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FLOW IN GEOTHERMAL WELLS:
PART III. CALCULATION MODEL FOR
SELF-FLOWING WELL

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1. Introduction

In this third part of our study of flows of geothermal fluids in wells we give a description of the proposed flow model and quote the results of a number of numerical calculations.

The flow of the brine starts with a segment at the lower end of the well where the temperature is lower than saturation so that, consequently, the fluid flows in a single, liquid phase. Two-phase flow begins at a level where the two temperatures become equal and the liquid flashes into vapor. In order to provide an analytic description of this two-phase flow, use is made of the one-dimensional (or "engineering") approximation discussed in Part II. It was shown there that the set of fundamental equations is not a closed one and that two supplementary phenomenological statements are required for completeness. These will be chosen in the form of an equation for wall stress, τ_w , and void fraction ϕ . The majority of alternative models differ among themselves by the analytic expressions chosen to define these two quantities. A short review of some of these alternative methods follows.

Elliott [22-24] proposed a method, known as the JPL method, which is based on the assumption that the slip ratio in a two-phase flow is constant. The numerical value of this slip ratio, β_s , in any particular calculation is left free and constitutes an adjustable parameter. On the basis of some comparisons between calculation and field data (well-head pressure *versus* flow rate) Elliott obtained optimum agreement for $\beta_s=1$ in the case of IID (Imperial Irrigation District) No. 1 [23]. In the case of the East Mesa well 16-29 [24] the optimum fit occurred for $\beta_s=6$.

The resistance to flow is calculated in Elliott's model with the aid

of the formula

$$\tau_w = C_f \left[\frac{1}{2} \rho' (1-\phi) w'^2 + \frac{1}{2} \rho'' \phi w''^2 \right], \quad (99)$$

with the constant value

$$C_f = 0.008. \quad (99a)$$

A different method was proposed by Karlsson [25] who postulated a slip ratio described by Zivi [26] and based on a minimum kinetic-energy criterion for the two-phase mixture. This implies that the slip ratio is a unique function of the two densities and is given by

$$\beta_s = (\rho'/\rho'')^{1/3}. \quad (100)$$

The wall stress in this algorithm was based on the correlation introduced into this subject by Martinelli and Nelson [27].

$$\phi_{1tt}^2 = \frac{(dP/dz)_{TPF}}{(dP/dz)_\ell}. \quad (101)$$

Here ϕ_{1tt} is the void fraction of the liquid in a turbulent-turbulent liquid-vapor flow. The subscript TPF refers to two-phase flow, and ℓ refers to liquid flow at the same specific rate of mass flow, ψ , as the two-phase stream. The quantity ϕ_{1tt} is defined by a graph whose independent variable is

$$x_{tt} = \left(\frac{\rho''}{\rho'} \right)^{0.5} \left(\frac{\eta'}{\eta''} \right)^{0.1} \left(\frac{1-y}{y} \right)^{0.9}. \quad (102)$$

The symbols η' and η'' the viscosity of the respective phase.

Dukler [28] recognized that the flow model ought to take into account the fact of the existence of several basic flow configurations, the slip ratio being given by a different correlation for each. The present model is based on the same principle.

C. Miller [30] described an algorithm for the calculation of transient

equilibrium flow using a finite-difference approximation with a partially implicit method for her numerical calculations. These are restricted to a homogeneous, slip-less flow, the frictional losses being evaluated with the aid of the Darcy-Weisbach formula [30].

2. The proposed model

The most essential characteristic of the model proposed here is the circumstance that the assumption for the void fraction is adjusted to the flow pattern, and that the algorithm keeps track of the changes in the structure of the flow. The particular choices made will be specified later.

2.1 Simplifying assumptions

The proposed model is based on the following simplifying assumptions:

- (1) The flow is steady and one-dimensional.
- (2) The liquid is in thermodynamic equilibrium with the vapor ($\mu' = \mu''$).
- (3) Above the flashing level, the flow becomes two-phase, and the sequence of the structures is:
  - (a) bubbly flow; (b) slug flow; (c) froth flow;
  - (d) annular-mist flow.
- (4) Each transition is described by a characteristic criterion, discussed in Sec. 2.3 below.
- (5) The void fraction for each structure is described by a specific correlation, see Sec. 2.4 below.
- (6) The shearing stress at the wall is described by Petrick's

method [19], namely

$$\tau_w = \frac{1}{2} f \frac{\psi^2}{\rho'} \left(\frac{1-\gamma}{1-\phi} \right)^2, \quad (103)$$

where

$$f = 0.08 \left(\frac{\psi' D}{\eta'} \right)^{-0.25}, \quad (104a)$$

$$\psi' = \psi \left(\frac{1-\gamma}{1-\phi} \right). \quad (104b)$$

(7) The wall of the well is impermeable above the flashing level; the perforations end below this level.

(8) The pressure drop between reservoir ("draw-down") and well is a linear function of the flow rate with a constant factor of proportionality

$$\Delta P_d = k_D \dot{m}, \quad (k_D = 4 \div 20 \text{ kPa s/kg})^\# \quad (105)$$

where k_D is constant for each case but differs from case to case.

(9) The void fraction varies along the channel but its variation must be a smooth function of z . This means that there is no jump in ϕ at a structure boundary, even though the derivative $d\phi/dz$ may change.

2.2 Sequence of flow patterns in the model

The model which is being proposed is restricted to a sequence of four structures in the conviction that they are the essential ones for our vertical-flow problem in that they exhibit the largest differences in the slip ratio. A much more detailed description of possible flow structures can be found in Hewitt's [6] treatise.

We now proceed to characterize the four structures in more detail:

(1) Bubbly flow in which the gas phase appears in the form of a

[#] 1.3 ÷ 6.4 lbf s/in² lbm.

dispersed array of vapor bubbles spread over a homogeneously contained liquid phase.

(2) Slug flow. (Also called plug flow) Here the gaseous phase appears in the form of an array of comparatively large bubbles which almost fill each cross-section. These bubbles emerge in the form of slugs. The space between such comparatively large slugs is filled with plugs of liquid which still contain dispersed small vapor bubbles in them.

(3) Froth flow is characterized by the fact that the liquid phase alternates between assuming an unstable, wavy, ring-like form which presses against the walls of the channel and a short plug which fills the whole cross-section.

(4) Annular-mist flow pattern. Here the liquid phase forms a film which wets the walls of the channel and the vapor phase fills the center carrying with it a mist of liquid particles which move with the vapor practically without slip.

2.3 Criteria for the appearance of a particular flow pattern

The limits which delineate the four flow patterns from each other follow from definite criteria, first formulated by Bilicki [29]. A detailed statement of these criteria is also given in [39].

The boundary between bubble flow and plug flow is defined by

$$a/d_s = l/d_s , \quad (106)$$

plug flow occurring for

$$a/d_s < l/d_s . \quad (106a)$$

Here a , a function of the void fraction ϕ , denotes the mean distance between the surfaces of the vapor bubbles. The symbol ℓ denotes the effective length of the wake which forms behind a bubble of diameter d_s (see Fig. 10). The magnitude of this diameter is related to the diameter of the well, D ($d_s \approx 0.01D$), as discussed in Part IV.

In turn, plug flow becomes unstable and undergoes a transition to froth flow when one of the following two conditions is satisfied:

$$\frac{L'_s}{D} = \frac{\ell_x}{D} \quad \text{or} \quad w'' + w'_b = \sqrt{\sigma/\rho''\delta}. \quad (107a,b)$$

Condition (107a) is analogous to that in eqn. (106) and states that plug flow undergoes a transition to froth flow when the mean length L'_s , Fig. 11, of the separation between two bullet-shaped bubbles becomes equal to the effective wake length, ℓ_x , behind the bubble (see Fig. 11). The second condition (107b) has been derived from a simplified flow model, Part IV. This is a condition of stability for the annular film (indicated in Fig. 11) which surrounds the bullet-shaped bubble (also known as a Taylor bubble). When the algebraic sum of the annular-film velocity, w'_b , and the bubble velocity, w'' , exceeds the quantity $\sqrt{\sigma/\rho''\delta}$, the annular film will become disrupted and froth flow sets in. This criterion is based on the theory of surface waves outlined in [39].

The alternating character of froth flow makes it difficult to describe it analytically. Numerous experiments show that in the presence of a sufficiently high vapor velocity, the liquid film flows upwards and its arrest constitutes the boundary between froth flow and a stable annular-mist pattern.

If one of the two criteria in eqns. (107a,b) fails to be satisfied,

Fig. 10. Mean distance between vapor bubbles, a , and effective wake length, l , illustrating transition criterion from bubble to slug flow.

Fig. 11. Notation for transition criterion between slug and froth flow.

froth flow ensues.

The condition for transition from froth flow to annular-mist flow has been assumed in relation to the characteristic number

$$K = w''_0 (\rho'')^{0.5} [g\sigma(\rho' - \rho'')]^{-0.25} \quad (108)$$

first suggested by Kutateladze [31] and later modified by Pushkin and Sorokin. Annular mist is present when

$$K > 3.2 . \quad (108a)$$

As already mentioned, full details are given in [39].

2.4 Working equations

The calculation of well flow has been based on the one-dimensional equations discussed in Part II. Closure is obtained by adding Petrick's [19] correlation for wall stress together with empirical correlations which determine the void fraction ϕ for each topological flow structure [32,33]. For ease of reference, we list here the complete set of equations in the integral form in which they have been employed for computer programming.

o Conservation of mass:

$$A(z_2) \psi(z_2) = A(z_1) \psi(z_1) \quad (109)$$

o Conservation of momentum:

$$\begin{aligned} A(z_2)P(z_2) = A(z_1)P(z_1) - \frac{\dot{m}^2}{A(z_2)\rho'(z_2)} \left[\frac{(1-y(z_2))^2}{1-\phi(z_2)} - \frac{y^2(z_2)\rho'(z_2)}{\phi(z_2)\rho'(z_2)} \right] + \\ + \frac{\dot{m}^2}{A(z_1)\rho'(z_1)} \left[\frac{(1-y(z_1))^2}{1-\phi(z_1)} - \frac{y^2(z_1)\rho'(z_1)}{\phi(z_1)\rho'(z_1)} \right] - g \int_{z_1}^{z_2} A(z)\rho(z)dz + \\ - \int_{z_1}^{z_2} C(z)\tau_w(z)dz \end{aligned} \quad (110)$$

o Conservation of energy in which

$$\dot{m}h = \dot{m}'h' + \dot{m}''h'' = \dot{m}(y h_{fg} + h') \quad (111)$$

has been substituted (here $h_{fg} = h'' - h'$):

$$y(z_2) = \frac{h(z_1) - h'(z_2) + \frac{1}{2} [w^2(z_1) - w^2(z_2)] + g(z_1 - z_2)}{h_{fg}(z_2)} \quad (111a)$$

o The Clausius-Clapeyron vapor-pressure equation:

$$P(T_2) = P(T_1) + \int_{T_1}^{T_2} \frac{1}{T} \frac{h_{fg}(T)}{v_{fg}(T)} dT \quad (112)$$

o The thermophysical properties of water and steam as functions of pressure and/or temperature have been taken from the 1967 IFC (International Formulating Committee) analytic formulation [34]

o Petrick's equation for wall stress:

$$\tau_w = \frac{1}{2} f \frac{\psi^2}{\rho'} \left(\frac{1-y}{1-\phi} \right)^2 \quad (113)$$

where

$$f = 0.08 \left[\frac{\psi(1-y)D}{\eta'(1-\phi)} \right]^{-0.25} \quad (114)$$

(D - pipe diameter; η' - viscosity of liquid [34])

o Void fraction

oo for bubbly flow [32]

$$\phi_b = \left\{ (w'_o + w''_o + \delta) - \left[(w'_o + w''_o + \delta)^2 - 4w''_o\delta \right]^{1/2} \right\} / 2\delta \quad (115)$$

where

$$\delta = \sqrt{\frac{200\sigma}{D\rho'} + \frac{gD}{200}} \quad (115a)$$

oo for slug flow [33]

$$\phi_s = \frac{1.06k_1}{1.2 \left(\frac{w'_o}{w''_o} + 1 \right) + \frac{0.25}{Fr}}, \quad (116)$$

where

$$Fr = w_o'' / \sqrt{gD} , \quad (116a)$$

and k_1 is a coefficient derived from the condition that $\phi_b = \phi_s$ at transition

oo For froth flow we assume a condition which amounts to a superposition of pure annular flow and no-slip flow:

$$\phi_f = \phi_a(1-k_2) + k_2\phi_{ns} \quad (117)$$

with

$$\phi_a = \frac{1}{1 + \frac{1-y}{y} \left(\frac{\rho''}{\rho'} \right)^{0.5}} \quad (117a)$$

and

$$\phi_{ns} = \frac{1}{1 + \frac{1-y}{y} \left(\frac{\rho''}{\rho'} \right)} \quad (117b)$$

