ELECTROMAGNETIC TWO-DIMENSIONAL ANALYSIS OF
TRAPPED-ION EIGENMODES

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ABSTRACT

A two-dimensional electromagnetic analysis of the trapped-ion instability for the tokamak case with $\beta \neq 0$ has been made, based on previous work in the electrostatic limit. The quasineutrality condition and the component of Ampere's law along the equilibrium magnetic field are solved for the perturbed electrostatic potential and the component of the perturbed vector potential along the equilibrium magnetic field. The general integro-differential equations are converted into a matrix eigenvalue-eigenfunction problem by expanding in cubic B-spline finite elements in the minor radius and in Fourier harmonics in the poloidal angle. A model MHD equilibrium with circular, concentric magnetic surfaces and large aspect ratio is used which is consistent with our assumption that $\beta \ll 1$. The effect on the trapped-ion mode of including these electromagnetic extensions to the calculation is considered, and the temperature (and β) scaling of the mode frequency is shown and discussed.

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1. INTRODUCTION

Since trapped-particle instabilities in tokamaks were first discovered theoretically by Kadomtsev and Pogutse¹ in 1966, efforts have been made not only to investigate theoretically how trapped particles make the plasma unstable and what the onset criteria would be, but also to demonstrate experimentally^{2,3} the existence of the instabilities. These drift-type instabilities should be understood because they can cause anomalous transport of plasma particles and energy even when macroscopic MHD instabilities, such as the interchange and kink instabilities, are stable.

For sufficiently high temperatures, since collision frequencies decrease with increasing temperatures, the plasma will enter the banana regime, where electrons and ions are trapped in the magnetic well at the outside of the tokamak. Much research has been done on the trapped-electron instability in the electrostatic ($\beta = 0$) limit⁴ and in the general electromagnetic ($\beta \neq 0$) case.⁵⁻⁷ Here, $\beta \equiv$ plasma pressure/magnetic pressure. As auxiliary heating methods such as radio frequency heating and neutral beam heating have been developed and applied on tokamaks, ion temperatures have been increased to the point where the ions are also in the banana regime. This has motivated many investigations of the trapped-ion instability.^{2,8-14}

There has been little previous work on the trapped-ion instability in the electromagnetic case. Marchand et al.^{13,14} made a two-dimensional analysis of the trapped-ion mode in the electrostatic limit. That analysis is extended here to include finite- β effects. The linearized gyrokinetic equation is solved by the method of characteristics (integration along unperturbed particle orbits) in the electromagnetic case. The perturbed electrostatic potential, ϕ , and the component of the perturbed vector potential along the equilibrium magnetic field line, $A_{\parallel}$, are retained in the calculation, but the

components of the perturbed vector potential perpendicular to the equilibrium magnetic field line are neglected, consistent with the assumption that $\beta \ll 1$. The solution of the gyrokinetic equation is then used in the quasineutrality condition, (which results from the assumption that $k\lambda_D \ll 1$, where k is the magnitude of the effective wave number and λ_D is the Debye length), and the component of Ampere's law along the equilibrium magnetic field lines. This procedure yields two integro-differential equations which involve ω , the mode frequency, r , the minor radius from the magnetic axis, and θ , the poloidal angle. The introduction of Fourier harmonics in θ and cubic B-spline finite elements¹⁵ in r converts these equations into a single, large matrix equation, which may be solved for eigenvalues and eigenvectors by standard numerical methods.

A model MHD equilibrium with circular, concentric magnetic surfaces and large aspect ratio is employed, also consistent with the assumption that $\beta \ll 1$.¹⁶ Other assumptions made to simplify the equation to be solved are that $k_\perp \rho_i \ll 1$ and that $k_\perp \rho_{bi} \ll 1$, where $k_\perp$ is the effective wave number perpendicular to the equilibrium magnetic field, ρ_i is the ion gyroradius, and ρ_{bi} is the trapped-ion banana width. These assumptions allow ϕ and $A_\parallel$ to be expanded in Taylor series in r , as will be discussed in more detail in the next section, so that the gyrokinetic equation becomes a purely differential equation in r .

In Sec. II, the derivation of the electromagnetically extended eigenmode equations will be presented, along with the methods used to solve them. Numerical results will be presented and discussed in Sec. III. In particular, the trapped-ion mode scaling with temperature (and β) at fixed electron-ion temperature ratio and fixed number density will be investigated in a particular case.

