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Confirming LEU in an LFUA Environment

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CONFIRMING LEU IN AN LFUA ENVIRONMENT

by

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ABSTRACT

A standard sequential procedure is adapted to a limited-frequency unannounced access (LFUA) inspection problem, where uranium enrichment is monitored using nondestructive assay instrumentation. If the enrichment is in the low-enriched uranium (LEU) range, rapid confirmation to that effect is provided under anticipated measurement conditions. Decision thresholds are derived based on the required confidence level in an LEU confirmation. Also, the procedure is easily automated and is such that the raw data need not be revealed.

I. INTRODUCTION

A measurement technique to be performed in a limited-frequency unannounced access (LFUA) inspection regime for centrifuge enrichment plants has recently been developed.¹ This technique, based on an integration of passive gamma-ray spectrometry and x-ray fluorescence, measures the enrichment of the gaseous UF_6 in the cascade pipework. In accordance with the conclusions² of the Hexapartite Safeguards Project, the enrichment measurements are to provide a quick "go/no go" confirmation that the enrichment is in the low-enriched uranium (LEU) range (that is, below 20%).

This report extends an earlier effort,³ advocating a standard decision procedure of sequential analysis for use in the LFUA monitoring problem. Confirmation of LEU is made with 95% confidence, and preliminary results indicate that fast, accurate "go/no go" decisions are reached under anticipated measurement conditions. In Secs. II and III, the decision

procedure is formally defined and background information concerning its use is given. Application to the LFUA problem is discussed in Sec. IV, and simulation work is presented in Sec. V to quantify performance.

II. BACKGROUND: THE SEQUENTIAL PROBABILITY RATIO TEST

Statistical techniques for the evaluation of data observed sequentially are in widespread use, particularly in the areas of time series and quality control. Safeguards applications are also increasing (see Ref. 4 and the bibliography contained therein). For the particular problem of enrichment monitoring, it is useful to review a procedure called the Sequential Probability Ratio Test (or SPRT) and attributed to Wald.⁵ A modified version of the SPRT is advocated in Sec. V of this report for LFUA work.

The generic decision problem of sequential analysis as related to LFUA monitoring is as follows. Individual enrichment measurements, corresponding to 30-minute counts, are made sequentially. The i^{th} measured value $m(i)$ is assumed to conform to the model

$$m(i) = \theta + e(i) \quad ,$$

where $e(i)$ is the measurement error and θ denotes the actual enrichment of the UF_6 stream. It is presumed that the actual enrichment changes only negligibly during the monitoring period. Measurement errors $\{e(i)\}$ are presumed independent and normally distributed with known standard deviation σ . In practice, of course, such modeling assumptions should be confirmed experimentally; alternative error structures are easily accommodated in the methodology outlined below.

For the moment, suppose that there are only two possible values for the actual enrichment θ , which are denoted by θ_0 and θ_1 and are known. The problem is to determine which of θ_0 and θ_1 is in fact the actual value. Moreover, an answer must be obtained quickly and with little chance of reaching an erroneous decision.

Wald's solution to this generic problem was to sequentially monitor the relative likelihood of θ_0 and θ_1 and to reach a decision when that relative likelihood strongly favored one value of θ over the other. How strongly one value was favored depended on the desired decision error rates. Specifically, Wald's SPRT proceeds as follows.

- (1) Establish the acceptable decision error rates α and β . Here, α denotes the acceptable probability that the decision favors θ_1 when θ_0 is the actual value. Similarly, β denotes the acceptable probability that the decision favors θ_0 when θ_1 is the actual value. Thus, α and β can be thought of as false-alarm rates.
- (2) Initialize the procedure by setting $n = 1$.
- (3) Observe the measured value $m(n)$ and compute the relative likelihood

$$\lambda(n) = \frac{\prod_{i=1}^n \exp -[m(i) - \theta_1]^2 / 2\sigma^2}{\prod_{i=1}^n \exp -[m(i) - \theta_0]^2 / 2\sigma^2} .$$

(Aside: Were the normal distribution inappropriate for characterizing measurement errors, the relative likelihood $\lambda(n)$ would be computed using a different expression from the above.)