The numerical value of the coefficient k_2 (which is here variable) satisfies two conditions. First at the point of transition we postulate

$$\phi_f = \phi_s$$

and deduce the transitional value k_{20} from which

$$k_2 = \frac{y-1}{y_0-1} k_{20} , \quad (118)$$

where y_0 is the dynamic dryness fraction at the point of transition.

oo For annular-mist flow, ϕ_{am} , is calculated in the same manner as ϕ_f , except that k_{20} must be calculated from

$$\phi_{am} = \phi_f \quad (119)$$

instead of (118).

2.5 Calculation of pressure distribution along the well

From the point of view of applications, and for ease of reference to field data, the most convenient output is to calculate the pressure distribution along the well. We begin by assuming the draw-down pressure loss

$$\Delta P_d = k_D \dot{m}$$

from eqn. (105). Thus, the working equation (110) is modified to

$$\begin{aligned} A(z_2)P(z_2) = & (P_b - \Delta P_b)A(z_1) - \left\{ \frac{\dot{m}^2}{A\rho'} \frac{(1-y)^2}{1-\phi} - \frac{y^2 \rho'}{\phi \rho''} \right\}_2 + \\ & + \left\{ \frac{\dot{m}^2}{A\rho'} \frac{(1-y)^2}{1-\phi} - \frac{y^2 \rho'}{\phi \rho''} \right\}_1 - g \int_{z_1}^{z_2} A \rho dz + \\ & - \int_{z_1}^{z_2} C \tau_w dz . \end{aligned} \quad (120)$$

It should be remembered that the dynamic dryness fraction has been defined in eqn. (111a).

2.6 Critical flow

Critical flow represents a singular solution of the system of ordinary nonlinear differential equations obtained from the conservation laws supplemented with the proper equations of state and closure equations. Regardless of the adopted model, this system is of the form

$$A_{ij} \frac{d\sigma_i}{dz} = B_j . \quad (121)$$

A detailed study of this system of equations is given elsewhere [35].

The theory of systems of equations of the type represented by (121)

leads to the conclusion that in upward flow through a pipe of constant

cross-section, choking can occur only at pipe exit. The mathematical condition for choking is the vanishing of the determinant, or

$$\Delta = |A_{ij}| = 0. \quad (122)$$

This is equivalent to the two conditions

$$\left(\frac{\partial z}{\partial P}\right)_\psi = \left(\frac{\partial z}{\partial h}\right)_\psi = 0, \quad (123)$$

if $\psi(z)$, $P(z)$ and $h(z)$ are chosen as mathematically dependent functions $\sigma_i(z)$ of the system (121). Numerical calculations can be conducted without difficulty by tracking the inverse functions $z(h)$ and $z(P)$ at $\psi=\text{const.}$ In the subcritical flow regime the derivatives (123) change sign when P and h fall below their critical values, and the coordinate z which corresponds to condition (123) determines the length of pipe for which choking occurs. Upon crossing the point of choking, the values of z decrease, because the solution turns on itself in the $h(z)$ and $P(z)$ planes.

2.7 The effect of heat transfer

The heat flux, q_w , at the wall of the well depends on the temperature difference $T(z)-T_0(z)$, between the temperature T of the geofluid and of that in the surrounding formation. The formation itself is not homogeneous and usually consists of a porous layer of high permeability ($z \leq z_1$) which is characterized by a very large thermal conductivity, λ_1 , and a layer of rock ($z > z_1$) of low permeability and very low conductivity. Hence the temperature gradient will be shallow in the wet sand and steep in the rock.

In approximate terms, it can be said that the temperature distribution $T_w(z)$ along the wall will be characterized by a shallow gra-

dient below the flashing level ($z \leq z_f$) and a steep gradient above it ($z \geq z_f$), because the coefficient of heat transfer is considerably higher than in single-phase flow. Hence, depending on the relation between z_1 and z_f and the slopes of the various temperature profiles, it is possible to encounter conditions where the heat-flux vector is directed outward (cooling of brine) as well as inward (heating of brine). In the latter case, boiling may occur. Two likely situations are shown sketched in Fig. 12. The calculations show, however, that the effect of heat transfer either way is small and this permits us to adopt the simplification that the flow is adiabatic.

2.8 Thermodynamic properties of brines

Brines are solutions of chloride, sulfate and carbonated salts in water. The salts are primarily NaCl, KCl, CaCl_2 and to a lesser extent K_2SO_4 , K_2CO_3 , Na_2CO_3 , KHCO_3 , NaHCO_3 , K_2SO_4 and Na_2SO_4 . Not all of the above salts are present in all the wells and in the great majority of the wells NaCl is by far the most abundant component.

An accurate determination of the thermodynamic properties of brines would have involved the calculation of the effect of all the salts on the properties, a work that is at least complicated if not impossible, given the numerous ionic interactions and chemical equilibria. For this purpose the "equivalent NaCl content" is used to obtain good approximations to the properties of the brines. The "equivalent NaCl content" is the amount of NaCl in the solution that will cause the same effect on the properties as the amount of all the salts combined. This assumption is supported by the fact that the major constituent of the salts is NaCl and the effect of the other chlorides on the properties is similar to the effect of NaCl.

(a)

(b)

Fig. 12. Temperature distribution in geofluid, T , and surrounding formation, T_0 .

(a) Cooling and heating of brine
(b) Cooling only

Hence the brines can be modeled as NaCl solutions and their properties identified with the corresponding properties of the NaCl solutions. For the case of geothermal fluids the necessary properties for the determination of the pressure distribution along the well are: the density, the viscosity, the enthalpy and saturation temperature at a given pressure.

2.8.1 The density

For the determination of the density of NaCl solutions the procedure followed by Haas was adopted: The density of a NaCl solution is given by the expression:

$$\rho = \frac{1000 + mM_2}{1000V_o + m\alpha} \quad \text{kg/m}^3, \quad (124)$$

where

$$\alpha = \alpha^* + k\sqrt{m}, \quad (124a)$$

$$\alpha^* = C_o + C_1V_o + C_2V_o^2, \quad (124b)$$

$$k = \frac{(C_3 + C_4V_o)V_c^2}{(V_c - V_o)^2}; \quad (124c)$$

V_o is the specific volume of pure water given by:

$$V_o = \frac{V_o + C_5\theta^{1/3} + C_6\theta + C_7\theta^4}{1 + C_8\theta^{1/3} + C_9\theta}; \quad (125)$$

θ is the difference of the saturation temperature of brine from the critical temperature of water:

$$\theta = 647.27 - 273.15 - T - \Delta T \quad (126)$$

(T in $^{\circ}\text{C}$ and for ΔT see Section 2.8.4).

In the above expressions m is the molality of the solution and

M_2 the molecular weight of the salt. The numerical constants C_j are:

$$\begin{array}{lll} C_0 = -167.219 & C_5 = -0.315154 & V_c = 3.1975 \\ C_1 = 448.55 & C_6 = -1.203374 \times 10^{-3} & \\ C_2 = -261.07 & C_7 = 7.48908 \times 10^{-13} & \\ C_3 = -13.644 & C_8 = 0.1342489 & \\ C_4 = 13.97 & C_9 = 3.946263 \times 10^{-3} & \end{array}$$

The accuracy of the results is claimed as 0.5% up to 300°C.

2.8.2 The viscosity

The correlation by J. Kestin and H.E. Khalifa [40] was adopted for the determination of viscosity of NaCl solutions. This consists of a correction to the viscosity of pure water which is given by the expression:

$$\log_{10} [\mu_w(T)/\mu_w(20)] = (20-T)[1.2378 - 1.303 \times 10^{-3}(20-T) + 3.06 \times 10^{-6}(20-T)^2 + 2.55 \times 10^{-8}(20-T)^2]/(g_6 + T) \quad (127)$$

with $\mu_w(20) = 1002 \mu\text{Pas}$;

T is in °C.

The viscosity of a solution of molality m is then:

$$\log_{10} [\mu_w(T,m)/\mu_w(T)] = A(m) + B(m) \log [\mu_w(T)/\mu_w(20)] \quad (128)$$

with $A(m) = 0.3324 \times 10^{-1}m + 0.3624 \times 10^{-2}m^2 - 0.1879 \times 10^{-3}m^3$

and $B(m) = 0.3961 \times 10^{-1}m + 0.102 \times 10^{-1}m^2 - 0.702 \times 10^{-3}m^3$.

The accuracy of the calculated viscosity is 1.5% up to 300°C.

2.8.3 The enthalpy

The enthalpy of both the liquid solution and its vapor are needed in the calculations for flashing into the well as well as their difference. The vapor was assumed pure water vapor, slightly superheated because the solvent is non-volatile and for a given pressure the saturation temperature is elevated (see Sec.2.8.4). The enthalpy of the vapor then is independent of the molality and is given by the equation:

$$h''(T) = 0.81227 \times 10^{-2} T^2 + 3.65228T + 2388.4 \quad \text{kJ/kg} \quad (129)$$

This equation is accurate to 0.5% up to 300°C.

