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THEORY OF ULTRA-LOW-FREQUENCY MAGNETIC PULSATIONS IN THE
EARTH'S MAGNETOSPHERE

BY

L. CHEN

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PRINCETON
PLASMA PHYSICS
LABORATORY

PRINCETON UNIVERSITY, PRINCETON, NEW JERSEY

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THEORY OF ULTRA-LOW-FREQUENCY MAGNETIC PULSATONS IN THE EARTH'S MAGNETOSPHERE*

Liu Chen

Plasma Physics Laboratory and Department of Astrophysical Sciences
Princeton University, Princeton, NJ 08543

ABSTRACT

Long-period ($T = 10\text{-}600$ s) geomagnetic pulsations are known to be associated with magnetohydrodynamic (MHD) perturbations in the Earth's magnetosphere. Broadly speaking, there are two categories of excitation mechanisms. The first category corresponds to impulsive/external excitations, where MHD waves exhibit the stable discrete as well as continuous spectra. The second category corresponds to spontaneous/internal excitations, where MHD instabilities are excited either reactively or via wave-particle interactions. In this tutorial lecture, we briefly review theories concerning both categories of excitation mechanisms and compare theoretical predictions with available satellite observations.

I. IMPULSIVE/EXTERNAL EXCITATIONS

I.1. In this category of excitations, the plasmas are locally MHD stable. Magnetic pulsations, therefore, correspond to plasma responses to either broad band impulsive or narrow band instability-type external sources. In terms of observations, these pulsations typically fall into the class of dayside Pc3-Pc4 ($T = 10\text{-}150$ s) events.

If we understand the spectrum of MHD waves in such a magnetospheric plasma, we can then, at least, theoretically construct the appropriate plasma responses. The success of the field-line resonance theory [1-3] indicates that one of the crucial physics elements is the spatial nonuniformity of the Alfvén velocity, $v_A = (B^2/4\pi\rho)^{1/2}$; where ρ is the plasma mass density and B is the background magnetic field.

The simplest theoretical model, which retains the important spectral properties, is the so-called rectangular box model. That is, we have a cold magnetized plasma with an uniform $B = B_0 \hat{z}$ but nonuniform mass density $\rho = \rho(x, z)$. In terms of magnetospheric coordinates, x , y , and z then correspond, respectively, to radial, azimuthal (East-West) and along the field line coordinates.

Adopting the one-fluid ideal MHD approximation, the linearized equations then become

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$$D_A \delta \tilde{\xi}_{\perp} = \tilde{\nabla}_{\perp} \delta \tilde{B}_{\parallel}, \quad (1)$$

$$\delta \tilde{B}_{\parallel} = -\tilde{\nabla}_{\perp} \cdot \delta \tilde{\xi}_{\perp}; \quad (2)$$

where

$$D_A = \omega^2 g + \frac{\partial^2}{\partial z^2}, \quad (3)$$

$g(x, z) = 1/v_A^2(x, z)$, $\delta \tilde{\xi}_{\perp}$ is the fluid displacement, $\delta \tilde{B}_{\parallel} = \delta B_{\parallel}/B_0$, $\exp[-i(\omega t + i k_y y)]$ dependence is assumed, and $\tilde{\nabla}_{\perp} = i k_y \hat{e}_y + \hat{e}_x \partial/\partial x$. Regarding D_A as an operator, Eqs. (1) and (2) can be combined into

$$\left[\frac{\partial}{\partial x} \frac{D_A}{k_y^2 - D_A} \frac{\partial}{\partial x} - D_A \right] \delta \xi_x = 0. \quad (4)$$

In the uniform limit, we readily recover from Eq. (4) the dispersion relations of transverse (shear) and compressional Alfvén waves given by, respectively, $D_A = 0$ and $k_x^2 + k_y^2 - D_A = 0$. Plasma nonuniformities, however, couple the two waves and one needs to carry out spectral analyses. We then note that two types of spectrum exist. One is the discrete compressional Alfvén spectrum and the other is the continuous shear Alfvén spectrum.

1.2. First, we discuss the discrete (or cavity) compressional Alfvén eigenmodes. Noting that the relevant eigenvector is $\delta B_{\parallel}$, Eqs. (1) and (2) can be recombined into

$$\left[\frac{\partial}{\partial x} D_A^{-1} \frac{\partial}{\partial x} - D_A^{-1} k_y^2 + 1 \right] \delta \tilde{B}_{\parallel} = 0. \quad (5)$$

