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**Losses in Superconducting Multifilament
Composites under Alternating Changing Fields**

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Bernard Turck*

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NOMENCLATURE

A	thickness of the slab equivalent to the round wire ($A^2 = \pi R^2$)
d	diameter of a filament
a	half thickness of the slab equivalent to a filament ($4d^2 = \pi d^2/4$)
J_c	critical current density
i	ratio of the transport current to the critical current
P	twist pitch length
R	radius of the MSC
R_f	radius of the filament bundle
η	filling factor of superconductor
η_f	filling factor of superconductor in the filament bundle only
ρ_L	effective transverse resistivity of the matrix
ρ_{Cu}	resistivity of the copper
ΔB	maximum variation of the changing field
B_0	amplitude of the sinewave changing field ($\Delta B = 2B_0$)
B_i	internal field
B_e	external field
$\dot{B}$	sweep rate of the changing field
τ	time constant of the changing field
θ	time constant of induced currents
x	ratio θ/τ
f	frequency
ω	pulsation of the changing field $\omega = 2\pi f$
w	thickness of the outer layer of saturated filaments
MSC	multifilament superconducting composite
J	current density
I_c	critical current

The expressions of overall losses to be used in a magnet design are

- for the small rate triangular wave
expression (6)
- for the single rise exponential wave
expression (13)

- for the sinewave alternating field
expressions (33) and (34)
or (35) when $\theta_2 \omega \gg 1$,
when the time constants θ_1 and θ_2 are given by expressions (3) and (4).
All other parameters are listed in the nomenclature.

LOSSES IN SUPERCONDUCTING MULTIFILAMENT COMPOSITES UNDER ALTERNATING CHANGING FIELDS

by

Bernard Turck

ABSTRACT

The first part of this report presents a brief review of the losses in superconducting composites in a changing external field. The losses are expressed for a single triangular shaped pulse and for a field varying exponentially with time. The losses are calculated as a function of two important time constants. The first is due to the coupling currents between the filaments, the second is due to the eddy currents induced in the matrix. In the second part of the report, the case of an oscillating sinusoidal field is considered. In particular, in a rapidly varying field, several layers of filaments can be saturated by returning currents. Complete expressions of the losses are provided together with simplified expressions which enable one to calculate the losses in most cases. In the third part, a comparison between several kinds of conductors shows that in some cases the losses are reduced by increasing the twist pitch of the conductor.

1. GENERAL AND REVIEW

1.1. Time Constants of Induced Currents in a Multifilamentary Superconducting Composite (MSC)

There are two kinds of induced currents in an MSC submitted to an external changing field. The first is called the coupling currents, which

flow from filament to filament across the matrix roughly perpendicular to the axis of the MSC and parallel to the external field. This distribution is imposed by the existence of an electric center line in the filament. The reacting field created by the induced current is homogeneous, and there exists an actual time constant for these currents.¹⁻⁴

In a round wire

$$\theta_{1R} = \frac{\mu_0}{8\pi^2} \frac{p^2}{\rho_e}$$

ρ_e is an equivalent resistivity given by $\frac{1}{\rho_e} \approx \frac{1}{\rho_l} + \frac{1}{\rho_{cu}} \log \frac{R}{R_f}$,

ρ_l is the average transverse resistivity in the multifilament bundle, ρ_{cu} is the resistivity of the outer copper shell, R and R_f are respectively the radius of the MSC and of the filament bundle, P is the twist pitch length.

The second currents are usually called eddy currents and flow in the matrix and the outer shell, parallel to the axis of the MSC and perpendicular to the external changing field. The induced currents decay with an infinite series of time constants. It has been shown^{5,6} that the overall behavior of all these currents can be averaged in a slab or in a round wire by

$$\theta_{2S} = \frac{\mu_0 A^2}{12\rho_{cu}} \quad \text{and} \quad \theta_{2R} = \frac{\mu_0 R^2}{8\rho_{cu}},$$

where A is the width of the slab.

1.2. The Case of a Layer of Round Wires

In a well cooled coil, that is, in a coil where the width of the channel is not negligible, or in a single-layer coil used for making loss measurements for instance, the internal field of each wire is not only due to the external field and to the field from its own induced currents but also to the field created by the induced currents in the other wires. We have shown that the slab model is a very good approximation of a layer of wires.⁷

Thus, two different time constants have to be used. These are

- (1) For the single round wire

$$\theta_{1R} = \frac{\mu_0 p^2}{8\pi^2 \rho_e} \quad (1)$$

and

$$\theta_{2R} = \frac{\mu_0 R^2}{8 \rho_{cu}} \quad (2)$$

(2) For the slab model

$$\theta_{1S} = \frac{\mu_0 p^2}{4\pi^2 \rho_e} \quad (3)$$

and

$$\theta_{2S} = \frac{\mu_0 A^2}{12 \rho_{cu}} \quad (4)$$

One can see that $\theta_S \approx 2 \theta_R$.

As a result, the calculation of the coupling losses given by $\int \rho j^2 dt$ can be changed to $\int \frac{\dot{B}_i^2 \theta_S}{\mu_0} dt$ for the slab and $2 \int \frac{\dot{B}_i^2 \theta_R}{\mu_0} dt$ for the round wire.

All the following calculations are derived for the slab model with the above time constants θ_{1S} and θ_{2S} . The results can be deduced for the round wire very simply, by changing the time constants to θ_{1R} and θ_{2R} , and multiplying the loss expression by 2.