For the enthalpy of the liquid phase of the solution a correction to the ideal solution enthalpy was adopted as follows:

$$h'(T,m) = x_1 h_1(T) + x_2 h_2(T) + m\phi(T,m) \quad \text{kJ/kg} \quad (130)$$

where $h'(T,m)$ is the enthalpy of the solution at temperature $T^\circ\text{C}$ and molality m ; x_1 is the mass fraction of water, $h_1(T)$ the enthalpy of pure water at temperature T , x_2 the mass fraction of NaCl, $h_2(T)$ the enthalpy of the pure salt and χ the correction factor which depends on temperature and molality and is given by the expression:

$$\chi(m,T) = \sum_{i=0}^3 \sum_{j=0}^2 a_{ij} T^i m^j \cdot \frac{4.184}{1000 + mM_2} \quad \text{kJ/kg} \quad (131)$$

where $M_2=58.44$ and the coefficients a_{ij} are:

$a_{00} = 9633.6$	$a_{01} = 4080.0$	$a_{02} = 286.49$
$a_{10} = 166.58$	$a_{11} = 68.577$	$a_{12} = 4.6865$
$a_{20} = -0.90963$	$a_{21} = -0.36524$	$a_{22} = 0.249667 \times 10^{-1}$
$a_{30} = 0.17965 \times 10^{-2}$	$a_{31} = 0.71924 \times 10^{-3}$	$a_{32} = -0.4900 \times 10^{-4}$

The datum point for this expression of enthalpy is 0°C and 0 molality. Enthalpy differences checked with other available data were accurate within 1.5%. The heat of vaporization h_{fg} is also important in the estimation of the quality of the produced vapor. This is given by the equation:

$$h_{fg} = h''(T) - h'(T,m) \quad (132)$$

The accuracy of the last expression is better than 1%. If the heat capacity of the solution or the vapor is needed, it may be obtained from the equations:

$$C'_p(T,m) = \frac{\partial h'(T,m)}{\partial T} \quad (133a)$$

$$C''_p(T) = \frac{\partial h''(T)}{\partial T} \quad (133b)$$

2.8.4 The elevation of the saturation temperature

The saturation pressure of a solution of given molality is higher by an amount

$$\Delta T = \frac{1.8RT^2}{Mh_{fg}} \frac{N_s}{N_w} \quad (134)$$

than for water. Here $R=8.315$ kJ/kmol°C, $M=18$ kg/kmol and ΔT is the temperature elevation, R the universal gas constant, T the thermodynamic temperature, h_{fg} the heat of vaporization of water, and N_s and N_w the moles of salt and water correspondingly. This is a modification of the well-known Raoult's law, the empirical constant 1.8 comes from the fact that the salt is ionized in the solution. Conversely, if the saturation temperature T is prescribed, the saturation pressure elevation is:

$$P = \frac{1.8RT}{v''_0 - v'_0} \cdot \frac{N_1}{N_0}, \quad (135)$$

where v''_0 and v'_0 are the specific volume of vapor and liquid water, and the other symbols are the same as above.

2.9 The effect of noncondensable gases on the properties of geothermal fluids

2.9.1 Amount of noncondensable gases

The proportion of noncondensable gases in a geothermal fluid depends on the time of operation and the location of the well. For dry steam wells it varies between 25 and almost 0 percent by weight, the higher numbers occurring only at wells in Italy and at the early stage of operation. In this country dry-steam wells contain less than 2% noncondensable gases by weight. In a pressurized liquid well the amount of noncondensibles is even lower and in most of the cases is less than 1%. The constituents of the gases are mainly carbon dioxide (more than 80%), hydrogen sulfide (about 1-10%), nitrogen, ammonia and methane.

2.9.2 Effect of the noncondensable gases on the properties of the brines

The geothermal liquid can be modeled as an ideal solution of the noncondensable gases and saline water. Any specific extensive property of such a solution may be computed on an average weight basis, as

$$\psi = \psi''\gamma'' + \psi'\gamma' , \quad (136)$$

where ψ is any specific extensive property of the ideal solution (enthalpy, entropy etc.), γ'' is the mass fraction of the gas, ψ'' is the specific property of the gas and ψ' the specific property of the saline water. Given that γ'' is less than 0.01 in most cases, we

can write to a good approximation:

$$\psi \approx \psi' \quad (137)$$

Of course in dry-steam wells γ'' may be higher and eqn. (136) should be used instead of (137). In this case, ψ is the mass average property for all the gases; given that the CO_2 is the predominant constituent of the gases, the property ψ'' can be given to a good approximation by the corresponding property of carbon dioxide.

The density of the geothermal liquid remains unaffected by the presence of such small amounts of noncondensable gases and is equal to the density of the saline water [36]. The presence of the gases affects the properties of the brines only near the critical point, but these conditions are not known to prevail in geothermal wells.

2.9.3 The effect of the noncondensibles on the flashing point

According to Raoult's law, dissolved volatile substances lower the temperature of the boiling point of the water, (keeping total pressure constant, and the non-volatile ones elevate the temperature of the boiling point. A form of "an extended Raoult's law" derived from the equation of the Gibbs functions for the vapor and liquid phases is:

$$T = T_o + 1.8 \frac{RT_o^2}{h_{fg}^p} \left[\sum_i \frac{N_i}{N_{ar}} - \sum_j \frac{N_j}{N_s} \right], \quad (138)$$

where T is the boiling temperature of the solution at pressure P , T_o is the saturation temperature of pure water at P , R the universal gas constant, h_{fg}^p the heat of vaporization for pure water at T_o , N_i the moles of solutes in the liquid phase of solution, N_w the moles of liquid water, N_s the moles of steam and N_j the moles of the solutes

in the gaseous phase.

The distribution of the noncondensable gases in the vapor and liquid phase obeys Henry's law:

$$K_H = \frac{f_g}{v'_g}, \quad (139)$$

where f_g is the fugacity of the gas, v'_g the mole fraction of the gas dissolved in the liquid phase, and K_H Henry's constant, a function of temperature and molality. The fugacity for an ideal solution is equal to the partial pressure of the gas:

$$f_g = P_g, \quad (140)$$

and the partial pressure is related to the total pressure through the mole fraction v_g in the vapor:

$$P_g = P_t v''_g. \quad (141)$$

Then eqn. (139) can be written:

$$K_H = P_t \frac{v''_g}{v'_g}. \quad (142)$$

The last equation determines the distribution of the gas in the liquid and vapor phase.

Given that the value of K_H is much higher than P_t [37] in the usual hydrothermal fields, the mole fraction of the gases in the vapor phase is higher than the mole fraction in the liquid phase. Then eqn. (138) will state that the saturation temperature of the solution is lower than the saturation temperature of the saline water, under the same pressure.

The fact that the gases distribute themselves in a liquid and vapor phase irrespective of the amount of steam present in the system,

leads to the conclusion that in geothermal systems we may encounter bubbles of a vapor phase well before the saturation temperature is reached. These bubbles will contain gas and steam in amounts predicted by Dalton's law:

$$P_t = P_s + P_g, \quad (143)$$

$$\frac{P_g}{P_s} = \frac{v_g''}{v_g'}, \quad (144)$$

where P_s is the partial pressure of steam (which is approximately equal to the saturation pressure prevailing at the fluid's temperature) and v_g'' is the mole fraction of steam in the bubble. Given though the small amount of the gas present in the system, it is not expected that these bubbles will alter significantly the density of the fluid, which may be taken equal to the density of the saline water.

3. Calculation method for the computer program

The geothermal well flow model which was presented in the preceding section is used to calculate the temperature distribution along the well and the thermodynamic properties of the fluid at the wellhead. The calculation is based on the step-by-step method with the following input data:

well flow rate: $\dot{m}$

static pressure at the well bottom: P_b

temperature at the well bottom: T_b

well depth: L

well diameter: D

drawdown factor: k_D

brine molality: b (note change of symbol from text)

calculation step: Δz

The calculation yields the values of the thermodynamic functions at all steps in the well and the regime transitions are indicated with an appropriate message. The printout for one set of input data is ended by the following information for the wellhead or for the level where the choking occurs:

Temperature: T

Pressure: P

Dynamic dryness fraction: y

Void fraction: ϕ

Ratio of pressure drop: $R = \Delta P_{TP} / \Delta P_L$

Mass flow rate of the liquid: $\dot{m}'$

Mass flow rate of the vapor: $\dot{m}''$.

A printout sample is included in Appendix A. A flow-chart of the computer program is shown in Fig. 13. The calculation procedure is also included in Appendix A in basic and Fortran IV languages. The equations employed in these programs are presented in Sec. 2.

4. Sensitivity of the self-flowing well model

The output of the computational procedure may be depicted in various figures, showing the distribution of thermodynamic variables such as the temperature, pressure, dryness fraction or void fraction along the well. A very useful graphic representation to the engineer is the mass-flow rate *versus* the wellhead pressure. The latter diagram may be used to obtain an assessment of the available power at the wellhead which is going to be transformed into electricity.

The distributions of the thermodynamic properties along the well depend on the well characteristics such as the ones listed below:

Fig. 13. Flow chart of computer program.

Well bottom temperature: T_b ,

Well bottom pressure: P_b ,

Drawdown factor: k_D ,

Salinity (or molality) of the fluid: b ,

Heat losses: q_o ,

Well diameter: D ,

Roughness of the well, or the skin friction coefficient.

The influence of the above parameters will be given separately in the following parts of this Section.

4.1 The influence of the wellbottom temperature and pressure

The influence of the wellbottom temperature and pressure is illustrated in Figs.14 and 15. It is seen that an increase in either quantity results in a corresponding increase of the mass-flow rate. The two diagrams are representative of a well with the following specifications:

$$L = 1600 \text{ m} ,$$

$$b = 1 \text{ molal (5.52\% by weight)} ,$$

$$D = 0.2 \text{ m} ,$$

$$k_D = 20 \text{ kPa}\cdot\text{s/kg} .$$

Figure 16 shows the distribution of pressure along the geothermal well for various values of the parameter T_b . When the temperature increases, the flashing level falls and the two-phase regime occupies a longer part of the pipe. This implies that bubbly flow starts at a higher density ratio ρ''/ρ' , and this affects the void fraction ϕ as shown in eqn. (66). The result is that ϕ becomes lower and the shear stress at the wall decreases (eqn. 113). Finally, choking conditions are expected for higher well flow rates. When the wellbottom pressure

Fig. 14. Mass-flow rate versus wellhead pressure for various well-bottom temperatures, T_b ($P_b=16$ MPa, $L=1600$ m, $D=0.2$ m, $k_D=20$ kPa s/kg, $b=1$ m)

Fig. 15. Mass-flow rate *versus* wellhead pressure for various well-bottom pressures, P_b ($T_b=250^\circ\text{C}$, $L=1600$ m, $D=0.2$ m, $k_D=20$ kPa s/kg, $b=1$ m)

Fig. 16. Pressure versus well-depth for various well-bottom temperatures, T_b ($P_b=16$ MPa, $\dot{m}=60$ kg/s, $b=5.5\%$, $D=0.2$ m, $k_D=20$ kPa s/kg, $L=1600$ m)

increases, the flashing level for a constant mass-flow rate moves upward, and a shorter part of the well is occupied by two-phase flow. The void fraction at the wellhead is smaller and choking conditions are averted.

4.2 The effect of drawdown factor

It was assumed in Section 2 that the pressure drop caused by the flow through the porous medium at the wellbottom is a linear function of the mass flow rate:

$$\Delta P_D = k_D \dot{m}$$

The drawdown factor k_D is taken as constant although at very high mass flow rates it becomes a function of $\dot{m}$. The increase of k_D acts like a decrease of the bottomhole pressure P_b and hence lowers the flashing level. The influence of the variation of k_D on the mass flow rate is shown in Fig. 17 for a well with the following specifications:

T_b : 200; 250; 300°C

P_b : 14; 16; 18 MPa

b : 5.52% = 1 molality

D : 0.2 m

L : 1600 m

k_D : 20,000 Pa/kg/s

Figure 18 shows the distribution of the void fraction ϕ for the same well.

4.3 Effect of brine concentration

The change of brine concentration affects the physical properties of the geothermal fluid, namely the enthalpy, density entropy and saturation temperature. Consequently, the flashing level is lower

Fig. 17. Mass-flow rate *versus* wellhead pressure for various drawdown pressure factors, k_D ($T_b = 295^\circ\text{C}$, $P = 15$ MPa, $L = 1520$ m, $D = 0.178$ m, $b = 0$).