We shall analyze Eq. (5) in the large- k_y limit; i.e., defining a smallness parameter ε as

$$1 \gg |\varepsilon| \equiv |k_y^{-2} \partial^2/\partial z^2|. \quad (6)$$

Equation (6) is equivalent to assuming large azimuthal wave numbers. While, in general, not necessary, we shall further assume, for simplicity,

$$g(x, z) = G(x) H(z) . \quad (7)$$

We can now proceed to analyze the corresponding eigenvalue problem via expansions in ϵ ; that is, we let

$$\delta \hat{B}_{\parallel} = \delta B_{\parallel}^{(0)} + \delta B_{\parallel}^{(1)} + \dots ,$$

and

$$\omega^2 = (\omega^2)_0 + (\omega^2)_1 + \dots .$$

In the zeroth order, we have

$$L_0 \delta B_{\parallel}^{(0)} = \left[\frac{\partial^2}{\partial x^2} - k_y^2 + (\omega^2)_0 H(z) G(x) \right] \delta B_{\parallel}^{(0)} = 0 . \quad (8)$$

Denoting $\Omega_0^2 \equiv (\omega^2)_0 H(z)$, Eq. (8) is an eigenvalue problem in x only. Note $G(x) \propto 1/v_A^2(x)$, the potential well $U(x) = k_y^2 - \Omega_0^2 G(x)$, therefore, tends to be centered about x_m , the magnetospheric boundary. Assuming boundary conditions $\delta B_{\parallel}$ or $\partial \delta B_{\parallel} / \partial x = 0$ at x_m and $\delta B_{\parallel} = 0$ at $x = 0$, we can then solve Eq. (8) yielding, for the n th eigenstate in x ,

$$\delta B_{\parallel}^{(0)} = \delta B_n(x) A_n(z) , \quad (9)$$

and

$$\Omega_0^2 = (\omega^2)_0 H(z) = \Omega_{0n}^2 . \quad (10)$$

Note that $\delta B_n(x)$ will be localized about $x = x_m$, consistent with numerical solutions [4, 5]. Also, at this order, z enters only as a parameter. In order to determine the z dependence, we need to go to the next order.

In the next order in ϵ , we have from Eq. (5),

$$L_{0n} \delta B_{\parallel}^{(1)} + \left[\omega^2 G(x) H(z) - \Omega_{0n}^2 G(x) - \frac{G'}{G} \frac{\partial}{\partial x} + \frac{\partial^2}{\partial z^2} \right] \delta B_{\parallel}^{(0)} = 0 . \quad (11)$$

Here, $G' \equiv \partial G(x) / \partial x$. Noting Eq. (9), we can operate $\int_0^{x_m} \delta B_n dx$ on Eq. (11) and obtain the following eigenmode equation for $A_n(z)$;

$$\left[\frac{\partial^2}{\partial z^2} + \omega^2 H(z) \bar{G} - \Omega_{0n}^2 \bar{G} + \overline{(G'/2G)} \right] A_n(z) = 0. \quad (12)$$

Here, $\bar{p} \equiv \int dx p \delta B_n^2(x) / \int \delta B_n^2(x) dx$ is just a constant representing $p(x)$ weighted over x by $\delta B_n^2(x)$. Equation (12) is, thus, a one-dimensional eigenvalue problem in z . Applying the boundary conditions that $A_n(z = \pm z_b) = 0$, we can then obtain, for the j th eigenstate,

$$A_n(z) = A_{nj}(z), \quad (13)$$

$$\omega = \omega_c(\underline{N}); \quad (14)$$

where $\underline{N} = (n, k_y, j)$. Noting, again, that $H(z) \propto v_A^{-2}(z)$, the potential well of Eq. (12) is thus typically centered at the $z = 0$ equator and $A_{nj}(z)$ will, hence, behave similarly.

As indicated by Eqs. (13) and (14), for a given k_y or m (the azimuthal wave number), the compressional Alfvén eigenmodes have a two-dimensional discrete spectrum indexed by n and j . Employing the large- k_y ($m \gg 1$) limit, we have presented a formulation where this intrinsic two-dimensional eigenvalue problem is reduced to two one-dimensional eigenvalue problems; first, in x and then in z . Our analysis indicates that the eigenmodes tend to be localized around the minimum of v_A^2 , i.e., typically, near the magnetospheric boundary and the equator. Also, from straightforward WKB arguments, one can see that, as k_y increases, ω also increases, but the radial localization width decreases. All the above features are consistent with direct numerical solutions [5].

Several issues remain when we extrapolate the present results to realistic magnetospheric plasmas. The more crucial one, I believe, is that, due to the imposed boundary conditions, the magnetospheric cavity is assumed to be lossless. Since the eigenmodes are global, wave energy may be dissipated by either the (mainly nightside) ionospheres or leaking through the outer magnetospheric boundary, or both. Such dissipations will damp the eigenmodes and may explain why few supporting observations have been found.

