the conclusion that depleted uranium is more valuable than thorium is valid. Thus, the lower bound for the tails price is the value of thorium. Recent work has reported a range for thorium unit costs of 5 to 25 \$/lb ThO_2 .⁽⁵⁾ A value of \$15/lb ThO_2 is representative.

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APPENDIX

Data in this appendix are supplementary to Sections 5.0 and 6.0. This appendix is intended to summarize the analysis and list key data and assumptions.

Applicable LMFBR and LWR system characteristics are shown in Tables A.1 and A.2. Assumed fuel cycle logistics and applicable unit costs are listed in Tables A.3 and A.4. Using the data listed in Tables A.1 through A.4, LWR annual fuel cycle costs can be calculated. These costs are shown in Table A.5 as a function of materials (natural uranium, plutonium, and ^{233}U) price. Table A.6 lists fissile material indifference prices derived from data in Tables A.1 through A.5. Tables A.7 through A.9 display the intermediate steps in deriving a functional relationship between tails price and ^{235}U assay. The comparison of thorium and depleted uranium as they perform in an LMFBR radial blanket is summarized in Tables A.10 through A.13.

TABLE A.1. LMFBR Design Parameters⁽²⁾

Electric Power (MWe)	1,200
Thermal Power (MWt)	3,736
Core Height (in.)	46.8
Core Radius (in.)	65.3
Axial Blanket Thickness (in.)	13.0
Radial Blanket Thickness (in.)	13.6
Number of Fuel Assemblies	380
Number of Control Assemblies	20
Number of Blanket Assemblies	184
Fuel Pin Diameter (in.)	0.286
Pins per Fuel Assembly	271
Cladding Thickness (in.)	0.012
Fuel Smear Density (% TD)	88
Core Composition (%)	
Fuel	41.43
Structure	14.96
Sodium	43.61
Number of Fuel Pins per Year ^(a)	51,490
Heavy Metal in Core per Year ^(a) kg	18,222
Isotopics of Charged Plutonium (wt%)	
²³⁹ Pu	58.9
²⁴⁰ Pu	25.7
²⁴¹ Pu	12.2
²⁴² Pu	3.2

(a) 315 Full Power Days

TABLE A.2. LWR Design Parameters

Electric Power (MWe)	1105
Thermal Power (MWt)	3400
Core Height (in.)	144
Core Radius (in.)	66.4
Number of Fuel Assemblies	193
Fuel Pin Diameter (in.)	0.374
Cladding Thickness (in.)	0.0225
Fraction of Core Discharged Annually	0.333
Heavy Metal Discharged Annually (MT)	28.9
²³⁵ U Enrichment, Steady State (wt%)	3.0

TABLE A.3. Fuel Cycle Logistics Data

Parameter	Value	Comment
Discount Factor	0.10	Representative utility value for weighted cost of capital
LWR Raw Materials Purchase ^(a)	-1.00	U ₃ O ₈ , MOx, or ²³³ U as appropriate
LWR Conversion	-0.75	Not appropriate for MOx or ²³³ U
LWR Enrichment	-0.50	Not appropriate for MOx or ²³³ U
LWR Fabrication	-0.25	--
LWR Exposure Period	3.00	--
LWR Reprocessing	4.00	--
LWR Materials Credit	4.00	Credit for recovery of Pu, U, etc.
LMFBR Raw Materials Purchase	-1.00	Plutonium purchase
LMFBR Fabrication	-0.50	--
LMFBR Exposure Period ^(b)	2.00	--
LMFBR Reprocessing	3.00	--
LMFBR Materials Credit	3.00	Credit for recovery of Pu, U, etc.

(a) Fuel cycle timing values are reported in years relative to fuel loading at the reactor.

(b) Radial blanket exposure periods are 2 to 3 years longer.

TABLE A.4. Fuel Cycle Unit Costs (1978 Dollars)^(a)

Parameter	Unit Cost \$/kg HM	Comment
Natural U ₃ O ₈	113	Current spot price
ThO ₂	38	Estimate based on reference 5
Conversion U ₃ O ₈ → UF ₆	5	--
Enrichment	86 per SWU	Current ceiling price
LWR ²³⁵ UO ₂ Fabrication	130	--
LWR ²³³ UO ₂ Fabrication	630	--
LWR MOx Fabrication	430	--
LMFBR Core and Axial Blanket Fabrication	650	--
UO ₂ Radial Blanket Fabrication	140	--
ThO ₂ Radial Blanket Fabrication	150	--
LWR Fuel Reprocessing	230	--
LMFBR Fuel Reprocessing	230	--
Waste Management	150	Applicable at reprocessing
Transportation	30	--

(a) All fabrication and reprocessing data are taken from Reference 6.

