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I. Introduction

The slot coupled TEM line has been employed at CERN¹ as a pick up electrode in the GHz range. It is a compact and broad band device, and will be quite attractive if the coupling efficiency is competitive with an array of quarter wave loops.² In this paper, we study various properties of such a structure.

Fig. 1 shows the geometry of a general slotted coupler. The bottom region is the beam pipe, and the top part is the TEM line which picks up the electromagnetic energy from the beam particles. Energy transfers from beam to line through a series of identical slots which connect the two regions.

FIG. (1) GEOMETRY OF A GENERAL SLOTTED COUPLED PICK UP.

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A slotted pickup structure is a special case of a general class of travelling wave couplers. In such a device, the signal voltages add in phase when the beam velocity v matches the line velocity v_L . The pickup impedance can thus be large if the structure is long. However, it turns out that the impedance contains a factor $\gamma_L^{-2} = 1 - \beta_L^2 = 1 - (v_L/c)^2$, and the signal in the synchronous case is weaker than the one obtained from an equal length array of loop couplers. In this paper, we derive the response of the slotted line in a general non-synchronous case, and explore the possible advantages of designing the coupler with reduced velocity to increase the coupling efficiency.

II. Theoretical Analysis

Some assumptions are necessary to analyze the system in a simple way. We assume that the slots are sufficiently far from each other so that they can be treated separately. We also assume that the slot dimensions are sufficiently small so that Bethe's small slot theory³ is applicable, which means that we can replace each slot by effective magnetic and electric dipole moments.

The phase velocity of the slot-loaded TEM line, v_L , is in general different from the smooth-line

velocity c . This is due to the wave scattering at slots, and the effect can be computed analytically from the results of Bethe's theory.

Our approximation treats the presence of slots as a perturbation on a smooth conducting wall. The quantities of interest in the smooth wall geometry are

D^{-1} = Electric field at the location of the slots on the TEM line wall per unit voltage on the line.⁴

E_c = Electric field at the location of the slots on the beam-pipe wall induced by the beam current.

Since the above quantities are defined in the absence of slots, they can be calculated by solving the two-dimensional field equations.

With the help of these quantities, the results of a detailed calculation⁵ are as follows: First, consider the case where only one slot is open. The voltage on the TEM line induced by the beam current is:

$$V_L = -ik_0 \cdot D \cdot E_c \cdot \frac{\bar{Z}(\alpha_m - \alpha_e)}{2D^2} \quad (1)$$

Here $k_0 = \omega/c = 2\pi f/c$ is the free-space wave number, $\bar{Z} = Z_L/377\Omega$, Z_L = characteristic impedance of the pickup line, α_m and α_e are the electric and the magnetic polarizabilities of the slots, respectively, and $\beta = v/c$, v being the velocity of the beam particles.

For the full slotted structure the results are as follows: the phase velocity of the pickup line is changed from c to v_L by the amount

$$\Delta\beta_L = 1 - \beta_L = 1 - v_L/c = \bar{Z}(\alpha_m - \alpha_e)/2D^2d \quad (2)$$

where d is the spacing between slots. We assume now that the pickup line is terminated properly at both ends to avoid reflections. The voltage induced in the line of length L is

$$V(L) = -i(k_0L)(DE_c) \frac{1}{2\gamma_L^2} e^{\frac{j}{2}(k_L + k) \cdot L} \frac{\sin \theta}{\theta} \quad (3)$$

Here $k = \omega/v$, $k_L = \omega/v_L$, $\theta = (k_L - k)L/2$,

$$\gamma_L^{-2} = (1 - \beta_L^2) - 2\Delta\beta_L$$

Thus, the pickup signal is a strong function of the line velocity v_L . When the line velocity matches exactly the beam velocity, $v_L = v$, $\sin\theta/\theta$ becomes unity. In such case, it is easy to show that $V(L)$ becomes simply V_L times the total number of the slots apart from the obvious phase factor. Because of the factor γ_L^{-2} in Eq. (3), however, the signal from a slotted coupler of a reasonable length is small in the extreme relativistic case. We shall return to the question of the optimization later in this paper.

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To compare with experiments, it is convenient to introduce the pickup impedance Z_p as

$$Z_p = \frac{V(L)}{I_b} = -i(k_0 L) Z' \frac{1}{2\gamma_L} e^{-i\theta} \frac{\sin \theta}{\theta} \quad (4)$$

Here $Z' = DE_c/I_b$ is a quantity independent of the beam current I_b , and is of the order of the line impedance.