1.3. We now discuss the continuous shear (transverse) Alfvén spectrum, or, more commonly known in the space physics community, the field line resonances (FLR). Following the approach of [6], we let, near the resonance,

$$|D_A| \sim O(\epsilon) \ll 1. \quad (15)$$

Expanding the solutions asymptotically as

$$\begin{aligned}
\delta\xi_y &= \delta\xi_y^{(-1)} + \delta\xi_y^{(0)} + \dots \\
\delta\xi_x &= \delta\xi_x^{(0)} + \delta\xi_x^{(1)} + \dots \\
\delta\hat{B}_{||} &= \delta B^{(0)} + \delta B^{(1)} + \dots,
\end{aligned}
\tag{16}$$

where the superscripts denote ordering in ϵ , Eqs. (1) and (2) then become, in the lowest order,

$$D_A^{(1)} \delta\xi_y^{(-1)} = i k_y \delta B^{(0)}, \tag{17}$$

and

$$\frac{\partial}{\partial x} \delta\xi_x^{(0)} = -i k_y \delta\xi_x^{(-1)}. \tag{18}$$

Let us define the orthonormal FLR eigenfunctions $\{a_l(z|x)\}$ and eigenfrequencies $\{\omega_l^2(x)\}$ as

$$D_{Al}^{(1)} a_l \equiv \left[\frac{\partial^2}{\partial z^2} + \omega_l^2 g(x, z) \right] a_l(z|x) = 0, \tag{19}$$

with boundary conditions $a_l(z = \pm z_b|x) = 0$, and

$$\langle\langle g a_l a_{l'} \rangle\rangle \equiv \int_{-z_b}^{z_b} g a_l a_{l'} dz = \delta_{ll'}. \tag{20}$$

We can then proceed to examine the x dependencies of the singular solutions near the resonant field lines. Letting

$$\delta\xi_y^{(-1)} = \sum_l \beta_{ly}(x) a_l(z|x), \tag{21}$$

we obtain, from Eq. (17),

$$\beta_{ly}(x) = i k_y \frac{\delta B_l(x)}{\omega^2 - \omega_l^2(x)}, \quad (22)$$

where

$$\delta B_l(x) = \langle\langle \delta B^{(0)} a_l \rangle\rangle. \quad (23)$$

Note that β_{ly} is singular (within the ideal MHD approximation) at $x = x_l$ where $\omega^2 = \omega_l^2(x)$. Meanwhile, taking

$$\delta \xi_x^{(0)} = \sum_l \beta_{lx}(x) a_l(x), \quad (24)$$

we have, from Eq. (18), near $x = x_l$,

$$\beta_{lx}(x) \equiv c_l - k_y^2 \frac{\delta B_l(x_l)}{(\omega_l^2)} \ln(x - x_l). \quad (25)$$

$\delta \xi_x$, thus, has a milder logarithmic singularity. The present analysis, therefore, would predict that, with a steady-state driving source, the y (azimuthal) component of the magnetic perturbation will be singularly enhanced when the driving frequency ω_0 equals to the particular FLR eigenfrequency ω_l . We emphasize that, according to Eq. (23), the excitation of FLR depends not only on the magnitude of the compressional component $\delta B^{(0)}$ but also on the projection of its field-aligned structure on the FLR eigenfunction a_l .

1.4. In the case of impulsive excitations, we are interested in the temporal evolutions of perturbations; in particular, their time asymptotic behaviors. For this discussion, we may assume $\delta B_l(x|\omega)$ is given by

$$\delta B_l(x|\omega) = s_l(\omega, x)/D(\omega), \quad (26)$$

where ω is properly understood as the Laplace transform variable, $s_l(\omega, x)$ corresponds to the initial perturbations, and $D(\omega)$ is the global "dispersion relation" containing the discrete spectrum; i.e. $D(\omega = \omega_{cN}) = 0$. From Eq. (22), we have

$$\begin{aligned} \delta B_{ly}(x, t) &\approx \beta_{ly}(x, t) = ik_y \int d\omega \frac{s_l(\omega, x) \exp(-i\omega t)}{D(\omega) [\omega^2 - \omega_l^2(x)]} \\ &= ik_y \left\{ \sum_N \frac{s_l(\omega_{cN}, x) \exp(-i\omega_{cN}t)}{D'(\omega_{cN}) [\omega_{cN}^2 - \omega_l^2(x)]} + \frac{s_l(\omega_l, x)}{2\omega_l(x) D(\omega_l)} \exp[-i\omega_l(x)t] \right\}. \end{aligned} \quad (27)$$

In Eq. (27), the first term consists of perturbations due to eigenmodes in the discrete spectrum. The second term, meanwhile, is associated with the continuous spectrum; i.e., the FLR eigenfrequencies, $\omega_l(x)$. We emphasize, again, that the discrete spectrum corresponds to cavity eigenmodes with spatially independent eigenfrequencies; i.e., $\nabla \omega_{cN} = 0$. The continuous spectrum, on the other hand, gives rise to perturbations whose oscillation frequencies $\omega_l(x)$ vary spatially. Meanwhile, from Eqs. (18) and (24), we have

$$\begin{aligned} \delta B_{lx}(x, t) &\approx \beta_{lx}(x, t) = k_y^2 \left\{ \sum_N \frac{\exp(-i\omega_{cN}t)}{D'(\omega_{cN})} \int^x dx' \frac{s_l(\omega_{cN}, x')}{\omega_{cN}^2 - \omega_l^2(x')} \right. \\ &\quad \left. + \int^x dx' \frac{s_l(\omega_l, x')}{2\omega_l(x') D(\omega_l)} \exp[-i\omega_l(x')t] \right\}. \end{aligned} \quad (28)$$

