8/24/87 (1)

(2)

DR-02925

31150

UCID--21123

DE87 013835

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MEASURE THE CURRENT PROFILE IN MTX

W. M. NEVINS
E. B. HOOPER
I. B. BERNSTEIN

JULY 14, 1987

Lawrence
Livermore
National
Laboratory

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Work performed under the auspices of the U.S. Department of Energy by the Lawrence Livermore National Laboratory under Contract W-7405-Eng-48.

Use of Polarimetry to
Measure the Current Profile in MTX

W.M. Nevins, E.B. Hooper
*Lawrence Livermore National Laboratory
Livermore, CA 94550*

and

I.B. Bernstein
*Mason Laboratory, Yale University
New Haven, CT 06520-2159*

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MASTER

1. Introduction

It is possible in principle to measure the poloidal magnetic field profile, and hence, the profile of the plasma current by measuring the change in the polarization of a sequence of microwave beams that pass through the plasma¹⁻³. Actual measurements of the plasma current profile would be very interesting in connection with Lower-Hybrid (or EC) current drive experiments since this would provide direct information on modification of the current profile by the application of RF power.

We are considering a microwave polarimetry diagnostic on MTX as part of the microwave interferometer. This diagnostic would be constructed in collaboration with Neville Luhmann and Tony Peebles at UCLA. The diagnostic would utilize the multicord far-infrared interferometer which is designed to operate at a base wavelength of 0.185 mm. In this paper we review our understanding of the physics issues raised by this diagnostic. We concur with Luhmann and Peebles' conclusion that the polarimetry measurements would be easier at longer wave lengths. An increase of only a factor of 2 in the wave length would make a substantial difference since the signal that we wish to measure goes as λ^4 . Hence, we will consider operation at longer wavelengths (0.337 mm and 0.447 mm) in this paper in addition to operation at 0.119 and 0.185 mm.

The theoretical basis of a microwave (or far infrared) polarimetry diagnostic has been developed in Refs. 1-3. Reference 1 provides a good description of the Poincaré sphere, which is a very useful in understanding polarization changes induced by the plasma. However, Segre's second paper³ appears to provide a more practical treatment of microwave polarimetry. Craig² claims that his result differ from those of De Marco and Segre¹ primarily through a more accurate treatment of higher order terms in ω_{p0}^2/ω^2 . We believe that this is a red herring. Craig's real contribution is to show that change in the power passed by crossed polarizers surrounding the plasma is higher order in the relative phase shift between X and O modes than the terms computed by De Marco and Segre in Ref. 1. The higher order terms in ω_{p0}^2/ω^2 that Craig² emphasizes and Segre³ retains are of the same size as other terms, arising from changes in the optical path through the plasma, which both authors ignore (see Appendix). These corrections will be small in the proposed MTX current-drive experiments since at the proposed microwave interferometer/polarimeter frequency $\omega_{pe}^2/\omega^2 \sim 10^{-3}$.

Segre's second paper³ addressed the problem of the power passed by crossed polarizers that surround the plasma. If one assumes perfect polarizers, then no power will be passed through the second polarizer in the absence of the plasma. Hence, the power passing through the second polarizer is a measure of the polarization changed in the microwave beam induced by the plasma. Clearly, the resolution of such a diagnostic will be limited by the quality of the polarizers, since some power will in fact be passed through any realizable system of crossed polarizers.

Modern experimental techniques⁴ allow direct measurements of the amplitude of the probe beam, thus effectively increasing the measured power and sensitivity. The fun-

damental problems, e.g., ellipticity generation and refraction, are not affected by this improvement, and must be considered carefully.

In section 2 we develop the theory of polarimetry using an eikonal representation of the wave fields and assuming that $\Omega_{ce}/\omega \gg 1$. In section 3 we relate our theoretical treatment of polarimetry to the work of De Marco and Segre³. In section 4 we consider the application of polarimetry to measuring the poloidal magnetic field in tokamaks. In section 4.1 we review previous polarimetry experiments in tokamaks, and in section 4.2 we consider some applications to MTX.

2. The Theory of the Polarization Experiment

The theory of the projected "Faraday Rotation" experiment proceeds via geometric optics. It will be taken from *Geometric Optics in Space and Time Varying Plasmas* by Ira B. Bernstein and Lazar Friedland, Handbook of Plasma Physics, Volume 1, p. 367, Edited by A.A. Galeev and R.N. Sudan, North Holland Publishing Company, New York 1983. This reference will be hereafter designated as I.

The electron conductivity tensor for a cold lossless magnetized plasma is

$$\sigma = \frac{i\omega_{pe}^2}{4\pi\omega} \left(\frac{\mathbf{I} - \hat{\mathbf{b}}\hat{\mathbf{b}}}{1 - \Omega^2/\omega^2} + i\frac{\Omega}{\omega} \frac{\hat{\mathbf{b}} \times \mathbf{I}}{1 - \Omega^2/\omega^2} + \hat{\mathbf{b}}\hat{\mathbf{b}} \right), \quad (2.1)$$

where the magnetic field $\mathbf{B}(\mathbf{r}) = B\hat{\mathbf{b}}$ and $\Omega = eB/mc$. The dielectric tensor,

$$\epsilon = \left(1 - \frac{k^2 c^2}{\omega^2} \right) \mathbf{I} + \frac{c^2 \mathbf{k} \mathbf{k}}{\omega^2} + \frac{4\pi i \sigma}{\omega}. \quad (2.2)$$

Consider an electric field of the eikonal form $\mathbf{E}(\mathbf{r}) = \text{Re}[\mathbf{a}(\mathbf{r})e^{i\psi(\mathbf{r})}]$ with $\mathbf{k} = \nabla\psi$. The Geometric optics equation for the amplitude $\mathbf{a}$, on using (I-B1), assuming a steady state, and noting that σ is independent of $\mathbf{k}$, can be written as

$$i\omega\epsilon \cdot \mathbf{a} = -\frac{c^2}{\omega} \mathbf{k} \times (\nabla \times \mathbf{a}) - c^2 \nabla \times \left(\frac{\mathbf{k} \times \mathbf{a}}{\omega} \right) \quad (2.3)$$

The ray equations are

$$\frac{d\mathbf{r}}{dt} = \frac{\partial\omega}{\partial\mathbf{k}} \quad (2.4)$$

$$\frac{d\mathbf{k}}{dt} = -\frac{\partial\omega}{\partial\mathbf{r}} \quad (2.5)$$

