**Ripple Transport and The Depletion of Energetic
Particles in Tokamaks**

by

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ABSTRACT

We use a local velocity-diffusion model to estimate the ripple transport-induced deviations from a Maxwellian distribution and the effects of such deviations on confinement time and reactivity.

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The principal contribution to ripple transport in tokamaks comes from suprathermal trapped particles, and it is accordingly these particles that are preferentially depleted when ripple transport is the dominant loss mechanism.¹ In such cases, we can model the particle and energy losses crudely as follows. Define a velocity-space loss region $|\lambda| < \lambda_c$ for $E > E_c$, where $\lambda = v_{\parallel}/v$ is the pitch angle, λ_c is the critical angle for trapping, and E is the energy in thermal units. Particles and energy may be lost from the system, either by ripple transport [on a confinement time scale $\tau_R(E_c)$], or by diffusion into the loss region [on a time scale $\tau_L(E_c)$] at which point the particles are assumed to leave the system immediately. The overall confinement time is then $\tau(E_c) \sim \{[\tau_R(E_c)]^{-1} + [\tau_L(E_c)]^{-1}\}^{-1}$. For given λ_c , τ_R will typically increase, and τ_L will decrease, with increasing E_c . For simple cases $\tau(E_c)$ will have a single maximum, which will give the confinement time subject to the limitations of this model.

One purpose of this paper is to present results of the calculation of $\tau_L(E_c, \lambda_c)$, and to make contact with earlier ripple-transport results. A second purpose is to calculate the reactivity $\langle \sigma v \rangle$ (for the D-T process) averaged over the distribution functions appropriate to this loss-region analysis. Since σv increases rapidly with relative velocity, one might a priori expect the depletion of the high-energy tail to have an important effect on $\langle \sigma v \rangle$ and hence on the Lawson criterion. We will show, however, that even if $E_c = 4$ (so that all particles with velocities greater than $2v_{th}$ are lost) $\langle \sigma v \rangle$ is only modestly reduced below $\langle \sigma v \rangle_{\text{Maxwellian}}$.

We consider the diffusion process

$$\frac{\partial f}{\partial t} = \frac{v_d}{2} \frac{\partial}{\partial \lambda} \left[(1 - \lambda^2) \frac{\partial f}{\partial \lambda} \right] + 2E^{-1/2} \frac{\partial}{\partial E} \left[E^{3/2} v_E \left(f + \frac{\partial f}{\partial E} \right) \right], \quad (1)$$

where

$$v_d(E) = \frac{3}{2} \left(\frac{\pi}{2}\right)^{1/2} v_B E^{-3/2} [\operatorname{erf}(E^{1/2}) - E(E^{1/2})] ,$$

$$v_E(E) = 3 \left(\frac{\pi}{2}\right)^{1/2} v_B E^{-1/2} E(E^{1/2}) ,$$

$$E(E^{1/2}) = \frac{\operatorname{erf}(E^{1/2})}{2E} - (\pi E)^{-1/2} \exp(-E) , \quad (2)$$

and v_B is the Braginskii collisionality. We introduce an isotropic source at low energy (e.g., $E = 1$) and compute f by a Monte Carlo process. After each time step Δt , the pitch λ' and energy E' are given in terms of the previous (λ, E) by

$$\begin{aligned} \lambda' &= \lambda(1 - v_d \Delta t) \pm [(1 - \lambda^2) v_d \Delta t]^{1/2} , \\ E' &= E - 2v_E \Delta t \left(E - \frac{3}{2} - \frac{E}{v_E} \frac{dv_E}{dE} \right) \pm 2(E v_E \Delta t)^{1/2} , \end{aligned} \quad (3)$$

where we have used the prescription given by Boozer and Kuo-Petravic.² (To ensure that $v_d \Delta t \ll 1$ for small E , where v_d diverges, we set $v_d(E) = v_d(E_0)$ for $E < E_0$; the results are insensitive to the choice of E_0 for $E_0 \leq 0.5$.)

The particle and energy confinement times τ_{LP} and τ_{LE} (in units of v_B^{-1}) are plotted versus λ_c in Fig. 1 for $E_c = 4, 5$, and 6. Confinement times for the case $\lambda_c = 0$ (here the loss cone becomes a "loss needle") are 4.7 to 6 times as long as those for the $\lambda_c = 1$ case, in which the distribution function is independent of λ . However, for local aspect

ratios over a wide range, e.g., $\lambda \lesssim 8$, the λ_c for trapping is essentially $(2/\lambda)^{1/2} \gtrsim 0.5$. For such λ_c the confinement times are given to within about 40% by their $\lambda_c = 1$ values, which can be calculated explicitly to be

$$v_B \tau_{LP} = \frac{1}{6} \left(\frac{2}{\pi}\right)^{1/2} \int_0^{\infty} d\eta \, \eta^{1/2} \exp(-\eta) \int_{\eta}^{\infty} d\eta' \, e^{-\eta'}$$

$$\left[\frac{1}{2} \operatorname{erf}(\eta^{1/2}) - \left(\frac{\eta}{\pi}\right)^{1/2} \exp(-\eta) \right]^{-1},$$

$$\tau_{LE} = \frac{3}{2} \frac{\tau_{LP}}{E_c}. \quad (4)$$

If one approximates $v_{d,E}$ by $1.5(\pi/2)^{1/2} v_B E^{-3/2}$, one obtains simply $v_B \tau_{LP} \approx 0.266 \exp(E_c)$, which is accurate to within 10 - 15%.