It goes without saying that all the expressions are exact as far as the coupling losses are concerned and must be considered as good approximations for the overall losses, including eddy current losses. This is simply due to the different origins of the two time constants θ_1 and θ_2 .

1.3. How to Calculate the Losses

The internal field is the sum of the external field and the field created by the induced currents.

$$B_i = B_e - B_f ,$$

with $B_f = \theta \dot{B}_i ,$

$$B_i = B_e - \theta \dot{B}_i .$$

This differential equation yields B_i and $\dot{B}_i$ for any external field variation.

The losses are derived from

$$\int_0^t \rho J^2 dt = \int_0^t \frac{\dot{B}_i^2 \theta}{\mu_0} dt . \quad (5)$$

The coupling losses are calculated in the multifilamentary region (radius R_f) and the eddy current losses in the whole conductor (radius R).

1.4. Case of a Slow-Rate Triangular Wave Variation (Fig. 1.)

A slow rate means simply that the risetime τ is much larger than any of the time constants θ_1 and θ_2 . The consequence is that $B_i \approx B_e$ and $\dot{B}_i = \Delta B / \tau$.

The energy loss over one cycle is $W = \frac{2\Delta B^2 \theta}{\mu_0 \tau} .$

The average power loss is $P = \frac{\Delta B^2 \theta}{\mu_0 \tau} .$

If we compare this expression to the case of a sinewave of the same amplitude ($\Delta B = 2B_0$), the loss in the latter case is only $\pi^2/8$ larger. This

Fig. 1.
Triangular wave field variation.

justifies calculating the losses in the high-rate case directly from a sinewave changing field.

To conclude, the overall losses per unit volume of MSC and per cycle are

$$W = \frac{2\Delta B^2}{\mu_0 \tau} \left(\theta_1 \left(\frac{R_f}{R} \right)^2 + \theta_2 \right) + \eta J_c a \Delta B (1+i^2) \quad (6)$$

1.5. Case of an Exponential External Field Variation (Fig. 2)

The calculations are derived during the period of one rise.

The external field is $B_e = \Delta B(1 - e^{-t/\tau})$.

This case has been evaluated with great attention to Ref. 7.

1.5.1 Nonsaturation of the Outer Layer of Filaments.

$$\text{That is } \frac{\theta_1}{\theta_1 + \theta_2} \Delta B x^{\frac{1}{1-x}} < \mu_0 J_c d(1-i), \text{ where } x = \frac{\theta_1 + \theta_2}{\tau}.$$

The eddy current losses are evaluated to be $W_E = \frac{\Delta B^2 \theta_2}{2\mu_0(\theta_2 + \tau)}$,

and the coupling losses are $W_C = \frac{\Delta B^2 \theta_1}{2\mu_0(\theta_1 + \tau)} \left(\frac{R_f}{R} \right)^2$.

The overall loss is the sum $W_E + W_C$, only if τ is much larger than θ_1 and θ_2 . In that case,

Fig. 2.
Exponential magnetic field variation.

$$W_E + W_C \approx \frac{\Delta B^2}{2\mu_0} \left(\frac{\theta_1}{\tau} \left(\frac{R_f}{R} \right)^2 + \frac{\theta_2}{\tau} \right).$$

For small values of the time constant τ , the overall loss tends towards the limit $W_E + W_C \approx \frac{\Delta B^2}{2\mu_0}$.

The following expression is a very good approximation of the overall loss for any value of the time constant τ .

$$W = \frac{\Delta B^2}{2\mu_0} \left(\frac{\theta_1 + \theta_2}{\theta_1 + \theta_2 + \tau} \left(\frac{R_f}{R} \right)^2 + \frac{\theta_2}{\theta_2 + \tau} \left(1 - \left(\frac{R_f}{R} \right)^2 \right) \right) + \frac{\eta J_c a \Delta B (1+i^2)}{2}. \quad (7)$$

1.5.2. Partial Saturation of the Outer Layers.

The maximum value for the current density induced in the filaments is

$$J_{ZM} = \frac{\theta_1 \dot{B}_{iM}}{\mu_0 d},$$

that is to say

$$J_{ZM} = \frac{\Delta B \theta_1}{d\mu_0(\theta_1 + \theta_2)} \times \frac{1}{1-x} \quad \text{when } x = \frac{\theta_1 + \theta_2}{\tau}.$$

Partial saturation appears when

$$\mu_0 J_c (1-i)d < \Delta B \frac{\theta_1}{\theta_1 + \theta_2} \times \frac{1}{1-x} < \mu_0 \eta_f J_c R_f (1-i).$$

a) The hysteretic losses are calculated only in the nonsaturated zone that is changing with time.

$$W_H = \frac{\eta J_c a}{2} \left(\Delta B (1+i^2) - \frac{4}{3} \mu_0 J_c a (1-i^3) \right) \cdot \left(1 - \frac{\delta \Delta B}{2(\theta_1 + \theta_2 + \tau)} \right), \quad (8)$$

with $\delta = \theta_1 / \mu_0 \eta_f J_c R_f (1-i^2)$.