Note that, consequent to (2.4) and (2.5), $(d\omega/dt) = (d\mathbf{r}/dt) \cdot (\partial\omega/\partial\mathbf{r}) - (d\mathbf{k}/dt) \cdot (\partial\omega/\partial\mathbf{k}) = 0$. As we shall see later, for the case of interest,

$$\omega = \sqrt{\omega_{pe}^2 + k^2 c^2}. \quad (2.6)$$

It is convenient to define

$$\epsilon_0 \equiv \left(1 - \frac{\omega_{pe}^2 + k^2 c^2}{\omega^2}\right) \mathbf{I} + \frac{c^2 \mathbf{k} \mathbf{k}}{\omega^2}, \quad (2.7)$$

and

$$\epsilon_1 \equiv -\frac{\omega_{pe}^2}{\omega^2} \left(\frac{\Omega_{ce}^2}{\omega^2} \frac{\mathbf{I} - \hat{\mathbf{b}} \hat{\mathbf{b}}}{1 - \Omega_{ce}^2/\omega^2} + i \frac{\Omega_{ce}}{\omega} \frac{\hat{\mathbf{b}} \times \mathbf{I}}{1 - \Omega_{ce}^2/\omega^2} \right), \quad (2.8)$$

whence

$$\epsilon = \epsilon_0 + \epsilon_1; \quad (2.9)$$

and it is convenient to write Eq. (2.3) as

$$i\omega\epsilon_0 \cdot \mathbf{a} = -\frac{c^2}{\omega} \mathbf{k} \times (\nabla \times \mathbf{a}) - \frac{c^2}{\omega} \nabla \times \left(\frac{\mathbf{k} \times \mathbf{a}}{\omega} \right) - i\omega\epsilon_1 \cdot \mathbf{a}. \quad (2.10)$$

Also,

$$\begin{aligned} \mathbf{k} \times (\nabla \times \mathbf{a}) + \nabla \times (\mathbf{k} \times \mathbf{a}) &= (\nabla \mathbf{a}) \cdot \mathbf{k} - \mathbf{k} \cdot \nabla \mathbf{a} + \mathbf{a} \cdot \nabla \mathbf{k} - \mathbf{k} \cdot \nabla \mathbf{a} + \mathbf{k} \nabla \cdot \mathbf{a} - \mathbf{a} \nabla \cdot \mathbf{k} \\ &= \nabla(\mathbf{k} \cdot \mathbf{a}) - (\nabla \mathbf{k}) \cdot \mathbf{a} + \mathbf{a} \cdot \nabla \mathbf{k} - 2\mathbf{k} \cdot \nabla \mathbf{a} + \mathbf{k} \nabla \cdot \mathbf{a} - \mathbf{a} \nabla \cdot \mathbf{k} \\ &= \nabla(\mathbf{k} \cdot \mathbf{a}) - \mathbf{a} \times (\nabla \times \mathbf{k}) - 2\mathbf{k} \cdot \nabla \mathbf{a} + \mathbf{k} \nabla \cdot \mathbf{a} - \mathbf{a} \nabla \cdot \mathbf{k} \end{aligned} \quad (2.11)$$

and Eq. (2.10) can be expressed as

$$\begin{aligned} i\frac{\omega^2}{c^2} \epsilon_0 \cdot \mathbf{a} &= \nabla(\mathbf{k} \cdot \mathbf{a}) - 2\mathbf{k} \cdot \nabla \mathbf{a} + \mathbf{k} \nabla \cdot \mathbf{a} - \mathbf{a} \nabla \cdot \mathbf{k} \\ &\quad - \frac{(\nabla \omega) \times (\mathbf{k} \times \mathbf{a})}{\omega} + i\frac{\omega^2}{c^2} \epsilon_1 \cdot \mathbf{a} \end{aligned} \quad (2.12)$$

since $\nabla \times \mathbf{k} = 0$. Moreover, if we introduce a right handed orthonormal set of unit vectors $\hat{\mathbf{e}}_1(\mathbf{r})$, $\hat{\mathbf{e}}_2(\mathbf{r})$, and $\hat{\mathbf{e}}_3(\mathbf{r}) \equiv \mathbf{k}/k$, then

$$\epsilon_0 = \left(1 - \frac{\omega_{pe}^2 + k^2 c^2}{\omega^2}\right) \hat{\mathbf{e}}_1 \hat{\mathbf{e}}_1 + \left(1 - \frac{\omega_{pe}^2 + k^2 c^2}{\omega^2}\right) \hat{\mathbf{e}}_2 \hat{\mathbf{e}}_2 + \left(1 - \frac{\omega_{pe}^2}{\omega^2}\right) \hat{\mathbf{e}}_3 \hat{\mathbf{e}}_3 \quad (2.13)$$

and in component form Eq. (2.12) yields

$$\begin{aligned} -i\frac{\omega^2}{c^2} \left(1 - \frac{\omega_{pe}^2 + k^2 c^2}{\omega^2}\right) a_1 &= \\ \hat{\mathbf{e}}_1 \cdot \left[\nabla(\mathbf{k} \cdot \mathbf{a}) - 2\mathbf{k} \cdot \nabla \mathbf{a} - \mathbf{a} \nabla \cdot \mathbf{k} + \mathbf{a} \frac{\mathbf{k} \cdot \nabla \omega}{\omega} - i\frac{\omega^2}{c^2} \epsilon_1 \cdot \mathbf{a} \right], \end{aligned} \quad (2.14)$$

$$\begin{aligned} -i\frac{\omega^2}{c^2} \left(1 - \frac{\omega_{pe}^2 + k^2 c^2}{\omega^2}\right) a_2 &= \\ \hat{\mathbf{e}}_2 \cdot \left[\nabla(\mathbf{k} \cdot \mathbf{a}) - 2\mathbf{k} \cdot \nabla \mathbf{a} - \mathbf{a} \nabla \cdot \mathbf{k} + \mathbf{a} \frac{\mathbf{k} \cdot \nabla \omega}{\omega} - i\frac{\omega^2}{c^2} \epsilon_1 \cdot \mathbf{a} \right], \end{aligned} \quad (2.15)$$

and

$$i \frac{\omega^2}{c^2} \left(1 - \frac{\omega_{pe}^2}{\omega^2} \right) a_3 = \hat{\mathbf{e}}_3 \cdot \left[\nabla(\mathbf{k} \cdot \mathbf{a}) - 2\mathbf{k} \cdot \nabla \mathbf{a} - \mathbf{k} \nabla \cdot \mathbf{a} - \mathbf{a} \nabla \cdot \mathbf{k} + \mathbf{k} \frac{\mathbf{a} \cdot \nabla \omega}{\omega} + \mathbf{a} \frac{\mathbf{k} \cdot \nabla \omega}{\omega} + i \frac{\omega^2}{c^2} \epsilon_1 \cdot \mathbf{a} \right] \quad (2.16)$$

The choice $\omega = \sqrt{\omega_{pe}^2 + k^2 c^2}$ corresponds to a doubly degenerate set of geometric optics modes, namely the transverse waves of a non-magnetized plasma. The coefficient of a_3 on the left hand side of Eq. (2.16) then reduces to ik^2 , and a_3 is formally of order $|1/kL| \ll 1$, where L is the length characterizing the spacial variation of a_1 and a_2 . Thus to lowest significant order in the small parameter $1/kL$ of geometric optics one can write

$$\mathbf{a} = a_1 \hat{\mathbf{e}}_1 + a_2 \hat{\mathbf{e}}_2 \quad (2.17)$$

and the wave is transverse.