II. DERIVATION OF THE EIGENMODE EQUATIONS

In this section, the eigenmode equation, which was derived by Marchand in the electrostatic case, is extended to include the perturbed parallel vector potential, so that we may investigate the trapped-ion instability in the electromagnetic case.

Our starting point is the linearized gyrokinetic equation in the electromagnetic case, governing the perturbed distribution function:

$$\begin{aligned} \frac{\partial g_j}{\partial t} + \mathbf{v}_j \cdot \nabla g_j + \mathbf{v}_d \cdot \nabla g_j + v_j g_j \\ = -e_j \frac{\partial}{\partial t} \langle \phi - \frac{\mathbf{v}}{c} \cdot \mathbf{A} \rangle \frac{\partial}{\partial E} F_{Mj} \\ - \frac{e_j}{m_j n_j} \hat{\mathbf{b}} \cdot \nabla \langle \phi - \frac{\mathbf{v}}{c} \cdot \mathbf{A} \rangle \times \nabla F_{Mj}, \end{aligned} \quad (1)$$

where

$$\mathbf{v}_d = \frac{\hat{\mathbf{b}} \times (\mu \nabla_B + m_j v_j^2 \hat{\mathbf{b}} \times \nabla \hat{\mathbf{b}})}{m_j n_j}, \quad (2)$$

$$\mu = \frac{m_j v_j^2}{2B}, \quad (3)$$

$$v_e = \frac{v_{e1} [1 + H(Y^{1/2})]}{Y^{3/2} (1 - \epsilon - \Lambda)}, \quad (4)$$

$$v_i = \frac{v_{i1}}{Y^{3/2}} \frac{H(Y^{1/2})}{(1 - \epsilon - \Lambda)}, \quad (5)$$

$$Y = \frac{E}{T}, \quad (6)$$

$$v_{\alpha i} = \frac{4\pi n_e e^4 \ln \lambda_{\alpha i}}{(2T_\alpha)^{3/2} m_\alpha^{1/2}} \quad (7)$$

$$H(x) = \frac{e^{-x^2}}{\pi^{1/2} x} + \left(1 - \frac{1}{2x^2}\right) \operatorname{erf}(x) \quad (8)$$

$$\Omega_j = \frac{e_j B}{m_j c} \quad (9)$$

In Eq. (1), $\hat{b}$ is the unit vector parallel to the equilibrium magnetic field, $\operatorname{erf}(x)$ is the standard error function, $\epsilon = r/R$, $\ln \lambda_{\alpha i}$ is the Coulomb logarithm,¹⁷ $\Lambda = \mu B_0/E$ is a pitch-angle variable, and F_{Mj} is the local Maxwellian distribution function of species j .

The spatial coordinate system used here is the usual toroidal coordinate system: θ is the poloidal angle, ϕ is the toroidal angle and r is the minor radius. Also, g_j is the nonadiabatic perturbed distribution function in the guiding center coordinate system. The relationship between g_j and f_j , the perturbed distribution function in the particle coordinate system, is

$$f_j = g_j \exp\left[\frac{+ik_\perp v_\perp \sin(\phi + \phi_0)}{\Omega_j}\right] - e_j \phi \frac{F_{Mj}}{T_j} \quad (10)$$

where $k_\perp$ is the component of the effective wave vector perpendicular to the equilibrium magnetic field. In our toroidal coordinate system,

$$k_\theta = \frac{(\ell q - m)}{R_0 q}, \quad k_r = -i \frac{\partial}{\partial r}, \quad k_\phi = \frac{-m}{r}$$

where q is the usual safety factor and ℓ and m are the toroidal and poloidal mode numbers, respectively. ϕ is the gyrophase angle in velocity space and ϕ_0 is an initial phase constant.

Collisions are modeled by a Krook collision operator whose energy and

pitch-angle dependence are chosen to reproduce the results of using a Lorentz operator in some limits.