b) The eddy current losses and coupling losses in the filament bundle per unit volume of composite are

$$W_{E+C} = \left(\frac{R_f}{R}\right)^2 \frac{\Delta B^2}{2\mu_0} \left(\frac{\theta_1 + \theta_2}{\theta_1 + \theta_2 + \tau} - \frac{4\delta\theta_1\Delta B}{3\tau^2(2+x)(1+2x)} \right), \quad (9)$$

with $x = (\theta_1 + \theta_2)/\tau$.

c) The eddy current losses in the outer copper shell per unit volume of composite are

$$W_E = \frac{\Delta B^2 \theta_2}{2\mu_0(\theta_2 + \tau)} \left(1 - \left(\frac{R_f}{R}\right)^2 \right). \quad (10)$$

d) The additional losses in the saturated outer layer are

$$W_S = \frac{1}{3} \left(\frac{R_f}{R}\right)^2 \frac{\Delta B^3 \theta_1^2 (1+i^2)}{\mu_0^2 \eta_f J_c R_f (1-i^2) \tau^2 (1+2x)(2+x)}, \quad (11)$$

and the overall losses are $W_T = W_{E+C} + W_S + W_E + W_H$,

$$W_T = \left(\frac{R_f}{R}\right)^2 \left(\frac{\Delta B^2 (\theta_1 + \theta_2)}{2\mu_0 (\theta_1 + \theta_2 + \tau)} - \frac{\Delta B^3 \theta_1^2 (1-3i^2)}{3\mu_0^2 \eta_f J_c R_f (1-i^2)^2 \tau^2 (1+2x)(2+x)} \right) + \left(1 - \left(\frac{R_f}{R}\right)^2 \right) \frac{\Delta B^2 \theta_2}{2\mu_0 (\theta_2 + \tau)} + W_H. \quad (12)$$

The expression looks very complicated. Thank Goodness the term in ΔB^3 between brackets is negative and generally can be neglected as long as the current remains smaller than $0.75 I_c$.

A very good approximation which overestimates the overall loss only a small amount is thus given by the following.

$$W_T = \frac{\Delta B^2}{2\mu_0} \left(\left(\frac{R_f}{R}\right)^2 \frac{\theta_1 + \theta_2}{\theta_1 + \theta_2 + \tau} + \left(1 - \left(\frac{R_f}{R}\right)^2 \right) \frac{\theta_2}{\theta_2 + \tau} \right) + \frac{\eta J_c a \Delta B (1+i^2)}{2}. \quad (13)$$

2. LOSSES IN AN MSC UNDER AN EXTERNAL OSCILLATING FIELD

The external alternating field is $B_e = B_0 \sin \omega t$

2.1. The Different "Critical" Frequencies

In a multifilament composite the time constant θ_1 of the coupling currents is usually larger than θ_2 , time constant of the eddy currents. For the case of a sinusoidal alternating field, three distinct frequency regions have to be identified with respect to critical frequencies fc_1 and fc_2 , or critical pulsation ω_{c1} and ω_{c2} defined by

$$\omega_{c1}(\theta_1 + \theta_2) = 1 \quad \text{and} \quad \omega_{c2} \theta_2 = 1 .$$

For $\omega < \omega_{c1}$, the interior field B_i inside the composite is shielded very little by the coupling currents. For $\omega_{c1} < \omega < \omega_{c2}$, the shielding plays a very important role. Nevertheless as long as ω remains smaller than ω_{c2} , induced currents flow across the matrix in the whole filament bundle, and the calculations can be derived in the usual way from

$$\int_0^T \frac{\dot{B}_i 2\theta}{\mu_0} dt .$$

The internal magnetic field is derived from the differential equation

$B_i = B_e - \theta \dot{B}_i$, where

$\theta = \theta_1 + \theta_2$ in the filament bundle,

$\theta = \theta_2$ in the outer copper shell.

B_i and $\dot{B}_i$ are given by

$$B_i = \frac{B_0}{(1 + \theta^2 \omega^2)^{1/2}} \sin (\omega t - \phi) , \quad (14)$$

and

$$\dot{B}_i = \frac{\omega B_0}{(1 + \theta^2 \omega^2)^{1/2}} \cos (\omega t - \phi) , \quad (15)$$

where $\text{Arctg } \phi = \theta \omega$.

For $\omega > \omega_{c2}$, a quite different behavior appears. The skin effect in the copper itself limits the penetration of the field to a depth of the order of

$$\left(\frac{\pi \mu_0 f}{\rho_{Cu}}\right)^{1/2} \text{ or } \left(\frac{2D}{\omega}\right)^{1/2},$$

where D is the magnetic diffusivity equal to

2.2. The Low-Frequency Region $\omega(\theta_1 + \theta_2) \ll 1$

2.2.1. The Hysteretic Losses. The amplitude of the internal field, $\frac{B_0}{(1+\theta^2\omega^2)^{1/2}}$, is assumed to be much larger than $\mu_0 J_c d$. Therefore, the hysteretic losses are

$$W_H = \frac{2\eta J_c a B_0}{(1+\theta^2\omega^2)^{1/2}}, \quad (16)$$

which decreases with the frequency.

The average power is $P_H = \frac{\eta J_c a B_0 \omega}{\pi(1+\theta^2\omega^2)^{1/2}}, \quad (17)$

which decreases with the time constant θ .

