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MASTER

SCALING RELATIONS FOR A BEAM-DEFLECTING TM₁₁₀ MODE IN AN ASYMMETRIC CAVITY*

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Abstract

A deflecting mode in an RF cavity caused by an aperture of the coupling hole from a waveguide is studied. If the coupling hole has a finite size, the RF modes in the cavity can be distorted. We consider the distorted mode as a sum of the accelerating mode and the deflecting mode. The finite size coupling hole can be considered as radiating dipole sources in a closed cavity. Following the prescription given by H. Bethe,¹ the relative strength of the deflecting mode (TM₁₁₁) to the accelerating (TM₁₁₀) mode is calculated by decomposing the impinging source field into cavity eigenmodes. Scaling relations are obtained as a function of the coupling hole radius.

Introduction

A coupling between a waveguide and a cavity excites two eigenmodes of the closed cavity. When the aperture of the coupling hole is finite, the incident accelerating eigenmode can have a beam-deflecting component. We study the TM₁₁₀ mode deflecting component as a function of coupling aperture. We apply the theory of cavity coupling developed by Bethe¹ to a rectangular waveguide coupled to the side of a pillbox cavity. We assume that a TE₁₀ mode in the waveguide couples to the cavity through a finite coupling hole.

The Bethe theory assumes a small coupling hole in the side of the waveguide and the radius of the coupling hole is much smaller than the cavity size. The coupling hole acts as a magnetic dipole and electric dipole sources. A closed pillbox cavity has two main eigenmodes of the cavity, the accelerating TM₁₁₀ mode and the deflecting TM₁₁₁ mode. The relative strength of the TM₁₁₁ mode to the TM₁₁₀ mode is calculated by decomposing the impinging source field into cavity eigenmodes. The scaling relations are obtained as a function of the coupling hole radius. The results are compared with the results of a three-dimensional finite element analysis.

Transverse Magnetic Modes in a Pillbox Cavity

The pillbox cavity is a cylindrical cavity with a radius a and a height $2b$. The coordinate system is defined as in Fig. 1. The transverse magnetic modes in the pillbox cavity are given by the following equations:

$$\nabla_{\perp}^2 \mathbf{A}_{\perp} = -k_{\perp}^2 \mathbf{A}_{\perp} \quad (1)$$

$$\nabla_{\parallel}^2 \mathbf{A}_{\parallel} = -k_{\parallel}^2 \mathbf{A}_{\parallel} \quad (2)$$

where $\nabla_{\perp}^2 = \nabla^2 - \nabla_{\parallel}^2$ and $\nabla_{\parallel}^2 = \partial^2 / \partial z^2$. The boundary conditions for the transverse magnetic modes are given by the following equations:

$$\mathbf{A}_{\perp} \cdot \mathbf{n} = 0 \quad \text{at } r = a \quad (3)$$

$$\mathbf{A}_{\parallel} = 0 \quad \text{at } z = \pm b \quad (4)$$

where $\mathbf{n}$ is the unit normal vector to the surface of the cavity. The solutions of the above equations are given by the following equations:

$$\text{and } H_{\theta} = \sum_{\mathbf{k}} P_{\mathbf{k}} \frac{e^{i\mathbf{k} \cdot \mathbf{r}}}{k^2} \quad (5)$$

where $\mathbf{k} = (k_{\perp}, k_{\parallel})$, $k_{\perp} = \sqrt{k^2 - k_{\parallel}^2}$ with an integer $k_{\parallel}$, $k_{\perp} = \sqrt{k^2 - k_{\parallel}^2}$ and $k_{\parallel}$ is the n^{th} root of J_0 .

