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ABSTRACT

Clearing of electrons in the vicinity of the axis of a proton beam in the dipoles of a separated function AG-focusing structure is accomplished by transverse crossed-field drift due to the magnetic dipole field and a weak longitudinal electric field. This electric field is generated by potential differences caused by cross section variations of the beam due to variations of the β -functions and dispersion. The resulting radial drift brings the electrons into the off-axis region where they are driven out of the magnets by the usual longitudinal cross-field drift caused by the radial space charge field. The neutralization in ISABELLE is then a factor 20 below former estimates, and transverse stability can be obtained by chromaticity adjustments well within the design strengths of the sextupole correction windings.

INTRODUCTION

It has been pointed out recently that neutralization of the proton beam due to trapped electrons can lead to a coherent transverse instability ("brick wall-effect") in ISABELLE.¹ The planned strength of sextupole and/or octupole correction coils then would be insufficient to counteract the detuning.² These calculations are based on an estimate of neutralization³ which establishes that a crossed-field drift, due to radial space charge fields, is the major contribution to clearing in the dipole magnets. Since the electric fields vanish at the beam axis, a small region around this axis remains uncleared and constitutes the main contribution to the expected average neutralization of slightly above 10^{-4} at high energy.

If the variations of the transverse beam dimensions due to changes of the β -functions combined with momentum dispersion are taken into account, we find that a longitudinal component of electric field of several V/m also exists on the beam axis. This leads to a radial drift of electrons across the otherwise uncleared region and the average neutralization is reduced to below 10^{-5} . The remaining detuning of the working line is then sufficiently small and can be corrected by sextupole/octupole coils with less strength than foreseen and coherent transverse oscillation can be stabilized.

The longitudinal electric field on the beam axis will be modulated by the sagitta of the beam in the long dipoles. However, by properly centering the beams such that the excursions from the magnet-axis are minimized (± 5 mm for 5 m magnets) - which is also required for minimizing the available aperture - the effect becomes sufficiently small to be of no further consequence.

BEAM POTENTIAL AND LONGITUDINAL ELECTRIC FIELD

The potential of a uniform, cylindrical beam of charged particles (current I_0) of elliptic cross section (half width w , half height h) situated inside a circular cylindrical vacuum chamber (radius b) is given in Ref. 4. The rather complicated expression becomes quite simple for the center of a centrally located beam, where it can be written

$$\Phi_c = \frac{Z_0 I_0}{2\pi} \left[\ln \frac{2b}{w+h} + \frac{1}{2} \right] \quad (1)$$

$Z_0 = 120 \pi (\Omega)$ is the free space impedance. If the beam is displaced from the chamber center (distance d), we find

$$\Phi_d = \frac{Z_0 I_0}{2\pi} \left[\ln \frac{d(D+E)}{b(w+h)} + D \frac{D-E}{w^2-h^2} \right] \quad (2)$$

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1. G. Parzen, BNL Report, BNL 50842, ISA 78-5, (1978).
 2. M. Month, G. Parzen, BNL Report, BNL 50843, ISA 78-6, (1978).
 3. J.C. Herrera, BNL Report, BNL 50533, ISA 76-12, (1976).
 4. E. Regenstreif, CERN Report, CERN/PS/DL 77-37, (1977).

where
$$D = \frac{b^2}{d} - d \quad \left. \vphantom{\frac{b^2}{d}} \right\} \quad (3)$$

and
$$E^2 = D^2 - w^2 + h^2$$

Similar expressions can be found for the potentials at the beam edges, which are always less than the potential at the beam axis. To first order, the reduction of beam potential at the edge $x=w$ is given by

$$\Phi_c - \Phi_e = \frac{Z_o I_o}{2\pi} \frac{w}{w+h} \quad (4)$$

If we neglect the fact that the beams in an AG-focusing machine are not quite cylindrical, we can evaluate the potentials at various positions along a Half-Cell of ISABELLE (see Table I).