2.2.2 The Eddy Current and Coupling Current Losses. The eddy current and coupling current losses in the filament bundle are derived from

$$\left(\frac{R_f}{R}\right)^2 \int_0^T \frac{B_i^* \theta}{\mu_0} dt.$$

For the energy

$$W_{E+C} = \left(\frac{R_f}{R}\right)^2 \frac{\pi(\theta_1 + \theta_2)\omega B_0^2}{\mu_0(1+(\theta_1 + \theta_2)^2\omega^2)}. \quad (18)$$

For the power

$$P_{E+C} = \left(\frac{R_f}{R}\right)^2 \frac{(\theta_1 + \theta_2) \omega^2 B_0^2}{2\mu_0 (1 + (\theta_1 + \theta_2)^2 \omega^2)} \quad (19)$$

2.2.3. The Eddy Current Losses in the Outer Copper Shell. They are

$$W_E = \left(1 - \left(\frac{R_f}{R}\right)^2\right) \frac{\pi \theta_2 \omega^2 B_0^2}{\mu_0 (1 + \theta_2^2 \omega^2)} \quad (20)$$

and

$$P_E = \left(1 - \left(\frac{R_f}{R}\right)^2\right) \frac{\theta_2 \omega^2 B_0^2}{2\mu_0 (1 + \theta_2^2 \omega^2)} \quad (21)$$

One can see that the power is maximum for $\theta \omega = 1$ and that for a given frequency the losses decrease with increasing values of θ . These points will be analyzed in detail after the following calculations.

2.3. Case of Partial Saturation of the Outer Layers

2.3.1. The Outer Saturated Layer (Fig. 3). The outermost layer saturates if the maximum density J_{zM} reaches J_c , that is, when

$$(1-i)\mu_0 d J_c < \theta_1 \dot{B}_{iM}.$$

Fig. 3.
Outer saturated layers.

The condition is

$$\frac{B_0 \theta_1 \omega}{(1 + (\theta_1 + \theta_2)^2 \omega^2)^{1/2}} > \mu_0 d (1-i) J_c .$$

One can see that for $\theta_1 \omega \gg 1$ this condition tends towards the penetration condition $B_0 > \mu_0 d (1-i) J_c$. The thickness of the saturated layer is averaged for both sides to give

$$\frac{W_1 + W_2}{2} = \frac{\theta_1 |\dot{B}_i|}{\mu_0 \eta_f J_c (1-i^2)} . \quad (22)$$

2.3.2. The Hysteretic Losses. The hysteretic losses are calculated only in the nonsaturated zone. Let us consider the case where the field amplitude is

much larger than the penetration depth in a filament such that $\frac{B_0}{(1 + \theta^2 \omega^2)^{1/2}} \gg \mu_0 J_c d$.

The instantaneous power per unit volume is

$$P = \frac{\eta J_c a}{2} |\dot{B}_i| (1 - \delta |\dot{B}_i|) , \quad \text{with } \delta = \frac{\theta_1}{\mu_0 \eta_f J_c (1-i^2) R_f} .$$

The hysteretic loss per cycle is thus

$$W_H = \frac{2 \eta J_c a B_0}{(1 + \theta^2 \omega^2)^{1/2}} \left(1 - \frac{\delta B_0 \omega \pi}{4 (1 + \theta^2 \omega^2)^{1/2}} \right) . \quad (23)$$

The average power is

$$P_H = \frac{\eta J_c a \omega B_0}{\pi (1 + \theta^2 \omega^2)^{1/2}} \left(1 - \frac{\delta B_0 \omega \pi}{4 (1 + \theta^2 \omega^2)^{1/2}} \right) . \quad (24)$$

2.3.3. Eddy Current Losses and Coupling Losses in the Filament Bundle.

These losses are calculated separately in the inner unsaturated region and in the saturated layer.

- In the inner region

$$W_{E+C} = \left(\frac{R_f}{R}\right)^2 \int \frac{\dot{B}_i^2 \theta}{\mu_0} (1 - \delta |\dot{B}_i|) dt ,$$

with $\theta = \theta_1 + \theta_2$.

- In the saturated region

$$W_{ES} = \left(\frac{R_f}{R}\right)^2 \int \frac{\dot{B}_i^2 \theta_2}{\mu_0} \delta |\dot{B}_i| dt .$$

The overall losses are

$$W_{E+C} + W_{ES} = \left(\frac{R_f}{R}\right)^2 \left(\int \frac{\dot{B}_i^2 \theta}{\mu_0} dt - \int \frac{\delta \dot{B}_i^2 |\dot{B}_i| \theta_1 dt}{\mu_0} \right) .$$

After performing some algebra

$$W_{E+C} + W_{ES} = \left(\frac{R_f}{R}\right)^2 \left(\frac{\pi \theta \omega B_0^2}{\mu_0 (1 + \theta^2 \omega^2)} - \frac{8 \delta \theta_1 \omega^2 B_0^3}{3 \mu_0 (1 + \theta^2 \omega^2)^{3/2}} \right) , \quad (25)$$

and the power is

$$P_{E+C} + P_{ES} = \left(\frac{R_f}{R}\right)^2 \left(\frac{\theta \omega^2 B_0^2}{2 \mu_0 (1 + \theta^2 \omega^2)} - \frac{4 \delta \theta_1 \omega^3 B_0^3}{3 \pi \mu_0 (1 + \theta^2 \omega^2)^{3/2}} \right) . \quad (26)$$

2.3.4. Additional Losses in the Saturated Layer. These losses appear in the

saturated layer where the local field sweep rate is $\dot{B}_S = \frac{\omega B_0}{(1 + \theta_2^2 \omega^2)^{1/2}} \cos(\omega t - \phi_2)$.