To express the vector potentials $\mathbf{A}$ in terms of the cavity modes, we define the orthogonal base vectors $\mathbf{d}_{\mathbf{k}}^{\perp}$ as

$$\mathbf{d}_{\mathbf{k}}^{\perp} = \frac{1}{\sqrt{2}} \left(\frac{k_{\theta}}{k_{\perp}} \nabla_{\theta} \mathbf{r} + \frac{k_{\phi}}{k_{\perp}} \nabla_{\phi} \mathbf{r} \right) \quad (6)$$

Then, any vector potential $\mathbf{A}_{\perp} = \sum_{\mathbf{k}} P_{\mathbf{k}} \mathbf{d}_{\mathbf{k}}^{\perp} e^{i\mathbf{k} \cdot \mathbf{r}}$ for the TM₁₁₀ expansion with the amplitude $P_{\mathbf{k}}$ as

$$\mathbf{A}_{\perp} = \sum_{\mathbf{k}} P_{\mathbf{k}} \mathbf{d}_{\mathbf{k}}^{\perp} e^{i\mathbf{k} \cdot \mathbf{r}} + P_{\mathbf{k}} \mathbf{d}_{\mathbf{k}}^{\parallel} e^{i\mathbf{k} \cdot \mathbf{r}} \quad (7)$$

We also define the normalization $M_{\mathbf{k}}$ as

$$M_{\mathbf{k}} = \frac{1}{\sqrt{2}} \left(\frac{k_{\theta}}{k_{\perp}} \frac{d_{\theta}}{d_{\theta}} + \frac{k_{\phi}}{k_{\perp}} \frac{d_{\phi}}{d_{\phi}} \right) \quad (8)$$

$$\text{where } \frac{d_{\theta}}{d_{\theta}} = \frac{1}{\sqrt{2}} \frac{k_{\theta}}{k_{\perp}} \nabla_{\theta} \mathbf{r}$$

Cavity Mode Equations and the Mode Component of Electric Current

The electric current density $\mathbf{J}$ in the cavity is given by the following equation:

$$\mathbf{J} = \frac{1}{\mu_0} \nabla \times \mathbf{A} \quad (9)$$

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$$\mathbf{J} = \frac{1}{\mu_0} \nabla \times \mathbf{A} \quad (13)$$

- Using the following relations: $\vec{B}_{\mathbf{k}} = \nabla \times \vec{A}_{\mathbf{k}}$, $\vec{E}_{\mathbf{k}} = -\nabla A_{\mathbf{k}}$, $\nabla \times \vec{E}_{\mathbf{k}} = -i\frac{\partial \vec{B}_{\mathbf{k}}}{\partial t} = \mp \vec{B}_{\mathbf{k}}$, and Eq. (9) – $\vec{u}$ is expressed with the electric field eigenvector $\vec{E}_{\mathbf{k}}$ as

$$\vec{u}_{\mathbf{k}} = \frac{e}{imN_{\mathbf{k}}} \int_V dV \vec{E}_{\mathbf{k}}^* \vec{A} \quad (10)$$

Applying the Theory of Diffraction Developed by H. Bethe

We apply the diffraction theory developed by H. Bethe to solve the hole coupling between the waveguide and the cavity. To solve the problem, Bethe obtained a set of boundary conditions that must be satisfied on a plane at the hole. In his small-hole approximation, the fields are approximately constant over the hole opening. Bethe showed that the radiated fields are generated as if they are from an electric dipole and a magnetic dipole located at the hole. The magnetic and electric dipole moments are given as¹

$$\vec{M} = -\frac{2}{3\pi} R^3 \vec{H} \quad \text{and} \quad \vec{P} = \frac{1}{3\pi} R^3 \vec{E}_0 \quad (11)$$

The magnetic dipole moment points along the z -axis, since the electric dipole moment points normal to the surface. The radius of the hole is R , $\vec{H}$ and $\vec{E}_0$ are the fields at the plane in the waveguide if the hole is closed. However, it is important in the cavity at the hole, both $\vec{H}$ and $\vec{E}$ are complicated, as the difference between the closed waveguide field and the radiated field.