Let s denote arc length measured along a ray. Then $\mathbf{k} \cdot \nabla = k(d/ds)$. Note that $(d/dt)f(\mathbf{r}) = (\partial\omega/\partial\mathbf{k}) \cdot \nabla f(\mathbf{r}) = (c^2/\omega)\mathbf{k} \cdot \nabla f(\mathbf{r}) = (c^2/\omega)k(df/ds)$, which can be used in the ray equations (2.4) and (2.5). In particular it follows from $d\omega/dt = 0$ that $\mathbf{k} \cdot \nabla \omega = 0$. Then, to lowest order in the ratio of wave length to scale length, Eqs. (2.14) and (2.15) become

$$\begin{aligned} & \frac{c^2}{\omega^2} \left(2k \frac{da_1}{ds} + 2ka_2 \hat{\mathbf{e}}_1 \cdot \frac{d\hat{\mathbf{e}}_2}{ds} + a_1 \nabla \cdot \mathbf{k} \right) \\ &= -i \hat{\mathbf{e}}_1 \cdot \frac{\omega_{pe}^2}{\omega^2} \left(\frac{\Omega_{ce}^2}{\omega^2} \frac{\mathbf{I} - \hat{\mathbf{b}}\hat{\mathbf{b}}}{1 - \Omega_{ce}^2/\omega^2} + i \frac{\Omega_{ce}}{\omega} \frac{\hat{\mathbf{b}} \times \mathbf{I}}{1 - \Omega_{ce}^2/\omega^2} \right) \cdot (a_1 \hat{\mathbf{e}}_1 - a_2 \hat{\mathbf{e}}_2) \quad (2.18) \\ &= -i \frac{\omega_{pe}^2}{\omega^2} \left(\frac{\Omega_{ce}^2}{\omega^2} \frac{a_1 - b_1(b_1 a_1 + b_2 a_2)}{1 - \Omega_{ce}^2/\omega^2} - i \frac{\Omega_{ce}}{\omega} \frac{b_3 a_2}{1 - \Omega_{ce}^2/\omega^2} \right) \end{aligned}$$

and

$$\begin{aligned} & \frac{c^2}{\omega^2} \left(2k \frac{da_2}{ds} + 2ka_1 \hat{\mathbf{e}}_2 \cdot \frac{d\hat{\mathbf{e}}_1}{ds} + a_2 \nabla \cdot \mathbf{k} \right) \\ &= -i \hat{\mathbf{e}}_2 \cdot \frac{\omega_{pe}^2}{\omega^2} \left(\frac{\Omega_{ce}^2}{\omega^2} \frac{\mathbf{I} - \hat{\mathbf{b}}\hat{\mathbf{b}}}{1 - \Omega_{ce}^2/\omega^2} + i \frac{\Omega_{ce}}{\omega} \frac{\hat{\mathbf{b}} \times \mathbf{I}}{1 - \Omega_{ce}^2/\omega^2} \right) \cdot (a_1 \hat{\mathbf{e}}_1 + a_2 \hat{\mathbf{e}}_2) \quad (2.19) \\ &= -i \frac{\omega_{pe}^2}{\omega^2} \left(\frac{\Omega_{ce}^2}{\omega^2} \frac{a_2 - b_2(b_1 a_1 + b_2 a_2)}{1 - \Omega_{ce}^2/\omega^2} + i \frac{\Omega_{ce}}{\omega} \frac{b_3 a_1}{1 - \Omega_{ce}^2/\omega^2} \right) \end{aligned}$$

In Eqs. (2.18) and (2.19) $b_1 = \hat{\mathbf{e}}_1 \cdot \hat{\mathbf{b}}$, etc. Note that in general the equations are coupled by the terms involving $\hat{\mathbf{e}}_1 \cdot (d\hat{\mathbf{e}}_2/ds) = -\hat{\mathbf{e}}_2 \cdot (d\hat{\mathbf{e}}_1/ds)$. One can find $\nabla \cdot \mathbf{k}$ as required for Eqs. (2.18) and (2.19) by taking the trace of the dyadic $\nabla \mathbf{k}$, which can be calculated by integration along a ray from equation (I-71), namely

$$\frac{kc^2}{\omega} \frac{d}{ds} \nabla \mathbf{k} = - \frac{\partial^2 \omega}{\partial r \partial \mathbf{r}} - (\nabla \mathbf{k}) \cdot \frac{\partial}{\partial \mathbf{k}} \frac{\partial \omega}{\partial \mathbf{r}} - \left[\frac{\partial}{\partial \mathbf{r}} \frac{\partial \omega}{\partial \mathbf{k}} + (\nabla \mathbf{k}) \cdot \frac{\partial^2 \omega}{\partial \mathbf{k} \partial \mathbf{k}} \right] \cdot \nabla \mathbf{k}. \quad (2.20)$$

where we have used Eq. (2.21) below to write $(\partial\omega/\partial\mathbf{k}) \cdot \nabla = (kc^2/\omega^2)(d/ds)$. Note that on using Eq. (2.6) we find

$$\frac{\partial\omega}{\partial\mathbf{k}} = \frac{c^2}{\omega} \mathbf{k} \quad (2.21)$$

and

$$\frac{\partial\omega}{\partial\mathbf{r}} = \frac{\nabla\omega_{pe}^2}{2\omega}, \quad (2.22)$$

whence,

$$\frac{\partial}{\partial\mathbf{k}} \frac{\partial\omega}{\partial\mathbf{r}} = 0, \quad (2.23)$$

$$\frac{\partial}{\partial\mathbf{r}} \frac{\partial\omega}{\partial\mathbf{k}} = 0, \quad (2.24)$$

$$\frac{\partial^2\omega}{\partial\mathbf{r}\partial\mathbf{r}} = \frac{\nabla\nabla\omega_{pe}^2}{2\omega} - \frac{(\nabla\omega_{pe}^2)(\nabla\omega_{pe}^2)}{4\omega^3}, \quad (2.25)$$

and

$$\frac{\partial^2\omega}{\partial\mathbf{k}\partial\mathbf{k}} = \frac{c^2}{\omega} \left[\mathbf{I} - \frac{c^2\mathbf{k}\mathbf{k}}{\omega^2} \right]. \quad (2.26)$$

In the idealized experiment, in spherical polar coordinates r, ϕ, z , one has $\hat{\mathbf{e}}_2 = \hat{\mathbf{e}}_\phi$, and $\hat{\mathbf{e}}_1 = \hat{\mathbf{e}}_\phi \times (\mathbf{k}/k)$, while $\mathbf{B} = B_\phi(r, z)\hat{\mathbf{e}}_\phi + [\nabla\Psi(r, z)] \times \hat{\mathbf{e}}_\phi/r$, where Ψ is the poloidal flux function.