TABLE I. Potential in Beam Center (centered beam) For
 $e_H + e_V = 15\pi \cdot 10^{-6}$, $\Delta p/p = 2.6 \times 10^{-4}$, $\gamma = 426$,
 $b = 44\text{mm}$, $I_o = 8\text{A}$

Position	$\beta_V(\text{m})$	β_H	$X_p(\text{m})$	$h(\text{mm})$	$w(\text{mm})$	$\Phi_c(\text{V})$	$\Phi_{\text{edge}}(\text{V})$
QF (center)	11.5	66.7	2.73	0.318	1.476	2108.5	1713.7
BF {	entrance	13.2	59.6	2.58	0.341	2124.4	1738.7
	exit	21.1	40.7	2.16	0.431	2166.2	1816.2
B {	entrance	23.6	37.0	2.07	0.456	2174.2	1834.3
	exit	36.9	23.6	1.73	0.570	2202.4	1909.2
BD {	entrance	40.6	21.2	1.66	0.598	2207.0	1922.1
	exit	59.4	13.2	1.39	0.723	2218.8	1994.7
QD (center)	66.5	11.5	1.32	0.765	0.661	2218.6	2000.1

Dividing the potential differences by the distances, we obtain an average longitudinal electric field on axis of about 6 V/m (2.6, 6.1 and 9.1 V/m in the three dipoles). The longitudinal electric fields are somewhat larger at the edges of the beam, but we will neglect this effect for simplicity.

Due to the sagitta of the beam in the long, straight dipoles, the center of the beam will be displaced less than the maximum value of

$$d = \frac{L}{\varphi} \left(1 - \cos \frac{\varphi}{2} \right) \approx \frac{\varphi L}{8} \quad (5)$$

where L is the length and φ the bending angle. Reduction of the potential for a small displacement (and $w^2 - h^2 \ll b^2$) can be found from Eqs. (1) and (2) and is approximated by

$$\Delta\Phi = \Phi_c - \Phi_d \approx - \frac{Z_o I_o}{2\pi} \frac{d^2}{b^2} \quad (6)$$

For ISABELLE dipoles with a length of 4.6 m and a bending angle of 17.2 mrad, the sagitta is almost 10 mm, corresponding to a potential reduction of about 25 V. Such a large reduction would cause a reversal of the

longitudinal electric field. By centering the beam such that its maximum displacement is ± 5 mm, the potential reduction is reduced to about 6 V and can be neglected (see Figure 1).

CROSSED FIELD DRIFT

Due to the combined action of the longitudinal electric and the vertical magnetic field, the electrons will drift radially with a velocity given by

$$v_x = \frac{E_z}{B} = \frac{Z_o I_o}{2\pi B L} S \quad (7)$$

where

$$S = \ln \frac{w_1 + h_1}{w_2 + h_2} \quad (8)$$

and subscript 1 refers to the entrance, subscript 2 to the exit of a magnet of length L and strength B . We neglect the initial velocity of electrons and the increase of longitudinal electric field from the axis, and integrate Eq. (7). We obtain

$$X = X_o + S \frac{F}{L} t \quad (9)$$

where

$$F = \frac{Z_o I_o}{2\pi B} \quad (10)$$

is written for short, and X_o is the initial radial position of an electron.

Once the electrons have drifted away from the axis they see the much stronger radial electric field due to space charge given by

$$E_x = \frac{Z_o I_o}{2\pi w^2} X \quad (11)$$

where w is the average half width of the beam. The longitudinal drift is then

$$v_z = -\frac{E_x}{B} = -\frac{F}{w^2} \left(X_o + \frac{SF}{L} t \right), \quad (12)$$

and a further integration with respect to time yields

$$Z = Z_o - \frac{F}{w^2} \left(X_o t + \frac{SF}{2L} t^2 \right) \quad (13)$$

For $S > 0$ (positive longitudinal electric field), electrons with $X_o > 0$ will travel to the left (decreasing Z), while electrons with $X_o < 0$ will at least initially travel to the right (see Figure 2). If X_o is large enough in magnitude and negative ($X_o < -w/\sqrt{2}$) they will reach the magnet edge $Z = L$ and go to the nearest clearing

Figure 1. Potential at center of beam in Isabelle lattice cell.

Figure 2. Beam cross-section, equi potentials and electron trajectories in dipole B (sagitta neglected)..

electrode. If X_0 is less negative, they will get to the upper half of the magnet, reverse their direction and leave at $Z = 0$. For $S < 0$, the whole picture is simply inverted.