Fig. 18. Void fraction *versus* well-depth for various drawdown pressure factors k_D ($T_b=295^\circ\text{C}$, $P_b=15\text{ MPa}$, $L=1520\text{ m}$, $D=0.178\text{ m}$, $b=0$).

because at higher salinity the density and the saturation temperature are higher at a given static pressure. The increase of the heat of vaporization lowers the dynamic dryness fraction along the well. The influence of the brine molality on the wellhead conditions is depicted in Figs. 19 and 20.

4.4 The effect of heat losses

It was mentioned in Section 2.7 that the heat transfer to the surroundings from a typical geothermal well is very low and amounts to 1-2% of the specific exergy of the geofluid. The heat losses become significant only in the case of low mass flow rates. The influence of the heat losses is depicted in Fig. 21 where it can be seen that the state of the fluid at the wellhead is sensitive to the heat losses at high wellhead pressures.

4.5 The effect of the well diameter - stepped diameter

The diameter of the well is the most important parameter on the critical flow rate through the well. This can be seen in Fig. 22 where the mass flow rate is shown plotted for three well diameters. It appears that critical conditions are avoided for a given mass-flow rate if the diameter is increased above the flashing level (stepped well). This is equivalent to saying that the flow rates through a given well are much higher in the case of stepped diameter. The higher mass flow rates from a stepped diameter are shown in Fig. 23.

4.6 The effect of the wall roughness

The shear stress at the wall is determined by the friction coefficient there which in turn is a function of the relative roughness of the wall. The effect of roughness was examined for three cases:

Fig. 19. Mass-flow rate *versus* wellhead pressure for various brine salinities, b ($T_b=295^\circ\text{C}$, $P_b=15$ MPa, $L=1520$ m, $D=0.178$ m, $k_b=23$ kPa s/kg).

Fig. 20. Temperature *versus* well-depth for various brine concentrations, $b(T_b=295^\circ\text{C}$, $P_b=15$ MPa, $L=1520$ m, $D=0.178$ m, $k_b=23$ kPa s/kg).

Fig. 21. Mass-flow rate *versus* wellhead pressure for various heat-loss values measured as a percentage of total energy transported through the well. ($T_b=168^\circ\text{C}$, $P_b=22.54$ MPa, $L=2433$ m, $D=0.391$ m, $k_D=21$ kPa s/kg, $b=0$).

Fig. 22. Mass-flow rate *versus* wellhead pressure for various well diameters, D ($T_b=200^\circ\text{C}$, $P_b=22$ MPa, $L=2300$ m, $k_D=20$ kPa s/kg, $b=0$).

Fig. 23. Comparison between constant diameter and scheduled casing characteristics. ($T_b=180^\circ\text{C}$, $P_b=15$ MPa, $L=1520$ m, $k_D=21$ kPa s/kg, $b=5\%$).

(a) $D=0.21$ m constant

(b) $D=0.391$ m from wellhead to a depth of 200 m with $D=0.21$ to bottom.

when $f=0.01$ and 0.03 and when f is determined by the Blasius equation (eqn. 100a). The results are depicted in Fig. 24 where it is seen that increasing f by a factor of 3 reduces the maximum flow rate by 25%.

5. Comparison of the theoretical models developed by the Denver Research Institute (DRI) and Brown University

There are two sets of data for the operation of geothermal wells which can be used for comparison with the theoretical models. These are No. 1 IID (Imperial Irrigation District) reported by Helgeson [38], and East Mesa 16-29 investigated by DRI.

The data for the well No. 1 IID are as follows:

$$\begin{aligned}T_b &= 295^\circ\text{C} , \\L &= 1520 \text{ m} , \\D &= 0.178 \text{ m} , \\k_D &= 22.8 \text{ kPa s/kg} , \\P_b &= 15 \text{ MPa} , \\b &= 30\% .\end{aligned}$$

A comparison of the flow rate *versus* wellhead pressure diagram for the model described and the actual operating conditions at this well is given in Fig. 25. The conditions at the wellbottom for East Mesa 16-29 are taken from Elliot [24] and the data of its operation were received from DRI. The wellbottom data are:

$$\begin{aligned}T_b &= 156^\circ\text{C} , \\L &= 2433 \text{ m} , \\D &= 0.391 \text{ m from the wellhead to 283 m depth} , \\D &= 0.276 \text{ m from 283 m to 1366 m} , \\D &= 0.201 \text{ m from 1366 m to the wellbottom} ,\end{aligned}$$

Fig. 24. Mass-flow rate *versus* wellhead pressure for various values of coefficient of friction, f ($T_b=250^\circ\text{C}$, $P_b=20$ MPa, $L=2000$ m, $D=0.2$ m, $k_D=10$ kPa s/kg, $b=0$).

Fig. 25. Comparison between calculation and measurement [IID No. 1 well]
 $(T_b=295^\circ\text{C}, P_b=15 \text{ MPa}, L=1520 \text{ m}, D=0.178, k_D=22.8 \text{ kPa s/kg}, b=15\%).$

$$k_D = 21.0 \text{ kPa s/kg} ,$$

$$P_b = 22.54 \text{ MPa} ,$$

$$b = 0\% .$$

A comparison of the operational characteristics (mass flow rate vs. well-head pressure) of the well and the model is given in Fig. 26. The next Fig. 27 gives a comparison of the temperature distribution along the same well and the theoretical model. Figure 28 compares pressure distribution predicted by our model and the DRI model. These distributions are shown for four different conditions.

The following conclusions may be drawn from the comparison of the two models:

- (a) Our model shows good agreement with operating data from both wells (IID #1 and East Mesa 16-29).
- (b) The two theoretical models (ours and DRI) give similar curves for the operational characteristics of the wells. The agreement is better in the region of bubbly and slug flow patterns. The discrepancy of the models becomes high in the case of high void fractions. No conclusions can be drawn from this, given that the existing well data do not cover this region of flow.
- (c) The theoretical model described in the present report may be used to calculate the thermodynamic properties of any two-phase vertical flow. Consequently, the model may be tested against experimental results where the investigator can create any flow regime and reach any desired conditions. Such experiments may result in an important test of the usefulness and the accuracy of all theoretical models.

Fig. 26. Comparison between calculation and measurement [East Mesa 16-29 well]. ($T_b=156^\circ\text{C}$, $P_b=22.54$ MPa, $L=2433$ m, $k_D=21$ kPa s/kg, $b=0$, $D=0.391$ m from wellhead to a depth of 281 m with $D=0.276$ m to 1355 m, and $D=0.201$ to bottom).

Fig. 27. Comparison between calculated and measured temperature distribution ($T_b=156^\circ\text{C}$, $P_b=22.54$ MPa, $L=2433$ m, $k_D=21$ kPa s/kg, $b=0$, $D=0.391$ m from wellhead to a depth of 281 m, $D=0.276$ m to 1355 m, and $D=0.201$ to bottom).

Fig. 28. Comparison between our model calculations and those of the Denver Research Institute. ($D=7.125$ m, $\lambda=0.012$).

K,1	$\dot{m}=200\ 000$ lb/hr, $T_b=500^\circ\text{F}$
J,2	$\dot{m}=300\ 000$ lb/hr, $T_b=500^\circ\text{F}$
H,3	$\dot{m}=200\ 000$ lb/hr, $T_b=450^\circ\text{F}$
G,4	$\dot{m}=300\ 000$ lb/hr, $T_b=450^\circ\text{F}$

6. Comparison between the available work of a self-flowing and a pumped well.

The assessment of the thermodynamic quality of a geothermal well is best done in relation to its specific available work (exergy), w^o , [1][#], or of its available power

$$\dot{W}^o = \dot{m} w^o . \quad (145)$$

For a given dead state, we have

$$w^o = h_1 - h_o - T_o(s_1 - s_o) , \quad (146)$$

where subscript 1 refers to the wellhead, and subscript o describes the dead state.

The net work of a geothermal power station is obtained when the utilization factor, η_T , is applied, so that

$$\dot{W}_n = \eta_T \dot{W}^o . \quad (147)$$

If a pump is installed below the flashing level, the pressure level above it will increase by ΔP , and the resulting net work is

$$\dot{W}_{np} = \eta_T \dot{W}_p^o - \frac{\dot{m}_p}{\rho_1} \frac{\Delta P}{\eta_p} , \quad (148)$$

where

$$\dot{W}_p^o = \dot{m}_p w_p^o \quad (149)$$

represents the power with the changed mass flow and modified conditions (subscript 1) at the wellhead stipulated in eqn. (146).

The effect of a number of factors on $\dot{W}^o$ is shown graphically in Figs. 29-35. Fig. 29 shows the effect of wellbottom temperature, Fig. 30 illustrates the effect of pressure at well bottom, Fig. 31 demonstrates the rather small effect of the draw-down factor, Fig. 32 takes into account brine salinity, Fig. 33 shows the rather modest effect of heat losses, Fig. 34 depicts the effect of the well diameter, and the last Fig. 35 characterizes

[#]See also "Sourcebook"

Fig. 29. Available power *versus* mass-flow rate for various well-bottom temperatures, T_b ($P_b=16$ MPa, $L=1600$ m, $D=0.2$ m, $k_D=20$ kPa s/kg, $b=1$ mol)

Fig. 30. Available power *versus* mass-flow rate for various well-bottom pressures P_b ($T_b=250^\circ\text{C}$, $L=1600 \text{ m}$, $D=0.2 \text{ m}$, $k_D=20 \text{ kPa s/kg}$, $b=1 \text{ mol}$)

Fig. 31. Available power *versus* mass-flow rate for various draw-down pressure factors, k_D ($T_b=295^\circ\text{C}$, $P_b=15\text{ MPa}$, $L=1520\text{ m}$, $D=0.178\text{ m}$, $b=0$).

Fig. 32. Available power *versus* mass-flow rate for various brine salinities, $b(T_b=295^\circ\text{C}, P_b=15\text{ MPa}, L=1520\text{ m}, D=0.178\text{ m}, k_D=23\text{ kPa s/kg})$

Fig. 33. Available power *versus* mass-flow rates for various heat losses expressed as percentage of total energy transported through well ($T_b=168^\circ\text{C}$, $P_b=22.54\text{ MPa}$, $L=2433\text{ m}$, $D=0.391\text{ m}$, $k_D=21\text{ kPa s/kg}$, $b=0$).

Fig. 34. Available power *versus* mass-flow rate for various well diameters, D ($T_b = 200^\circ\text{C}$, $P_b = 22$ MPa, $L = 2300$ m, $k_D = 20$ kPa s/kg, $b = 0$).

Fig. 35. Available power *versus* mass-flow rate for various values of friction factor, f ($T_b=250^\circ\text{C}$, $P_b=20$ MPa, $L=200$ m, $D=0.2$ m, $k_D=10$ kPa s/kg, $b=0$).

the effect of the friction factor. The diagrams illustrate the fact that the available power is most sensitive to the well-bottom temperature and salinity, in that order.

The diagrams in Figs. 36 illustrate the considerable increase in the mass flow rate produced by pumping in the examples considered here. Fig. 37 shows the effect of pumping on the net available power for various efficiency levels. With $\eta_T = \eta_p = 1$ it is possible to achieve considerably higher levels of power due to the increase in mass-flow rate before the onset of choking shown earlier. With decreased efficiencies, the net available power may decrease in the presence of pumping if mass-flow rates below choking are considered. However, the delay in choking produced by pumping leads to considerably larger levels of power if advantage is taken of the ability of a pumped well to supply a considerably larger flow output.

