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TITLE: CONSTRAINTS OF OBSERVATIONS OF SOLAR OSCILLATIONS ON SOLAR
MODELS WITH MIXING BY TURBULENT DIFFUSION

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CONSTRAINTS OF OBSERVATIONS OF SOLAR OSCILLATIONS ON
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ABSTRACT

The pulsational behavior of solar models with mixing produced by turbulent diffusion has been investigated, for both p (pressure) modes and g (gravity) modes. It is found that while a small amount of mixing may be allowed by the observed data on solar oscillations, values of the pseudo-Reynolds number large enough to reduce the predicted neutrino counting rate to the level seen by the ^{37}Cl experiment are in conflict with published solar oscillation data.

INTRODUCTION

Schatzman and Maeder¹ have investigated solar models with mixing induced by turbulent diffusion with diffusion coefficient

$$D_T = R_e^* \nu \quad , \quad (1)$$

where ν is the microscopic molecular viscosity and R_e^* is a pseudo-Reynolds number. They found, as shown in Table I, that increasing the value of the pseudo-Reynolds number increased the amount of mixing induced and consequently increased the amount of hydrogen remaining at the center of the sun in the present epoch, decreased the central temperature, and reduced the counting rate predicted for the ^{37}Cl experiment, as is typical for solar models with significant mixing of the central regions. Models with pseudo-Reynolds number as large as about 100 or greater produce counting rates for the ^{37}Cl experiment in agreement with the observations.

Table I
Central conditions for solar models with
mixing from Reference 1

R_e^*	X_c	$T_c, 10^6 \text{K}$	N_ν, SNU
0	0.359	15.65	11.66
100	0.569	14.73	2.38
200	0.616	14.46	1.43

P MODE OBSERVATIONS

We have constructed solar models of the Schatzman and Maeder type by matching the composition profiles given in Fig. 2 of Reference 1, for $R_e^* = 0, 100, \text{ and } 200$, and have performed pulsational stability analyses for such models. In Fig. 1 we display the difference in μHz between the theoretical frequency calculated from our models and the measured² frequency for p modes of degree $\ell = 1$ through 5 as a function of order n . A satisfactory solar model would produce a horizontal line at zero deviation of theory from observation on this plot. We see from Fig. 1 that although there is some drift from good agreement with experiment for the standard unmixed model $R_e^* = 0$, the discrepancy between theory and observation becomes much worse as the pseudo-Reynolds number R_e^* is increased.

Figure 1. Deviation of theoretical frequency from measured frequency for low degree p modes.

Simple pulsation theory predicts that the frequency splitting

$$\Delta\nu(\ell, n) = \nu_{\ell, n} - \nu_{\ell+2, n-1} \quad (2)$$

for low degree p modes differing by 1 in n and by 2 in ℓ should be approximately zero. Thus the deviation of this quantity from zero is a sensitive probe of the detailed structure of a solar model. Fig. 2 shows the frequency splitting for the p modes of Fig. 1. For $\ell = 1$ and $\ell = 3$ modes in Fig. 2a, even the standard model shows too much splitting at higher orders, and mixed models lie much higher than the observations in this plot. For $\ell = 2$ and $\ell = 4$ frequency splitting in Fig. 2b the standard model $R_e^* = 0$ is much closer to the observed splitting, and for $\ell = 3$ and $\ell = 5$ splitting in Fig. 2c it begins to appear that small intermediate values of pseudo-Reynolds number may offer a better fit to the observations than the standard model, although values of R_e^* as large as 100 still predict much too much frequency splitting for these modes.

Figure 2a. p mode frequency splitting from $\ell = 1$ to $\ell = 3$.

Figure 2b. p mode frequency splitting from $l = 2$ to $l = 4$.

Figure 2c. p mode frequency splitting from $l = 3$ to $l = 5$.

Thus the p mode observations of solar oscillations do not seem to allow sufficient mixing by the Schatzman and Maeder mechanism to produce neutrino counting rates compatible with the ^{37}Cl experiment. However, it is well known that p mode oscillations are more sensitive to conditions nearer the solar surface, and that g mode oscillations are more sensitive to conditions deep in the solar interior, as shown schematically in Fig. 3.

Figure 3. Schematic amplitude of oscillation for $\ell = 3$, $n = 17$ p and g modes. The center horizontal line indicates zero amplitude of these modes. Deep amplitudes for the p modes are typically 0.001 of the surface amplitude.

Here the amplitude of a typical p mode and a typical g mode are shown plotted on an arbitrary scale as a function of zone number J in our standard model, with the mass fraction interior to zone J for comparison. On this scale the p modes show most of their structure exterior to most of the mass of the sun, and the g modes show most of their structure interior to most of the mass of the sun.

