198
6/18/79

DR. 2677

LA-7814-MS

Informal Report

MASTER

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Cold, Relativistic Electron Beam and a Collisional
Plasma-Astron Experiment**

University of California

LOS ALAMOS SCIENTIFIC LABORATORY

Post Office Box 1663 Los Alamos, New Mexico 87545

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Barry S. Newberger
Lester E. Thode

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A NONLINEAR TWO-STREAM INTERACTION BETWEEN A
COLD, RELATIVISTIC ELECTRON BEAM AND A COLLISIONAL
PLASMA-ASTRON EXPERIMENT

by

Barry S. Newberger and Lester E. Thode

ABSTRACT

Experiments on the two-stream instability of a relativistic electron beam propagating through a neutral gas, carried out with the Lawrence Livermore Laboratory Astron beam, have been analyzed using a nonlinear saturation model for a cold beam. The behavior of the observed microwave emission due to the instability is in good agreement with that of the beam energy loss. Collisions on the plasma electrons weaken the nonlinear state of the instability but do not stabilize the mode. The beam essentially acts as if it were cold, a result substantiated by linear theory for waves propagating along the beam. In order to predict the effect of both beam momentum scatter and plasma electron collisions on the stability of the mode in future experiments a full two-dimensional linear theory must be developed.

I. INTRODUCTION

A major issue in the transport of energy at high densities by relativistic electron beams is the stability of the beam as it passes through a background plasma. This plasma may be produced by some external mechanism or by the beam itself. As new experiments are proposed, it is imperative that a detailed understanding of the scaling of the properties of the unstable modes with various parameters be obtained. A potentially important mode is the two-stream instability, and thus this became the subject of a fairly intensive theoretical research effort.^{1,2}

It has long been felt, however, that collisions on the background plasma electrons would stabilize the beam if there were sufficient velocity spread on the beam electrons. The theoretical underpinning of this conclusion has been established by balancing a phenomenological phase mixing due to the velocity spread against some linear growth rate. The growth rate is not calculated self-consistently. There are two immediate difficulties associated with such an approach. While there is some intuitive appeal to such a model, since physically the growth rate must depend on the position of the wave phase velocity in the beam distribution, the value of the phase velocity is a function of both the beam scatter and the collision frequency and must be calculated self-consistently. Furthermore, recent results we have obtained from the electrostatic Vlasov theory of the instability in a scattered beam and collisional plasma show that the scaling of this phenomenological model is in substantial error in the limit where the perpendicular wave number, $k_{\perp}$, is zero. Although the implications for the finite $k_{\perp}$ case are not absolute, it is certainly clear that the linear theory in two dimensions ultimately must be carried out.

The credence of the model has been based largely on experiments performed with the Lawrence Livermore Laboratory (LLL) Astron beam. Experiments in which this beam was injected into a background gas were interpreted as evidence that the beam was unstable to the two-stream mode until electron-neutral collisions (proportional to the ambient pressure) on the background electrons were sufficiently frequent to stabilize the instability.³ The pressures at which the apparent stabilization occurred were in reasonable agreement with those expected on the basis of the phenomenological models. In what follows, we provide an alternative interpretation of those experimental results. We will see that, in fact, the Astron beam used in those experiments was essentially cold and that a nonlinear saturation mechanism for a cold beam in a collisional plasma⁴ yields results that are in good agreement with the experiment. Furthermore, we have examined the role of the parallel mode ($k_{\perp} = 0$) and have found that, indeed, the linear spatial growth length is too long for this mode to be observed. This does not contradict the earlier interpretation; neither does it imply the validity of the phenomenological model. We also found that the effects of finite beam radius in the linear theory alone are not sufficient to explain the experimental results.

We summarize our findings thus: the LLL Astron experiments on the two-stream instability are well explained by nonlinear saturation in a cold beam. Furthermore, the implications of the theory, in the absence of beam "temperature" for experiments at Experimental Test Accelerator (ETA) and Advanced Test Accelerator (ATA) parameters, are very serious. The linear theory for these beams in the presence of finite momentum scatter and the consequent nonlinear states of the instability are the subject of current and proposed research.