In the above, the effect of the waveguide modes in the beam pipe are neglected. In a general case, the waveguide modes could be excited by the pickup line wave and vice versa. We have studied such a case and the results are as follows: When the phase velocity of the waveguide mode is sufficiently different from the pickup line wave, the coupling of the two can be neglected. The case where the two velocities are close to each other can be treated by a degenerate perturbation theory. One then decomposes the waves into the sum mode and the difference mode. For the sum mode, the velocity reduction becomes twice that given by Eq. (2), while for the difference mode it is zero. Hence, the increased k_2 would be used in Eq. (3) to obtain the signal voltage.

Eq. (2) states that the line velocity v_L is independent of the frequency. This behavior is a direct consequence of Bethe's small slot coupling theory. In a realistic case, the slot has a non-zero aperture and the theory breaks down as the frequency becomes large. In particular one expects a resonance behavior as the wave length approaches twice the aperture width. The resulting dispersion is one of the limiting factors of the pickup performance.

It is instructive to illustrate the results of this section by a simple example. Consider the case of a coaxial line coupled to a round beam pipe. Let $a(b)$ = the radius of the inner (outer) conductor of the coaxial line and g = radius of the beam pipe. Then,

$$D = b \ln(b/a) \text{ and } Z' = Z_L \frac{b}{g} \quad (5)$$

Thus Eq. (4) becomes

$$Z_p = -ik_0 L Z_L \frac{b}{g} \frac{1}{2\gamma_L} e^{-i\theta} \frac{\sin \theta}{\theta} \quad (6)$$

For a rectangular aperture the electric and magnetic polarizabilities are

$$\alpha_m = .216 w^2 z + .044 w^3, \quad \alpha_e = \frac{\pi}{16} w z^2 \quad (7)$$

In the above, w and z are the lengths of the aperture along the direction transverse to and parallel to the beam direction, respectively.

III. A 50-Slot Coupler

We have built and tested a model coupler shown in Fig. 2. The dimensions in the figure are in cm. Two pickup lines, top and bottom, are coupled to the rectangular beam pipe. Pickup response of up to 50 slots was measured.

Let us first compute the theoretical prediction. In the model, the slot width w is made large to increase the coupling. Therefore the quantity $1/D^2$ varies over the slot aperture. In the present

FIG. (2) 50-SLOT COUPLER

analysis, this is computed by solving the two dimensional Laplace equation and we take the average over the aperture. We obtain

$$\left\langle \frac{1}{D^2} \right\rangle = .65 / (.5)^2 \text{ cm}^{-2} \quad (8)$$

Next we compute the polarizabilities. For a rectangular slot, one obtains $\alpha_m = \alpha_e = .516 \text{ cm}^3$ from Eq. (7) using the values $w = 1.9 \text{ cm}$, $z = .326 \text{ cm}$. However, the slots in the model coupler are rounded at both ends, for which the results of electrolytic tank measurements yields $\alpha_m = \alpha_e = .4776 \text{ cm}^3$. One then obtains, from Eq. (2) and Eq. (3) the results

$$\Delta \beta_L = .066, \quad (9)$$

and

$$Z_p = 58.6 \times \sin(1.462 \frac{N}{50} f) \text{ ohms.} \quad (10)$$

Here f is the frequency in GHz, N is the number of the slots exposed.

Tests carried out using a modulated electron beam at ANL⁷ showed the expected dependence on frequency and on the number of slots exposed as given by Eq. (10). In particular, the maximum response for 50 slots appeared around 3 GHz, to be compared with Eq. (10) which has a peak at 3.4 GHz. However the first minimum was at 4.6 GHz, instead of 6 GHz, indicating a considerable dispersion in the slotted line. The experiment lacked a calibration of overall response sensitivity because of the severe beam divergence. Thus the coefficient in Eq. (10) could not be compared directly with experiment.

The amount of the phase velocity change was measured by another experiment in which the pickup line is coupled to another identical line. Measurement of the phase lag shows a dispersion of $\Delta \beta_L$ from $\Delta \beta_L = .04$ at $f = 1 \text{ GHz}$ to $\Delta \beta_L = .065$ at 3 GHz. This is in reasonable agreement with Eq. (9) considering the crude nature of the approximations.

Finally the pickup response was also measured by replacing the beam by a 0.050" diameter wire. We obtained a maximum $Z_p = 43 \Omega$ at 2.75 GHz to be compared with the prediction of Eq. (10) which gives 56.0 Ω . There are many possible reasons for this discrepancy. In particular, the wire may not behave the same as the beam current.