Note that Eqs. (27) and (28) exhibit the familiar singular behaviors at $x = x_{lN}$ where $\omega_{cN}^2 = \omega_l^2(x)$. This is expected since the cavity eigenmodes may be regarded as steady-state driving sources. In the following section, we shall discuss the effects of finite FLR coupling on the cavity eigenmodes. As to the continuous spectrum, while δB_{ly} exhibits undamped oscillations at the local FLR eigenfrequency $\omega_l(x)$, δB_{lx} decays time asymptotically as $1/t$ due to phase mixing. To see this, we note that, with $F(x)$ being analytic,

$$\lim_{t \rightarrow \infty} \int^x dx' F(x') \exp[-i\omega_l(x')t] \rightarrow \left[\omega_l' t \right]^{-1} \exp[-i\omega_l(x)t].$$

Thus, for both the discrete and the continuous spectra, the y (azimuthal) component dominates over the x (radial) component. Such a property is consistent with satellite observations of FLR oscillations; i.e., the shear Alfvén continuous spectrum [7].

1.5. We now discuss the effects of finite FLR coupling on the discrete/cavity eigenmodes. The singular behaviors at $x = x_{lN}$ where

$\omega_{lN}^2 = \omega_l^2(x)$ indicate that x_{lN} is the shear Alfvén singular resonance layer and, therefore, resonant wave energy absorption is expected. Following Chen and Hasegawa [8], the rate of energy absorption dW/dt is given by

$$\begin{aligned} \frac{dW_N}{dt} &\propto \text{Re} \int dx \int dz \delta \mathbf{I} \cdot \delta \mathbf{E} \Rightarrow L_z k_y \sum_l \text{Re} \int dx \delta B_l(x) \beta_{ly}^*(x) \\ &= k_y^2 L_z \text{Im} \int dx \sum_l |\delta B_l|^2 [\omega_l^2(x) - \omega_{lN}^2] \\ &\Rightarrow \sum_l |\delta B_l(x_{lN})|^2 / (\omega_l^2)' . \end{aligned} \quad (29)$$

In realistic plasmas, this resonant energy absorption will occur via either collisional or collisionless dissipations and finite FLR coupling will lead to damping of the discrete eigenmodes. Quantitatively, since $\delta B_l(x)$ is evanescent at $x = x_{lN}$, the damping rate, at least in the large k_y limit, is therefore exponentially small. That is, the wave energy must first tunnel through a potential barrier before it is absorbed. This prediction of weak damping is consistent with one-dimensional numerical calculations [4].

1.6. Finally, we remark that the above theoretical analyses can be more or less straightforwardly extended to the dipole $\mathbf{B}$ geometry. In fact, the shear Alfvén continuous spectrum and the associated FLR properties have been recently analyzed [6]. Similarly, assuming large azimuthal wave numbers $|m| \gg 1$, the two-dimensional discrete compressional eigenmode analysis can be reduced to two one-dimensional eigenvalue problems. One is in the radial direction and the other is along $\mathbf{B}$. One can be readily convinced that the qualitative features remain, however, similar to those considered in the present work.

II. SPONTANEOUS/INTERNAL EXCITATIONS

In this category of excitations, the plasmas within the local magnetic flux tube contain internal free energy. The system is thus linearly unstable and MHD-like instabilities can be spontaneously excited. In terms of satellite observations, the present analysis is applicable to Pc4-5 MHD waves associated with geomagnetic storms; i.e., the so-called storm-time geomagnetic pulsations occurred during either the main or the recovery phase.

II.1. Before we discuss the specific theoretical analysis, we shall make some general remarks on the appropriate free energy sources and instability mechanisms. These remarks will help to delineate the physics underlying the theoretical results.

Since the magnetospheric plasma can be classified as a collisionless magnetized plasma, its equilibrium distribution function F_0 can, therefore, be expressed as

$$F_0 = F_0(\mu, \epsilon, \psi). \quad (30)$$

Here, $\mu = v_\perp^2/2B$ is the first adiabatic constant of motion, $\epsilon = v^2/2$ where we have, for simplicity, neglected any electrostatic potential, and ψ is the magnetic flux function; that is $\mathbf{B} = \nabla\psi \times \nabla\phi$. It is then well known that the free energy can be associated with either the velocity space or the configuration space. The velocity-space free energy can be further classified into (i) anisotropy $\partial F_0/\partial\mu|_{\epsilon,\psi} \neq 0$ which gives rise to the mirror and firehose instabilities, and (ii) "inverted-energy" distribution $\partial F_0/\partial\epsilon|_{\mu,\psi} > 0$ which leads to the so-called bump-in-tail instability. The configuration-space free energy, meanwhile, is associated with $\partial F_0/\partial\psi|_{\mu,\epsilon} \neq 0$; i.e., finite density and pressure gradients which can drive the familiar drift instabilities as well as the ballooning/interchange instabilities.