II. THEORY OF THE ASTRON EXPERIMENT

The caveats inherent in the phenomenological models of the phase-mix damping of the two-stream mode have led us to consider alternative interpretations of the experimental results for the propagation of the Astron beam in low-pressure hydrogen.³ Although the gross qualitative features of that experiment are not inconsistent with plausible expectations based on the model, we will see below that at least one other theory can give results that not only agree qualitatively with the experiment overall, but also agree with the behavior of the microwave signal data as a function of background pressure. The microwave signal was the fundamental evidence for the existence of the two-stream instability in the Astron experiments. We acknowledge that it is difficult to argue for the confirmation of a particular theory on the basis of a single, limited experiment. We only point out that this proviso holds equally for any competing theory, and in this regard our results (described below) are far more satisfying than those arising from a phenomenological linear theory. It is therefore not unlikely that the gross predictions based on linear stability due to phase mixing were simply fortuitous.

The theoretical model we adopted is based on the nonlinear saturation amplitude of the two-stream mode in a cold beam as it propagates through a cold collisional plasma.⁴ This model predicts that the beam loses, as a result of the instability, an energy per unit volume of

$$\delta W_b = \frac{1.5 S_G}{(1+1.5 S_G)^{5/2}}, \quad (1)$$

where δW_b is in units of the beam energy density $\gamma_0 n_b m c^2$. S_G is a measure of the strength of the beam-plasma interaction and is in general a function of γ_0 , the beam-to-plasma density ratio n_b/n_p , the collision frequency ν_c , and

the angle of propagation, θ . This quantity is determined from the linear theory and is related to the shift in the parallel wave phase velocity relative to the beam velocity. Specifically,

$$S_G = 2\beta_\theta^2 \gamma_\theta^2 \Omega(\gamma_0, n_b/n_e, v_c, \theta) \quad , \quad (2)$$

where Ω is defined by

$$\Omega = 1 - \frac{\text{Re}(w)}{k_z c \beta_0 \cos \theta} \quad (3)$$

and $\text{Re}(w)$ is found from the solution of the linear dispersion relation for the fastest growing mode in the system. The linear theory for the cold beam propagating through a collisional plasma is given in Ref. 4. Although the interest in Ref. 4 was in fully ionized plasmas, the results are applicable to our problem with the simple replacement $v_{ei} \rightarrow v_c$, where v_c is the electron-neutral momentum transfer collision frequency, and is proportional to the ambient gas pressure, p . In the above, $\gamma_\theta = (1 - \beta_\theta^2)^{-1/2}$ and $\beta_\theta = \beta_0 \cos \theta$.

We have calculated δW_b appropriate to the Astron experiment. The finite beam radius induces a quantization of $k_\perp$ with the lowest mode having $k_\perp \cong 2.4/r_b$, where r_b is the beam radius. The propagation angle, θ , is then $\theta = \arctan(k_\perp/k_z)$. We tested the full effect of finite beam radius (finite geometry) in the linear limit⁵ and have found it to be small, approximately ten per cent at most. Furthermore, two-dimensional particle-in cell simulations of the two-stream instability indicate that the nonlinear evolution of the mode is treated adequately by restricting the oblique wave propagation to quantized angles if the linear dispersion relation is satisfied approximately by the zeros of the function $J_0(z)$. This requirement is consistent with our results from the linear theory. The behavior of δW_b is shown in Fig. 1. The solid line is a curve through the experimental data and is reproduced from Fig. 3 of Ref. 3. The experimental parameter measured is simply the output voltage from a crystal detector and is a relative measure of the X-band microwave noise induced by the two-stream turbulence. The noise signal is plotted as a function of the ambient gas pressure. The dashed curve is a plot of the fractional loss δW_b . For the values of S_G obtained, the energy lost by the beam is essentially proportional to S_G . Below 0.1 torr, the beam does not produce sufficient ionization for a plasma to exist and the microwave signal

drops rapidly. Because the plasma density is changing dramatically in this region and we had no reliable data for its value, we did not extend our curve into this region. Using plausible values of n_e , however, we confirmed that the strength parameter drops significantly in this regime.

Note that we did not normalize our values of δW_b to the crystal diode output voltage although the microwave signal measurements are relative. Furthermore, although the assumption that the microwave signal energy generated by two-stream turbulence is proportional to the energy lost by the beam is quite plausible (indeed implied by the experiment), there is no theory known to us which provides an absolute calibration.