These additional losses are given by

$$W_S = \left(\frac{R_f}{R}\right)^2 \int \frac{1}{2R_f} \left(\int_{R_f-w_1}^{R_f} E_1 n_f J_c dx + \int_{R_f-w_2}^{R_f} E_2 n_f J_c dx \right) dt ,$$

where

$$E_1 = |\dot{B}_S| (x - R_f + w_1) .$$

$$E_2 = |\dot{B}_S| (x - R_f + w_2) .$$

$$w_1 = \frac{\theta_1 |\dot{B}_i|}{\mu_0 n_f J_c (1-i)} , \text{ and } w_2 = \frac{\theta_1 |\dot{B}_i|}{\mu_0 n_f J_c (1+i)}$$

By integrating over space, these losses are

$$\left(\frac{R_f}{R}\right)^2 \int \frac{\theta_1^2 (1+i^2) \dot{B}_i^2 |\dot{B}_S|}{2\mu_0 n_f J_c R_f (1-i^2)^2} dt ,$$

which gives

$$W_S = \frac{4}{3} \left(\frac{R_f}{R}\right)^2 \frac{\theta_1^2 (1+i^2) \omega^2 B_0^3}{\mu_0 (1-i^2) (1+\theta_1^2 \omega^2) (1+\theta_2^2 \omega^2)^{1/2}} \quad (27)$$

2.3.5. Total Losses. Total losses are obtained after adding the eddy current losses in the outer copper shell. The resulting equation is

$$W_T = W_H + W_{E+C} + W_{ES} + W_S + W_E ,$$

$$\begin{aligned}
W_T = W_H + \left(\frac{R_f}{R}\right)^2 & \left(\frac{\pi \theta \omega B_0^2}{\mu_0 (1 + \theta^2 \omega^2)} + \frac{4 \theta_1 \delta \omega^2 B_0^3}{3 \mu_0 (1 + \theta^2 \omega^2)} \left(\frac{1 + i^2}{(1 - i^2)(1 + \theta_2^2 \omega^2)^{1/2}} - \frac{2}{(1 + \theta^2 \omega^2)^{1/2}} \right) \right) \\
& + \left(1 - \left(\frac{R_f}{R}\right)^2 \right) \frac{\pi \theta_2 \omega B_0^2}{\mu_0 (1 + \theta_2^2 \omega^2)} .
\end{aligned} \quad (28)$$

2.3.6. Important Remarks on the Saturated Layer. In the preceding calculations the saturated layer thickness is assumed to change continuously. This is due to the definition of the thickness

$$w = \frac{\theta_1 \dot{B}_i}{\mu_0 J_C (1 \pm i)} .$$

However, one must satisfy all the boundary conditions and mainly the total longitudinal current and the local magnetic field variation. This implies that in the saturated layer two regions of opposite currents may exist provided the overall return current is such that $\mu_0 \bar{J} w = \theta_1 \dot{B}_i$.

In the following, the transport current is taken as zero.

The return current is made of two parts according to the expression

$$\theta_1 \dot{B}_i = \eta_f \mu_0 J_C (w - w') .$$

Figure 4 shows that the boundary R_s does not move during the cycle, but the separation line s does.

The maximum thickness of the saturated layer is

$$w + w' = \frac{\theta_1 \dot{B}_i M}{\mu_0 \eta_f J_C} = \frac{\theta_1 \omega B_0}{\mu_0 \eta_f J_C (1 + \theta^2 \omega^2)^{1/2}} .$$

The thickness of unsaturated volume is proportional to $1 - \frac{w + w'}{R_f}$.

Fig. 4.
Current distribution and field profile.

The hysteretic loss per cycle is

$$W_H = \frac{2\eta J_c a B_0}{(1+\theta^2\omega^2)^{1/2}} \left(1 - \frac{\delta\omega B_0}{(1+\theta^2\omega^2)^{1/2}} \right). \quad (29)$$

The eddy current and coupling current losses are

$$W_{E+C} + W_{ES} = \left(\frac{R_f}{R} \right)^2 \left(\frac{\pi\theta\omega B_0^2}{\mu_0(1+\theta^2\omega^2)} - \frac{\pi\delta\theta_1\omega^2 B_0^3}{\mu_0(1+\theta^2\omega^2)^{3/2}} \right). \quad (30)$$

The additional losses in the saturated layer are derived from the equivalent compact superconductor with critical density ηJ_c in the partial penetration mode when submitted to a changing field $\theta_1 \dot{B}_{iMAX}$ in amplitude. The result is

$$W_S = \frac{2}{3} \left(\frac{R_f}{R} \right)^2 \frac{\omega^3 B_0^3 \theta_1^3}{R_f \mu_0^2 \eta J_c (1+\theta^2\omega^2)^{3/2}}. \quad (31)$$

The overall losses in the composite are

$$W = \left(\frac{R_f}{R}\right)^2 \frac{\pi \theta \omega B_0^2}{\mu_0 (1 + \theta^2 \omega^2)} \left[1 - \frac{\theta_1 \delta \omega B_0}{\theta (1 + \theta^2 \omega^2)^{1/2}} \left(1 - \frac{2}{3\pi} \omega \theta_1 \right) \right] + \left(1 - \left(\frac{R_f}{R}\right)^2 \right) \frac{\pi \theta_2 \omega B_0^2}{\mu_0 (1 + \theta_2^2 \omega^2)} + W_H . \quad (32)$$

Usually, in this expression (32) as well as the expression (28), the terms coming from the saturated portion are less than 10% of the main term.