TABLE A.5. LWR Fuel Cycle Costs

System	Annual Fuel Cycle Costs (\$ millions)
Low Enriched Uranium	$23.25 + 0.1742 \text{ NU} - 0.0001877 \text{ P}$
Mixed Oxide	$19.98 + 0.03032 \text{ NU} + 0.0009736 \text{ P}$
^{233}U in Natural Uranium	$25.91 + 0.03124 \text{ NU} - 0.0001837 \text{ P} + 0.0005523 \text{ U3}$
^{233}U in Thorium	$26.94 + 0.01224 \text{ Th} + 0.0005539 \text{ U3}$

where:

NU, Natural uranium price, \$/kg U
P, LEU discharge plutonium price, \$/kg HM
Th, Thorium price, \$/kg Th
U3, ^{233}U price, \$/kg U

TABLE A.6. LWR Fissile Material Indifference Prices
for Natural Uranium at \$113/kg U(a)

LMFBR Charge Plutonium - Core	\$16.8/gram
58.9 wt% ^{239}Pu	
25.7 wt% ^{240}Pu	
12.2 wt% ^{241}Pu	
3.2 wt% ^{242}Pu	
LMFBR Discharge Plutonium - Core and Axial Blanket	\$11.9/gram
64.8 wt% ^{239}Pu	
25.4 wt% ^{240}Pu	
6.4 wt% ^{241}Pu	
3.4 wt% ^{242}Pu	
LMFBR Discharge Plutonium - Depleted Uranium Radial Blanket	\$27.8/gram
94.8 wt% ^{239}Pu	
5.2 wt% ^{240}Pu	
<.1 wt% ^{241}Pu	
LMFBR Discharge ^{233}U - Thorium Radial Blanket	\$24.4/gram

(a) At these price levels the LWR operator is indifferent to operating with ^{235}U enriched uranium (for the systems summarized in Table A.2 and A.3) or purchasing the above fissile materials. The power costs are identical.

TABLE A.7. LMFBR Fuel Cycle Costs as a Function of Tails Assay

<u>Tails Feed Assay, wt% ^{235}U</u>	<u>Fuel Cycle Costs, Discounted 1978 Dollars (\$ millions)</u>
0.0	$50.7889 + 0.014676 T (0.0)^{(a)}$
0.2	$50.3596 + 0.038652 T (0.2) - 0.018331 T (0.1367) - 0.0056231 T (.1670)$
0.3	$50.1810 + 0.038672 T (0.3) - 0.018341 T (0.2097) - 0.0056235 T (0.2513)$
0.5	$49.7324 + 0.038712 T (0.5) - 0.018360 T (0.3429) - 0.0056211 T (0.4193)$
0.711 (Natural)	$49.2738 + 0.038751 T (0.711) - 0.018380 T (0.488) - 0.0056179 T (0.595)$

(a) $T (0.0)$ is the value of depleted uranium at a ^{235}U assay of 0.0 wt%.

TABLE A.8. LMFBR Fuel Cycle Costs as a Function of "a" and "b"(a)

<u>Tails Feed Assay, wt% ^{235}U</u>	<u>Fuel Cycle Costs, Discounted 1978 Dollars (\$ millions)</u>
0.0	$50.7889 + 0.014676 a$
0.2	$50.3596 + 0.014698a + 0.00428549b$
0.3	$50.1810 + 0.014708a + 0.00641571b$
0.5	$49.7324 + 0.0147309a + 0.0107034b$
0.711 (Natural)	$49.2738 + 0.0147531a + 0.0152340b$

(a) This table is derived from Table A.7 by formulating the tails price as $T = a + bA$ where A is the ^{235}U assay.

TABLE A.9. Indifference Cost Equation as a Function of "a" and "b"

Data Combination	Equation	Solution ^(a)	
		a	b
0.0 wt% and Natural	$1.5051 = 7.71\text{E-}5 \text{ a} + 0.015240 \text{ b}$	47.95	98.52
0.0 wt% and 0.5 wt%	$1.0464 = 5.49\text{E-}5 \text{ a} + 0.010703 \text{ b}$	48.66	97.52
0.0 wt% and 0.3 wt%	$0.5979 = 3.20\text{E-}5 \text{ a} + 0.0064157 \text{ b}$	51.92	92.93
0.0 wt% and 0.2 wt%	$0.4193 = 2.00\text{E-}5 \text{ a} + 0.0042855 \text{ b}$	48.61	97.59
0.2 wt% and Natural	$1.0858 = 5.31\text{E-}5 \text{ a} + 0.010955 \text{ b}$	47.70	98.87
0.2 wt% and 0.5 wt%	$0.6272 = 3.29\text{E-}5 \text{ a} + 0.0064179 \text{ b}$	48.69	97.48
0.2 wt% and 0.3 wt%	$0.1786 = 1.00\text{E-}5 \text{ a} + 0.0021303 \text{ b}$	58.59	83.56
0.3 wt% and Natural	$0.0972 = 4.51\text{E-}5 \text{ a} + 0.0088243 \text{ b}$	45.07	102.58
0.3 wt% and 0.5 wt%	$0.4486 = 2.29\text{E-}5 \text{ a} + 0.0042877 \text{ b}$	43.78	104.39
0.5 wt% and Natural	$0.4586 = 2.22\text{E-}5 \text{ a} + 0.0045366 \text{ b}$	46.29	100.86

a) Obtained by the boundary condition at natural; $118 = a + 0.711b$.