- (4) If $\lambda(n) < \beta/(1 - \alpha)$, decide that the actual value is θ_0 .
If $\lambda(n) > (1 - \beta)/\alpha$, decide that the actual value is θ_1 .
- (5) If $\beta/(1 - \alpha) \leq \lambda(n) \leq (1 - \beta)/\alpha$, the relative likelihood is not yet one-sided enough to reach a decision. Set n to one more than its current value and return to Step 3.

The above procedure is very simple to implement in an automated fashion. And, although a comprehensive discussion of the statistical theory is beyond the scope of this report, SPRT has the desirable properties that

- the actual decision error rates can be shown⁵ not to exceed the stated acceptable error rates α and β , and

- decisions are reached very quickly--in fact, no other test procedure, sequential or otherwise, with the same decision error rates reaches decisions, under either θ or θ_1 , with a lower average number of measurements.⁶⁻⁸

Because of these desirable properties for the "pure" two-state decision problem, SPRT has become a very widely used procedure.

III. THE MODIFIED SPRT

Despite its low average number of measurements required for decisions, the SPRT can be very slow in isolated cases. The run-length distribution is skewed (in the present application) so that numerous measurements may occasionally be required. Also, if the actual enrichment θ were between the values θ_0 and θ_1 specified by SPRT, the procedure could have a hard time deciding between the two. Indeed, the relatively poor properties of SPRT at certain intermediate values have prompted work^{9,10} in developing alternative procedures.

A simple modification to SPRT overcomes the open-endedness of the above five-step iteration. This modification sets a maximum number, $n_{\max}$, of measurements that can be obtained. The modified SPRT proceeds as the unmodified version, and, if no decision has been reached after $(n_{\max} - 1)$ measurements, one final measurement is made and a decision is then reached on the basis of the available data.

For the LFUA monitoring problem, the value $n_{\max} = 8$ is advocated so that a decision is sure to be forthcoming without undue delay. Because an inspector's time at the facility is limited, prompt decisions are important. Given one half-hour counting per measurement, $n_{\max} = 8$ means that a maximum of 4 hours per decision is involved. The number "8" is somewhat arbitrary, as would be any other choice, but it provides a reasonable upper limit to the number of measurements.

Other suggested parameter values for the modified SPRT are $\theta = 20\%$ (corresponding to the LEU/HEU threshold), $\theta_0 = 10\%$, and decision error rates $\alpha = 0.1\%$ and $\beta = 5\%$. If all eight measurements are required, the

decision to "confirm LEU" is made, provided the relative likelihood $\lambda(\theta) \leq \beta/(1 - \alpha)$; otherwise, the decision "unable to confirm" is reached. This termination rule guarantees that the decision procedure maintains the error rate $\beta < 5\%$. In other words, if the actual enrichment is in the highly enriched uranium (HEU) range, then the decision "unable to confirm" is forthcoming with $>95\%$ probability. Thus, following from what logicians call the converse contrapositive, decisions to "confirm LEU" are made with high confidence.

Properties of the advocated procedure are discussed in Sec. V, though one point of general interest is made here. It is important to recognize that the acceptable decision error rates α and β established for use in the modified SPRT may or may not be simultaneously achievable. If individual measurements are not of sufficient accuracy, it is not possible to reach high-quality decisions based on eight or fewer such values. In such a case, α is sacrificed in order to maintain $\beta < 5\%$. The situation is as described in Table I. If measured values allow for high confidence in LEU, the "confirm LEU" decision is reached. If measured values allow high confidence in HEU, the "unable to confirm" decision is reached. Finally, if confidence is not sufficiently high in either LEU or HEU, as could be the case if measurement uncertainties were large, then the "unable to confirm" decision is reached.

TABLE I
DECISIONS UNDER VARIOUS CONDITIONS

<u>Condition</u>	<u>Decision</u>
High confidence in LEU	"Confirm LEU"
High confidence in HEU	"Unable to confirm"
Confidence is not high	"Unable to confirm"

IV. THE LFUA MONITORING PROBLEM

The task is to determine whether the actual enrichment does or does not exceed 20%. The decision procedure based on observed measurement data must generate a binary result--either a "yes" or a "no"--to that effect; in other words, the ideal detection probability "curve" is a step function, with the step at 20% enrichment. Moreover, it is desired that the decision be as timely as possible, be easily automated, and be such that the raw data need not be revealed.

A decision procedure having the ideal detection probability curve is not attainable. Using a modified SPRT, however, leads to decisions that are reached quickly and with relatively little chance of a mistake when

- (1) the actual enrichment is in the anticipated range (below 5%) or
- (2) the actual enrichment is in the anomalous range (20% or above).