We can write Eqs. (2.18) and (2.19) in the form

$$\begin{aligned} & \frac{c^2}{\omega^2} \left(2k \frac{da_1}{ds} + a_1 \nabla \cdot \mathbf{k} \right) \\ &= -i \frac{\Omega_{ce}^2}{\omega^2} \frac{\omega_{pe}^2}{\omega^2} \frac{[a_1 - b_1(b_1a_1 + b_2a_2)]}{1 - \Omega_{ce}^2/\omega^2} - \frac{\Omega_{ce}}{\omega} \frac{\omega_{pe}^2}{\omega^2} \frac{b_3a_2}{1 - \Omega_{ce}^2/\omega^2} \end{aligned} \quad (2.27)$$

and

$$\begin{aligned} & \frac{c^2}{\omega^2} \left(2k \frac{da_2}{ds} + a_2 \nabla \cdot \mathbf{k} \right) \\ &= -i \frac{\Omega_{ce}^2}{\omega^2} \frac{\omega_{pe}^2}{\omega^2} \frac{[a_2 - b_2(b_1a_1 + b_2a_2)]}{1 - \Omega_{ce}^2/\omega^2} + \frac{\Omega_{ce}}{\omega} \frac{\omega_{pe}^2}{\omega^2} \frac{b_3a_1}{1 - \Omega_{ce}^2/\omega^2} \end{aligned} \quad (2.28)$$

These equations must be integrated subject to suitable initial conditions. For example one can note that on entering the plasma as shown in Fig. 2.1, $\mathbf{B} = B\hat{\mathbf{e}}_\phi$, and a reasonable choice is $\mathbf{k} = (\omega/c)\hat{\mathbf{e}}_z$, $\nabla\mathbf{k} = 0$.

It is readily seen from Eq. (2.12), since ϵ is Hermitian, that

$$\nabla \cdot \left(\frac{|\mathbf{a}|^2 \mathbf{k}}{\omega} \right) = 0 \quad (2.29)$$

Fig. 2.1. Schematic Diagram of Cross Section of Tokamak

which represents conservation of energy. Since $\mathbf{k} \cdot \nabla \omega = 0$ this simplifies to $\nabla \cdot (|a|^2 \mathbf{k}) = 0$.

3. Previous Theoretical Work

Previous theoretical work (DeMarco and Segre, Segre) has describes the polarization of the microwave or far infrared beam in terms of the Stokes parameters,

$$s_0 \equiv |a_1|^2 + |a_2|^2, \quad (3.1)$$

$$s_1 \equiv \frac{|a_1|^2 - |a_2|^2}{|a_1|^2 + |a_2|^2}, \quad (3.2)$$

$$s_2 \equiv \frac{2\text{Re}(a_1^* a_2)}{|a_1|^2 + |a_2|^2}, \quad (3.3)$$

and

$$s_3 \equiv \frac{2\text{Im}(a_1^* a_2)}{|a_1|^2 + |a_2|^2}; \quad (3.4)$$

where we have followed De Marco and Segre in the definitions of s_1 , s_2 , and s_3 .

We may relate the analysis presented in the previous section to that of De Marco and Segre by using Eqs. (2.27) and (2.28) to derive equations for the evolution of the Stokes

parameters along the ray path. An equation for s_0 is obtained by multiplying Eq. (2.27) by a_1^* and taking the real part. This procedure yields

$$\frac{c^2}{\omega^2} \left(k \frac{d}{ds} + \nabla \cdot \mathbf{k} \right) |a_1|^2 = - \frac{\Omega_{ce}^2}{\omega^2} \frac{\omega_{pe}^2}{\omega^2} \frac{b_1 b_2 \text{Im}(a_1^* a_2)}{1 - \Omega_{ce}^2/\omega^2} - \frac{\Omega_{ce}}{\omega} \frac{\omega_{pe}^2}{\omega^2} \frac{b_3 \text{Re}(a_1^* a_2)}{1 - \Omega_{ce}^2/\omega^2}. \quad (3.5)$$

while a similar procedure applied to Eq. (2.28) yields

$$\frac{c^2}{\omega^2} \left(k \frac{d}{ds} + \nabla \cdot \mathbf{k} \right) |a_2|^2 = - \frac{\Omega_{ce}^2}{\omega^2} \frac{\omega_{pe}^2}{\omega^2} \frac{b_1 b_2 \text{Im}(a_1 a_2^*)}{1 - \Omega_{ce}^2/\omega^2} - \frac{\Omega_{ce}}{\omega} \frac{\omega_{pe}^2}{\omega^2} \frac{b_3 \text{Re}(a_1 a_2^*)}{1 - \Omega_{ce}^2/\omega^2}. \quad (3.6)$$

Adding Eq. (3.5) to Eq. (3.6) yields

$$\frac{c^2}{\omega^2} \left(k \frac{d}{ds} + \nabla \cdot \mathbf{k} \right) s_0 = 0, \quad (3.7)$$

which is equivalent to Eq. (2.29) above.

Before going on to derive equations of evolution for the remaining Stokes parameters (which, following De Marco and Segre, we have normalized such that $s_1^2 + s_2^2 + s_3^2 = 1$) we note that it follows from Eq. (3.7) that for any function $u(s)$,

$$\frac{1}{s_0} \left(k \frac{d}{ds} + \nabla \cdot \mathbf{k} \right) u(s) = k \frac{d}{ds} \left(\frac{u}{s_0} \right). \quad (3.8)$$

An equation for s_1 may be obtained by subtracting Eq. (3.6) from Eq. (3.5), yielding

$$\frac{k c^2}{\omega^2} \frac{d}{ds} s_1 = - \frac{\Omega_{ce}^2}{\omega^2} \frac{\omega_{pe}^2}{\omega^2} \frac{b_1 b_2 s_3}{1 - \Omega_{ce}^2/\omega^2} - \frac{\Omega_{ce}}{\omega} \frac{\omega_{pe}^2}{\omega^2} \frac{b_3 s_2}{1 - \Omega_{ce}^2/\omega^2}. \quad (3.9)$$

We must go back to Eqs. (2.27) and (2.28) in order to obtain equations for the evolution of s_2 and s_3 along the ray path. Multiplying the complex conjugate of Eq. (2.27) by a_2 we have

$$\begin{aligned} & \frac{c^2}{\omega^2} \left(2k a_2 \frac{d a_1^*}{ds} + a_1^* a_2 \nabla \cdot \mathbf{k} \right) \\ &= i \frac{\Omega_{ce}^2}{\omega^2} \frac{\omega_{pe}^2}{\omega^2} \frac{[a_1^* a_2 - b_1(b_1 a_1^* a_2 + b_2 |a_2|^2)]}{1 - \Omega_{ce}^2/\omega^2} - \frac{\Omega_{ce}}{\omega} \frac{\omega_{pe}^2}{\omega^2} \frac{b_3 |a_2|^2}{1 - \Omega_{ce}^2/\omega^2}. \end{aligned} \quad (3.10)$$