TABLE A.10. LMFBR Parameter Analysis - Annual Fuel Cycle Costs, Discounted 1978 Dollars (\$000)

Sensitivity Parameter ^(a)	Depleted Uranium System		Thorium System	
	Core and Axial Blanket	Radial Blanket	Core and Axial Blanket	Radial Blanket
Zero Materials Value	27101.87	2669.92	27220.48	2461.30
²³³ U Credit	---	---	---	2562.91
Th Feed	---	---	---	9743.30
Th Credit	---	---	---	6096.51
Pu Feed	29971.77	---	30095.87	---
Pu Credit	29283.57	2788.46	29405.93	---
Tails Feed	55391.67	10818.72	55701.68	---
Tails Credit	45435.46	6712.68	45541.29	---

a) Sensitivites were evaluated by holding all material feed costs and credits at zero except the parameter of interest. The parameter evaluated was set at \$1000/kg to evaluate the effect on the system.

TABLE A.11. LMFBR Sensitivity Parameters

<u>Sensitivity Parameter^(a)</u>	<u>Depleted Uranium System</u>		<u>Thorium System</u>	
	<u>Core and Axial Blanket</u>	<u>Radial Blanket</u>	<u>Core and Axial Blanket</u>	<u>Radial Blanket</u>
Zero Materials Cost ^(a)	27101.97	2669.92	27220.48	2461.30
²³³ U Credit ^(b)	---	---	---	0.10161
Th Feed	---	---	---	7.2820
Th Credit	---	---	---	3.6352
Pu Feed	2.8699	---	2.8754	---
Pu Credit	2.1817	0.11854	2.1855	---
Tails Feed	28.2898	8.1488	28.4812	---
Tails Credit	18.3336	4.0428	18.3208	---

a) This value is the base fuel cycle costs in thousands of dollars for zero materials cost.

b) Parameter units are \$1000's .
\$/kg

TABLE A.12. LMFBR Annual Fuel Cycle Cost Equations

$$\begin{aligned}
 \text{Depleted Uranium System} & & 29771.8 + 2.8699 P_{F_{CAB}} + 28.2898 T_{F_{CAB}} + 8.1488 T_{F_{RB}} - 2.1817 P_{CR_{CAB}} \\
 \text{Annual Cost, Discounted 1978} & = & - 0.11854 P_{U_{CR_{RB}}} - 18.3336 T_{CR_{CAB}} - 4.0428 T_{CR_{RB}} \\
 \text{1978 Dollars ($000)} & & \\
 \\
 \text{Thorium System Annual} & & 29681.8 + 2.8754 P_{F_{CAB}} + 28.4812 T_{F_{CAB}} + 7.2820 T_{F_{RB}} - 2.1855 P_{CR_{CAB}} \\
 \text{Cost, Discounted 1978} & = & - 18.3208 T_{CR_{CAB}} - 3.6352 T_{CR_{RB}} - 0.10161 U_{CR_{RB}} \\
 \text{Dollars ($000)} & &
 \end{aligned}$$

Materials costs are in \$/kg and are defined as

- P , Plutonium price
- T , Tails price
- Th, Thorium price
- U , ²³³U price
- F , Feed material
- CR, Credit material
- CAB, Core and axial blanket
- RB, Radial blanket

Example: $P_{F_{CAB}}$ is the plutonium price for the material charged to the core and axial blanket.

TABLE A.13. LMFBR Tails Valuation - Comparison With Thorium

$$\begin{aligned} \text{Tails Price (\$/kg U)} = & -23.07 + 0.001410 P_{F_{CAB}} - 0.0009779 P_{CR_{CAB}} + 0.03038 P_{CR_{RB}} \\ & + 0.9346 Th - 0.02604 U \end{aligned}$$

Applying the ratios of relative plutonium values shown in Table A6 the above equation can be expressed as a function of only the LMFBR charge plutonium. Thus the equation can be simplified to:

$$\text{Tails Price (\$/kg U)} = -23 + 0.935 Th + 0.0510 P - 0.026 U$$

where,

Th, Thorium price, \\$/kg Th

P, Plutonium price as charged to the LMFBR, \\$/kg Pu

U, ^{233}U price, \\$/kg U

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