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Simple Computer Model for the Nonlinear*

Beam-Beam Interaction in ISABELLE

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SUMMARY

The beam-beam interaction for two counter-rotating continuous proton beams crossing at an angle can be simulated by a 1-dimensional nonlinear force. The model is applicable to ISABELLE as well as to the ISR. Since the interaction length is short compared with the length of the beam orbit, the interaction region is taken to be a point. The problem is then treated as a mapping with the remainder of the system taken to be a rotation of phase given by the betatron tune of the storage ring. The evolution of the mean square amplitude of a given distribution of particles is shown for different beam-beam strengths. The effect of round-off error with resulting loss of accuracy for particle trajectories is discussed.

I. Introduction

We consider a simple computer model for the nonlinear beam-beam interaction in ISABELLE. The collision geometry, similar to the ISR, consists of two counter-rotating continuous proton beams crossing at a horizontal angle. The difference between ISABELLE and the ISR is essentially the magnitude of the crossing angle, the former being about 10 mrad, with the ISR having a crossing angle of about 25 times this value.

A simple discussion of the beam-beam force and the equation for the motion of a particle about its central orbit around the storage ring is presented in Section 2. Our model is a weak-strong one, meaning that one beam, the "strong" beam, is taken to be fixed while we study the motion of the particles in the weak beam passing through the strong one. The proper self consistent problem is not treated. Because a given particle passes through the strong beam from one side to the other, the net force experienced by a "weak" particle crossing the "strong" beam is only

in the vertical direction. The horizontal force averages to zero. In other words, the beam-beam interaction for beams crossing at a non-zero angle is 1-dimensional.

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Some simulation results are given in Section 3. We show in particular the evolution of the mean square amplitude of a distribution of particles for different beam-beam strengths. We observe that the mean square amplitude tends to grow if the beam-beam strength is sufficiently large. This growth is associated with the presence of round-off error, with resulting loss of accuracy for particle trajectories. We give a brief discussion of this loss of accuracy problem and we consider whether by taking averages over a sufficiently large particle distribution in the phase space we can obtain meaningful results in spite of the errors in the individual particle trajectories.

II. Motion of a Particle Crossing an Unbunched Beam

Throughout the following discussion we shall consider the collisions in the weak-strong case, that is, we shall formulate the problem of a single proton crossing a continuous proton beam in the horizontal plane at an angle (α_0) and then making one pass through a perfectly linear machine before colliding again with the continuous beam. Under these circumstances the particle trajectory will suffer a

net angular deflection only in the vertical plane and this change in slope per intersection is given by

$$\Delta\left(\frac{dy}{ds}\right) = \frac{2\pi r_p I}{ec\gamma\beta^2 \tan\frac{\alpha_0}{2}} \mathfrak{F}\left(\frac{y}{\sqrt{2}\sigma_v}\right) \quad , \quad (1)$$

where $\mathfrak{F}$ is the error function defined by

$$\mathfrak{F}(x) = \frac{2}{\sqrt{\pi}} \int_0^x dt e^{-t^2} \quad . \quad (2)$$

[The significance of each of the symbols in these equations is presented in Paper 1 of this compilation].

In discussing the change in particle coordinates for one revolution through such a system, it is convenient to change variables: from the transverse vertical coordinates $y(s)$ and the slope $y'(s)$ [$\equiv dy/ds$], dependent on the azimuthal distance s , to the normalized coordinates $\eta(\psi)$ and $\dot{\eta}(\psi)$ [$\equiv d\eta/d\psi$], dependent on the betatron phase coordinate ψ . The matrix equation yielding such a transformation is

$$\begin{bmatrix} y(s) \\ y'(s) \end{bmatrix} = \begin{bmatrix} \beta^{-\frac{1}{2}}(s) & 0 \\ \beta^{-\frac{1}{2}}(s)\alpha(s) & \beta^{\frac{1}{2}}(s) \end{bmatrix} \begin{bmatrix} \eta(\psi) \\ \dot{\eta}(\psi) \end{bmatrix} \quad , \quad (3)$$

where $\alpha(s)$ and $\beta(s)$, which appear in the transformation matrix are the conventional Twiss parameters for the linear machine. If we introduce the complex variable, $z = \eta(\psi) + i\dot{\eta}(\psi)$, to describe the proton position and angle at any azimuth (see Fig. 1), we can then write the variable Z_2 after the intersection in terms of its initial value Z_0 and the increment $\Delta\dot{\eta}$ experienced at the collision point. Thus, we have

$$Z_2 = Z_0 e^{-i\psi_1} + i\Delta\eta \quad , \quad (4)$$

or, using Eqs. (1) and (3),

$$Z_2 = Z_0 e^{-i\psi_1} + i \frac{(2\pi)^{3/2} \sigma_V^* |\Delta v_V|}{\beta_V^{*2}} e^{i\left(\frac{\beta_V^{*2} \eta_1}{\sqrt{2}\sigma_V^*}\right)} \quad . \quad (5)$$

Here σ_V^* is the rms size of the strong beam at the intersection point and β_V^* is the beta function at the same point for the linear machine. The quantity $|\Delta v_V|$ is the beam-beam strength parameter, which is written as [see Paper 1]

$$|\Delta v_V| = \frac{\beta_V^{*2} r_p I}{\sqrt{2\pi} e c \gamma \beta_V^{*2} \sigma_V^* \tan \frac{\alpha}{2}} \quad (6)$$