Given the free energy, instabilities can be excited via either reactive or dissipative processes. In the MHD regime, reactive instabilities correspond to perturbations which lead to a decrease in the potential energy. This is, of course, just the famous ideal MHD energy principle and its variations [9-12]. Noting the system is self-adjoint, one then has

$$\omega^2 \propto \delta W, \quad (31)$$

where δW is the change in the potential energy due to wave perturbations. Thus, if there exists perturbation(s) for which $\delta W < 0$, the system is then reactively unstable. On the other hand, if for all perturbations satisfying appropriate boundary conditions $\delta W \geq 0$, we then have a stable plasma.

A reactively stable plasma may still become unstable by releasing its free energy via dissipative processes. In collisionless plasmas, such energy transfer occurs through wave-particle interactions. The corresponding resonance condition for a magnetically trapped particle is

$$\omega - \bar{\omega}_d - p \omega_b = 0, \quad (32)$$

where $p = \text{integers}$, $\omega_b = 2\pi/\tau_b$, $\tau_b = \oint dl/|v_{\parallel}|$ is the bounce period between the magnetic turning point, $\bar{\omega}_d = \mathbf{k}_{\perp} \cdot \mathbf{v}_d$, $\mathbf{k}_{\perp}$ is the perpendicular (to $\mathbf{B}$) wave vector, $\mathbf{v}_d$ is the ∇B and curvature drift, and $\bar{A}$ denotes the bounce averaged value of A .

II.2. Here, we shall present highlights of the theoretical model and analyses. The details are given in Chen and Hasegawa [13]. Since storm associated pulsations are of interest here, the plasma is taken to be at the outer edge of the ring current; i.e., $L \sim 6-7 R_E$. Consistent with satellite data, we further assume the plasma to be composed of two components. One is the core component with $T_c \sim O(10^2 \text{ eV})$. The other is the energetic component with $T_E \sim O(10 \text{ keV})$. With $n_c/n_e \sim O(10)$, the core component provides the inertia for the Alfvén waves. Meanwhile, $\beta_E \sim O(1)$ and $\beta_c \sim O(10^{-1})$. Here, $\beta \equiv \text{plasma pressure/magnetic pressure}$. The energetic component, thus, dominates the pressure contribution and provides the free energy for the instability. The theoretical model also allows a realistic plasma equilibrium with an anisotropic pressure $\underline{P} = P_{\perp} \underline{\underline{1}} + (P_{\parallel} - P_{\perp}) \underline{\underline{e}}_{\parallel} \underline{\underline{e}}_{\parallel}$. Here, $\underline{\underline{e}}_{\parallel} = \mathbf{B}/B$, $P_{\perp} = P_{\perp}(\psi, B)$, and $P_{\parallel} = P_{\parallel}(\psi, B)$. The B dependencies of $P_{\parallel}$ and $P_{\perp}$ are due to the pressure anisotropy. The equilibrium condition of force balance then yields

$$B \left. \frac{\partial P_{\parallel}}{\partial B} \right|_{\psi} = P_{\parallel} - P_{\perp}, \quad (33)$$

and

$$\tau \nabla_{\perp} \ln B + \frac{4\pi}{B^2} \nabla \psi \left. \frac{\partial P_{\perp}}{\partial \psi} \right|_B = \sigma \kappa. \quad (34)$$

Here, $\tau = 1 + 4\pi(\partial P_{\perp}/B \partial B)$ is the mirror criterion, $\sigma = 1 + 4\pi(P_{\perp} - P_{\parallel})/B^2$ is the firehose criterion, and $\kappa = (\underline{\underline{e}}_{\parallel} \cdot \nabla) \underline{\underline{e}}_{\parallel} = \partial \underline{\underline{e}}_{\parallel} / \partial l$ is the curvature.

The theoretical analysis is based on the low-frequency linear gyrokinetic equations [14, 15], which adopt the following ordering scheme

$$|\omega| \sim |\omega_i| \sim |\omega_d| \sim |\omega_b| \ll |\Omega_d|, \\ |\mathbf{k}_{\perp} \rho| \sim O(1), \quad (35)$$

and

$$|k_{\parallel}/k_{\perp}| \ll 1.$$

Here ω_* and Ω_e are, respectively, the diamagnetic drift and cyclotron frequencies, ρ is the cyclotron radius. The analysis can be further simplified by taking the following subsidiary expansions. First, we assume $|\omega/\omega_{b,e}| \ll 1$ and electrons mainly respond to bounce averaged dynamics. For ions, we assume $|\omega/\omega_{b,i}| \gg 1$; i.e., core ions only respond locally. The energetic ions, however, have $|\omega/\omega_{b,iE}| < 1$; i.e., their dynamics is bounce averaged. In addition, there are hot energetic ions with $T_{iH} \sim O(10^2 \text{ keV})$ such that $|\omega_{d,iH} - \bar{\omega}_{d,iH}|$ and they may experience the important drift-bounce resonances.