Performance of the modified SPRT depends on the underlying measurement conditions. In the next section, that performance is quantified.

One unavoidable aspect of decisionmaking in the presence of measurement noise is that intermediate enrichments--say, from 12% to 18%--produce intermediate results. It is simply not possible to construct a reasonable decision procedure for which the detection probability curve is discontinuous. If, for example, the probability of the decision "unable to confirm" is equal to 0.1% when the actual enrichment is 5% and is equal to 95% when the actual enrichment is 20%, then the continuity of the detection probability curve implies that there is an intermediate enrichment for which a 50% detection, say, ensues. In some respects, this is disturbing since certain sub-20% enrichments generate "unable to confirm" decisions with relatively large probability. The situation is an inevitable consequence, however, of achieving fast and accurate decisions under (1) and (2) above.

Problems at intermediate enrichments would tend to disappear if information could become more perfect so that the detection probability curve would approach the desired step function. In the present case, though, the steepness of the detection probability curve at intermediate enrichments is determined by the quality of the individual measurements (reflected by the standard deviation σ) and the quantity of measurements

(limited to a maximum of eight per decision). Under anticipated conditions, the steepness is sufficient to provide excellent discrimination between cases (1) and (2) above; if still better discrimination is desired, it appears that timeliness must be sacrificed.

V. A SUGGESTED PROCEDURE

Consider defining a modified SPRT by establishing the parameter values

$$\theta_1 = 20\%, \quad \theta_0 = 10\%, \quad \alpha = 0.1\%, \quad \beta = 5\%, \quad \text{and } n_{\max} = 8.$$

The termination rule, as noted in Sec. III, compares $\lambda(8)$ to $\beta/(1 - \alpha)$ in the event that eight measurements are required. As will be shown, this SPRT gives relatively fast and accurate decisions for anticipated enrichments. More generally, parameter values can be altered to yield a sequential test procedure with somewhat different properties, and the class of modified SPRT tests is quite useful for LFUA monitoring.

To illustrate, suppose that the standard deviation of each individual measurement is $\sigma = 4\%$. This standard deviation corresponds to good--but under some circumstances, expected--measurement conditions. The modified SPRT defined using the above parameter values is depicted in Fig. 1.

For this procedure, if the first measurement is less than 10.2%, the decision "confirm LEU" is reached immediately. If the first measurement exceeds 26.0%, the decision "unable to confirm" is reached immediately. If the first measurement falls between 10.2% and 26.0%, then a second measurement is made.

Upon observation of the second measurement, the average m_2 is formed. Because the second measurement is, loosely speaking, no better and no worse than the first one, the decision is based on the average. At the second step, if the average m_2 of the first two measurements is $<12.6\%$, the decision "confirm LEU" is reached; if m_2 exceeds 20.5%, the decision "unable to confirm" is reached. If m_2 falls between 12.6% and 20.5%, then a third measurement is made.

The decision process continues, as indicated in Fig. 1, at the n^{th} step monitoring the average m_n of the n individual measurements observed to that point. If no decision has been reached after the 7th measurement, then the 8th and last measurement is made. If m_8 exceeds 14.4%, the decision "unable to confirm" is reached; otherwise, the decision "confirm LEU" is reached. Because the values of $\{m_n\}$ should tend to converge to the actual enrichment, it is clear from Fig. 1 that when measurement conditions are good, decisions of high quality are reached quickly for actual enrichments in the anticipated range. Only when the observed measurements are "ambiguous" do decisions require a longer time.

Figure 2 depicts the modified SPRT, using the same parameter values as above, when the standard deviation of each measurement is $\sigma = 10\%$. This standard deviation corresponds to poor--but under some circumstances, expected--measurement conditions. Because of the increased uncertainty, the "continue measuring" bands are wider than those in Fig. 1. More measurements are required, on the average, to reach decisions and those decisions are not of the same quality as those for the $\sigma = 4\%$ case.

From Figs. 1 and 2, a quick termination with the "unable to confirm" decision can be characterized. Reaching that decision relatively quickly can happen either as the result of an initial, very large measured value or as the result of several measured values whose average is large enough. Either way, sufficient doubt is cast on the credibility of LEU to preclude confirmation with high confidence. Once a strong doubt has been created, it is not adequately removed by observing a small number of additional measured values internally consistent with those already available.