IV. Optimization

The factor $\sin \theta / \theta$ in Eq. (3) is maximum when $v_L = v$. On the otherhand, the factor $1/D^2$ then becomes very small for a realistic beam. Therefore the optimization of the coupler parameters needs to be studied more carefully.

A sensible way to optimize the parameters is to maximize power per unit $1/L$, i.e., the quantity

$$R = \frac{P}{1/L}, \text{ with } P = \frac{V^2(L)}{Z_L} \quad (11)$$

From Eq. (3), we obtain

$$\sqrt{R} = \sqrt{\frac{Z^2 k}{2 Z_L}} u(x, L), \quad x = \frac{v}{V_L} = \frac{k_L}{k} \quad (12)$$

Here u is the function

$$u(x, L) = \frac{x^2 - \beta^2}{x\sqrt{x-1}} \frac{\sin \theta}{\theta}, \quad \theta = \frac{\pi(x-1)L}{\lambda g} \quad (13)$$

where λ is the free-space wavelength. For a given velocity mismatch x , the maximum value of u is

$$u = u_m(x) = .8514 \frac{x^2 - \beta^2}{x\sqrt{x-1}} \quad (14)$$

and this occurs when

$$L = L_m(x) \equiv \frac{1.1655 \beta \lambda}{\pi(x-1)} \quad (15)$$

Now consider the corresponding expression for the loop coupler pickup which produces a voltage $V = Z_L I_L/2$ in a length $L = \lambda/4$. Here β is a geometric factor of order unity. Computing R using the definition (11), one obtains

$$\sqrt{R_{\text{loops}}} = \alpha \sqrt{\frac{Z_L k}{2}} \frac{1}{\sqrt{\pi}} \quad (16)$$

Thus we may identify

$$u_{\text{loops}} = \frac{1}{\sqrt{\pi}} = 0.5642 \quad (17)$$

Table I gives values of u_m , the line velocity, and $L_m(x)$ for a range of x for a relativistic beam with $\beta = 0.9945$. A synchronous coupler would depend on length according to

$$u_{\text{syn}} = u(x=1, L) = (1-\beta^2) \sqrt{\frac{\pi}{\lambda g}} \quad (18)$$

and values of this are also given for the lengths L_m . Even for the longest case, the detuned pickup is stronger and improves as line velocity decreases. Beyond $x = 1.11$, the slotted coupler becomes superior to the loop coupler, but since some space is necessary between the shorter loop couplers, the slot coupler may become attractive when $x \geq 1.07$. It appears then that a series of short, velocity-detuned slot couplers can be competitive with an array of loop couplers in a stochastic cooling system. However, the dispersion effect of the loaded TEM line will alter the response of the slot coupler. The dispersion increases as x increases because the slot aperture must increase accordingly. Our tests indicate that for β less than .95, dispersive effects may limit the useful bandwidth.

Conclusion

In this paper, we have analyzed the slot-coupled TEM line using simplifying assumptions. The results are in rough agreement with the tests carried out with a 50-slot model coupler. The experiments are still in a preliminary stage, and a more thorough investigation is necessary to understand the limitations of the simple theory.

A pickup device based on the slot coupled TEM line is attractive because of the structural simplicity, especially at higher frequencies where the system of the loop couplers becomes very intricate. Optimization of the theoretical response formula of the slot-coupled device indicates that its coupling could be made comparable to the loop couplers.

However, as one increases the size of the slot to increase the coupling, the slot-loaded TEM line becomes dispersive because the simple derivation based on Bethe's theory fails. Dispersion is apparent in our model coupler. More information on line dispersion is needed to proceed with design optimization.

Table I
Performance of Maximized and Synchronous Couplers

x	$\beta_L = \beta/x$	L_m/λ	u_m	u_{syn}
1.01	.9846	36.9	.2622	.1188
1.02	.9750	18.4	.3034	.0840
1.03	.9655	12.3	.3431	.0686
1.04	.9562	9.22	.3790	.0594
1.05	.9471	7.83	.4116	.0531
1.06	.9382	6.15	.4414	.0485
1.07	.9294	5.26	.4489	.0449
1.08	.9208	4.61	.4944	.0420
1.09	.9124	4.10	.5184	.0396
1.10	.9041	3.69	.5409	.0376
1.11	.8959	3.35	.5622	.0358
1.12	.8879	3.08	.5824	.0343

$\gamma = 9.5$

$\beta = .9945$

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