Besides the ideal MHD approximation $\delta E_{\parallel} = 0$, the resultant eigenmode equations exhibit a finite coupling between the compressional component $\delta B_{\parallel}$ and the transverse component $\delta \psi$ where $\partial \delta \psi / \partial \ell = i \omega \delta A_{\parallel} / c$. That is, we have

$$\begin{aligned} \tau \delta \hat{B}_{\parallel} = & \frac{4\pi}{B^2} (\mathbf{e}_k \cdot \tilde{\nabla} P_{\perp}) \delta \psi + 4\pi \sum_{j=e,i} m_{jE} \langle QF_0 P(\omega - \bar{\omega}_d)^{-1} \overline{\mu \Omega_d \delta \psi + \mu \delta \hat{B}_{\parallel}} \rangle_{jE} \\ & - 4i \pi^2 \sum_{j=e,i} m_j \left\langle \sum_K QF_0 \delta(\omega - \bar{\omega}_d - K\omega_b) S_B \overline{S_{\psi} \delta \psi + S_B \delta \hat{B}_{\parallel}} \right\rangle_j, \end{aligned} \quad (36)$$

and

$$\begin{aligned} B \frac{\partial}{\partial \ell} \left(\frac{k_{\perp}^2}{B} \sigma \frac{\partial \delta \psi}{\partial \ell} \right) + \frac{\omega^2 k_{\perp}^2}{v_A^2} \delta \psi = & - \frac{4\pi}{B^2} (\mathbf{e}_k \cdot \tilde{\nabla} P_{\perp}) (\delta \hat{B}_{\parallel} + \Omega_B \delta \psi) \\ & - \frac{4\pi}{B^2} (\mathbf{e}_k \cdot \tilde{\nabla} P_{\parallel}) \Omega_K \delta \psi - 4\pi \sum_{j=e,i} m_{jE} \langle QF_0 P(\omega - \bar{\omega}_d)^{-1} \Omega_d \overline{\Omega_d \delta \psi + i \delta \hat{B}_{\parallel}} \rangle_{jE} \\ & + 4i \pi^2 \sum_{j=e,i} m_j \left\langle \sum_K QF_0 \delta(\omega - \bar{\omega}_d - K\omega_b) S_{\psi} \overline{S_{\psi} \delta \psi + S_B \delta \hat{B}_{\parallel}} \right\rangle_j; \end{aligned} \quad (37)$$

where $\delta \hat{B}_{\parallel} = \omega \delta B_{\parallel} / c$, $\mathbf{e}_k = \mathbf{k} \times \mathbf{e}_{\parallel}$, $\tilde{\nabla} P_{\perp} = \nabla \psi \partial P_{\perp} / \partial \psi$, $\Omega_d = \mu \Omega_B + v_{\parallel}^2 \Omega_K / B$, $\Omega_B = \mathbf{e}_k \cdot \nabla \ln B$, $\Omega_K = \mathbf{e}_k \cdot \kappa$.

$$QF_0 = \left(\omega \frac{\partial}{\partial \varepsilon} + \frac{1}{\Omega} \mathbf{e}_k \cdot \nabla \right) F_0. \quad (38)$$

$S_\psi \propto \Omega_d$ and $S_B \propto \mu$ are, meanwhile, forces with finite Larmor-radius corrections and phase information. Equations (36) and (37) may be regarded, respectively, as equations of perpendicular pressure balance and vorticity generalized to include finite ion Larmor radii and wave-particle interactions.

II.3. We first consider the stability properties of Eqs. (36) and (37) in the reactive limit. Thus, ignoring the wave-particle resonance terms, we can construct the quadratic forms and obtain the following low-frequency kinetic energy principle [12]

$$\omega^2 \delta I = \delta W_f + \delta W_{kT}, \quad (39)$$

where

$$\delta I = \left[k_\perp^2 |\delta \psi|^2 / v_A^2 \right],$$

$$\delta W_f = \left[\pi \delta \widehat{B}_\parallel - \frac{4\pi}{B^2 \tau} (\mathbf{e}_k \cdot \widetilde{\nabla} P_\perp) |\delta \psi|^2 \right] + \left[k_\perp^2 \sigma \left| \frac{\partial \delta \psi}{\partial \ell} \right|^2 \right] \quad (40)$$

$$- 4\pi \left[\frac{1}{B^2} \Omega_k |\delta \psi|^2 \mathbf{e}_k \cdot (\widetilde{\nabla} P_\parallel + \frac{\sigma}{\tau} \widetilde{\nabla} P_\perp) \right], \quad (41)$$