The angular bracket in Eq. (1) represents an average over the gyrophase angle in velocity space. After performing the average over the gyrophase angle and neglecting the perpendicular components of $\mathbf{A}$, we get

$$\left\langle \phi - \frac{\mathbf{x} \cdot \mathbf{A}}{c} \right\rangle = \left(\phi - \frac{v_{\parallel} A_{\parallel}}{c} \right) J_0 \left(\frac{k_{\perp} v_{\perp}}{\Omega_j} \right) \quad (11)$$

where we have assumed that $\phi = \exp[+ i \mathbf{k} \cdot \mathbf{x}]$ and $\mathbf{A} = \exp[+ i \mathbf{k} \cdot \mathbf{x}]$. Now assuming that $k_{\parallel} \ll k_{\perp}$, $k_{\perp} \rho_j \ll 1$, that is to say, the short wavelength across the magnetic field and the long wavelength along the magnetic field follow the drift-kinetic ordering, Eq. (11) can be rewritten as follows:

$$\left\langle \phi - \frac{\mathbf{x} \cdot \mathbf{A}}{c} \right\rangle = \phi - \frac{v_{\parallel} A_{\parallel}}{c} \quad (12)$$

to first order in $k_{\perp} \rho_j$. Here we used

$$J_0(x) = 1, \quad |x| \ll 1. \quad (13)$$

We will assume that $\beta \ll 1$, in order to be consistent with the model MHD equilibrium employed, which has circular, concentric magnetic surfaces and large aspect ratio.¹⁶ Note that Eq. (12) is evaluated in the guiding center coordinate system. Substituting Eq. (12) into Eq. (1) yields

$$\begin{aligned} \frac{\partial}{\partial t} g_j + (\mathbf{v}_{\parallel} + \mathbf{v}_d) \cdot \nabla g_j + v_{\perp} g_j \\ = -i \left(\frac{e_j}{T_j} \right) F_{Mj} (\omega - \omega_{*Tj}) \left(\phi - \frac{v_{\parallel} A_{\parallel}}{c} \right) \end{aligned} \quad (14)$$

where

$$\omega_{*Tj} = - \left(\frac{m}{r} \right) \left(\frac{c T_j}{e_j B_0} \right) \left(\frac{d \ln n_j}{dr} \right) \left[1 + \eta_1 \left(\gamma - \frac{3}{2} \right) \right] , \quad (15)$$

and

$$\eta_j = \frac{(d \ln T_j / dr)}{(d \ln n_j / dr)} . \quad (16)$$

Equation (14) can be solved by the method of characteristics; since $d/dt = \partial/\partial t + (\underline{v}_i + \underline{v}_d) \cdot \nabla$,

$$\frac{d}{dt} [g_j \exp(v_j t)] = -i \left(\frac{e_j}{T_j} \right) F_{Mj} (\omega - \omega_{*Tj}) \left(\phi - \frac{v_{||} A_{||}}{c} \right) \exp [v_j t] ,$$

so that

$$g_j = \exp[-v_j t] \int_{-\infty}^t dt' \left[-i \left(\frac{e_j}{T_j} \right) F_{Mj} \right] (\omega - \omega_{*Tj}) \left(\phi - \frac{v_{||} A_{||}}{c} \right) \exp[v_j t'] . \quad (17)$$

Since we assume axisymmetric geometry and our problem is linear and stationary in time, we assume all perturbations to have the form $\exp [i (l \zeta - m \theta - \omega t)]$. Furthermore, in order to explain the poloidal structure, we expand all perturbations as Fourier series in θ :

$$\phi = \sum_m \phi_m(r) \exp [i (l \zeta - m \theta - \omega t)] , \quad (18)$$

$$A_{||} = \sum_m A_{||m}(r) \exp [i (l \zeta - m \theta - \omega t)] .$$

Substituting Eq. (18) into Eq. (17), we get

$$\begin{aligned}
g_j = & \exp [i(l\zeta - \omega t)] \sum_m \exp[-im\theta] \\
& \times \int_{-\infty}^t dt' \left(\frac{-ie_j F_{Mj}}{T_j} \right) (\omega - \omega_{*Tj}) \left(\phi_m - \frac{v_{\parallel} A_{\parallel m}}{c} \right) \\
& \times \exp \{ i[l(\zeta' - \zeta) - m(\theta' - \theta) + (\omega + iv_j)(t' - t)] \}, \quad (19)
\end{aligned}$$

where the primes on ζ' and θ' mean the evaluation is at time t' rather than at time t .