Other qualitative features are well explained by our theory. In particular, the fairly abrupt loss in signal in the fundamental channel at the plasma frequency for pressures in excess of a few torr may, in part, be due to the nonlinear effects described above; but it is principally due simply to the shift in the fundamental eigenfrequency out of the channel bandpass. We estimate down shifts in frequency on the order of 3 to 10 percent as the pressure is increased into the 3- to 10-torr range, and this could account for the observation. In the previous work,³ similar "tuning" effects due to variations in the equilibrium were invoked to explain signal loss in the 26- to 28-GHz channel. Because both effects exist simultaneously, interpretation is complicated. Indeed, the phenomenological model for the collisional stabilization of the mode in the linear regime applies only for the fundamental mode, which is the highest observed frequency in the system. Thus, any interpretation of experimental observation that demands that the highest frequency waves be stabilized first is inconsistent with that theory. The loss of signal in the center channel (16-18 GHz) is not explained easily as the noise in this part of the spectrum necessarily is generated by strong nonlinear processes (turbulence) or perhaps in the lower density plasma in the wings of the column. We only remark that no other theory known to us provides an interpretation either.

The theory presented above is in good agreement with the LLL Astron beam experiments on the two-stream interaction for a cold relativistic electron beam. In fact, this experiment may be the only one reported to date in which the beam had sufficiently small momentum spread to behave as a cold beam. (For discussion of other experiments see Ref. 6.) Indeed, a comparison of the growth rates for parallel propagation in a beam with 18 mrad scatter with

those for a beam of 0.1 mrad scatter supports our contention that the beam is cold. Specifically, Fig. 2 displays the absolute value of the fractional difference in growth rates for the two cases as a function of collision frequency. The other parameters are as appropriate to the Astron experiment. The differences are obviously very small, with the largest occurring only for ν in excess of $\omega_p/2$, and there we find only a few percent effect. We will now digress briefly and discuss the scaling of the linear theory for the collisional stabilization of the two-stream instability in energetic beams of high current where the velocity spread is increased. The implications for future experiments will be discussed in the concluding section.

III. LINEAR THEORY FOR PARALLEL PROPAGATION

It has long been recognized that if a charged particle beam propagating through a cold plasma has sufficient velocity spread, and the background plasma is sufficiently collisional, the high-frequency, two-stream mode is stable.⁷ This result was established rigorously only for wave propagation along the beam direction and for a nonrelativistic beam. Relativistic effects were introduced in an ad hoc fashion by replacing the rest electron mass for the beam particles, m_0 , by $\gamma_0^3 m_0$.

Attempts to generalize the linear theory to oblique wave propagation have, so far, relied on a phenomenological approach in which a phase-mix damping, measured by $\vec{k} \cdot \Delta \vec{v}$, is balanced against a collisional growth rate calculated for a cold beam. The velocity spread is $\Delta \vec{v}$, and $\vec{k}$ the instability wave number.^{3,8} It is a disconcerting feature of these models that as $k_\perp \rightarrow 0$, the results pass, within numerical factors of order unity, continuously to those of Ref. 7. We shall see that the scaling of this theory is substantially different from a rigorous, relativistic theory we have recently carried out.

The relativistic electron beams used in propagation experiments in the past, as well as those proposed for future experiments, can be reasonably described by a scattered distribution in velocity space.⁶ Such beams have a fixed energy γ_0 , but the individual velocity vectors of the beam particles have a spread in angle about the mean direction of propagation. This spread is characterized by an angle $\bar{\theta}$, typically rather smaller than one radian. In terms of $\bar{\theta}$, the velocity spread $\Delta \vec{v}$ is $\Delta v_\perp/c \cong \bar{\theta}$ and $\Delta v_\parallel/c \cong \bar{\theta}^2/2$. For self-pinched beams, $\bar{\theta}$ is a function of $(I_B/I_A)^{3/2}$, where I_B is the net beam current