Performing a similar operation on Eq. (2.28) yields

$$\begin{aligned} & \frac{c^2}{\omega^2} \left(2k a_1^* \frac{d a_2}{ds} + a_1^* a_2 \nabla \cdot \mathbf{k} \right) \\ &= i \frac{\Omega_{ce}^2}{\omega^2} \frac{\omega_{pe}^2}{\omega^2} \frac{[a_1^* a_2 - b_2(b_1 |a_1|^2 + b_2 a_1^* a_2)]}{1 - \Omega_{ce}^2/\omega^2} + \frac{\Omega_{ce}}{\omega} \frac{\omega_{pe}^2}{\omega^2} \frac{b_3 |a_1|^2}{1 - \Omega_{ce}^2/\omega^2}. \end{aligned} \quad (3.11)$$

Summing Eqs. (3.10) and (3.11) we obtain

$$\begin{aligned} \frac{kc^2}{\omega^2} \frac{d}{ds} \left(\frac{2a_1^* a_2}{s_0} \right) = & - \frac{\Omega_{ce}^2}{\omega^2} \frac{\omega_{pe}^2}{\omega^2} \frac{(b_1^2 - b_2^2)}{1 - \Omega_{ce}^2/\omega^2} \left(\frac{ia_1^* a_2}{s_0} \right) \\ & + \frac{\Omega_{ce}}{\omega} \frac{\omega_{pe}^2}{\omega^2} \frac{b_3 + i(\Omega_{ce}/\omega)b_1 b_2}{1 - \Omega_{ce}^2/\omega^2} \left(\frac{|a_1|^2 - |a_2|^2}{s_0} \right). \end{aligned} \quad (3.12)$$

Equations for the evolution of s_2 and s_3 are obtained by taking the real and imaginary parts of Eq. (3.12):

$$\frac{kc^2}{\omega^2} \frac{d}{ds} s_2 = + \frac{1}{2} \frac{\Omega_{ce}^2}{\omega^2} \frac{\omega_{pe}^2}{\omega^2} \frac{(b_1^2 - b_2^2)}{1 - \Omega_{ce}^2/\omega^2} s_3 + \frac{\Omega_{ce}}{\omega} \frac{\omega_{pe}^2}{\omega^2} \frac{b_3}{1 - \Omega_{ce}^2/\omega^2} s_1, \quad (3.13)$$

and

$$\frac{kc^2}{\omega^2} \frac{d}{ds} s_3 = - \frac{1}{2} \frac{\Omega_{ce}^2}{\omega^2} \frac{\omega_{pe}^2}{\omega^2} \frac{(b_1^2 - b_2^2)}{1 - \Omega_{ce}^2/\omega^2} s_2 + \frac{\Omega_{ce}}{\omega^2} \frac{\omega_{pe}^2}{\omega^2} \frac{b_1 b_2}{1 - \Omega_{ce}^2/\omega^2} s_1. \quad (3.14)$$

Finally, we may write Eqs. (3.9), (3.13), and (3.14) in the form

$$\frac{d}{ds} \vec{s} = \vec{\Omega} \times \vec{s}, \quad (3.15)$$

where $\vec{s}$ is a unit vector with components (s_1, s_2, s_3) , and $\vec{\Omega}$ has components

$$\Omega_1 = - \frac{1}{2} \frac{\omega}{c} \frac{\omega}{kc} \frac{\Omega_{ce}^2}{\omega^2} \frac{\omega_{pe}^2}{\omega^2} \frac{(b_1^2 - b_2^2)}{1 - \Omega_{ce}^2/\omega^2}, \quad (3.16)$$

$$\Omega_2 = - \frac{\omega}{c} \frac{\omega}{kc} \frac{\Omega_{ce}^2}{\omega^2} \frac{\omega_{pe}^2}{\omega^2} \frac{b_1 b_2}{1 - \Omega_{ce}^2/\omega^2}, \quad (3.17)$$

and

$$\Omega_3 = \frac{\omega}{c} \frac{\omega}{kc} \frac{\Omega_{ce}}{\omega} \frac{\omega_{pe}^2}{\omega^2} \frac{b_3}{1 - \Omega_{ce}^2/\omega^2}. \quad (3.18)$$

We may compare our results, Eqs. (3.16) thru (3.18), to those of De Marco and Segre by following De Marco and Segre in adopting a spherical polar co-ordinate system with the axis parallel to $\mathbf{k}$. Using θ to denote the angle between $\mathbf{k}$ and $\mathbf{b}$, and β to denote the angle between the component of $\mathbf{b}$ perpendicular to $\mathbf{k}$ and $\hat{\mathbf{e}}_2$, we have

$$b_1 = \sin \theta \sin \beta, \quad (3.19)$$

$$b_2 = \sin \theta \cos \beta, \quad (3.20)$$

and

$$b_3 = \cos \theta. \quad (3.21)$$

The components of $\vec{\Omega}$ may be written in terms of these angles as

$$\Omega_1 = \frac{1}{2} \frac{\omega}{c} \frac{\omega}{kc} \frac{\Omega_{ce}^2}{\omega^2} \frac{\omega_{pe}^2}{\omega^2} \frac{\sin^2 \theta \cos 2\beta}{1 - \Omega_{ce}^2/\omega^2}, \quad (3.22)$$

$$\Omega_2 = -\frac{1}{2} \frac{\omega}{c} \frac{\omega}{kc} \frac{\Omega_{ce}^2}{\omega^2} \frac{\omega_{pe}^2}{\omega^2} \frac{\sin^2 \theta \sin 2\beta}{1 - \Omega_{ce}^2/\omega^2} \quad (3.23)$$

and

$$\Omega_3 = \frac{\omega}{c} \frac{\omega}{kc} \frac{\Omega_{ce}}{\omega} \frac{\omega_{pe}^2}{\omega^2} \frac{\cos \theta}{1 - \Omega_{ce}^2/\omega^2}. \quad (3.24)$$

We see that our results differ from Eq. (14) of De Marco and Segre in that De Marco and Segre have taken $N = kc/\omega \approx 1$ (this is part of the content of De Marco and Segre's approximation of ignoring refraction effects), while we are (somewhat inconsistently) retaining the difference between N and 1.