With $g_j = \tilde{g}_j \exp[i(l\zeta - \omega t)]$ and likewise for f_j , ϕ , and $A_{\parallel}$, from Eq. (10)

$$\tilde{f}_j = \tilde{g}_j \exp \left[+i \frac{k_{\perp} v_{\perp}}{\Omega_j} \sin(\phi + \phi_0) \right] - \frac{e_j \tilde{\phi} F_{Mj}}{T_j}, \quad (20)$$

where

$$\begin{aligned}
\tilde{g}_j = & \sum_m \exp[-im\theta] \int_{-\infty}^t dt' \left(\frac{-ie_j F_{Mj}}{T_j} \right) (\omega - \omega_{*Tj}) \left(\phi_m - \frac{v_{\parallel} A_{\parallel m}}{c} \right) \\
& \times \exp \{ i[l(\zeta' - \zeta) - m(\theta' - \theta) + (\omega + iv_j)(t' - t)] \}. \quad (21)
\end{aligned}$$

Assuming that the wavelength is long compared to the Debye shielding length in the plasma, the quasineutrality condition, $\sum_j \tilde{n}_j e_j = 0$, can be used to obtain the dispersion relation. Since Eq. (21) contains the parallel component of the perturbed vector potential, we need another equation which enables us to relate $A_{\parallel}$ and ϕ , and this is just the parallel component of Ampere's law:

$$v_{A\parallel}^2 = - \sum_j \frac{4\pi}{c} \int d^3v v_{\parallel} e_j \tilde{f}_j ,$$

or

$$k_{A\parallel}^2 = \sum_j \frac{4\pi}{c} \int d^3v v_{\parallel} e_j \tilde{f}_j , \quad (22)$$

where

$$k^2 = k_r^2 + k_\theta^2 + k_z^2 = \frac{m^2}{r^2} - \frac{\partial^2}{\partial r^2} ,$$

with the assumption that $k_z \sim k_{\parallel} \ll k_{\perp}$.

For the quasineutrality condition,

$$\begin{aligned} 0 &= \sum_j e_j \tilde{n}_j = \sum_j e_j \int d^3v \tilde{f}_j \\ &= \sum_j e_j \int d^3v \left\{ \tilde{g}_j \exp\left[i \frac{k_{\perp} v_{\perp}}{\Omega_j} \sin(\phi + \phi_0)\right] - \frac{e_j F_{Mj} \tilde{\phi}}{T_j} \right\} \\ &= \sum_j e_j \left[- \frac{e_j n_j \tilde{\phi}}{T_j} + \int d^2v \tilde{g}_j J_0 \left(\frac{k_{\perp} v_{\perp}}{\Omega_j} \right) \right] \\ &= \sum_j e_j \left(\frac{-e_j n_j \tilde{\phi}}{T_j} + \int d^2v \tilde{g}_j \right) \end{aligned} \quad (23)$$

to first order in the small quantity $k_{\perp} \rho_j \ll 1$, where

$$\int d^2v \equiv 2\pi \int_{-\infty}^{\infty} dv_{\parallel} \int_0^{\infty} dv_{\perp} v_{\perp} . \quad (24)$$

Likewise, for the parallel component of Ampere's law,

$$k^2 \tilde{A}_j = \frac{4\pi}{c} \int_j e_j \int d^2v v_{\parallel} \tilde{g}_j ,$$

or

$$0 = \sum_j e_j \int d^2v v_{\parallel} \tilde{g}_j - \frac{c}{4\pi} k^2 \tilde{A}_j . \quad (25)$$

Basically we follow the method of Ref. 13 to evaluate the perturbed distribution function in detail which then enables us to evaluate Eqs. (23) and (25). The difference comes from the parallel velocity term in Eqs. (22) and (25). Detailed calculations are presented in the appendix. The results are as follows: from Eq. (23),

$$\sum_m \left[A_{pm}^{Q\phi} \phi_m'' + A_{pm}^{QA} A_{\parallel m}'' + B_{pm}^{Q\phi} \phi_m' + B_{pm}^{QA} A_{\parallel m}' + C_{pm}^{Q\phi} \phi_m + C_{pm}^{QA} A_{\parallel m} \right] = 0 , \quad (26)$$

and from Eq. (25),

$$\sum_m \left[A_{pm}^{A\phi} \phi_m'' + A_{pm}^{AA} A_{\parallel m}'' + B_{pm}^{A\phi} \phi_m' + B_{pm}^{AA} A_{\parallel m}' + C_{pm}^{A\phi} \phi_m + C_{pm}^{AA} A_{\parallel m} \right] = 0 , \quad (27)$$