The inverse energy confinement time $\tau_{RE}^{-1}(E_c)$ due to ripple transport alone (i.e., in the absence of the loss-region diffusion mechanism treated earlier in this paper) can be estimated as cK/na^2 where K is the thermal conductivity, a is the minor radius, n is the density, and c is a number of order unity. For definiteness, we will consider the banana drift diffusion regime in which $K \propto \nu^{-1}$, and we shall use the expression for K that is consistent with ambipolarity (see Linaker and Boozer¹). Then we have [Eq. (26) of Ref. 1]

$$\tau_{RE}^{-1}(E_c) = \alpha v_B \left[I_{10}(E_c) - \frac{I_8^2(E_c)}{I_6(E_c)} \right], \quad (5)$$

where

$$\alpha \equiv 0.204 c K \langle \delta^2 \rangle_I \left(\frac{\rho_0}{a} \right)^2 \frac{v_T}{R} \frac{v_*}{(Nq)^2 v_B^2}, \quad (6)$$

$$I_k \equiv \int_0^{\infty} dx \frac{x^{k+3} \exp(-x^2)}{\operatorname{erf}(x) - \Psi(x)},$$

ρ_θ is the poloidal gyroradius, $\langle \delta^2 \rangle_I$ is a measure of the ripple amplitude (see Ref. 1), N is the number of toroidal field coils, and $v_* = c^{3/2} v_T / qR$. In a reactor, α could be typically 10^{-3} .

Figure 2 shows the maximum in τ_E resulting from the combined effects of the ripple transport and loss-region diffusion mechanisms. Here we take

$$v_B \tau_E(E_c) \approx \left[\alpha \left(I_{10} - \frac{I_8^2}{I_6} \right) + \frac{\exp(-E_c)}{0.266} \right]^{-1}. \quad (7)$$

We find that for $\alpha = 10^{-3}$, $v_B \tau_E$ has a maximum value of 40 when $E_c \approx 6$. If the loss-region mechanism had been ignored, one would have found $v_B \tau_{RE}(E_c = 6) \approx 62$. Note that assuming an exact Maxwellian distribution ($E_c = \infty$) would yield a substantially more pessimistic estimate of confinement, $v_B \tau_E(\infty) \approx 15$.

We calculate finally the effect of a loss region $E > E_c$ on the reactivity of a D-T plasma. (Since the effect is found to be modest, it suffices to treat the case $\lambda_c = 1$.) Assume that both species are at the same temperature T , and for simplicity assume that the cutoff energy $E_c = T y_c$ is the same for both species. Then (defining $y \equiv E/T$)

$$\left(\frac{m_D}{2T} \right)^{1/2} \langle \sigma v \rangle = \frac{1}{2} \left[\int_0^{y_c} dy_D \int_0^{y_c} dy_T \int_{-1}^1 d\lambda \sigma(T y_{\text{rel}}) y_{\text{rel}}^{1/2} f(y_D) f(y_T) (y_D y_T)^{1/2} \right] \left[\int_0^{y_c} f(y) y^{1/2} dy \right]^{-2}, \quad (8)$$

where

$$y_{rel} = y_D + Sy_T + 2(Sy_D y_T)^{1/2} \lambda ,$$

$$S = \frac{n_D}{n_T} \approx \frac{2}{3} .$$

We obtain σ from Miley et al.,³ and take

$$f(y) = \exp(-y) - \exp(-y_c) ,$$

which is the steady-state solution to the diffusion Eq. (1) when v_E is approximated as $\propto y^{-3/2}$. Figure 3 shows $\langle \sigma v \rangle$ versus y_c for $T = 10$ and 20 keV. We note that as long as $y_c > 5$, $\langle \sigma v \rangle$ is within 35% of the value it takes when f is exactly Maxwellian.

To summarize: Ripple transport in tokamaks is strongly affected by even slight deviations from the Maxwellian in the high-energy tail. We provide a self-consistent way of estimating the behavior of this tail and the resulting confinement, assuming that ripple transport of trapped particles at high energies ($E > E_c$) and velocity diffusion at lower energies are the dominant loss mechanisms. We find that the loss rate due to velocity diffusion is approximately $v_E \exp(-E_c)/0.266$ for typical local aspect ratios of practical interest. We show how E_c is to be estimated for the purpose of calculating ripple transport coefficients (the $1/v$ banana drift regime, where the dependence on E_c can lead to an order-of-magnitude change in thermal conductivity,¹ is used as an example). We finally show that even for fairly low E_c (poor confinement) the Maxwellian-averaged $\langle \sigma v \rangle$ gives a reasonable estimate of the actual reactivity for a D-T system.

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FIGURE CAPTIONS

Fig. 1. Logarithm of the particle and energy confinement times $v_B \tau_{Ip,E}$ (due to velocity-space diffusion alone) versus λ_c , for $E_c = 4, 5$, and 6 times thermal energy. The loss region is $|\lambda| < \lambda_c$ for $E > E_c$. Solid lines refer to particle confinement; broken lines refer to energy confinement.

Fig. 2. Logarithm of the overall energy confinement time $v_B \tau_E$ (including velocity-space diffusion and ripple transport) versus E_c ; the ripple-transport parameter α is defined by Eq. (6).

Fig. 3. Reactivity $\langle \alpha v \rangle$ versus $y_c = E_c/T$, for the case $\lambda_c = 1$.

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Fig. 1

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Fig. 2

Fig. 3