4. Application of Polarimetry to Tokamak Experiments

We wish to use measurements of the change in the polarization state of a probe beam that passes through the plasma to infer the radial profile of the plasma current. The discussion of the previous two sections may be applied to this problem by considering an experiment in which the probe beam stays within the poloidal plane. It is launched from the top (or bottom) of the plasma parallel to the symmetry axis of the tokamak. In a tokamak the magnetic field is nearly in the toroidal direction, so that $\vec{b} = \vec{e}_2 + O(\epsilon)$, i.e., $b_2 \approx 1$, while b_1 and b_3 are of order ϵ . We will assume that the frequency of the probing beam is high enough that ω_{pe}/ω and Ω_{ce}/ω are also of order ϵ . At leading order $\vec{\Omega}$ is then given by

$$\Omega_1 \approx \frac{1}{2} \frac{\Omega_{ce}^2}{\omega^2} \frac{\omega_{pe}^2}{\omega^2}, \quad (4.1)$$

$$\Omega_2 \approx 0, \quad (4.2)$$

and

$$\Omega_3 \approx \frac{\Omega_{ce}}{\omega} \frac{\omega_{pe}^2}{\omega^2} b_3. \quad (4.3)$$

We follow Segre³ in considering the case where

$$\int_{-z_0}^{z_0} |\vec{\Omega}| dz \ll 1. \quad (4.4)$$

The change in the polarization state of the probe on crossing the plasma may then be approximated by

$$\vec{s} \approx \vec{s}_0 + \int_{-z_0}^{z_0} \vec{\Omega} dz, \quad (4.5)$$

or

$$s_1 \approx s_{10} - s_{20} P \frac{x}{a} \frac{q(a)}{n(0)} \frac{1}{a} \int_{-z_0}^{z_0} \frac{n(r)}{q(r)} dz, \quad (4.6)$$

$$s_2 \approx s_{20} + s_{10} P \frac{x}{a} \frac{q(a)}{n(0)} \frac{1}{a} \int_{-z_0}^{z_0} \frac{n(r)}{q(r)} dz - s_{30} M \frac{1}{n(0)a} \int_{-z_0}^{z_0} n(r) dz, \quad (4.7)$$

and

$$s_3 \approx s_{30} + s_{20} M \frac{1}{n(0)a} \int_{-z_0}^{z_0} n(r) dz; \quad (4.8)$$

where x is the displacement of the vertical cord from the magnetic axis, and $r^2 = x^2 + z^2$. We have assumed a tokamak equilibrium in the form of circular concentric flux surfaces. The safety factor at the plasma edge is denoted by $q(a)$, while $n(0)$ is the density at the magnetic axis. We have followed Segre³ in defining the dimensionless parameters M and P . In fact, Segre's theory involves four dimensionless parameters,

$$N_0 = \frac{\omega_{p0}^2}{\omega^2} \quad (4.9)$$

$$P = \frac{a\omega}{c} \frac{\omega_{p0}^2}{\omega^3} \frac{eB_I}{mc} \quad (4.10)$$

$$Q = B_T/B_I \quad (4.11)$$

$$M = \frac{a\omega}{2c} \frac{\omega_{p0}^2}{\omega^2} \frac{\Omega_{ce}^2}{\omega^2} \quad (4.12)$$

where ω_{p0} is the plasma frequency at the center of the plasma, a is the minor radius of the plasma, B_T is the toroidal magnetic field, and $B_I \equiv 2I_p/ac$ is the poloidal magnetic field at the plasma edge.

We see that there are two competing effects; a rotation of a linearly polarized probe beam's direction of polarization through an angle proportional to $\int dz n/q$, and a relative phase shift between the linear components, a_1 and a_2 proportional to $\int n dz$. The polarization vector will rotate at most through an angle of order P , while the phase shift has a maximum value of M . Segre's remaining parameters, N_0 and Q are only important when one retains higher order corrections to Eqs. (4.1) - (4.3). Hence, Segre's parameters can be used to compare the expected polarization changes in different experiments, and access the difficulty of a Faraday rotation experiment in MTX relative to other tokamaks.

4.1. Review of Polarimetry Experiments on Tokamaks

Polarimetry has been used to measure the poloidal magnetic field in several tokamak experiments.^{4,5,7-12} The relevant parameters for computing Segre's dimensionless parameters are given in Table 4.1.I, while Segre's parameters are given in Table 4.1.II. The signal associated with the line integral of $nB_{\parallel}$ is proportional to P . One thing that stands out in comparing the parameters for the proposed MTX experiment and other tokamak experiments is that P is substantially smaller at the shorter wavelengths in the proposed MTX experiment. At the largest (0.337 mm) wavelength in the MTX experiment P is

Table 4.1.I. Plasma Parameters in Tokamak Polarimetry Experiments

	TEXTOR ⁴	TFTR ⁵	TEXT ⁶	MicroTor ⁷	TFR ⁸	ISX-B ⁹
λ (mm)	0.337	0.119	0.337	1.0	0.337	0.381
n_0 (10^{14} cm ⁻³)	0.45*	1.0	0.5	0.55	1.0	0.65
I_p (kA)	334	2500	400	100	200	140
a (cm)	45	85	28	10	23	27
B_T (Tesla)	2.0	5.2	1.6	2.0	4.3	1.2
$\Delta\phi$ (error)	0.1°	0.26°	—	0.25°	0.06°	0.3°
τ_{avg} (ms)	10	—	—	—	10	4

Plasma parameters for polarimetry experiments various tokamak plasmas

*Ref. 4 quotes a line averaged density of 3×10^{13} cm⁻³. We have assumed that the peak density is larger by a factor of 1.5.

comparable with the values in other experiments, but M is now larger than one. Such large values of M may complicate the interpretation of the experimental data since the phase shift of the X-mode relative to the O-mode will be dominated by a term involving the line density.

Table 4.1.II. Dimensionless Parameters Characterizing Polarimetry Experiments

	TEXTOR	TFTR	TEXT	MicroTor	TFR	ISX-B
N_0	0.0045	0.0013	0.005	0.031	0.01	0.0084
P	0.118	0.367	0.235	0.363	0.235	0.137
Q	13.5	8.84	5.6	10.0	24.7	11.6
M	0.05	0.0094	0.033	0.34	0.39	0.034

Values of Segre's four dimensionless parameters that characterize polarimetry for various polarimetry experiments in tokamaks. Plasma parameters are those quoted in Table 4.1.I

Although there have been numerous demonstrations of polarimetry^{4,5,7-12} in tokamak

plasmas, the best results have been obtained by Soltwisch and coworkers on TEXTOR^{4,11,12}. In these experiments a heterodyne system was used with the reference beam (0.337 mm - HCN) frequency shifted by 10 kHz by a rotating grating. The detectors are room temperature pyroelectric devices. The nine channel polarimeter doubles as an interferometer.

The detected signal is averaged for (typically) 10 ms; tests with a half-wave plate yield a noise level much less than 0.1° . The signal during a discharge in TEXTOR shows very little increase in noise. The resulting accuracy in $q(r)$ is claimed to be $\pm 15\%$ at $r = 0$; $\pm 5\%$ at $r = a/2$; and $\pm 4\%$ at $r = a$ for a discharge with $n_0 = 6 \times 10^{13} \text{ cm}^{-3}$, $B = 2 \text{ T}$, $I_p \approx 300 \text{ kA}$, $a = 50 \text{ cm}$, and $R = 174 \text{ cm}$. The corresponding Segre parameters are $P = 0.21$ and $M = 0.10$ so that ellipticity is small.

An example of data from TEXTOR¹² is shown in Fig. 4.1.1a-d. The current density, found as a derivative of B_p , shows structure at the rational- q surfaces. It is difficult to assess the accuracy of this evaluation from the published data.