Reverting to Fig. 34 we can conclude that an increase in the well diameter seems to provide us with the greatest possibility of securing a high mass-flow rate prior to choking, and hence a high level of power. The simplest way to achieve this is to use stepped wells in view of the very high cost of drilling large-diameter wells to a large depth. This point is illustrated in Fig. 38. In this example, the maximum increase in power in a self-flowing stepped well is 30%. The addition of pumping in a well of constant diameter increases the power by 80%, that is by about three times as much as has been achieved through steps. Which of these alternatives is economically preferable must be analyzed separately. The opinion is expressed here that the use of stepped wells is probably more advantageous, because it is certainly simpler in day-to-day operation, and possible damage to the substrate is avoided.

Fig. 36. Well flow-rate *versus* wellhead pressure for self-flowing and pumped wells ($T_b=295^\circ\text{C}$, $D=0.178$ m, $P_b=14.9$ MPa, $L=1500$ m, $b=0$, $k_D=23$ kPa s/kg).

Fig. 37. Net available power *versus* mass-flow rate for self-flowing and pumped wells ($T_b=295^\circ\text{C}$, $D=0.178$ m, $P_b=14.9$ MPa, $L=1500$ m, $b=0$, $k_D=23$ kPa s/kg):

Fig. 38. Net available power *versus* mass-flow rate for self-flowing and pumped wells ($T_b=295^\circ\text{C}$, $L=1500$ m, $b=0$, $k_D=23$ kPa s/kg)
 (a) self-flowing $P_b=15$ MPa, $D=0.178$ m(const)
 (b) self-flowing $P_b=15$ MPa,
 $D=0.391$ m to 200 m
 $D=0.275$ m to depth of 300 m
 $D=0.210$ m to depth of 500 m
 $D=0.178$ m to well-bottom
 (c) pumped, $P_b=15$ MPa, $\Delta P=3.5$ MPa, $D=0.178$ m(const)

The preceding examples serve to illustrate typical trends in well design, but the authors wish to caution the reader that broad generalizations without specific numerical calculations in individual cases is not justified.

7. Conclusions

(a) The theoretical model described in this report predicts the temperature, pressure, dynamic dryness fraction and void fraction along the vertical channel of two-phase flow.

(b) The existing data from operating wells (No. 1 IID and 16-29 East Mesa) indicate good agreement with the model.

(c) The complete justification of this theoretical model will require laboratory experimental verification.

(d) The need for laboratory experiments results from the fact that in most cases the accuracy of the measurements in actual wells is inadequate to match the sensitivity of the model.

(e) The model may be used not only for geothermal wells but for all vertical two-phase flow configurations.

(f) The model should be improved in order to predict better the magnitude of shear stresses, τ_w , for different flow patterns.

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Figure Captions

- Figure 10. Mean distance between vapor bubbles, a , and effective wake length, ℓ , illustrating transition criterion from bubble to slug flow.
- Figure 11. Notation for transition criterion between slug and froth flow.
- Figure 12. Temperature distribution in geofluid, T , and surrounding formation, T_o .
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- Figure 13. Flow chart of computer program.
- Figure 14. Mass-flow rate *versus* wellhead pressure for various well-bottom temperatures, T_b ($P_b=16\text{MPa}$, $L=1600\text{m}$, $D=0.2\text{m}$, $k_D=20\text{kPa s/kg}$, $b=1\text{m}$).
- Figure 15. Mass-flow rate *versus* wellhead pressure for various well-bottom pressures, P_b ($T_b=250^\circ\text{C}$, $L=1600\text{m}$, $D=0.2\text{m}$, $k_D=20\text{kPa s/kg}$, $b=1\text{m}$).
- Figure 16. Pressure *versus* well-depth for various well-bottom temperatures, T_b ($P_b=16\text{MPa}$, $\dot{m}=60\text{kg/s}$, $b=5.5\%$, $D=0.2\text{m}$, $k_D=20\text{kPa s/kg}$, $L=1600\text{m}$).
- Figure 17. Mass-flow rate *versus* wellhead pressure for various drawdown pressure factors, k_D ($T_b=295^\circ\text{C}$, $P_b=15\text{MPa}$, $L_b=1520\text{m}$, $D=0.178\text{m}$, $b=0$).
- Figure 18. Void fraction *versus* well-depth for various drawdown pressure factors k_D ($T_b=295^\circ\text{C}$, $P_b=15\text{MPa}$, $L=1520\text{m}$, $D=0.178\text{m}$, $b=0$).
- Figure 19. Mass-flow rate *versus* wellhead pressure for various brine salinities, b ($T_b=295^\circ\text{C}$, $P_b=15\text{MPa}$, $L=1520\text{m}$, $D=0.178\text{m}$, $k_b=23\text{kPa s/kg}$).
- Figure 20. Temperature *versus* well-depth for various brine concentrations, b ($T_b=295^\circ\text{C}$, $P_b=15\text{MPa}$, $L=1520\text{m}$, $D=0.178\text{m}$, $k_b=23\text{kPa s/kg}$).
- Figure 21. Mass-flow rate *versus* wellhead pressure for various heat-loss values measured as a percentage of total energy transported through the well. ($T_b=168^\circ\text{C}$, $P_b=22.54\text{MPa}$, $L=2433\text{m}$, $D=0.391\text{m}$, $k_D=21\text{kPa s/kg}$, $b=0$).
- Figure 22. Mass-flow rate *versus* wellhead pressure for various well diameters, D ($T_b=200^\circ\text{C}$, $P_b=22\text{MPa}$, $L=2300\text{m}$, $k_D=20\text{kPa s/kg}$, $b=0$).
- Figure 23. Comparison between constant diameter and scheduled casing characteristics. ($T_b=180^\circ\text{C}$, $P_b=15\text{MPa}$, $L=1520\text{m}$, $k_D=21\text{kPa s/kg}$, $b=5\%$).
 (a) $D=0.21\text{m}$ constant
 (b) $D=0.391\text{m}$ from wellhead to a depth of 200m with $D=0.21$ to bottom.

- Figure 24. Mass-flow rate *versus* wellhead pressure for various values of coefficient of friction, f ($T_b=250^\circ\text{C}$, $P_b=20\text{MPa}$, $L=2000\text{m}$, $D=0.2\text{m}$, $k_D=10\text{kPa s/kg}$, $b=0$).
- Figure 25. Comparison between calculation and measurement [IID No. 1 well] ($T_b=295^\circ\text{C}$, $P_b=15\text{MPa}$, $L=1520\text{m}$, $D=0.178$, $k_D=22.8\text{kPa s/kg}$, $b=15\%$).
- Figure 26. Comparison between calculation and measurement [East Mesa 16-29 well] ($T_b=156^\circ\text{C}$, $P_b=22.54\text{MPa}$, $L=2433\text{m}$, $k_D=21\text{kPa s/kg}$, $b=0$, $D=0.391\text{m}$ from wellhead to a depth of 281m with $D=0.276\text{m}$ to 1355m , and $D=0.201$ to bottom).
- Figure 27. Comparison between calculated and measured temperature distribution ($T_b=156^\circ\text{C}$, $P_b=22.54\text{MPa}$, $L=2433\text{m}$, $k_D=21\text{kPa s/kg}$, $b=0$, $D=0.391\text{m}$ from wellhead to a depth of 281m , $D=0.276\text{m}$ to 1355m , and $D=0.201$ to bottom).
- Figure 28. Comparison between our model calculations and those of the Denver Research Institute.
 $(D=7.125\text{m}, \lambda=0.012)$
 $K,1 \quad \dot{m} = 200\ 000\ \text{lb/hr}, T_b=500^\circ\text{F}$
 $J,2 \quad \dot{m} = 300\ 000\ \text{lb/hr}, T_b=500^\circ\text{F}$
 $H,3 \quad \dot{m} = 200\ 000\ \text{lb/hr}, T_b=450^\circ\text{F}$
 $G,4 \quad \dot{m} = 300\ 000\ \text{lb/hr}, T_b=450^\circ\text{F}$
- Figure 29. Available power *versus* mass-flow rate for various well-bottom temperatures, T_b ($P_b=16\text{MPa}$, $L=1600\text{m}$, $D=0.2\text{m}$, $k_D=20\text{kPa s/kg}$, $b=1\ \text{mol}$).
- Figure 30. Available power *versus* mass-flow rate for various well-bottom pressures P_b ($T_b=250^\circ\text{C}$, $L=1600\text{m}$, $D=0.2\text{m}$, $k_D=20\text{kPa s/kg}$, $b=1\ \text{mol}$).
- Figure 31. Available power *versus* mass-flow rate for various drawdown pressure factors, k_D ($T_b=295^\circ\text{C}$, $P_b=15\text{MPa}$, $L=1520\text{m}$, $D=0.178\text{m}$, $b=0$).
- Figure 32. Available power *versus* mass-flow rate for various brine salinities, b ($T_b=295^\circ\text{C}$, $P_b=15\text{MPa}$, $L=1520\text{m}$, $D=0.178\text{m}$, $k_D=23\text{kPa s/kg}$).
- Figure 33. Available power *versus* mass-flow rates for various heat losses expressed as percentage of total energy transported through well ($T_b=168^\circ\text{C}$, $P_b=22.54\text{MPa}$, $L=2433\text{m}$, $D=0.391\text{m}$, $k_D=21\text{kPa s/kg}$, $b=0$).
- Figure 34. Available power *versus* mass-flow rate for various well diameters, D ($T_b=200^\circ\text{C}$, $P_b=22\text{MPa}$, $L=2300\text{m}$, $k_D=20\text{kPa s/kg}$, $b=0$).
- Figure 35. Available power *versus* mass-flow rate for various values of friction factor, f ($T_b=250^\circ\text{C}$, $P_b=20\text{MPa}$, $L=2000\text{m}$, $D=0.2\text{m}$, $k_D=10\text{kPa s/kg}$, $b=0$).

Figure 36. Well flow-rate *versus* wellhead pressure for self-flowing and pumped wells ($T_b=295^\circ\text{C}$, $D=0.178\text{m}$, $P_b=14.9\text{MPa}$, $L=1500\text{m}$, $b=0$, $k_D=23\text{kPa s/kg}$).

Figure 37. Net available power *versus* mass-flow rate for self-flowing and pumped wells ($T_b=295^\circ\text{C}$, $D=0.178\text{m}$, $P_b=14.9\text{MPa}$, $L=1500\text{m}$, $b=0$, $k_D=23\text{kPa s/kg}$).

Figure 38. Net available power *versus* mass-flow rate for self-flowing and pumped wells ($T_b=295^\circ\text{C}$, $L=1500\text{m}$, $b=0$, $k_D=23\text{kPa s/kg}$)

(a) self-flowing $P_b = 15\text{MPa}$, $D=0.178\text{m}(\text{const})$

(b) self-slowing $P_b = 15\text{MPa}$,

$D = 0.391\text{m}$ to 200m

$D = 0.275\text{m}$ to depth of 300m

$D = 0.210\text{m}$ to depth of 500m

$D = 0.178\text{m}$ to well-bottom

(c) pumped, $P_b = 15\text{MPa}$, $\Delta P = 3.5\text{MPa}$

$D = 0.178\text{m}$ (const)

Appendix A

This appendix contains the print-outs of the algorithms which are written in two different languages. The first algorithm is written in the Hewlett-Packard BASIC language for a user who has access to H.P. computers.