G MODE OBSERVATIONS

The difficulty in using gravity modes to diagnose the solar interior is that pressure mode observations have been on a much sounder observational footing for many years. Consequently most analyses of constraints of solar oscillation observations on models of solar structure have been in terms of p modes. Recently, however, published accounts of g mode observations have begun to appear in the astrophysical literature. For example, Delache and Scherrer³ have made identifications of oscillations with periods from 171.9 minutes to 328.6 minutes with g modes of $\ell = 1$ and $\ell = 2$ and n ranging from 6 to 20. Such modes are conveniently analyzed in terms of the asymptotic formula

$$P = P_0 \frac{(n + \ell/2 - 1/4)}{\sqrt{\ell(\ell + 1)}} \quad (3)$$

Thus if $P\sqrt{\ell(\ell + 1)}$ is plotted as a function of $n + \ell/2 - 1/4$, a straight line of slope P_0 should result. A plot of this type for the Delache and Scherrer identifications is shown in Fig. 4.

Figure 4. g mode periods from Reference 3.

The best fit to the data shown in Fig. 4 gives a period spacing $P_0 = 38.6$ minutes. However, Gabriel⁴ has reinterpreted the Delache and Scherrer data to find $P_0 = 35.98$ minutes or 35.64 minutes, depending on the particular mode assignments chosen, in good agreement with the results for his standard solar model $P_0 = 35.52$ minutes. The ACRIM experiment on the Solar Maximum satellite produced data which has been interpreted to give $P_0 = 38.9$ minutes. Our standard solar model with $R_e^* = 0$ gives $P_0 = 37$ minutes.

Berthomieu, Provost, and Schatzman⁵ have calculated g mode period spacings for solar models with mixing induced by turbulent diffusion, and display P_0 as a function of pseudo-Reynolds number R_e^* . They find that P_0 increases from 33.9 minutes for their $R_e^* = 0$ model to 54.6 minutes at $R_e^* = 100$ and 62.9 minutes at $R_e^* = 200$.

Table II
Period spacing (P_0) for g modes; Theory vs Observation

Observed (min.)				
Gabriel	Severny	Delache and Scherrer	ACRIM	van der Raay ⁷ et al.
36.0	37.4	38.6 ± 0.5	38.9	41.2
Theory (min.)				
	Berthomieu et al.	Gough ⁸	Gabriel	This work
$R_e^* = 0$	33.9	34.5	35.5	37
$R_e^* = 100$	54.6	-	-	58
$R_e^* = 200$	62.9	-	-	-

DISCUSSION

The observational and theoretical situation as regards g modes is summarized in Table II. Berthomieu et al.⁵ have argued that since their $R_e^* = 0$ model produces a period spacing smaller than the observations, small intermediate values of R_e^* are indicated. However, the observed period spacing seem to lie in the range of about 36 to 39 minutes, and the theoretical period spacing for $R_e^* = 0$ lies in the range of about 34 to 37 minutes. The two ranges overlap, and there seems to be no serious disagreement between the standard solar models without mixing and the g mode observations reported in the literature. On the other hand, values of the pseudo-Reynolds number large enough to significantly reduce the counting rate predicted for the ^{37}Cl solar neutrino experiment are not compatible with the period spacings reported for g modes.

The interpretation of g mode observations (their ℓ values) is controversial at present, and indeed the very existence of gravity modes in the solar oscillation spectrum is still under question in some circles. Thus the g mode observations discussed above may not provide a strong constraint for solar models with mixing produced by turbulent diffusion. Nonetheless, it is perhaps significant that both the well established p mode oscillations of the sun and the g mode oscillations which have been reported support the same conclusion. Although small values of R_e^* cannot be ruled out by the data, mixing produced by turbulent diffusion in amounts large enough to produce neutrino counting rates consistent with the ^{37}Cl experiment does not seem to be allowed by observations of solar oscillations.

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Fig. 1 QMC results for CHAB at fields $h_z = 0.0(o)$, $3.3(\Delta)$, $6.5(+)$, $10.0(x)$ kG. The data points in (a) were found from expectation values of the variable $\frac{\partial}{\partial \beta}(\beta E^{(m)}(l,r))$, while those in (b) were the fluctuations in this variable. The curves in (a) are weighted least squares polynomial fits to the data points, while the curves in (b) are derivatives of the fitted curves in (a). In (c) we also show the difference $\Delta C(T)$ -- the specific heat with a field minus the zero field result, obtained directly from (b). Finally (d) shows the susceptibility in the direction of the applied field (in the easy plane); the curves are polynomial fits to the reciprocal susceptibility.