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S-Matrix Parameterizations
in NN-Scattering*

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A Comment on S-Matrix Parameterizations in NN-Scattering

P. J. Mulders

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A COMMENT ON S-MATRIX PARAMETERIZATIONS IN NN-SCATTERING

by

P. J. Mulders

ABSTRACT

The parameterization of the S-matrix used for the elastic part of the NN-scattering matrix in, for example, the Virginia Polytechnic Institute interactive nucleon-nucleon program SAID, is not general enough to parameterize any 2 by 2 submatrix of a unitary matrix.

The choice of parameters in parameterizations of the S-matrix in NN-scattering is significant. When only the NN channels are considered, the S-matrix is one- or two-dimensional. For N channels, the N by N unitary symmetric S-matrix is determined by $N(N+1)/2$ parameters. The commonly used parameterizations in NN-scattering are

$$\underline{N=1}: \quad S = e^{2i\delta} \quad (1)$$

To avoid ambiguities, the range of the parameter δ , the phase shift, is restricted to $0 \leq \delta < \pi$.

$$\underline{N=2}: \quad S = \begin{pmatrix} \cos \epsilon & -\sin \epsilon \\ \sin \epsilon & \cos \epsilon \end{pmatrix} \begin{pmatrix} e^{2i\delta_1} & 0 \\ 0 & e^{2i\delta_2} \end{pmatrix} \begin{pmatrix} \cos \epsilon & \sin \epsilon \\ -\sin \epsilon & \cos \epsilon \end{pmatrix} \quad (2a)$$

$$= \begin{pmatrix} \cos 2\bar{\epsilon} e^{2i\bar{\delta}_1} & i \sin 2\epsilon e^{i(\bar{\delta}_1 + \bar{\delta}_2)} \\ i \sin 2\bar{\epsilon} e^{i(\bar{\delta}_1 + \bar{\delta}_2)} & \cos 2\bar{\epsilon} e^{2i\bar{\delta}_2} \end{pmatrix} \quad (2b)$$

In Eq. (2a), δ_1 and δ_2 are the eigenphase shifts. The phase shifts in Eq. (2b) are the nuclear bar phase shifts.¹ The relations between the parameters are

$$\bar{\delta}_1 + \bar{\delta}_2 = \delta_1 + \delta_2,$$

$$\sin 2\bar{\epsilon} = \sin 2\epsilon \sin (\bar{\delta}_1 + \bar{\delta}_2), \text{ and}$$

$$\sin (\bar{\delta}_1 - \bar{\delta}_2) = \frac{\tan 2\bar{\epsilon}}{\tan 2\epsilon}. \quad (3)$$

The range for the δ 's is $0 \leq \delta < \pi$; for the angles ϵ it is $-\pi/4 < \epsilon \leq \pi/4$.

When scattering into several inelastic channels occurs, the one- or two-dimensional elastic part of the symmetric S-matrix is a 1 by 1 or 2 by 2 complex symmetric matrix. Then the unitarity of the whole S-matrix requires at least

$$|S_{11}| \leq 1 \quad (4)$$

for the uncoupled NN-channel, or

$$|S_{11}|^2 + |S_{12}|^2 \leq 1 \quad (5)$$

$$|S_{21}|^2 + |S_{22}|^2 \leq 1$$

for the coupled NN-channel.

For the uncoupled channel, the parameterization

$$S = \eta e^{2i\delta} = \cos \rho e^{2i\delta} = \lambda^2 e^{2i\delta} \quad (6)$$

can be used. The range of δ is $0 \leq \delta < \pi$; the elasticity η , $\cos \rho$, or λ^2 is restricted by the unitarity condition $0 \leq \eta = \cos \rho = \lambda^2 \leq 1$.

For the case of coupled channels, a common parameterization is²

$$\begin{pmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{pmatrix} = \begin{pmatrix} \cos \rho_1 \cos 2\varepsilon e^{2i\delta_1} & i \sin 2\varepsilon e^{i(\delta_1+\delta_2+\alpha)} \\ i \sin 2\varepsilon e^{i(\delta_1+\delta_2+\alpha)} & \cos \rho_2 \cos 2\varepsilon e^{2i\delta_2} \end{pmatrix} \quad (7)$$

In this parameterization, the choice of $\cos \rho_1$ and $\cos \rho_2$ is justified by the unitarity condition in Eq. (5). The range for the parameters is $0 \leq \delta_1 < \pi$, $0 \leq \delta_2 < \pi$, $-\pi/4 < \varepsilon \leq \pi/4$, $0 \leq \alpha \leq \pi$, $0 \leq \rho_1 < \pi/2$, and $0 \leq \rho_2 < \pi/2$, although not every choice of parameters allows one to find a (larger than 2 by 2) unitary matrix. When inelastic channels do not contribute, $\rho_1 \rightarrow 0$, $\rho_2 \rightarrow 0$, and $\alpha \rightarrow 0$, and the parameters are the nuclear bar phase shifts.

Another way of parameterizing is to make ε complex,

$$\begin{pmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{pmatrix} = \begin{pmatrix} \lambda_1^2 \cos 2\varepsilon' e^{2i\delta_1} & i\lambda_1\lambda_2 \sin 2\varepsilon' e^{i(\delta_1+\delta_2)} \\ i\lambda_1\lambda_2 \sin 2\varepsilon' e^{i(\delta_1+\delta_2)} & \lambda_2^2 \cos 2\varepsilon' e^{2i\delta_2} \end{pmatrix}, \quad (8)$$

with

$$\varepsilon' = \varepsilon + i\phi.$$

The ranges for δ_1 , δ_2 , ε , and ϕ are $0 \leq \delta_1 < \pi$, $0 \leq \delta_2 < \pi$, $-\pi/4 < \varepsilon \leq \pi/4$, and $\phi \geq 0$ to parameterize a complex symmetric matrix. The unitarity conditions from Eq. (5) give

$$\lambda_1^2\lambda_2^2 + (\lambda_1^4 - \lambda_1^2\lambda_2^2) \cos^2 2\varepsilon + (\lambda_1^4 + \lambda_1^2\lambda_2^2) \sinh^2 2\phi \leq 1, \text{ and}$$

$$\lambda_1^2\lambda_2^2 + (\lambda_2^4 - \lambda_1^2\lambda_2^2) \cos^2 2\varepsilon + (\lambda_2^4 + \lambda_1^2\lambda_2^2) \sinh^2 2\phi \leq 1. \quad (9)$$

By adding these two inequalities, we see that unitarity requires as a necessary condition $\lambda_1^2 \lambda_2^2 \leq 1$. It does not, however, require λ_1^2 and λ_2^2 both to be smaller than unity. In fact, it is easy to find a case in which, for example, $\lambda_1 > 1$, so it is not possible to express the λ 's as cosines of angles, as is done in Ref. 3, without loss of generality. In the absence of inelastic scattering, $\lambda_1 \rightarrow 1$, $\lambda_2 \rightarrow 1$, and $\phi \rightarrow 0$, and the nuclear bar phase shifts remain.

If the nuclear bar phase shifts in Eq. (2b) are complex ($\delta + i\rho_1$, $\delta_2 + i\rho_2$, and $\varepsilon + i\phi$), Eq. (2b) becomes identical to Eq. (8) with $\lambda_i = \exp(-\rho_i)$. Using this parameterization, values of λ larger than unity are possible. In that case $\rho \leq 0$. Because $\lambda_1^2 \lambda_2^2 \leq 1$, we always have $\rho_1 + \rho_2 \geq 0$. When there is no elasticity, all angles become real.

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