As a result, the overall losses per cycle in a MSC submitted to an oscillating field, for which $\omega \theta_2 < 1$, are

$$W = \left(\frac{R_f}{R}\right)^2 \frac{\pi \theta \omega B_0^2}{\mu_0 (1 + \theta^2 \omega^2)} + \left(1 - \left(\frac{R_f}{R}\right)^2 \right) \frac{\pi \theta_2 \omega B_0^2}{\mu_0 (1 + \theta_2^2 \omega^2)} + \frac{2 \eta J_c a B_0}{(1 + \theta^2 \omega^2)^{1/2}} , \quad (33)$$

where $\theta = \theta_1 + \theta_2$.

And the average power loss is

$$P = \left(\frac{R_f}{R}\right)^2 \frac{\theta \omega^2 B_0^2}{2 \mu_0 (1 + \theta^2 \omega^2)} + \left(1 - \left(\frac{R_f}{R}\right)^2 \right) \frac{\theta_2 \omega^2 B_0^2}{2 \mu_0 (1 + \theta_2^2 \omega^2)} + \frac{\eta J_c a \omega B_0}{\pi (1 + \theta^2 \omega^2)^{1/2}} . \quad (34)$$

2.4 The High-Frequency Region $\omega \theta_2 \gg 1$ (Fig. 5)

The skin effect in the outer copper shell limits the penetration of the field in the filament bundle. The hysteretic losses are zero, the same is true for the coupling losses. The eddy current losses are calculated directly.

The diffusion equation is

$$\frac{\partial^2 B}{\partial x^2} = \frac{j \omega}{D} B .$$

Fig. 5.
Field distribution and skin effect.

If the skin depth is small with respect to R , then $B = B_0 e^{\alpha(x-R)} e^{j\omega t}$, with

$$\alpha = \sqrt{\frac{\omega}{2D}} (1+i) .$$

The current density is $J = \frac{\alpha B_0}{\mu_0} e^{\alpha(x-R)} e^{j\omega t}$

which can be written as

$$J = \sqrt{\frac{\omega}{2D}} \frac{B_0}{\mu_0} e^{\sqrt{\frac{\omega}{2D}} (x-R)} e^{j(\omega t + \sqrt{\frac{\omega}{2D}} (x-R) + \frac{\pi}{4})} .$$

The losses over a cycle are

$$W = \frac{B_0^2 \rho \pi \sqrt{2}}{2R \mu_0^2 \sqrt{\omega D}} .$$

This expression can be written in a different way as

$$W = \frac{B_0^2 \pi}{2R \mu_0} \sqrt{\frac{2D}{\omega}} = \frac{B_0^2 \pi \Delta_S}{2\mu_0 R} ,$$

where Δ_S is the skin depth.

The power is

$$P = \frac{B_0^2}{2R\mu_0} \sqrt{\frac{\omega D}{2}} = \frac{B_0^2 \omega \Delta}{4\mu_0 R} \quad (35)$$

2.5. Discussion of the Losses versus Frequency and Time Constants

2.5.1. Hysteretic losses. The hysteretic losses tend towards a maximum value with frequency, and they decrease for a given frequency as a function of increasing θ (Fig. 6).

2.5.2. Eddy current losses and coupling losses. These losses depend highly on the ratio $\left(\frac{R_f}{R}\right)^2$. Nevertheless, in high frequency the limit for the coupling loss is much less than the theoretical limit for the eddy current losses. This is due to the fact that $\theta = \theta_1 + \theta_2$ is usually much larger than θ_2 . At high frequency, the skin effect introduces a decrease of the coupling current losses, together with an increase of overall losses with an increase in $\omega^{1/2}$ (Fig. 7).

The main points are:

- (1) There is a limit for the coupling current losses.

$$P_L = \left(\frac{R_f}{R}\right)^2 \frac{E_0^2}{2\mu_0 \theta} \quad .$$

These losses can be noticeably decreased by increasing the time constant, that is, by increasing the twist pitch of the composite. This conclusion is also valid for the hysteretic losses.

Fig. 6.
Hysteretic losses.

Fig. 7.
Coupling losses and eddy current losses.

2) There is no limit for the eddy current losses. However, at high frequency ($\omega\theta_2 > 1$), the loss increases as

$$\frac{\theta_0^2}{2R\mu_0} \sqrt{\frac{\omega D}{2}} \text{ or } \frac{B_0^2}{4\mu_0} \sqrt{\frac{\omega}{2\theta_2}},$$

which shows that one can expect a decrease of the losses by increasing the time constant θ_2 . All the same, the improvement is limited by the square root of θ_2 . On the other hand, θ_2 is not a quite free parameter. θ_2 is directly proportional to the cross-sectional area of the composite, which is usually imposed by other considerations.