For example, Fig. 1 indicates that for the $\sigma = 4\%$ case, an "unable to confirm" decision would follow the fourth measurement if the average m_4 of those measurements were equal to 18% (and if no decision had been reached earlier). That the "unable to confirm" decision came quickly did not mean that high confidence could be placed in an HEU claim; after all, $m_4 = 18\%$ is slightly more supportive of LEU than HEU.

To quantify the performance of the modified SPRT, a simulation exercise was conducted. Taken as representative of potential measurement conditions based on GCEP-related activities are the following:

Fig. 1. Graphical representation of the suggested decision procedure when $\sigma = 4\%$.

Fig. 2. Graphical representation of the suggested decision procedure when $\sigma = 10\%$.

- (a) a "clean pipe," where the standard deviation σ of each individual measurement varies (linearly) from 4% for 3% actual enrichment to 6% for 20% actual enrichment; and
- (b) a "dirty pipe," where the standard deviation σ of each individual measurement varies (linearly) from 10% for 3% actual enrichment to 12% for 20% actual enrichment.

This situation is depicted in Fig. 3; note that the extremes correspond to Figs. 1 and 2 when the enrichment is 3%. (Asides: (1) In practice, standard deviations are not known exactly and are estimated based on counting statistics. It is important that count rates are such that the propagated standard deviations are very good; otherwise, the decision error rates are adversely affected. As mentioned in Sec. II, experimental work on this subject should be undertaken. (2) If measurement uncertainties were far better than indicated for the clean pipe, then different parameter values for θ_0 and α in the modified SPRT should be used. At present, such a circumstance seems unlikely.)

Results from the simulation are presented in Table II. Under good measurement conditions, decisions are fast and accurate. For sub-5% enrichments, only one or two measurements are usually needed to make virtually error-free decisions. For 20% enrichment, four to seven measurements are usually needed and there is less than a 2% chance of erroneously concluding "confirm LEU." As always, the slowest decisions correspond to intermediate enrichments, but only when the enrichment is 14-18% are all eight measurements frequently (two-fifths of the time) needed.

At the other extreme of measurement conditions, performance of the modified SPRT is not nearly as good. For 3% enrichment, decisions require three to four measurements, on the average, and the decision error rate is 0.7%. For 20% enrichment, eight measurements per decision are common. As would be expected, increased measurement uncertainty means that "confirm LEU" decisions cannot be made with high confidence as frequently as when uncertainties are smaller. Note from Fig. 4 that the probability of an "unable to confirm" decision is uniformly higher over the LEU range for the dirty pipe than for the clean one.

Fig. 3. Measurement standard deviations under anticipated conditions.

Fig. 4. Detection probability curve for the example of Sec. V.

TABLE II

PERFORMANCE* OF THE SUGGESTED MODIFIED SPRT PROCEDURE

<u>"Clean Pipe"</u>	Actual Enrichment						
	<u>3%</u>	<u>5%</u>	<u>10%</u>	<u>15%</u>	<u>18%</u>	<u>20%</u>	<u>25%</u>
Probability of deciding "unable to confirm"	0%	0%	0.3%	56.1%	92.2%	98.0%	99.9%
Average number of measurements required	1.0	1.2	2.2	5.9	6.2	5.3	3.7

<u>"Dirty Pipe"</u>	Actual Enrichment						
	<u>3%</u>	<u>5%</u>	<u>10%</u>	<u>15%</u>	<u>18%</u>	<u>20%</u>	<u>25%</u>
Probability of deciding "unable to confirm"	0.7%	3.4%	36.5%	83.0%	95.8%	98.4%	99.9%
Average number of measurements required	3.3	3.9	6.1	7.6	7.9	7.9	7.8

*Based on 10,000 simulated sequences. The "0%" means that no simulated sequence resulted in an "unable to confirm" decision; in other words, "0%" should not be interpreted as implying there is absolutely no chance of an "unable to confirm" decision.

VI. SUMMARY

As reviewed in this report, the modified SPRT is an effective decisionmaking tool in the sequential measurement framework. The theoretical foundation for this tool has been well established in the statistical literature for many years. Simulation results applicable to the LFUA monitoring problem indicate that fast, accurate decisions are reached under anticipated conditions.

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