We can consider Eq. (13) a quadratic for the traversal time T

$$T^2 + \frac{2LX_0}{SF} T + \frac{2Lw^2}{SF^2} (Z-Z_0) = 0 \quad (14)$$

with the solutions

$$T = -\frac{LX_0}{SF} + \sqrt{\left(\frac{LX_0}{SF}\right)^2 - \frac{2Lw^2}{SF^2} (Z-Z_0)} \quad (15)$$

Assuming $S > 0$, we have to take $Z = 0$ for $X_0 > 0$. The longest traversal time is for particles with $Z_0 = L$ and $X_0 = 0$ and becomes

$$T_1 = \frac{WL}{F} \sqrt{\frac{2}{S}} \quad (16)$$

For $X_0 < -w\sqrt{2S}$, we have to use $Z = L$. The longest traversal time now is for particles with $Z_0 = 0$, $X_0 = -w\sqrt{2S}$ and is again T_1 . For particles with $0 < X_0 < -w\sqrt{2S}$, we have to use $Z = 0$, and the maximum traversal time is found for $Z_0 = 0$, $X_0 = -w\sqrt{2S}$ and is

$$T_{\max} = 2T_1 = \frac{2WL}{F} \sqrt{\frac{2}{S}} \quad (17)$$

For $S < 0$, we can use the same expression with S replaced by $|S|$ in the square root.

AVERAGE NEUTRALIZATION

For equilibrium conditions, the number of electrons produced equals the number of electrons cleared

$$R_p N_p = R_c N_e \quad (18)$$

where R_p is the electron production rate

R_c is the electron clearing rate

N_p the total number of protons

N_e the total number of electrons.

The average neutralization is

$$\eta = \frac{N_e}{N_p} = \frac{R_p}{R_c} \quad (19)$$

The production rate has been calculated in Ref. 3 and is approximately 1.2×10^{-2} for typical ISABELLE parameters. The clearing rate is the inverse of the traversal time, for which we take (on the conservative side) the maximum value given by Eq. (17). Substituting for all abbreviations,

we obtain

$$\eta = \left(\frac{4\pi BL}{Z_0 I_0} R_p \right) w \sqrt{\frac{2}{\ln \frac{w_1+h_1}{w_2+h_2}}} \quad (20)$$

For $B = 5T$, $L = 5m$, $I_0 = 8A$ the factor in parenthesis is about 1.25×10^{-3} . The actual values of the neutralization in the three lattice magnets of ISABELLE can be found in Table II,

TABLE II. Average Neutralization In Dipoles
 $B = 5T$, $L = 5m$, $I_0 = 8A$, $R_p = 1.2 \times 10^{-2}$

Magnet	$S = \ln \frac{w_1+h_1}{w_2+h_2}$	$\langle w \rangle$ (mm)	η
BF	0.087	1.28	7.6×10^{-6}
B	0.059	1.01	7.6×10^{-6}
BD	0.025	0.78	8.6×10^{-6}

and are 8×10^{-6} on average. Because we have neglected the reduction of fields due to the sagitta of the beam, we shall use $\eta = 10^{-5}$ as our estimate of the neutralization in the ISABELLE dipole magnet of high field.

This value is 20 times smaller than the one previously used for estimating the sextupole and/or octupole fields required to correct the detuning of the working line. While the octupoles remain still marginal, the sextupoles should be more than sufficient for this purpose, provided the other regions in the machine can be cleared equally well. The neutralization in the long straight section in the insertions has been estimated to be about 3×10^{-6} , but this puts more stringent limitations on the cross section variations of the vacuum chamber wall,⁵ and may necessitate an increase in the number of clearing electrodes in the field free regions of ISABELLE.

TRANSVERSE IMPEDANCE

The transverse dispersion relation coefficients U and V are simply related to the tune shifts

$$U + (1-j)V = \Omega_0 (\Delta\nu_{inc} - \Delta\nu_{coh}) \quad (21)$$

The tune shifts have been calculated for a wide variety of geometries, but generally under the assumption of perfectly conducting walls. They are usually expressed by the image coefficients ξ and ϵ in the form

$$\Delta\nu_{coh} = - \frac{N\nu_0 R}{\pi\beta^2 \gamma\nu} \left[\frac{\epsilon_1}{b^2} + \frac{\xi_1}{\gamma^2 b^2} + \beta^2 \frac{\epsilon_2}{g^2} \right] \quad (22)$$

$$\Delta\nu_{inc} = - \frac{N\nu_0 R}{\pi\beta^2 \gamma\nu} \left[\frac{\epsilon_1}{b^2} + \beta^2 \frac{\epsilon_2}{g^2} + \frac{\epsilon_{sc}}{\gamma^2 h^2} \right] \quad (23)$$

5. J.C. Herrera, Proceedings 1976 ISABELLE Summer Workshops, Brookhaven, BNL 50611, 236.

where $\epsilon_{sc} = h/h+w$ for elliptic beams,
 ϵ_1 is the electrostatic,
 ϵ_2 is the magnetic,
 ξ_1 is the coherent image coefficient,
 b is the chamber radius,
 g is the pole-piece distance,
 R is the machine radius,
 r_0 is the classical proton radius, and
 N is the total number of protons.