2.5.3 Influence of the time constant. One can plot the losses as a function of the time constant for a given frequency. There is a maximum $(\omega B_0^2)/4\mu_0$ for $\theta_1 = 1/\omega$ in the coupling current losses (Fig. 8). A given loss can be obtained in two different ways, with a small twist pitch (thin wires) assembled in a cable or a large twist pitch (monolithic composite).

The same is true for the eddy current losses. But the decrease with θ_2 is less important (Fig. 9).

As a result, regarding the eddy current losses, the size of the composite must be limited to a value such that the time constant θ_2 is less than $1/\omega$. For 1-Hz frequency, the radius R must be typically less than 10 mm and, for 50 Hz, R must be less than 1.4 mm. One can see that 50 Hz is just in the bad region. In higher frequencies ($\theta_2 \omega \gg 1$), problems can be solved more easily. Now the coupling current losses can be kept at a low value with either $\theta_1 \omega \ll 1$ or $\theta_1 \omega \gg 1$.

Fig. 8.
Coupling losses versus the coupling current time constant.

Fig. 9.
Eddy current losses versus the eddy current time constant.

The limit between the two losses is given by a twist pitch of 30 mm at 1 Hz and 4.5 mm at 50 Hz.

3. LOSSES IN FOUR TYPICAL CONDUCTORS

It is interesting to compare the losses in several kinds of conductors such as cables and monolithic composite for the two typical frequencies 1 Hz and 50 Hz. Four conductors are investigated. The volume of superconductor and copper are the same in all cases.

The diameter of the filaments is 10 μm .

Conductor 1 (Fig. 10)

- Subbundle 6 copper wires around one MSC. The ratio copper to superconductor is 2 in the MSC.
- Cable of 6 subbundles around a copper wire.
- Diameter of the MSC: 0.5 mm.
- Twist pitch two cases 10 x diam (5 mm) and 50 X diam (25 mm).
- Matrix and bonding between MSC and copper wires.
 - Two cases - copper and good bonding
 - Cu + CuNi and intermediate bonding.
- Each subbundle is insulated.

Conductor 2

The same as above, but the subbundles are not insulated. The bonding is supposed to be good or intermediate. The twist pitch of the cable is 10 cable diameters.

Conductor 3 (Fig. 11)

The conductor is a monolithic MSC with the same overall superconductor and copper volumes.

- The twist pitches are 10 x diam (45 mm) and 50 x diam (225 mm).
- The matrix is copper and Cu-CuNi.
- The radius of the filament bundle is small.

$$R_f = 0.61 \text{ mm when } R = 2.25 \text{ mm.}$$

Conductor 4 Fig. 12)

The same as above except the radius of the filament bundle $R_f = 2 \text{ mm}$ when $R = 2.25 \text{ mm}$. All losses are given in watts per cubic meter of whole conductor.

Fig. 10.
Conductor 1.

Fig. 11.
Conductor 3.

Fig. 12
Conductor 4.

Conductor 1

Twist Pitch Length Time constants			5 mm	50 mm
	θ_1 (s)	Cu	$4. \times 10^{-3}$	0.1
		CuNi	1.1×10^{-3}	0.028
	θ_2 (s)	Cu	0.6×10^{-3}	0.6×10^{-3}
		CuNi	0.4×10^{-3}	0.4×10^{-3}
Losses				
1	Matrix	P _H	144	119
		P _{C+E}	53	839
		P _E	74	74
		P _{Cu}	13	13
		P _T	284	1045
	CuNi	P _H	141	139
50	Matrix	P _{C+E}	17	319
		P _E	48	48
		P _{Cu}	13	13
		P _T	219	519
	Cu	P _H	4050	225
		P _{C+E}	43400	295
50	Matrix	P _E	175000	175000
		P _{Cu}	32000	32000
		P _T	255450	211150
	CuNi	P _H	6440	790
		P _{C+E}	35600	10300
		P _E	120000	120000
50	Matrix	P _{Cu}	32000	32000
		P _T	194000	163000
	Cu	P _H	6440	790
		P _{C+E}	35600	10300
		P _E	120000	120000
		P _{Cu}	32000	32000
		P _T	194000	163000

At 1 Hz, it is worthwhile reducing the twist pitch length and by using a Cu, CuNi matrix.

At 50 Hz, the solution is to increase the twist pitch length and to use a CuNi barrier just inside the outer copper shell.

Conductor 2

Twist Pitch length Time Constants			5 mm	50 mm
(cable)	θ_1 (S)	Cu	4	0.1
		CuNi	1.1×10	0.028
	θ_2 (S)	Cu	5.3×10	5.3×10
		CuNi	3.5×10^{-3}	3.5×10^{-3}
	θ_3 (S)	Cu	0.32	0.32
		CuNi	0.08	0.08
1	Losses f Hz Cu	P	62	349
		P ^H	4364	3649
		P ^{C+E}	458	458
		P ^E _T	4884	4446
	CuNi	P	125	813
		P ^H	4606	5216
		P ^{C+E}	305	305
		P ^E _T	5036	6335
50	Cu	P	71	53
		P ^H	5550	4175
		P ^{C+E}	307100	307100
		P ^E _T	312800	311400
	CuNi	P	266	202
		P ^H	20870	15860
		P ^{C+E}	617000	617000
		P ^E _T	638136	633062

There is apparently no interest in a soldering of the whole cable.