$$\delta W_{kT} = (4\pi)^2 \sum_j \left\langle \left\langle \frac{m \mathbf{e}_k \cdot \nabla F_0}{\Omega} \tau_b \left[\Omega_d \delta \psi + \mu \delta \widehat{B}_\parallel \right]^2 \right\rangle \right\rangle. \quad (42)$$

$$[A] \equiv \int_{\ell_S}^{\ell_N} A d\ell / B \quad (43)$$

with ℓ_S and ℓ_N being the end points of the field line and $\delta \psi(\ell = \ell_S, \ell = \ell_N) = 0$, and

$$\langle A \rangle \equiv \int d\mu \int d\varepsilon (A). \quad (44)$$

δI , thus corresponds to the inertia of core ion transverse motion. δW_f is the well-known δW of the ideal MHD energy principle [9]. The first term in δW_f corresponds to perpendicular compression and its sign depends on τ . For $P_\perp \geq P_\parallel$, as of interest here, $\sigma > 0$ and the second term represents the positive-definite field line bending stabilization. The third term, meanwhile, is the well-known ballooning/interchange driving term. As to δW_{kT} , we note that at the outer edge of the ring current, we have $\epsilon_k \cdot \nabla F_0 / \bar{\omega}_d \sim \omega_w / \bar{\omega}_d > 0$ and, thus,

$$\delta W_{kT} \geq 0; \quad (45)$$

i.e., the trapped-particle compression is stabilizing.

For $\tau < 0$, the perpendicular compression is negative and, hence, we readily obtain $\delta W_f < 0$ by taking the $|\delta\psi| \rightarrow 0$ limit. On the other hand, since δW_{kT} involves bounce averaging, we can make $\delta W_{kT} = 0$ by taking $\delta B_\parallel$ to be antisymmetric (odd with respect to the equator). We then conclude that the odd- $\delta B_\parallel$ perturbations yield the most unstable reactive mirror instabilities. The corresponding growth rate, meanwhile, is of $O(|q|^{1/2} k_\phi v_A)$ and is extremely fast.

For $\tau > 0$, the positive-definite stabilizing perpendicular compression term can be made to vanish with

$$\delta \hat{B}_\parallel = \frac{4\pi}{B^2 \tau} \left(\epsilon_k \cdot \widetilde{\nabla} P_\perp \right) \delta \psi. \quad (46)$$

The instability is, thus, driven by the ballooning/interchange term, i.e., the third term in Eq. (41). Maximizing the driving term then stipulates that $|k_\phi| \gg |k_\psi|$; i.e., $|\delta B_\psi| \gg |\delta B_\phi|$. We note this property is opposite to that of external/impulsive excitations (c.f., Sections I.3 and I.4). δW_{kT} has a stabilizing effect toward the ballooning term. It, however, can be made to vanish with odd (antisymmetric) $\delta\psi$ and $\delta B_\parallel$. Since the odd (e.g., the second-harmonic) $\delta\psi$ also experiences more field line bending stabilization than the even (fundamental) $\delta\psi$, the net stability properties in principle must be evaluated numerically with self-consistent finite- β equilibria. Noting that the plasma is interchange stable and the driving term is proportional to β which is localized about the equator, one expects that, for the fundamental mode, the ballooning term to be nearly cancelled by δW_{kT} . The second-harmonic odd $\delta\psi$ (even δB_ψ) perturbation is, therefore, expected to be the most unstable mode for this reactive Alfvén ballooning-mirror instability. This expectation has been verified numerically by Chan and Chen

[16] in the isotropic limit. Assuming a localized pressure bump, $P(\psi) = L^{-\alpha}$, in an otherwise dipole B , it is found that for $\alpha = 0.67$, the critical vacuum β value for the second-harmonic antisymmetric mode $\beta_{v,crit} = 0.848$ is lower than that of the fundamental symmetric mode $\beta_{v,crit} = 1.318$. Studies employing anisotropic pressure equilibria are also in progress.

II.4. We now consider the final topic of this tutorial lecture; that is, instabilities excited via wave-particle interactions. Recalling the resonance condition in Eq. (32), these "dissipative"-type instabilities are important when the plasma is either reactively stable and/or near marginal stability such that particle dynamics become important.

Including wave-particle interactions, the corresponding quadratic form then becomes

$$\omega^2 \delta I = \delta W_f + \delta W_{KT} + i \delta W_R,$$

where

$$\delta W_R = \omega \delta W_\omega + \delta W_*, \quad (48)$$

and

$$\begin{pmatrix} \delta W_\omega \\ \delta W_* \end{pmatrix} = 16\pi^3 \sum_K \sum_{j=c,i} \left\langle \left\langle m \begin{pmatrix} \partial F_0 / \partial \varepsilon \\ \underline{e}_K \cdot \nabla F_0 / \Omega \end{pmatrix} \right\rangle \delta(\omega - \bar{\omega}_d - K\omega_b) \tau_b \left[S_\psi \delta\psi + S_B \delta \hat{B}_\parallel \right]^2 \right\rangle_j \quad (49)$$