In analogy to the longitudinal coupling impedance, it has become customary to define a transverse coupling impedance (per unit displacement, hence Ω/m), related to U and V by

$$Z_{\perp} = j Z_0 \frac{2\pi R \gamma \gamma}{N \gamma_0 \beta c} \left[U + (1-j)V \right] \quad (24)$$

or, using Eqs. (20) - (22), we get

$$Z_{\perp} = j Z_0 \frac{2R}{\beta^2 \gamma^2} \left[\frac{\xi_1 - \epsilon_1}{b^2} - \frac{\epsilon_{sc}}{h^2} \right] \quad (25)$$

If we include the effect of neutralization, the coherent tune shift remains unchanged, while the electrostatic terms in the incoherent are reduced by η

$$\Delta \nu_{inc} = - \frac{N \gamma_0 R}{\pi \beta^2 \gamma \gamma} \left[(1-\eta) \frac{\epsilon_1}{b^2} + \beta^2 \frac{\epsilon_2}{g^2} + \left(\frac{1}{\gamma^2} - \eta \right) \frac{\epsilon_{sc}}{h^2} \right] \quad (26)$$

and

$$Z_{\perp} = Z_{\perp}^{\eta=0} + j Z_0 \frac{2R\eta}{\beta^2} \left(\frac{\epsilon_1}{b^2} + \frac{\epsilon_{sc}}{h^2} \right) \quad (27)$$

For $\eta \gamma^2 \gg 1$ and $b \gg h$, the last term in Eq. (25) dominates and we find with Eq. (23) and $\beta \approx 1$

$$Z_{\perp} \approx j Z_0 \frac{2R\eta}{h(w+h)} \quad (28)$$

This value of $Z_{\perp}$ increases with energy because h and w become smaller, and is directly proportional to the neutralization. For ISABELLE parameters ($R = 612$ m, $\langle h \rangle = 0.54$ mm, $\langle w \rangle = 1.1$ mm) we find

$$Z_{\perp} = 5.3j \text{ M}\Omega/\text{m} \text{ (for } \eta = 10^{-5}\text{)}$$

while it was over $100 j \text{ M}\Omega/\text{m}$ for $\eta = 2 \times 10^{-4}$ used in Ref. 2.

STABILITY CRITERION

Rather than using the exact criterion which requires $|Z_{\perp}| < \bar{F} Z_{\perp \max}$, where the form factor $\bar{F}$ depends on the distribution functions in momentum and/or amplitude as well as on the ratio of real to imaginary parts of $Z_{\perp}$, we use the simplified criterion that the frequency shift $\Delta \omega$

be less than the half-spread $\delta\omega$. The real frequency shift is equal to $(U+V)$, hence from Eq. (24) we find

$$\Delta\omega = \frac{c I_0}{4\pi E_0/e} \frac{\text{Im}Z_{\perp}}{\gamma\gamma} \quad (29)$$

while the frequency half-spread is

$$\delta\omega = |\chi + (n-\gamma)\eta_p| \Omega_0 \left(\frac{\Delta p}{p} \right)^{\frac{1}{2}} \quad (30)$$

where

$$\chi = \frac{d\gamma}{dp/p} \text{ is the chromaticity,}$$

and

$$\eta_p = \frac{-d\Omega/\Omega}{dp/p} \text{ the "frequency slip factor".}$$

η_p is positive above transition energy, hence, we want $\chi > 0$ there in order to avoid cancellation of the two terms inside the absolute sign for the unstable modes $n > \gamma$. We can then neglect the second term, which is usually quite small for the lowest mode numbers anyhow, and find the stability criterion

$$\text{Im}Z_{\perp} < 4\pi \frac{E_0 \gamma \beta \gamma}{e R I_0} \left(\frac{\Delta p}{p} \right)^{\frac{1}{2}} \chi \quad (31)$$

The RHS is essentially energy independent as $p/\beta\gamma = m_0 c$ is an invariant. For ISABELLE parameters ($\gamma = 22.6$, $\beta\gamma = 426$, $R = 612$ m, $I_0 = 8$ A, $\Delta p/p = 2.6 \times 10^{-4}$) we obtain

$$\chi > \frac{\text{Im}Z_{\perp}}{6.10^6} = 0.9$$

For the former value of $\eta = 2 \times 10^{-4}$ which led to $Z_{\perp} > 100 M\Omega/m$, we needed $\chi > 17$ which was beyond the capabilities of the correction sextupoles. The present value is comfortably small, however, and correction well within the design strength of the sextupoles.

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