Conductor 3

Twist Pitch Time Constants			45 mm	225 mm
	θ_1 (s)	Cu	0.34	8.5
	θ_2 (s)	CuNi	0.224	5.6
		Cu	5.3×10^{-3}	5.3×10^{-3}
		CuNi		
Losses				
f Hz		P		
		H		
1	Cu	pCE	79	3
		pE	675	33
		T	773	773
			1527	809
1	CuNi	P	109	5
		H	838	50
		pCE	773	773
		pE	1720	828
50	Cu	T		
		P	87	3.5
		H	819	33
		pCE	513000	513000
50	CuNi	pE	514000	513000
		T		
		P	132	5.3
		H	1247	50
50	CuNi	pCE	513000	513000
		pE	514000	513000
50	CuNi	T		

Conductor 4

Twist Pitch length Time Constants			45 mm	225 mm	
θ_1 (s)		Cu	0.089	2.24	
		CuNi	0.038	0.944	
θ_2 (s)		Cu	5.3×10^{-3}	5.3×10^{-3}	
		CuNi	3.5×10^{-3}	3.5×10^{-3}	
Losses f Hz	1	Cu	P	139	13
			P _H	8661	1408
			P _{C+E}	346	346
			P _E T	9140	1260
	CuNi	P	177	31	
		P _H	4836	3280	
		P _{C+E}	230	230	
		P _E T	5240	3540	
50	Cu	P _H	317	13	
		P _{C+E}	33250	1400	
		P _E	231400	231400	
		P _T	265000	233000	
	CuNi	P _H	724	31	
		P _{C+E}	75840	3320	
		P _E	261000	261000	
		P _T	337500	264000	

As regards the low-frequency case, the smallest losses are obtained in the insulated strand conductor with a small twist pitch length and CuNi barriers. In for the high-frequency case, the smallest losses are obtained also in the insulated strand conductor with CuNi barrier, but with a large twist pitch length. Besides, all the results and especially those of the monolithic conductors show that the eddy current losses are not at all negligible, and can be the main part of the overall losses. One can see that the hysteretic losses can be reduced to a very low value by the shielding effect of the coupling current. At 50 Hz the main losses are the eddy current losses in the whole conductor, because the time constant θ_2 is not large enough to provide additional shielding. Nevertheless, there is some reduction of the overall losses by locating the outer layer of filaments close to the outer radius of the composite to provide the maximum shielding effect.

4. ADDENDUM: INFLUENCE OF THE LOCATION OF THE FILAMENTS IN THE MSC

Where must the filaments be located in the composite to provide the lowest overall losses?

Only the coupling current and eddy current losses are considered here.

The answer depends on the time variation of the external field.

4.1. Single-Shot Exponential Wave

The expression of the overall losses is

$$W = \frac{\Delta B^2}{2\mu_0} \left(\left(\frac{R_f}{R} \right)^2 \frac{\theta_1 + \theta_2}{\theta_1 + \theta_2 + \tau} + \left(1 - \left(\frac{R_f}{R} \right)^2 \right) \frac{\theta_2}{\theta_2 + \tau} \right) \text{ or}$$

$$W = \frac{\Delta B^2}{2\mu_0} \left(\left(\frac{R_f}{R} \right)^2 \frac{\theta_1 \tau}{(\theta_1 + \theta_2 + \tau)(\theta_2 + \tau)} + \frac{\theta_2}{\theta_2 + \tau} \right).$$

The smallest losses are always obtained by the smallest value of the ratio $\left(\frac{R_f}{R} \right)^2$ that is, for the filaments concentrated near the axis of the component with the largest outer copper shell.

4.2. Sinewave External Field

For intermediate frequencies ($\theta_2 \omega < 1$) the losses are

$$P = \frac{B_0^2 \omega^2}{2\mu_0} \left(\left(\frac{R_f}{R} \right)^2 \frac{\theta_1 + \theta_2}{(1 + (\theta_1 + \theta_2)^2 \omega^2)} + \left(1 - \left(\frac{R_f}{R} \right)^2 \right) \frac{\theta_2}{1 + \theta_2^2 \omega^2} \right)$$

or

$$P = \frac{B_0^2 \omega^2}{2\mu_0} \left(\left(\frac{R_f}{R} \right)^2 \frac{\theta_1 (1 - \theta_2^2 \omega^2 (\theta_1 + \theta_2))}{(1 + (\theta_1 + \theta_2)^2 \omega^2)(1 + \theta_2^2 \omega^2)} + \frac{\theta_2}{1 + \theta_2^2 \omega^2} \right)$$

For low frequencies, when $\theta_1 \theta_2 \omega^2 < 1$, the first term between brackets is positive, and the smallest losses are obtained as previously by the smallest

ratio $\left(\frac{R_f}{R} \right)^2$.

On the contrary, for high frequencies ($\theta_1\theta_2\omega^2 > 1$) the first term is negative and the shielding effect of the coupling currents also reduces the eddy

current losses. There is an interest to increase the ratio $\left(\frac{R_f}{R}\right)^2$.

However, on an average it does not seem that this improvement can be very important, for the effect is only limited to the region $\theta_1\theta_2\omega^2 > 1$ and $\theta_2\omega < 1$. As a matter of fact, for $\theta_2\omega > 1$ the skin effect prevents the filaments from shielding the interior of the component. As a result, for very high frequencies ($\theta_2\omega > 1$), the ratio R_f/R does not play any role and nothing is to be expected from the position of the filaments inside the composite.

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