where

$$A_{pm} = \begin{pmatrix} A_{pm}^{Q\phi} & A_{pm}^{QA} \\ A_{pm}^{A\phi} & A_{pm}^{AA} \end{pmatrix} \\ = \begin{pmatrix} 0 & 0 \\ 0 & - (1/4\pi)(c/v_e)(T_e/n_e e^2) \delta_{pm} \end{pmatrix} \\ + \sum_j \left(\frac{n_j}{n_e} \right) Z_j^2 \tau_j \left(\frac{\epsilon^{3/2} \rho_{\theta j}^2}{2\pi^{9/2}} \right) \int_{1-\epsilon}^{1+\epsilon} \frac{d\lambda}{L_b^3} \int_0^\infty dy y^{3/2} \exp(-y)$$

$$\times \sum_{n=-\infty}^{\infty} \left(\frac{\omega - \omega_{*Tj}}{\omega - \omega_{Dj}^{(0)} - n\omega_{bj} + i\nu_j} \right) \sum_{\substack{n_1, n_2 \\ \text{odd}}} I_{n_1} I_{n_2} \left(\frac{D^{Q\Phi}}{D^{A\Phi}} \middle| \frac{D^{QA}}{D^{AA}} \right) \quad (28)$$

where

$$D^{Q\Phi} \equiv J_{n+n_1+n_2, p} J_{n, m} - 2J_{n+n_1, p} J_{n-n_2, m} + J_{n, p} J_{n-n_1-n_2, m}$$

$$D^{QA} \equiv -Y^{1/2} \frac{\nu_j}{c} (J_{n+n_1+n_2, p} \bar{J}_{n, m} - 2J_{n+n_1, p}$$

$$\cdot \bar{J}_{n-n_2, m} + J_{n, p} \bar{J}_{n-n_1-n_2, m})$$

$$D^{A\Phi} \equiv -Y^{1/2} \frac{\nu_j}{v_e} (\bar{J}_{n+n_1+n_2, p} J_{n, m} - 2\bar{J}_{n+n_1, p}$$

$$\cdot J_{n-n_2, m} + \bar{J}_{n, p} J_{n-n_1-n_2, m})$$

$$D^{AA} \equiv Y \frac{\nu_j^2}{v_e c} (\bar{J}_{n+n_1+n_2, p} \bar{J}_{n, m} - 2\bar{J}_{n+n_1, p}$$

$$\cdot \bar{J}_{n-n_2, m} + \bar{J}_{n, p} \bar{J}_{n-n_1-n_2, m})$$

$$B_{pm} = \left(\frac{B_{pm}^{Q\Phi}}{B_{pm}^{A\Phi}} \middle| \frac{B_{pm}^{QA}}{B_{pm}^{AA}} \right) = \sum_j \left(\frac{n_j}{n_e} \right) \frac{1}{2} \tau_j \left(\frac{e p \theta_j}{\pi^{1/2}} \right) \int_{1-\epsilon}^{1+\epsilon} \frac{d\lambda}{L_b^2} \int_0^\infty dY Y \exp(-Y)$$

$$\times \sum_{n=-\infty}^{\infty} \left(\frac{\omega - \omega_{*Tj}}{\omega - \omega_{Dj}^{(0)} - n\omega_{bj} + i\nu_j} \right) \sum_{\substack{n_1 \\ \text{odd}}} I_{n_1} \left(\frac{G^{Q\Phi}}{G^{A\Phi}} \middle| \frac{G^{QA}}{G^{AA}} \right) \quad (29)$$

where

$$G^{Q\Phi} \equiv J_{n, p} J_{n-n_1, m} - J_{n+n_1, p} J_{n, m}$$

$$G^{QA} \equiv -Y^{1/2} \frac{v_j}{c} (J_{n,p} \bar{J}_{n-n_1,m} - J_{n+n_1,p} \bar{J}_{n,m}) ,$$

$$G^{A\Phi} \equiv -Y^{1/2} \frac{v_j}{v_e} (\bar{J}_{n,p} J_{n-n_1,m} - \bar{J}_{n+n_1,p} J_{n,m}) ,$$

$$G^{AA} \equiv Y \left(\frac{v_j^2}{v_e c} \right) (\bar{J}_{n,p} \bar{J}_{n-n_1,m} - \bar{J}_{n+n_1,p} \bar{J}_{n,m}) ,$$