Since we are interested in the excursions of the intersecting particle trajectory on successive revolutions with respect to the fixed strong beam, it is best to normalize the particle phase space variable Z to the square root of the linear invariant W_s of the strong beam. We choose W_s such that the corresponding boundary curve contains 95% of the particles for a beam which has a Gaussian distribution in phase space, that is to say,

$$W_s = \frac{6\sigma_V^{*2}}{\beta_V^*} \quad . \quad (7)$$

Using such a normalization, we can then express Eq. (5) as

$$\hat{Z}_2 = \hat{Z}_0 e^{-i\psi_1} + i4\pi |\Delta v_V| e^{i(\hat{\eta}_1)} \quad . \quad (8)$$

We have transformed the Z variables appearing in Eq. (5) according to

$$Z \rightarrow \hat{Z} = \frac{\eta + i\hat{\eta}}{\sqrt{W_s}} = \hat{\eta} + i\hat{\eta} \quad (9)$$

and have introduced the Θ function

$$\Theta(\hat{\eta}) = \frac{\sqrt{\pi}}{2\sqrt{3}} \exp(-\sqrt{3}\hat{\eta}_1) \quad . \quad (10)$$

If we multiply the phase space variable $\hat{Z}_2$ by its conjugate, $\hat{Z}_2^*$, we obtain the invariant associated with the unperturbed linear system. Thus,

$$W = \hat{Z}_2 \hat{Z}_2^* \quad . \quad (11)$$

We emphasize that this quantity is not an invariant of the perturbed system, but it does equal the square of the amplitude of the motion executed by the particle on successive revolutions. It is the mean value of W , over a distribution of N particles, which is investigated numerically in the next section. In general it can be written at any step, i , in the calculation as

$$\langle W \rangle_i = \frac{1}{N} \sum_{n=1}^N \hat{Z}_i(n) \hat{Z}_i^*(n) \quad . \quad (12)$$

III. Some Simulation Results

The behavior of a distribution of particles in phase space under repeated application of the mapping, given in Eq. (8), was studied numerically. The mean square amplitude of the particles for the distribution, $\langle W \rangle$, defined in (12), was computed initially and at regular intervals during the sequences of iterated mappings.

Using a phase space normalized to the dimensions of the "strong" beam, the initial distribution was chosen to be uniform within the unit circle, corresponding to an actual radius, $\sqrt{6}\sigma_v$. The expectation value of the initial mean square amplitude for this distribution is, $\langle W_0 \rangle = 0.5$.

The actual value of $\langle W_0 \rangle$ differed from 0.5 due to fluctuations associated with the finite size of the sample. Several different sample sizes were used. For purposes of comparison and in order to investigate growth rates, the ratio $\langle W \rangle / \langle W_0 \rangle$ is plotted in the figures.

There is concern about the effects of rounding errors in computations involving many iterations of a nonlinear mapping. To observe the cumulative effect of these rounding errors, both single (16 digits) and double precision (32 digits) arithmetic were used.

The two parameters are the betatron tune (the phase advance per collision/ 2π) and the beam-beam strength parameter, $|\Delta v_V|$. A tune of 3.77 (as in ISABELLE) was chosen, and a range of values of $|\Delta v_V|$ from 0.001 to 0.03 was tried. The figures show the results for the mean square amplitude for $|\Delta v_V| = 0.01, 0.01$ and 0.03, plotted at every 5000 iterations up to maximum of 100,000 iterations. Figure 2 contrasts the results obtained using single precision arithmetic with that using double precision arithmetic. Figure 3 shows the effect of the use of a finite sample, specifically the reduction in the finite sample fluctuations as the sample size is increased from 10 to 1000 particles.

The results for $|\Delta v_V| = 0.001$ show no change in $\langle W \rangle$ after 1.2×10^6 iterations and there was no detectable rounding error.

The following conclusions can be drawn:

(1) The mapping appears to be stable for $|\Delta v_V| = 0.001$, although it is not clear whether the stability is genuinely due to a threshold for growth (at some value of $|\Delta v_V|$ above 0.001) or just that the growth rate

is too slow to be observable in a reasonable number of iterations. In contrast, it is interesting that the model being used is a good approximation of the beam-beam interaction at the ISR and that beam observation at the ISR shows no observable beam-beam effects under normal operating conditions, i.e. $|\Delta v_V| \leq 0.001$, for up to $\sim 10^{11}$ interactions.

(2) For $|\Delta v_V| \geq 0.01$, the mean square amplitude shows growth. In the case of $|\Delta v_V| = 0.03$, there is a doubling of $\langle W \rangle$ in about 25,000 iterations. The growth rate is a strong function of $|\Delta v_V|$.

(3) The comparison of single and double precision computations demonstrates that complete "memory loss" due to rounding error can occur rather quickly, in less than 1000 iterations. There seems to be a correlation between growth and memory loss. The discrepancy due to rounding error increases with increasing $|\Delta v_V|$. There is some indication that the apparent growth rate is reduced by the rounding error.

Fig. 1. Pictorial representation of a particle trajectory in a linear machine with a non-linear point perturbation.

Fig. 2. Evolution of mean square amplitude for Beam-Beam Strengths, $|\Delta v_V| = 0.001, 0.01$ and 0.03 . Comparison of single and double precision arithmetic.

Fig. 3. Evolution of mean square amplitude for Beam-Beam Strength,
 $|\Delta v_V| = 0.01$ and 0.03 . Comparison for various sample sizes.