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SCALING RELATIONS FOR A BEAM-DEFLECTING TM_{110} MODE IN AN ASYMMETRIC CAVITY

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SCALING RELATIONS FOR A BEAM-DEFFACTING $TM_{110}$ MODE IN AN ASYMMETRIC CAVITY

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Abstract

A deflection mode in an RF cavity caused by an aperture of the coupling hole from a waveguide-studied. If the coupling hole has a finite size, the RF modes in the cavity can be detected. We consider the deflection mode as a sum of the accelerating mode and the deflection mode. The finite size coupling hole can be considered as radiating dipole sources in an open cavity. Following the prescription given by H. Biedenharn, the relative strength of the deflection mode $TM_{110}$ to the accelerating $TM_{111}$ mode is computed by decomposing the dipole source field into cavity eigenmodes. Scaling relations are obtained as a function of the coupling hole radius.

Introduction

A comparison between the accelerating and deflection modes of a cavity is made, and the deflection mode is derived as a free sum of an accelerating mode and a deflection component. We study the $TM_{110}$ deflection component as a function of coupling aperture size. We derive the field distribution of the coupling aperture of the cavity to the deflection component with the help of dipole source theory. The spatial distribution of the $TM_{110}$ component is approximated with a dipole source. The deflection modes of a cavity can be approximated as the superposition of the accelerating and deflection components. The uniformity of the field distribution in the cavity is shown to be the same as that of the accelerating and deflection components.

$TM_{110}$ Mode of a Cavity

The field distribution of the $TM_{110}$ component of the cavity is approximated with the dipole source

$$\mathbf{E}_{TM_{110}} = \sum_{n} I_n \overline{E}_{TM_{110}} \delta(\mathbf{r} - \mathbf{r}_n)$$

where $I_n$ is the amplitude of the dipole source, $\overline{E}_{TM_{110}}$ is the electric field of the $TM_{110}$ mode, and $\mathbf{r}_n$ is the location of the dipole source.

To express the electric field $\mathbf{E}$ in terms of the cavity modes, we define the orthogonal basis vectors $\mathbf{e}_{TM_{110}}$ for the $TM_{110}$ mode of the cavity,

$$\mathbf{E} = \sum_{n} I_n \mathbf{e}_{TM_{110}}$$

Then, the vector potential $\mathbf{A}$ for the $TM_{110}$ mode is obtained with the dipole source

$$\mathbf{A} = \sum_{n} I_n \mathbf{A}_{TM_{110}}$$

We consider the field distribution of the $TM_{110}$ mode of the cavity.

Cavity Mode Equations and the Mode Component of Electric Current

The electric field of the cavity $\mathbf{E}$ can be expressed as

$$\mathbf{E} = \sum_{m} \mathbf{E}_{TM_{110}}$$

where $\mathbf{E}_{TM_{110}}$ is the electric field of the $TM_{110}$ mode.
Applying the Theory of Diffraction Developed by H. Bethe

We apply the diffraction theory developed by H. Bethe to solve the scattering problem between the waveguide and the cavity. Because the system is linear, Bethe obtained a set of boundary conditions that must be satisfied on a plane at the hole. In his small hole approximation, the fields are approximately constant over the waveguide. Bethe showed that the scattered fields are generated as if they are from an electric dipole and a magnetic dipole located at the edge. The magnetic and electric dipole moments are given as

\[ M = \sum R_i N_i \]  
\[ E = \sum R_i N_i \] 

The magnetic dipole moment is given by the sum of the magnetic dipole moment for each point source, and the electric dipole moment is given by the sum of the electric dipole moment for each point source. However, the magnetic and electric dipole moments depend on the incident wave and the scattering properties of the cavity. The incident wave is known, but the scattering properties of the cavity are unknown. This problem is typically solved using numerical methods, such as the finite difference time domain (FDTD) method.

The expressions for the electric and magnetic fields are

\[ E = \sum R_i N_i \] 
\[ B = \sum R_i N_i \] 

Where \( R_i \) are the electric and magnetic dipole moments, and \( N_i \) are the normal modes of the cavity.

For a small cavity, the cavity modes are negligible, and the electric and magnetic fields are

\[ E = E_0 \] 
\[ B = B_0 \] 

Mode Equations for a Small Bethe Hole

For a small cavity, the equations for the electric and magnetic fields are

\[ \nabla \times E = -\frac{1}{c} \frac{\partial B}{\partial t} \] 
\[ \nabla \times B = \mu_0 \frac{\partial E}{\partial t} \] 

Analytical Expression of Mode Amplitudes

Neglecting Intracavity Coupling

\[ E_i = E_0 \] 
\[ B_i = B_0 \]
Example of the Bethe Hole Radiation