The second program is written in FORTRAN IV and is intended for use on large-scale digital computers.

The procedures used for both programs are described in narrative. They are shown underlined for BASIC and in the lines labeled by asterisks for FORTRAN IV.

Also enclosed is a sample of the printout of the calculations at the end of this appendix.

The algorithm written
in H.P. BASIC language

THERMODYNAMIC CONDITIONS ALONG A GEOTHERMAL WELL

INPUT DATA:

WELL FLOW RATE= 63 KG/S
 STATIC PRESSURE AT THE BOTTOM= 15 MPA
 TEMPERATURE AT THE BOTTOM= 295 GRAD C
 WELL DEPTH= 1520 M
 DIAMETER OF THE WELL= 0.178 M
 DRAWDOWN PRESSURE FACTOR K= 22800 PA PER KG/S
 BRINE CONCENTRATION= 15 PERCENT
 STEP OF CALCULATION= 40 M

DEPTH M	PRESSURE MPA	QUALITY	VOID FRACTION	FLOW PATTERNS
1520.0	13.56	0	0	LIQUID
885.1055				
755.0	6.92	0	0	FLASHING POINT
635.0	6.05	0.0290	0.4741	BUBBLY-SLUG TRANSITION
195.0	3.89	0.0974	0.8524	SLUG-FROTH TRANSITION
155.0	3.60	0.1069	0.8702	FROTH-ANNULAR TRANSITION

CONDITIONS AT WELLHEAD
 T= 201.490 C P= 1.441 MPA X= 0.155 DEL PH/DEL PF= 0.049
 DEL PA TOTAL= 81048.412 PA R= 131.003 W2= 41.927 M/S K= 0.936
 W1= 30.237 M/S VOID FRACTION= 0.926 R8= 1.151
 BRINE FLOW = 53.216 KG/S STEAM FLOW = 9.784 KG/S

THERMODYNAMIC CONDITIONS ALONG A GEOTHERMAL WELL

INPUT DATA:

WELL FLOW RATE= 30 KG/S
 STATIC PRESSURE AT THE BOTTOM= 15 MPA
 TEMPERATURE AT THE BOTTOM= 295 GRAD C
 WELL DEPTH= 1520 M
 DIAMETER OF THE WELL= 0.178 M
 DRAWDOWN PRESSURE FACTOR K= 22800 PA PER KG/S
 BRINE CONCENTRATION= 15 PERCENT
 STEP OF CALCULATION= 40 M

DEPTH M	PRESSURE MPA	QUALITY	VOID FRACTION	FLOW PATTERNS
1520.0	14.32	0	0	LIQUID
885.1055				
668.3	6.92	0	0	FLASHING POINT
548.3	6.06	0.0285	0.4512	BUBBLY-SLUG TRANSITION

CONDITIONS AT WELLHEAD
 T= 260.862 C P= 4.191 MPA X= 0.087 DEL PH/DEL PF= 1.888
 DEL PA TOTAL= 7830.938 PA R= 24.369 W2= 6.824 M/S K= 1.000
 W1= 6.474 M/S VOID FRACTION= 0.815 R8= 1.167
 BRINE FLOW = 27.402 KG/S STEAM FLOW = 2.598 KG/S

```
5 DISP "      GEOTHERMAL WELL THEORETICAL MODEL"
10 DISP "MAX Y-SCALE";
20 INPUT Y8
30 DISP "M-KG/S ";
40 INPUT M
50 DISP "PB-MPA";
60 INPUT C3
70 DISP "WELL DEPTH FOR DIAMETER D1 ";
80 INPUT Y1
90 DISP "D1-M";
100 INPUT D
110 DISP "STEPEd  CASING DEPTH FOR D2";
120 INPUT Y5
130 DISP "D2-M";
140 INPUT R5
150 DISP "STEPEd  CASING DEPTH FOR D3";
160 INPUT Y6
170 DISP "D3-M";
180 INPUT R6
190 DISP "STEPEd  CASING DEPTH FOR D4";
200 INPUT Y7
210 DISP "D4-M";
220 INPUT R7
230 DISP "TB-C.DEG.";
240 INPUT T0
250 DISP "RESERVOIR PERMEABILITY CONSTANT K=";
260 INPUT A
270 DISP "BRINE CONCENTRATION PERCENT";
280 INPUT C5
290 DISP "STEP Z-M";
300 INPUT Z
310 SCALE 0,Y1,0,Y8
320 XAXIS 0,Y1/10,0,Y1
330 YAXIS 0,Y8/10,0,Y8
340 K3=Y1
350 K4=C3
360 K5=A
370 R0=X=0
380 PRINT
390 PRINT
400 PRINT
410 PRINT "      THERMODYNAMIC  CONDITIONS ALONG A GEOTHERMAL WELL"
420 PRINT
430 PRINT "INPUT DATA:"
440 PRINT "WELL FLOW RATE="M"KG/S"
450 PRINT "STATIC PRESSURE AT THE BOTTOM="C3"MPA"
460 PRINT "TEMPERATURE AT THE BOTTOM="T0"GRAD C"
470 PRINT "WELL DEPTH="Y1"M"
480 PRINT "DIAMETER OF THE WELL="D"M"
490 IF Y5=0 THEN 530
500 PRINT "FROM DEPTH="Y5"M DIAMETER="R5"M"
510 PRINT "FROM DEPTH="Y6"M DIAMETER="R6"M"
520 PRINT "FROM DEPTH="Y7"M DIAMETER="R7"M"
530 PRINT "DRAWDOWN PRESSURE FACTOR K="A"PA PER KG/S"
540 PRINT "BRINE CONCENTRATION="C5"PERCENT"
550 PRINT "STEP OF CALCULATION="Z"M"
560 PRINT
570 PRINT
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580 PRINT
590 PRINT "      DEPTH      PRESSURE      QUALITY      VOID FRACTION";
600 PRINT "      FLOW PATTERNS"
610 PRINT "      M              MPA              -              -"
620 T1=T0
630 FIXED 4
640 C3=C3*106
650 FORMAT X,F9.1,5X,F6.2,10X,F2.0,14X,F2.0,7X,"LIQUID"
660 WRITE (15,650)Y1,(C3-M*A)/106,X,R0
670 GOTO 720
671 DISP "C7 MEANS SATURATION PRESSURE FOR GIVEN TEMP."
680 C7=EXP(2.1913E-07*T23-1.7816E-04*T22+6.53665E-02*T2-4.96081)*105
690 IF T2<270 THEN 710
700 C7=EXP(-3.6529+4.604E-02*T2-8.3728E-05*T22+6.7725E-08*T23)*105
710 RETURN
711 DISP "F1 MEANS MOLALITY OF THE BRINE"
712 DISP "H0-HEAT LATENT FOR A PURE WATER ;T2-SAT.TEMP.FOR THE BRINE"
720 F1=1000*C5/(5800-58*C5)
730 H0=2467.7-1.33292*T1-0.006762*T12
740 T2=T1-1.8*8.314*(T1+273)2*F1/(H0*1000)
750 GOSUB 680
760 C0=C7
770 FORMAT X,F9.1,5X,F6.2,10X,F2.0,14X,F2.0,7X,"FLASHING POINT"
780 GOSUB 970
781 DISP "Y1-FLASHING LEVEL OR A GIVEN DEPTH"
790 Y1=Y1-(C3-M*A-C0)/(9.81*G1)
800 WRITE (15,770)Y1,C0/106,X,R0
810 Y4=R0=C4=0
820 GOTO 930
821 DISP "S0-ENTHALPY OF THE PURE WATER;S1-OF THE BRINE"
830 S0=1.2453E-05*T13-4.5137E-03*T12+4.81155*T1-29.578
840 F1=1000*C5/(5800-58*C5)
850 F3=(-0.83623*104(-3)*T13+0.16792*T12-25.9293*T1)*4.184/58.44
860 F2=(-0.49*104(-4)*F12+0.71924*104(-3)*F1+0.17965*104(-2))*T13
870 F2=F2+(0.024967*F12-0.36524*F1-0.90963)*T12
880 F2=F2+(-4.68651*F12+68.577*F1+166.58)*T1
890 F2=F2+(286.49*F12-4080*F1-9633.6)
900 F2=F2*4.184/(1000+F1*58.44)
910 S1=(1-C5/100)*S0+C5*F3/100+F1*F2
920 RETURN
930 I=R3=K2=1
940 GOSUB 830
941 DISP "S2-ENTHALPY OF THE BRINE AT THE BOTTOM"
950 S2=S1
960 GOTO 1050
961 DISP "G1 INTO LINE 970 MEANS DENSITY OF THE PURE WATER"
962 DISP "G1 INTO 1020 LINE - DENSITY OF THE BRINE"
970 G1=(1003.8-2.47524E-03*T12-0.203*T1)/1000
980 H0=2467.7-1.33292*T1-0.006762*T12
990 T2=T1-1.8*8.314*(T1+273)2*F1/(H0*1000)
1000 H6=-167.219+448.55/G1-261.07/G12
1010 H7=(-13.644+13.97/G1)*3.19752/(3.1975-1/G1)2
1020 H8=H6+H7*F1+0.5
1030 G1=(1000+F1*58.44)*1000/(1000/G1+H8*F1)
1040 RETURN
1050 GOSUB 970
1060 T2=T1

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1070 GOSUB 680
1080 C6=C7
1090 H0=2467.7-1.33292*T1-0.006762*T1^2
1100 T2=T1-1.8*8.314*(T1+273)^2*F1/(H0*1000)
1110 GOSUB 680
1120 C8=C7-C6
1130 C9=C7/C6
1131 DISP "G2- DENSITY OF THE VAPOR"
1132 DISP " G0- TWO PHASE FLOW AVERAGE DENSITY"
1140 G2=C7*10^(-3)/(121.05+0.6325*T1-1.296E-03*T1^2)*C9
1150 G0=G2*R0+(1-R0)*G1
1160 C0=C0-9.81*G0*Z
1170 GOTO 1340
1171 DISP " T1- SAT. TEMP. OF THE PRESSURE C0"
1180 IF C0>38580 THEN 1210
1190 T1=1.66E-12*C0^3-1.39311E-07*C0^2+4.5194E-03*C0+12.731
1200 GOTO 1310
1210 IF C0>270100 THEN 1240
1220 T1=3.759E-15*C0^3-2.4861E-09*C0^2+6.8454E-04*C0+52.4542
1230 GOTO 1310
1240 IF C0>1553800 THEN 1270
1250 T1=2.15317E-17*C0^3-8.5614E-11*C0^2+1.47848E-04*C0+96.3286
1260 GOTO 1310
1270 IF C0>5500000 THEN 1300
1280 T1=3.3886E-19*C0^3-5.6954E-12*C0^2+4.38556E-05*C0+144.5435
1290 GOTO 1310
1300 T1=170.23+2.73873E-05*C0-2.2533E-12*C0^2+1.184E-19*C0^3-2.61E-27*C0^4
1301 DISP "Q1- AN ELEVATION OF A SATURATION TEMP. FOR THE BRINE"
1310 Q1=14.9652*F1/(H0*1000)*(T1+273)^2
1320 T1=T1+Q1
1330 RETURN
1340 GOSUB 1180
1350 GOSUB 1180
1360 GOSUB 830
1370 H0=2467.7-1.33292*T1-0.006762*T1^2
1371 DISP "H9- ENTHALPY OF THE VAPOR; H1- OF THE BRINE"
1380 H1=-0.81257*10^(-2)*T1^2+3.65228*T1+2388.4
1390 H1=H1-S1
1391 DISP "P- SURFACE TENSION; V2- BRINE AND GAS VISCOSITIES"
1400 P=((647-T1+273)/647.15)^1.25*(1-0.625*((647-T1+273)/647.15))^2.358E-01
1410 V1=-6.608E-11*T1^3+4.55705E-08*T1^2-1.069048E-05*T1+9.73E-04
1420 V2=2.8571E-11*T1^2+2.45714E-08*T1+9.66E-06
1430 H4=0.03324*F1+0.003624*F1^2-0.0001879*F1^3
1440 H5=-0.03961*F1+0.0102*F1^2-7.02E-04*F1^3
1450 V1=V1*10^(H4+H5*LGT(V1/1002*10^6))
1451 DISP "X- QUALITY"
1460 X=(S2-S1)/H1
1461 DISP "C1-PRESSURE DROP OF THE LIQUID PHASE"
1470 C1=0.16*(1.27*M/(G1*D^2))^1.75*G1^0.75*(V1/D)^0.25*Z/D
1480 R3=1
1490 IF X <= 0 THEN 2140
1500 IF X>1 THEN 2490
1501 DISP "W1,W2- LIQUID AND GAS SUPERFICIAL VELOCITY"
1510 W1=4*M*(1-X)/(G1*3.14*D^2)
1520 W2=4*M*X/(G2*3.14*D^2)
1530 IF I#1 THEN 1730
1531 DISP "R0 AT LINE 1580 - VOID FRACTION FOR BUBBLES FLOW"
1540 O1=1.41*((9.81*P)/G1)^0.24
1550 O1=(2*P/(D*G1/100)+9.81*D/200)^0.5

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156 O2=W1+W2+O1
157 O3=O2+2-4*W2*O1
1580 R0=(O2-O3+0.5)/(2*O1)
1581 DISP "A- AVERAGE DISTANCE BETWEEN BUBBLES"
1590 A=0.8/R0+0.333-1
1600 W4=O1
1610 O4=1.53+3.5*EXP(-W1+1.5/1.58)
1611 DISP "A<L- BUBBLY-SLUG TRANSITION CRITERION"
1620 L=0.42*(0.1*W4*(1-R0)+(O4/2)/(W1*(0.023*(W1*D*G1/V1)+(-0.2))+0.5))+1.5
1630 IF A>L THEN 2130
1640 I=2
1650 Y1=Y1-Z
1660 R8=R0
1670 FORMAT X,F9.1,5X,F6.2,5X,F7.4,9X,F7.4,7X,"BUBBLY-SLUG TRANSITION"
1680 WRITE (15,1670)Y1,C0*10+(-6),X,R0
1690 F0=W2/(9.81*D)+0.5
1700 R8=R8/(1.06/(1.2*(W1/W2+1)+0.25/F0))
1710 Y1=Y1+Z
1720 GOTO 2130
1730 IF I#2 THEN 2000
1740 F0=W2/(9.81*D)+0.5
1741 DISP "R0 AT LINE 1750- VOID FRACTION FOR SLUG FLOW"
1750 R0=1.06/(1.2*(W1/W2+1)+0.25/F0)*R8
1760 W4=W2/R0-W1/(1-R0)
1770 IF W4 <= 0 THEN 1900
1780 L=0.42*(0.5*W4*(1-R0)+0.7/(W1*(0.023*(W1*D*G1/V1)+(-0.2))+0.5))+1.5
1790 A=7*(1/R0-1)
1800 IF W1+W2 >= 0.3 THEN 1840
1810 W3=5.5*(W1+W2)
1820 E0=0
1830 GOTO 1860
1840 W3=1.12*(W1+W2)
1850 E0=1.06
1860 W3=-W3*LOG(R0*D/(3*(3*V1/(G1*9.81))+0.5))+E0
1870 T2=(ABSW3)+0.5*1.5*(3*V1/(9.81*G1))+0.5
1880 IF W2/R0+W3>(P/(T2*G2))+0.5 THEN 1900
1881 DISP "AT LINE 1890 SLUG-FROTH TRANSITION CRITERION"
1890 IF A>L THEN 2130
1900 I=3
1910 Y1=Y1-Z
1920 FORMAT X,F9.1,5X,F6.2,5X,F7.4,9X,F7.4,7X,"SLUG-FROTH TRANSITION"
1930 WRITE (15,1920)Y1,C0*10+(-6),X,R0
1940 B1=1/(1+(W1/W2)*(G1/G2)+0.5)
1950 B2=1/(1+(1-X)/X*(G2/G1))
1960 K0=(R0-B1)/(B2-B1)
1970 K1=X
1980 Y1=Y1+Z
1990 GOTO 2130
2000 IF I#3 THEN 2110
2010 K2=K0*X/(K1-1)-K0/(K1-1)
2011 DISP "R0 AT LINE 2020 -VOID FRACTION FOR FROTH FLOW"
2020 R0=(1-K2)/(1+(W1/W2)*(G1/G2)+0.5)+K2/(1+(1-X)*G2/(X*G1))
2021 DISP "AT LINE 2021 FROTH-ANNULAR FLOW TRANSITION CRITERION"
2030 IF W2*G2+0.5*(9.81*P*(G1-G2))+(-0.25)<3.2 THEN 2130
2040 I=4
2050 Y1=Y1-Z
2060 FORMAT X,F9.1,5X,F6.2,5X,F7.4,9X,F7.4,7X,"FROTH-ANNULAR "

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2070 WRITE (15,2060)Y1,C0*10+(-6),X,R0;
2080 PRINT "TRANSITION"
2090 Y1=Y1+Z
2100 GOTO 2130
2110 K2=K0*X/(K1-1)-K0/(K1-1)
2111 DISP "R0 AT 2120 LINE- VOID FRACTION FOR ANNULAR-MIST FLOW"
2120 R0=(1-K2)/(1+(W1/W2)*(G1/G2)+0.5)+K2/(1+(1-X)*G2/(X*G1))
2121 DISP "R3=(DELTA P)T.P.F./(DELTA P)LIQUID"
2130 R3=((1-X)/(1-R0))+2
2140 C0=C0+C4
2150 IF R0=0 THEN 2170
2151 DISP "C4- ACCELERATIONAL PRESSURE DROP"
2160 C4=1.62*M+2/(G1*D+4)*((1-X)+2/(1-R0)+X+2*G1/(R0*G2))-1
2170 C0=C0-C4
2171 DISP "C0-PRESSURE AT A GIVEN LEVEL"
2180 C0=C0-C1*R3
2190 Y1=Y1-Z
2200 IF C0<0 THEN 2480
2210 GOSUB 1180
2220 PLOT Y1,R0
2230 IF Y1<Y7 THEN 2310
2240 IF Y1<Y6 THEN 2290
2250 IF Y1<Y5 THEN 2270
2260 GOTO 2320
2270 D=R5
2280 GOTO 2320
2290 D=R6
2300 GOTO 2320
2310 D=R7
2320 IF I<3 THEN 2380
2330 Y3=(G1/G2)+0.5
2340 Y2=S1*(1/G2-1/G1)*(T1+273)/(T1*H1+2)
2350 Y3=31.62*(2/G1*Y2*((1-X)+Y3*X)*(Y3-1))+(-0.5)
2360 Y9=3.14*D+2/4*Y3
2370 IF M>Y9 THEN 2490
2380 IF Y1>Z THEN 960
2390 PRINT "CONDITIONS AT WELLHEAD"
2400 FIXED 3
2410 PEN
2420 PRINT "T="T1"C      P="C0*10+(-6)"MPa      X="X"DEL PH/DEL PF=";
2430 PRINT G0*Z*9.81/(C1*R3)
2440 PRINT "DEL PA TOTAL="C4"PA      R="R3"W2="W2/R0"M/S      K="K2
2450 PRINT "W1="W1/(1-R0)"M/S      VOID FRACTION="R0"R8="R8
2460 PRINT "BRINE FLOW ="M*(1-X)"KG/S      STEAM FLOW ="M*X"KG/S"
2470 STOP
2480 C0=0
2490 PRINT "CHOKING AT DEPTH ="Y1"AND OTHER CONDITIONS"
2500 GOTO 2400
2510 END