We assume a cylindrical TM₀₁₀ mode is contained in the rectangular waveguide. We use the magnetic field H_{ϕ} of the closed waveguide to express the magnetic dipole moment $\vec{M}$. The magnetic field H_{ϕ} is maximum at the center of the waveguide, so $\vec{M}$ is located at the center of the waveguide cross section, and is parallel to the z -axis. The dipole moment $\vec{M}$ can be written as $\vec{M} = R_{\text{eff}}^3 \vec{H}_{\phi}|_{z=0}$

$$\vec{M} = \frac{1}{2} R^3 (H_{\phi}|_{z=0} + R_{\text{eff}}^3 H_{\phi}|_{z=0}) \quad (12)$$

where R_{eff} is the effective radius of the hole. In the limit of the small hole, the dipole moment $\vec{M}_{\text{eff}}$, which radiates into the waveguide, is the same as $\vec{M}$ due to the presence of the magnetic field H_{ϕ} at the center of the waveguide cross section. In this case, $R_{\text{eff}} = R$. The dipole moment of the radiated wave is proportional to R_{eff}^3 and

$$R_{\text{eff}}^3 = \frac{1}{2} R^3 (1 + \frac{1}{2} R_{\text{eff}}^3)$$

where R_{eff} is the effective radius of the hole. The effective radius R_{eff} is determined by the boundary conditions at the hole. The boundary conditions at the hole are that the tangential component of the magnetic field is continuous and the normal component of the electric field is zero. The boundary conditions at the hole are

$$\vec{H}_{\text{wg}} = \vec{H}_{\text{cav}} + \vec{H}_{\text{rad}} \quad (13)$$

where $\vec{H}_{\text{wg}}$ is the magnetic field in the waveguide, $\vec{H}_{\text{cav}}$ is the magnetic field in the cavity, and $\vec{H}_{\text{rad}}$ is the magnetic field of the radiated wave. The boundary conditions at the hole are that the tangential component of the magnetic field is continuous and the normal component of the electric field is zero. The boundary conditions at the hole are

Field of TM Cavity Modes and the Mode Couplings by the Currents from Electric and Magnetic Dipoles

The field of the TM cavity modes is given by

$P_{\mathbf{k}}$ correspond to the measurable quantities instead of the real part of complex fields.

The magnetic and electric dipole moments in the driving currents (Eq. (14) and Eq. (15)) depend on the fields present at the hole surface. However, the fields can be known only after the mode coefficients $P_{\mathbf{k}}$ are obtained. If no fields are present in the cavity, only the incident field from the waveguide drives the moments. Because the effect of currents from the electric dipole describing the radiation from the cavity mode into the waveguide is much smaller than the effect of magnetic TE₀₁₀ mode into the cavity, we neglect the electric dipole in the following argument. Expanding the surface field in terms of the mode coefficients, the current that is due to the magnetic dipole is expressed as

$$\vec{u}_{\text{cm}} = \frac{e}{N_{\mathbf{k}}} \vec{H}_{\mathbf{k}}^* \propto \frac{2}{3\pi} R^3 \vec{H}_{\mathbf{k}}^* \quad (16)$$

$$\propto \left(\frac{2k_0 a}{\pi} R_{\text{eff}}^3 \right)^{1/2} (P_{\text{wg}}^{1/2} e^{i\phi_{\text{wg}}} + P_{\text{cav}}^{1/2} e^{i\phi_{\text{cav}}} + P_{\text{rad}}^{1/2} e^{i\phi_{\text{rad}}})$$

The mode coefficients P_{wg} and P_{cav} correspond to $\vec{u}_{\text{wg}}$ and the wave reflecting mode TM₀₁₀, respectively. The vector $\vec{u}_{\text{rad}}$ is

$$\vec{u}_{\text{rad}} = \frac{4\pi e^2}{N_{\mathbf{k}}} \vec{H}_{\mathbf{k}}^* \propto \frac{4}{3\pi} R^3 \vec{H}_{\mathbf{k}}^*$$

The magnetic dipole moments $\vec{u}_{\text{cm}}$ and $\vec{u}_{\text{rad}}$ are computed as

$$\vec{u}_{\text{cm}} = \frac{1}{2} R^3 (H_{\phi}|_{z=0} + R_{\text{eff}}^3 H_{\phi}|_{z=0}) \quad (17)$$