4.2 RF Current Drive Experiments in MTX

One important application of polarimetry in MTX is the measurement of the q profile during RF current drive experiments. For such experiments we expect $B_T = 9 \text{ T}$, $a = 16 \text{ cm}$, $n_0 \approx 1 \times 10^{14} / \text{cm}^3$, and a plasma current of about 200 kA. We consider polarimetry at wavelengths of 0.119, 0.185, 0.337 mm, and 0.447 mm. Reasonable estimates of the four dimensionless parameters characterizing polarimetry for the MTX current drive experiments are given in Table 4.2.I at each wave length. Note that $1/Q^2 \sim 10^{-3}$ is small in all cases so that the approximation discussed in Sec. 5 of Ref. 3, in which this term is neglected compared to one is quite accurate (although the approximation made in discussing the density diagnostic based on ellipticity, $(M/P)^2 \gg 1$, is not satisfied).

The MTX experiment proposes to use techniques similar to TEXTOR^{4,10-12}, with the reference frequency difference ($\approx 1 \text{ MHz}$) generated by using a second laser locked to the first. This technique, pioneered by Wolfe¹⁴ for an interferometer on Alcator A and used on the combined interferometer-polarimeter on TFTR¹³, is expected to provide an accuracy at least as good as the TEXTOR technique. The primary problem on MTX, therefore, is the large ellipticity developed in the plasma. Current drive experiments at lower density will maintain the ratio of M/P but require greater sensitivities. Densities as low as $2 \times 10^{13} \text{ cm}^{-3}$ have been considered.

This ellipticity could be handled by one of several techniques:

- 1) Operate at short enough wavelength that it can be neglected. As shown in Table 4.2.I, however, the resulting sensitivity to Faraday rotation is significantly reduced.
- 2) Use the theoretical result and the interferometer data to eliminate the ellipticity. This is satisfactory only if it cannot be directly measured.

Fig. 4.1.1 Example of data from TEXTOR¹². (a) Electron density at 200, 700, and 1000 ms from interferometer. (b) Measured Faraday rotation angle of probe beam. (c) $q(r)$ and $j(r)$ at 200 ms. The dotted line is from a tearing mode stability calculation. (d) $q(r)$ and $j(r)$ at 700 ms.

- 3) Measure the ellipticity and direction of polarization separately. This can be done, in principle, by comparing the phases of the two amplitudes. The accuracy to which this can be done requires careful analysis of the polarimeter design.

To make further progress we are using a ray-tracing analysis to determine accurate Faraday rotations and elliptizations for plasmas of interest. Non-ideal effects such as refraction, absorption, scattering from edge fluctuations, etc., will be compared with similar analysis at UCLA and combined with the analysis of possible hardware designs. The goal is to characterize polarimetry for MTX sufficiently by October, 1987, to permit a decision on its development.

4.3 Profile Modification Experiments on MTX

A second important application of polarimetry in MTX is the measurement of changes

Table 4.2.I

	CH_3OH	CH_2F_2	$HCOOH$	CH_3I
λ	0.119 mm	0.185 mm	0.393 mm	0.447 mm
Ω_{ce}/ω	0.11	0.16	0.35	0.40
N_0	1.3×10^{-3}	3.0×10^{-3}	1.4×10^{-2}	1.8×10^{-2}
P	2.93×10^{-2}	7.09×10^{-2}	0.320	0.414
Q	37.1	37.1	37.1	37.1
M	5.3×10^{-2}	0.20	1.91	2.82

Dimensionless parameters characterizing polarimetry for RF current drive in the MTX experiment. Plasma parameters are $B_T = 9$ T, $a = 16$ cm, $n_0 = 1 \times 10^{14}/\text{cm}^3$, and a plasma current of 200 kA.

in the $q(r)$ profile in MTX during lower-hybrid or ECR experiments on stabilization of sawtooth oscillations¹⁵⁻¹⁸, of $m = 2/n = 1$ (MHD) modes¹⁹, or on similar behavior which limits the achievable plasma parameters. In particular, lower-hybrid sawtooth-stabilization experiments have been proposed in MTX²⁰. The plasma parameters for this sawtooth stabilization experiment are $n_0 = 1.5 \times 10^{14} \text{ cm}^{-3}$, $B = 6$ T, $a = 16$ cm, and $I_p = 300$ kA. The Segre parameters are given in Table 4.3.I.

Table 4.3.I

	CH_3OH	CH_2F_2	$HCOOH$	CH_3I
λ	0.119 mm	0.185 mm	0.393 mm	0.447 mm
Ω_{ce}/ω	0.067	0.10	0.22	0.25
N_0	1.9×10^{-3}	4.6×10^{-3}	2.1×10^{-2}	2.7×10^{-2}
P	0.066	0.16	0.72	0.93
Q	16.0	16.0	16.0	16.0
M	0.036	0.14	1.31	1.92

Dimensionless parameters characterizing polarimetry for sawtooth suppression in the MTX experiment. Plasma parameters are $B_T = 6$ T, $a = 16$ cm, $n_0 = 1.5 \times 10^{14}/\text{cm}^3$, and a plasma current of 300 kA.

Compared to the current drive experiment which has $B = 9$ T and $I_p = 200$ kA, the Segre parameters for this experiment show a larger Faraday rotation and a smaller elliptization. The basic measurement is thus easier. However, both the experimental and the theoretical considerations^{21,22} indicate that the stabilization of sawtooth oscillations at high densities, like those in the proposed MTX experiment, is due to small changes in the shape of the $q(r)$ profile near the $q = 1$ surface. Consequently, the goal of the experiment is to measure relatively small changes in the plasma current profile, and hence, the required sensitivity is increased.

We estimate the required sensitivity from Eqs. (4.6) and (4.7) to be

$$|\delta \vec{s}| = P \frac{x}{a} \frac{q(a)}{n_0} \delta \int_{-z_0}^{z_0} \frac{n(r)}{q(r)} dz. \quad (4.3.1)$$

so that

$$\left| \frac{\delta \vec{s}}{\vec{s} - \vec{s}_0} \right| \approx \frac{n_0}{n} \frac{\bar{q} \delta q}{q^2(0)} \frac{\delta z}{2a}. \quad (4.3.2)$$

Here δq is the perturbation in q , about 0.05, and δz is the distance along the ray path across the perturbation, about 2 cm for effects near the plasma center. Thus $|\delta \vec{s}|/(|\vec{s} - \vec{s}_0|) \approx 10^{-2}$. This corresponds to a change in the rotation of the polarization of the probe beam of about 10 mrad at a wavelength $\lambda = 185 \mu$. The change will take place over a few ns, permitting differential techniques to be used to extract the data from the noise. A response time of about 1 ns will, however, be required. Further analysis will be required to determine whether this sensitivity can be obtained.

Appendix A. Dielectric Effects on the Optical Path Length

In this appendix we compute the change in the wave phase due to changes in the optical path induced by the plasma. We model the plasma as a cylinder with a dielectric constant N . The microwave beam from the interferometer is emitted from a point that is displaced radially from the axis of the plasma by a distance L , and vertically displaced by a distance h (see Fig. A.1).