```

The algorithm written
in FORTRAN IV

11 72

the first 19 lines give the boundary conditions at the bottom and characteristics of the stratum. Some values are redundantly specified in case the program runs many times. Rmf is the mass flow rate, sdhl the initial enthalpy loss, q heat loss, b1 and b2 the temperature slopes of the stratum, sq the total heat loss, pstep the incremental step in pressure, t the bottom temperature, d the diameter, qcr the value for critical flow, k the number denoting the flow regime, z the height, x the quality, acc and acco the acceleration components of the pressure drop.

DZ=0.0

M=0.

RMF=40.

2 SDHL=0.0

4 Q=0.0

B1=0.022222

B2=0.3

SQ=0.0

DEPTH=2433

PSTEP=1.

T=168

D=0.391

GCR=1000.

K=0

P=225.4-0.21*RMF

225.4 is the bottomhole pressure and 0.21 the drawdown

these values are subject to alteration for different wells

Z=0.0

X=0.0

RM=0.

ACC=0.0

the next 46 lines determine the properties of the fluid

5 CP=((0.2796E-6*T-0.122771E-3)*T+0.20121E-1)*T+3.1357

the specific heat of the liquid

DQT=DQTO*DZ/(CP*DEPTH)

drop of temperature due to heat losses

DHL=DHLO*DZ/DEPTH

drop of enthalpy

12 SDHL=SDHL+DHL

IF (K.GE.1.) GO TO 7

T=T-DQT

DHL=0.0

7 PSAT=EXP(((0.21913E-6*T-0.17816E-3)*T+0.653665E-1)*T-4.96087)

saturation pressure

C P AND PSAT ARE IN BARS

R2=PSAT*100./((-0.1296E-2*T+0.6325)*T+121.05)

9 R1=(-2.47524E-3*T-0.2030)*T+1003.8

density of vapor and liquid

HFG={(-0.3797E-4*T+0.11799E-1)*T-4.04181)*T+2582

latent heat

TO=273.15+T

DP=(8.315E-5*1.8*TO*RM)/((1./R2)-(1./R1))

depression of saturation pressure

PBS=PSAT-DP

DT=(1.4967E-2*TO**2.)*RM/HFG

elevation of saturation temperature

TH=647.27-T-273.15-DT

VO=(3.1975-0.315154*TH**0.3333-1.203374E-3*TH+7.48908E-13*TH**4
C/(1.+0.1342489*TH**0.3333-3.946263E-3*TH)

RK=(-13.644+13.97*VO)*(3.1975**2.)/(3.1975-VO)**2.

PHIST=-167.219+448.55*VO-261.07*VO**2.