Mode Equations for a Small Bethe Hole

The boundary conditions at the hole are that the tangential component of the magnetic field is continuous and the normal component of the electric field is zero. The boundary conditions at the hole are

$$\vec{H}_{\text{wg}} = \vec{H}_{\text{cav}} + \vec{H}_{\text{rad}} \quad (18)$$

The boundary conditions at the hole are that the tangential component of the magnetic field is continuous and the normal component of the electric field is zero. The boundary conditions at the hole are

Analytical Expression of Mode Amplitudes Neglecting Intramode Coupling

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The acceleration amplitude, whose frequency is equal to the driving wave, is calculated from

$$P_0 + \frac{\gamma_m}{Q_0} P_0 + \omega_{me}^2 P_0 = \alpha_0 B_0 e^{-i\omega_0 t} \quad (11)$$

where we used a binomial expansion for $\gamma_m \ll 1$. The stationary solution is written as

$$P_0 = \alpha_0 B_0 \left(\frac{e^{-i(\omega_0 - t)}}{1 + B_0 \omega_{me} - \frac{\gamma_m}{Q_0}} \right) \quad (12)$$

Examining the time dependence $e^{-i\omega_0 t}$, the imaginary parts P_0^I and P_0^R amplitudes are obtained from $\text{Im}[P_0 e^{-i\omega_0 t}]$ and $\text{Re}[P_0 e^{-i\omega_0 t}]$

$$P_0^I = \alpha_0 B_0 \frac{\frac{\gamma_m}{Q_0}}{1 + B_0 \omega_{me} - \frac{\gamma_m}{Q_0}} e^{-\frac{\gamma_m}{Q_0} t} \quad (13)$$

$$P_0^R = \alpha_0 B_0 \frac{1 + B_0 \omega_{me}}{1 + B_0 \omega_{me} - \frac{\gamma_m}{Q_0}} e^{-\frac{\gamma_m}{Q_0} t} \quad (14)$$

For $\omega_0 = 1$, the real and imaginary parts of the acceleration P_0^I and P_0^R are shown in Fig. 1 and Fig. 2.

$$P_0^I = \alpha_0 B_0 \frac{\frac{\gamma_m}{Q_0}}{1 + B_0 \omega_{me} - \frac{\gamma_m}{Q_0}} e^{-\frac{\gamma_m}{Q_0} t} \quad (15)$$

$$P_0^R = \alpha_0 B_0 \frac{1 + B_0 \omega_{me}}{1 + B_0 \omega_{me} - \frac{\gamma_m}{Q_0}} e^{-\frac{\gamma_m}{Q_0} t} \quad (16)$$

Example of the Bethe-Hole Radiation

Let us take an example of the Bethe-Hole radiation [10] as a typical example of the radiation from a hole in a plasma.

$$\alpha_0 = \frac{1}{2} \frac{1}{\omega_{pe}^2} \frac{d^2 \phi}{dt^2} = \frac{1}{2} \frac{1}{\omega_{pe}^2} \frac{d^2}{dt^2} \left(\frac{1}{\omega_{pe}^2} \frac{d^2 \phi}{dt^2} \right) \quad (17)$$

where ϕ is the electrostatic potential. The acceleration α_0 is calculated from the electrostatic potential ϕ and the electron plasma frequency ω_{pe} .

Let us consider a hole in a plasma with a uniform magnetic field B_0 and a uniform electric field E_0 in the x - y plane.

$$\mathbf{E} = E_0 \hat{x} + E_0 \hat{y} \quad (18)$$

$$\mathbf{B} = B_0 \hat{z} \quad (19)$$

$$\mathbf{E} = E_0 \hat{x} + E_0 \hat{y} \quad (20)$$

$$\mathbf{B} = B_0 \hat{z} \quad (21)$$

$$\mathbf{E} = E_0 \hat{x} + E_0 \hat{y} \quad (22)$$

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$$\mathbf{E} = E_0 \hat{x} + E_0 \hat{y} \quad (24)$$

$$\mathbf{B} = B_0 \hat{z} \quad (25)$$

$$\mathbf{E} = E_0 \hat{x} + E_0 \hat{y} \quad (26)$$

$$\mathbf{B} = B_0 \hat{z} \quad (27)$$

The acceleration amplitude, whose frequency is equal to the driving wave, is calculated from