Fig. A.1 Sketch of the probe beam passing through the plasma. The plasma is approximated by a sharp boundary cylinder. The probe beam is launched from a plane displaced by a distance L from the magnetic axis. The probe beam is emitted a distance h above the magnetic axis. The receiver is placed symmetrically on the opposite side of the magnetic axis.

The total phase change on passing through the plasma is

$$\Delta\phi = 2\frac{a\omega}{c} [N \cos\theta_1 + (L/a - \cos\theta_1)/\cos(\theta_1 - \theta_2)]$$

where θ_1 and θ_2 are the angles between the microwave beam and the normal to the plasma surface inside and outside the plasma respectively.

It follows from Snell's law,

$$N \sin\theta_1 = \sin\theta_2,$$

and the fact that $\omega_{pe}^2/\omega^2 \ll 1$ that $\theta_1 \approx \theta_2$. Writing $N \approx 1 - \frac{1}{2} \frac{\omega_{pe}^2}{\omega^2}$, we have

$$\theta_1 - \theta_2 \approx \frac{1}{2} \frac{\omega_{pe}^2}{\omega^2} \tan \theta_1.$$

This approximation breaks down as θ_1, θ_2 approach $\sin^{-1}(N)$, i.e., our approximation for $\theta_1 - \theta_2$ breaks down on chords that pass through the edge of the plasma.

Through second order in (ω_{pe}/ω^2) we may write

$$\phi \approx \frac{2a\omega}{c} \left[\frac{L}{a} - \frac{1}{2} \frac{\omega_{pe}^2}{\omega^2} \cos \theta_1 + \frac{1}{8} \left(\frac{L}{a} - \cos \theta_1 \right) \frac{\omega_{pe}^4}{\omega^4} \tan^2 \theta_1 \right].$$

Evidently, it is necessary to compute θ_1 correct to first order in ω_{pe}^2/ω^2 in order to obtain ϕ through second order in ω_{pe}^2/ω^2 .

We obtain an expression for θ_1 by holding the height at which the microwave beam is launched fixed:

$$h = a \sin \theta_1 + (L - a \cos \theta_1) \tan(\theta_1 - \theta_2).$$

Writing $\theta_1 = \theta_1^{(0)} + \delta\theta_1$, we find

$$\theta_1^{(0)} = \sin^{-1}(h/a)$$

and

$$\delta\theta_1 \approx -\frac{1}{2} \frac{\omega_{pe}^2}{\omega^2} \frac{h}{a^2 - h^2} \left(L - \sqrt{a^2 - h^2} \right).$$

We may now write the phase change of the microwave beam in passing through the plasma as $\phi = \phi_0 + \delta\phi$, where

$$\phi_0 = 2 \frac{a\omega}{c} \left(\frac{L}{a} - \frac{1}{2} \frac{\omega_{pe}^2}{\omega^2} \cos \theta_1^{(0)} \right)$$

is the phase change along a straight line connecting the emitter and receiver; i.e., the result usually assumed when analyzing data from microwave interferometers. The correction term is

$$\delta\phi = -\frac{1}{4} \frac{a\omega}{c} \frac{\omega_{pe}^4}{\omega^4} \frac{h^2}{a^2 - h^2} \left(\frac{L}{a} - \frac{\sqrt{a^2 - h^2}}{a} \right).$$

Hence, we find that there is a correction to the phase due to changes in the ray path that is smaller than the signal (i.e., the term in ϕ_0 that is proportional to ω_{pe}^2/ω^2) by a factor of ω_{pe}^2/ω^2 . When other terms of this order are kept (as suggested by both Craig² and Segre³), then these terms must be kept as well.

In analyzing actual interferometer data it would be necessary to compute the ray paths in the assumed density profile. This would greatly complicate the analysis. Hence, I would recommend that these corrections be ignored in both the interferometry and the polarimetry.

Appendix B. Model Amplitude Equations

It is instructive to consider the solution to the amplitude equations for a model which neglects the spatial dependence of density and magnetic field. Neglecting refraction and approximating $b_2 = 1$ and $b_1 = 0$, Eqs. (2.27) and (2.28) become

$$\frac{da_1}{dz} = \frac{1}{2}Pa_2 + iMa_1 \quad (B.1)$$

$$\frac{da_2}{dz} = -\frac{1}{2}Pa_1 \quad (B.2)$$

with $z \equiv s/a$ and P and M given by Eqs. (4.10) and (4.12).

The solution to these equations with initial amplitudes a_{10} and a_{20} is

$$a_1 = e^{iMz/2} \left[\left(\cos \phi + i \frac{M}{\sqrt{M^2 + P^2}} \sin \phi \right) a_{10} - \frac{P}{\sqrt{M^2 + P^2}} \sin \phi a_{20} \right] \quad (B.3)$$

$$a_2 = e^{iMz/2} \left[\left(\cos \phi - i \frac{M}{\sqrt{M^2 + P^2}} \sin \phi \right) a_{20} + \frac{P}{\sqrt{M^2 + P^2}} \sin \phi a_{10} \right] \quad (B.4)$$

with the phase given by

$$\phi = \frac{1}{2} \sqrt{M^2 + P^2}. \quad (B.5)$$

This solution demonstrates the properties shown by Segre using Stokes parameters. Consider the case of initially linear polarization so that a_{10} and a_{20} are real. For small ellipticity, the ratio of minor to major axes of the polarization ellipse, b/a , can be written as

$$\begin{aligned} \frac{b}{a} &\approx \text{Im}(a_1^* a_2) \\ &\approx -\frac{M}{\sqrt{M^2 + P^2}} \sin \phi \left[2a_{10}a_{20} \cos \phi + \frac{P}{\sqrt{M^2 + P^2}} (a_{10}^2 - a_{20}^2) \sin \phi \right] \end{aligned} \quad (B.6)$$

We see that if either a_{10} or a_{20} is zero, the ellipticity is of order $M \times P$; however, if they are comparable the ellipticity is larger and of order M . In particular, an initial polarization at 45° yields no Faraday rotation and only elliptization.

In the usual case, ϕ is small. Assuming $a_{10} = 0$ and $a_{20} = 1$, Eqs. (B.3)-(B.6) can be written (with the common phase retained):

$$a_1 \approx e^{iMz/2} (-Pz/2) \quad (B.7)$$

$$a_2 \approx e^{iMz/2} (1 - iMz/2) \quad (B.8)$$

$$b/a \approx -MPz^2/4 \quad (B.9)$$

A measurement which only gave the amplitudes of a_1 and a_2 would have yielded an estimated polarization of $(Pz/2)/(1 + M^2z^2/4)^{1/2}$. The fractional error is therefore $\sim M^2z^2/8$ and small except to the extent that the phase, $\phi = (M^2 + P^2)^{1/2}z/2$, is not small.

The solution of the full equations differs from the above in that the assumptions that the field and density are constant in space is violated. However, the first term in Segre's expansion is essentially the leading term in the above model with Pz replaced by the integral. It is thus only at higher orders in the expansion that the solution will demonstrate different character than the above.

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