PPHI=PHIST+RK*SQRT(RM)

DM=(1000.+RM*58.44)*1000./(1000.*VO+RM*PPHI)

density of solution

30 CORR=4.184/(1000.+RM*58.44)

X1=1000./(1000.+RM*58.44)

X2=1.-X1

PSI={(-0.49E-4*RM**2.+0.71924E-3*RM+0.17965E-2)*T**3.+(0.24967E-
C*RM**2.-0.36524*RM-0.90963)*T**2.+(-4.685607*RM**2.+68.577*RM+
C166.58)*T+(286.96*RM**2.-4080*RM-9633.6))*CORR

HLS={(-0.83624E-3*T+0.16792)*T-25.9293)*T*4.184/58.44

HLW={(0.12453E-4*T-0.45137E-2)*T+4.81155)*T-29.578

enthalpy of salt and water

HL=X1*HLW+X2*HLS+RM*PSI-SDHL

enthalpy of solution

HG=(-0.81257E-2*T+3.65228)*T+2388.4

enthalpy of vapor

DH=HG-HL

SIGMA=((0.5779-T/647.15)**1.256)*(0.6388-T/1035.4)*0.2358

surface tension


```

20    AC=0.3324E-1*RM+0.3624E-2*RM**2.-0.1879E-3*RM**3.
      BC=((-0.702E-3*RM+0.102E-1)*RM-0.3961E-1)*RM
      VIST=(((0.138598E-6*T-0.148587E-3)*T+0.595723E-1)*T-11.0489)*T
      C+927.7)*1E-6
*viscosity of water*
      VC=AC+BC*ALOG(VIST/1002E-6)/2.30258
      V=EXP(2.30258*VC)*VIST
*viscosity of solution*
C V IS IN PA*SEC
      IF (K.GE.1) GO TO 55
*liquid flow*
77      X=0.0
      PHI=0.0
*void fraction*
      IF (PBS.GT.P) GO TO 50
      HLTO=HL+0.1
      GOTO 190
50      K=1
      WRITE(2,51)
51      FORMAT(4X,'TRANSITION TO BUEBLE FLOW')
55      X=(HLTO-HL)/DH
      WS1=RMF*(1.-X)/(DM*AR)
*liquid superficial velocity*
      IF (X.GE.1.) GOTO 216
      WS2=RMF*X/(R2*AR)
*vapor superficial velocity*
      GO TO (99,115,135,155),K

```

goes to the appropriate flow regime

bubbly flow

```

99      F1=(2.*SIGMA/(D*DM)+9.81*D/2.)**0.5
        F2=(1.-X)*F1*R2/(X*WS1*DM)
        F3=(1.+(1.-X)*R2/(X*DM))
        PHI=(-F3+(F3**2.+4.*F2)**0.5)/(2.*F2)
        AL=0.886/(PHI)**0.5-1.
        XI=0.023*(V/(D*WS1*DM))**0.25
        WS3=WS2/PHI-WS1/(1.-PHI)
        EL=1.2*(0.05*ABS(WS3)*(1.-PHI)**0.7/(WS1*(XI)**0.5))**1.5

```

criterion for transition

```

      IF (EL.GT.AL) GO TO 114
      GO TO 190

```

slug flow

```

114      K=2
          WRITE (2,113)
          FROUDE=WS2/(9.81*D)**0.5
113      FORMAT (4X,'BUEBLE TO SLUG TRANSITION')
          PHIF=PHI/(1.06/(1.2*(WS1/WS2+1.))+0.25/FROUDE)
115      FROUDE=WS2/(9.81*D)**0.5
          PHIR=1.06/(1.2*(WS1/WS2+1.))+0.25/FROUDE
          PHI=PHIR*PHIF
          PHISL=PHI
          XSL=X
          WB=((0.3014*D*ABS(WS2/PHI-WS1-WS2)*(DM)**0.5/V**0.5)**0
C.666667)
          DELTA=1.5*(3*V*WB/(DM*9.81))**0.5

```

first criterion

CRIT1=SQRT(SIGMA/(DELTA*R2))+WE-WS2

IF (CRIT1.LE.0.0) GO TO 130

WS3=ABS(WS2/PHI-WS1/(1.-PHI))

XI=0.023*(V/(WS1*D*DM))**0.25

EL=1.2*(0.05*WS3*(1.-PHI)**0.7/(WS1*(XI)**0.5))**1.5

AL=7.*(1.-PHI)/PHI

second criterion for transition

CRIT2=AL-EL

IF (CRIT2.LE.0.0) GO TO 130

GO TO 190

froth flow

130 K=3

WRITE (2,131)

131 FORMAT (4X,'SLUG TO FROTH TRANSITION')

135 PHIAN=1./((R1**0.5*WS1)/(R2**0.5*WS2)+1.)

PHINS=1./(((1.-X)*R2)/(X*DM)+1.)

CONNS=(PHISL-PHIAN)/(PHINS-PHIAN)

PHI=(1.-CON)*PHIAN+CON*PHINS

XNS=X

CRITFA=PHI*(9.81*D*(DM-R2)/R2)**0.5

criterion for transition

IF(CRITFA.LE.WS2) GO TO 150

GO TO 190

annular flow

150 K=4

WRITE (2,151)

```
151  FORMAT (4X, 'FROTH TO ANNULAR TRANSITION')
155  PHIAN=1./((DM**0.5*WS1)/(R2**0.5*WS2)+1.)
      PHINS=1./(((1.-X)*R2)/(X*DM)+1.)
      CON2=CONNS*X/(XSL-1.)-CONNS/(XSL-1.)
      PHI=(1.-CON2)*PHIAN+CON2*PHINS
160  FORMAT (1H+,4X, 'ANNULAR FLOW')
      GO TO 190
190  R=({1.-X}/{1.-PHI})**2
*the two phase friction factor ratio*
      AR=0.7854*D**2
      VEL=(RMF)/(AR*DM)
      ACC=VEL**2.*DM
      IF (K.LT.1) GO TO 196
      ACC=VEL**2.*DM*({1.-X})**2./({1.-PHI})+X**2.*R2/(PHI*DM)
196  RE=(D*VEL*DM/V)
*reynolds number*
      FR=R*0.158*{VEL**2.}*DM/(D*(RE**0.25))
*friction factor from Blasius solution*
      DENS=(1.-PHI)*DM+PHI*R2
*density of geothermal fluid at given point*
      PF=(DENS*9.81+FR+ACC-ACCO)*1.E-5
C ACC-ACCO IS THE CHANGE OF MOMENTUM
C PF IS PRESSURE DROP PER METER IN BAR/M
      ACCO=ACC
      DZ=PSTEP/PF
      Z=Z+DZ
      IF (Z.GE.DEPTH) GO TO 210
```

```

P=P-PSTEP
TSTEP=PSTEP*TO*((1./R2)-(1./R1))*100./HFG
*temperature change from Clapeyron equation*
IF (K.LT.1) GO TO 191
T=T-TSTEP
*the next 8 lines calculate the critical flow for water*
10      VL=1/((-2.47524E-3*T-.2030)*T+1003.8)
      VG=((-.0.1296E-2*T+0.6325)*T+121.05)/(100.*P)
IF (VG.LE.0.0) GO TO 200
      Y=SQRT(VG/VL)
      GAMA=(CP*(VG-VL)*(T+ 273.15))/(HFG**2)
      BETA=(VG-Y*VL)*Y*(Y*X+1-X)+Y*(Y-1)*(Y*VL+Y*X*VL+X*VG)
      GCR=3.14159*D**2.*SQRT(1000*Y**2/(BETA*GAMA))/4.
      IF (GCR.LT.RMF) GO TO 200
*the next 7 lines calculate the heat losses in dz*
191     TW=313.8-B1*Z
      IF (Z.LT.620) GO TO 230
      TW=300-B2*(Z-620.)
230     THICK=0.0508
      U=2*1.2*ALOG((D+THICK+0.0127)/(D+0.0127))/(D+0.0127)
      Q=U*DZ*3.14*(D+THICK+0.0127)*(T-TW)
      SQ=SQ+Q
C Q AND SQ ARE IN J/SEC
      WRITE (2,192) Z,P,T,X,FR,PF,DENS,PHI,DHL,SDHL,RMF,D
192     FORMAT(1X,F8.2,2X,F8.3,2X,F8.3,2X,F8.5,2X,F8.1,2X,
      CF8.4,2X,F8.3,2X,F8.4,1X,F10.4,1X,F10.3,1X,F5.1,1X,F9.5)
*this is the output of the program*
```

additional variables can be added

GC TO 5

200 WRITE (2,302)

302 FORMAT (4X,'CRITICAL FLOW,CALCULATION STOPS')

210 WRITE (2,215)

215 FORMAT (4X,'WELL ENDS',/)

GO TO 4

216 WRITE (2,217)

217 FORMAT (4X,'SUPERHEATED STEAM')

220 CCNTINUE

STOP

END

Sample of a printout
of the calculation from H.P. 9835

THERMODYNAMIC CONDITIONS ALONG A GEOTHERMAL WELL

INPUT DATA:

WELL FLOW RATE= 60 KG/S
 STATIC PRESSURE AT THE BOTTOM= 15 MPA
 TEMPERATURE AT THE BOTTOM= 295 GRAD C
 WELL DEPTH= 1500 M
 DIAMETER OF THE WELL= 0.178 M
 DRAWDOWN PRESSURE FACTOR K= 0 PA PER KG/S
 BRINE CONCENTRATION= 0 PERCENT
 STEP OF CALCULATION= 10 M

DEPTH M	PRESSURE MPA	QUALITY -	VOID FRACTION -	FLOW PATTERNS
1500.0	15.00	0	0	LIQUID
785.7	7.99	0	0	FLASHING POINT
555.7	6.61	0.0430	0.4649	BUBBLY-SLUG TRANSITION
25.7	3.47	0.1446	0.8523	SLUG-FROTH TRANSITION
15.7	3.46	0.1451	0.8560	FROTH-ANNULAR TRANSIT

CONDITIONS AT WELLHEAD

T= 241.877 C P= 3.337 MPA X= 0.146 DEL PH/DEL PF= 0.123
 DEL PA TOTAL= 44889.285 PA R= 35.543 W2= 23.779 M/S K= 0.908
 W1= 17.765 M/S VOID FRACTION= 0.857 R8= 1.105
 BRINE FLOW = 51.266 KG/S STEAM FLOW = 8.734 KG/S

THERMODYNAMIC CONDITIONS ALONG A GEOTHERMAL WELL

INPUT DATA:

WELL FLOW RATE= 60 KG/S
 STATIC PRESSURE AT THE BOTTOM= 15 MPA
 TEMPERATURE AT THE BOTTOM= 295 GRAD C
 WELL DEPTH= 1520 M
 DIAMETER OF THE WELL= 0.178 M
 DRAWDOWN PRESSURE FACTOR K= 0 PA PER KG/S
 BRINE CONCENTRATION= 0 PERCENT
 STEP OF CALCULATION= 10 M

DEPTH M	PRESSURE MPA	QUALITY -	VOID FRACTION -	FLOW PATTERNS
1520.0	15.00	0	0	LIQUID
805.7	7.99	0	0	FLASHING POINT
575.7	6.61	0.0430	0.4649	BUBBLY-SLUG TRANSITION
45.7	3.47	0.1446	0.8523	SLUG-FROTH TRANSITION
35.7	3.46	0.1451	0.8560	FROTH-ANNULAR TRANSIT

CONDITIONS AT WELLHEAD

T= 237.717 C P= 3.085 MPA X= 0.154 DEL PH/DEL PF= 0.109
 DEL PA TOTAL= 47580.776 PA R= 39.373 W2= 25.967 M/S K= 0.899
 W1= 18.625 M/S VOID FRACTION= 0.865 R8= 1.105
 BRINE FLOW = 50.743 KG/S STEAM FLOW = 9.257 KG/S