$$P_0 + \frac{\gamma_m}{Q_0} P_0 + \omega_{me}^2 P_0 = \alpha_0 B_0 e^{-i\omega_0 t} \quad (28)$$

where we used a binomial expansion for $\gamma_m \ll 1$. The stationary solution is written as

$$P_0 = \alpha_0 B_0 \left(\frac{e^{-i(\omega_0 - t)}}{1 + B_0 \omega_{me} - \frac{\gamma_m}{Q_0}} \right) \quad (29)$$

Examining the time dependence $e^{-i\omega_0 t}$, the imaginary parts P_0^I and P_0^R amplitudes are obtained from $\text{Im}[P_0 e^{-i\omega_0 t}]$ and $\text{Re}[P_0 e^{-i\omega_0 t}]$

$$P_0^I = \alpha_0 B_0 \frac{\frac{\gamma_m}{Q_0}}{1 + B_0 \omega_{me} - \frac{\gamma_m}{Q_0}} e^{-\frac{\gamma_m}{Q_0} t} \quad (30)$$

$$P_0^R = \alpha_0 B_0 \frac{1 + B_0 \omega_{me}}{1 + B_0 \omega_{me} - \frac{\gamma_m}{Q_0}} e^{-\frac{\gamma_m}{Q_0} t} \quad (31)$$

For $\omega_0 = 1$, the real and imaginary parts of the acceleration P_0^I and P_0^R are shown in Fig. 3 and Fig. 4.

$$P_0^I = \alpha_0 B_0 \frac{\frac{\gamma_m}{Q_0}}{1 + B_0 \omega_{me} - \frac{\gamma_m}{Q_0}} e^{-\frac{\gamma_m}{Q_0} t} \quad (32)$$

$$P_0^R = \alpha_0 B_0 \frac{1 + B_0 \omega_{me}}{1 + B_0 \omega_{me} - \frac{\gamma_m}{Q_0}} e^{-\frac{\gamma_m}{Q_0} t} \quad (33)$$

Let us take an example of the Bethe-Hole radiation [10] as a typical example of the radiation from a hole in a plasma.

$$\alpha_0 = \frac{1}{2} \frac{1}{\omega_{pe}^2} \frac{d^2 \phi}{dt^2} = \frac{1}{2} \frac{1}{\omega_{pe}^2} \frac{d^2}{dt^2} \left(\frac{1}{\omega_{pe}^2} \frac{d^2 \phi}{dt^2} \right) \quad (34)$$

where ϕ is the electrostatic potential. The acceleration α_0 is calculated from the electrostatic potential ϕ and the electron plasma frequency ω_{pe} .

Let us consider a hole in a plasma with a uniform magnetic field B_0 and a uniform electric field E_0 in the x - y plane.

$$\mathbf{E} = E_0 \hat{x} + E_0 \hat{y} \quad (35)$$

$$\mathbf{B} = B_0 \hat{z} \quad (36)$$

$$\mathbf{E} = E_0 \hat{x} + E_0 \hat{y} \quad (37)$$

$$\mathbf{B} = B_0 \hat{z} \quad (38)$$

$$\mathbf{E} = E_0 \hat{x} + E_0 \hat{y} \quad (39)$$

$$\mathbf{B} = B_0 \hat{z} \quad (40)$$

$$\mathbf{E} = E_0 \hat{x} + E_0 \hat{y} \quad (41)$$

$$\mathbf{B} = B_0 \hat{z} \quad (42)$$

$$\mathbf{E} = E_0 \hat{x} + E_0 \hat{y} \quad (43)$$

$$\mathbf{B} = B_0 \hat{z} \quad (44)$$

$$\mathbf{E} = E_0 \hat{x} + E_0 \hat{y} \quad (45)$$

$$\mathbf{B} = B_0 \hat{z} \quad (46)$$

$$\mathbf{E} = E_0 \hat{x} + E_0 \hat{y} \quad (47)$$