Note that with $\partial F_0 / \partial \varepsilon < 0$, $\delta W_\omega < 0$ and $\delta W_* > 0$; that is, the instabilities are driven by configuration-space free energy. Also, only the $K = 0$ term enters the electron contribution. Near marginal stability, we may assume $\omega = \omega_r + i\gamma$ and $|\gamma/\omega_r| < 1$. Equation (47) then becomes

$$\omega_r^2 \delta I = \delta W_f + \delta W_{KT} - \gamma \delta W_\omega, \quad (50)$$

and

$$2\gamma\omega_r \delta I = \omega_r \delta W_\omega + \delta W_*. \quad (51)$$

For $\tau < 0$, the drift-mirror instability is operative [17]. Thus, again, letting $|\delta\psi| \rightarrow 0$, Eq. (50) reduces to

$$\gamma \delta W_{\omega} = \delta W_f + \delta W_{KT}. \quad (52)$$

Since $\delta W_{\omega} < 0$, instabilities set in if

$$\delta W_f + \delta W_{KT} \leq 0. \quad (53)$$

Here, again, the stabilizing δW_{KT} vanishes for odd $\delta B_{\parallel}$ and δW_f becomes negative-definite with $\tau < 0$. That is, for $\tau < 0$, odd- $\delta B_{\parallel}$ perturbations become unstable first. The real frequencies, meanwhile, are given by Eq. (51), $\omega_r \delta W_{\omega} + \delta W_{*} \approx 0$; i.e., $\omega_r \sim \omega_{*iE}$. We note that these features are consistent with numerical results of Cheng and Lin [18].

On the other hand, the drift Alfvén ballooning-mirror instability is operative in the $\tau > 0$ regime [13]. Here, Eq. (50) becomes

$$\omega_r^2 \delta I = \delta W_f + \delta W_{KT}, \quad (54)$$

and $\omega_r^2 > 0$ since the plasma is reactively stable. From Eq. (51), we see instabilities ($\gamma > 0$) require $|\omega_r \delta W_{\omega}| < |\delta W_{*}|$; i.e., $|\omega_r| < |\omega_{*}|$. Here, we have assumed, without any loss of generality, $\omega_r > 0$. Noting $|\omega_r| \sim |\omega_d|$, the resonance condition Eq. (32) then indicate that the destabilizing wave-particle interaction is the drift-bounce resonance ($\omega_d \approx \omega_b$) of hot energetic ions [19, 20]. The instability, furthermore, has the following features: (i) with $K = -1$, the perturbations are odd in $\delta\psi$; (ii) $\omega_r \propto \omega_{*iE}$; (iii) maximizing γ corresponds to minimization of ω_r^2 or δW_f ; noting Eq. (52) and $\delta W_{KT} = 0$ for odd $\delta\psi$. Thus, the most unstable perturbations also have features very similar to those of the reactive Alfvén ballooning-mirror instability such as (1) coupled compressional and transverse components [c.f., Eq. (46)], (2) second-harmonic antisymmetric $\delta\psi$ structure along B , (3) $|k_{\phi}| \gg |k_{\psi}|$ and, hence, $|\delta B_{\psi}| \gg |\delta B_{\phi}|$. Again, we emphasize this property is opposite to that associated with the continuous shear Alfvén spectrum. Maximizing the destabilizing drift-bounce resonance, meanwhile, predicts $k_{\phi} \rho_{iE} \sim O(1)$; i.e., large azimuthal wave numbers, $m \sim O(10^2)$ [c.f., Ref. 20]. Finally, we remark that warm-ion bounce resonances ($\omega_r \approx \omega_b$) and ionospheric collisional dissipations both contribute to stabilizing effects. The instability, therefore, requires $\beta_i > \beta_{i,crit} \sim O(10^{-1})$, typically.

III. CONCLUSION

In conclusion, we note that theories of impulsive/external excitations are primarily based on the concept of field line resonances of the continuous shear Alfvén spectrum. The theories have been fairly successful in explaining satellite

as well as ground observations of the dayside geomagnetic pulsations. One can further improve the theories by incorporating effects due to, e.g., ionosphere, plasma pressure, azimuthal asymmetries, etc. Since the theoretical formulation and approach have been established on a fairly comprehensive foundation, these improvements should be straightforward.

In the case of spontaneous/internal excitations, the theories are more complicated. For $\tau < 0$, we have the well-known mirror instabilities driven by pressure anisotropy. On the other hand, the $\tau \geq 0$ regime is dominated by the drift Alfvén ballooning-mirror (DABM) instabilities driven by pressure inhomogeneities and bad field line curvature. Since many wave features predicted by the DABM instability are consistent with available satellite observations, we are encouraged to further develop the theory. In particular, we would like to consider effects such as the finite FLR coupling, d.c. electric field, nonlinear saturation due to mode couplings and particle phase-space diffusion as well as to develop a self-consistent picture of associated particle flux modulation.

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