and I_A the Alfvén current. We have generalized the linear theory for such beams^g to include the effects of collisions on the background plasma electrons and used the results to calculate the critical collision frequencies necessary to stabilize the mode for propagation along the beam direction ($k_{\perp} = 0$). Some of our striking results are summarized in Table I. Here the critical collision frequency for stabilization ν_c is tabulated for three values of γ_0 . For comparison, the value based on results from Ref. 7, ν_s , are included. The beam-to-plasma density ratio is fixed at $n_b/n_p = 0.01$, a reasonable value for plasmas produced by the beam itself. Frequencies are in units of the plasma frequency. The scattering angles are based on a current of 10 kA. Note that if there is partial current neutralization, the scattering angles would be smaller still. The difference in scaling between the two theories is obvious. The values of ν_c correspond to ambient pressures in the 5- to 10-torr range ($p \propto \nu_c$). Furthermore, for beams in the quasi-hydrodynamic regime, it is well known that the oblique modes grow faster than those on the beam axis. Thus, the results presented here may be optimistic, and, in view of the results of the previous section, there currently exists no satisfactory theory to predict the stability of these modes for a collisional plasma.

That the oblique modes were more unstable than the parallel modes was appreciated, of course, when the Astron experiments were done.³ The hope was

TABLE I
STABILIZING COLLISION FREQUENCY FOR
TWO-STREAM INSTABILITY IN A SCATTERED BEAM

Relativistic Factor, γ_0	Mean Scattering Angle, $\bar{\theta}$	Beam-to-Plasma Density Ratio, n_b/n_p	Singhaus Collision Frequency, ^a ν_s	Stabilizing Collision Frequency, ^b ν_c
11	9.29°	0.01	3.3×10^{-2}	8.7×10^{-2}
100	3.10°	0.01	3.6×10^{-3}	5.8×10^{-2}
1000	0.98°	0.01	3.6×10^{-4}	4.7×10^{-2}

^aCollisions to stabilize based on Singhaus theory $\nu_s = 3.04/\gamma_0^3 n_b/n_p \beta_0^2/\bar{\theta}^4$.

^bAll frequencies are in units of the plasma frequency.

that because the perpendicular velocity spread was larger than the parallel velocity spread ($\Delta v_{\perp}/\Delta v_{\parallel} \approx 2/\bar{\theta}$), the oblique modes would phase mix away faster than they would grow, while the parallel modes simply would not have adequate room to grow to large amplitude over the length of the system. Our results show, however, that the observations are due to a weakening of the nonlinear state by collisions. The beam was behaving as if it were cold. It is unlikely that linear stabilization of the two-stream instability by collisions was ever observed in this experiment.

IV. CONCLUSION

We have reexamined the LLL Astron two-stream experiments in the context of the nonlinear saturation of the mode. The theoretical results are in good qualitative agreement (there is no absolute calibration available) with the experimental measurements of the microwave emission due to the instability. It is likely, therefore, that the beam was behaving essentially as a cold beam. This observation is confirmed by comparing the linear growth rates for the Astron beam with those of a cold beam. The relative difference was less than 2%.

In light of these results, it is improbable that linear stabilization of the two-stream mode due to collisions on the plasma electrons occurred in the Astron experiment. The phenomenological models used to establish stability boundaries for this experiment must, therefore, be considered suspect. By using such theories to design new experiments, one may well be courting untoward consequences.

To determine the scaling of future planned and proposed experiments, rigorous theoretical models must be developed and compared with available experimental data. We currently are extending the linear theory of a scattered beam propagating through a collisional plasma to include the oblique modes. Furthermore, in the intense beams of current interest, the nonlinear states are likely to be much stronger than those in the Astron beam and existing nonlinear theories need to be extended as well. Without an active theoretical research effort addressed to the two-stream instability, propagation experiments to be performed over the next several years may provide us with unexpected results.

ACKNOWLEDGMENTS

We wish to thank Brendan Godfrey for many useful discussions.

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Fig. 1.

The solid curve is the experimentally observed X-band microwave signal as a function of the ambient pressure from Ref. 3, Fig. 3. The measure of the signal is the output voltage from a crystal diode. The dashed curve is the calculated relative beam energy loss due to the two-stream instability. The horizontal bars indicate the span of values obtained at fixed pressure as the plasma density varies between experimental limits.

Fig. 2.

The fractional difference between linear growth rates for the Astron beam ($\bar{\theta} = 18$ mrad) and a cold beam for waves propagating parallel to the beam. The growth rates are calculated as a function of the collision frequency.