and

$$C_{pm} = \begin{pmatrix} C_{pm}^{Q\Phi} & C_{pm}^{QA} \\ C_{pm}^{A\Phi} & C_{pm}^{AA} \end{pmatrix} = \begin{pmatrix} -\sum_j \left(\frac{n_j}{n_e} \right) z_j^2 \tau_j \delta_{pm} & 0 \\ 0 & \delta_{pm} (c/v_e) (m^2/4\pi r^2) (T_e/n_e e^2) \end{pmatrix} + \sum_j \left(\frac{n_j}{n_e} \right) z_j^2 \tau_j \frac{e^{1/2}}{\pi^{5/2}} \int_{1-\epsilon}^{1+\epsilon} \frac{d\lambda}{L_b} \int_0^\infty dY Y^{1/2} \exp(-Y) \sum_{n=-\infty}^\infty \left(\frac{\omega - \omega_{*Tj}}{\omega - \omega_{Dj}(\omega) - n\omega_{bj} + i\nu_j} \right) \times \begin{pmatrix} J_{n,p} J_{n,m} & -Y^{1/2} (v_j/c) J_{n,p} \bar{J}_{n,m} \\ -Y^{1/2} (v_j/v_e) \bar{J}_{n,p} J_{n,m} & Y (v_j^2/v_e c) \bar{J}_{n,p} \bar{J}_{n,m} \end{pmatrix} + \int_0^{1-\epsilon} \frac{d\lambda}{L_t} \int_0^\infty dY Y^{1/2} \exp(-Y) \sum_{n=-\infty}^\infty \left(\frac{\omega - \omega_{*Tj}}{(\omega + i\nu_j)^2 - \{[n+s^{(m)}]\omega_{tj}\}^2} \right) \begin{pmatrix} H^{Q\Phi} & H^{QA} \\ H^{A\Phi} & H^{AA} \end{pmatrix} ,$$

(30)

$$H^{Q\Phi} \equiv (\omega + i\nu_j) K_{n,p,m} K_{n,m,m} ,$$

$$H^{QA} \equiv -Y^{1/2} \frac{v_j}{c} [n + s^{(m)}] \omega_{tj} K_{n,p,m} \bar{K}_{n,m,m} ,$$

$$H^{A\Phi} \equiv -Y^{1/2} \frac{v_j}{v_e} [n + s^{(m)}] \omega_{tj} \bar{K}_{n,p,m} K_{n,m,m} ,$$

$$H^{AA} \equiv Y \frac{v_j^2}{v_e c} (\omega + i v_j) \bar{K}_{n,p,m} \bar{K}_{n,m,m} ,$$

$$\tau_j \equiv \frac{T_j}{T_e} , \quad (31)$$

$$\rho_{\theta j} \equiv v_{thj} \left(\frac{c T_j}{e_j B_0} \right) , \quad (32)$$

$$v_{thj} \equiv \left(\frac{2T_j}{m_j} \right)^{1/2} (\equiv v_j) , \quad (33)$$

$$L_b \equiv \left[\frac{e^{1/2}}{2\pi} \right] \int_{-\theta_0}^{\theta_0} d\theta \left(1 - \frac{\Lambda}{h(\theta)} \right)^{-1/2} , \quad (34)$$

$$h(\theta) \equiv + \epsilon \cos \theta , \quad (35)$$

$$\omega_{Dj}^{(o)} \equiv \frac{\ell \rho_{\theta j} v_j^2}{R^2} \left[\frac{E(\kappa)}{K(\kappa)} - \frac{1}{2} + 2s \left(\kappa^2 - 1 + \frac{E(\kappa)}{K(\kappa)} \right) \right] , \quad (36)$$

$$\hat{s} \equiv \frac{q'r}{q} \text{ and } \Lambda \equiv 1 + \epsilon (1 - 2\kappa^2) , \quad (37)$$

$$\omega_{bj} \equiv 2\pi \left[4 \left(\frac{R_o q}{Y^{1/2} v_{thj}} \right) \int_0^{\theta_0} d\theta \left(1 - \frac{\Lambda}{h(\theta)} \right)^{-1/2} \right]^{-1} , \quad (38)$$

$$\omega_{tj} \equiv 2\pi \left[\left(\frac{2R_o q}{Y^{1/2} v_{thj}} \right) \int_0^{\pi} d\theta \left(1 - \frac{\Lambda}{h(\theta)} \right)^{-1/2} \right]^{-1} ,$$

$$J_{n,m} \equiv \int_0^{\theta_0} d\theta \left(1 - \frac{\Lambda}{h(\theta)} \right)^{-1/2} \left[\cos^2 \left(\frac{n\pi}{2} \right) \cos(s^{(m)} \theta) \cos(n\omega_b \hat{t}) \right]$$

$$+ \sin^2 \left(\frac{n\pi}{2} \right) \sin(s^{(m)} \theta) \sin(n\omega_b \hat{t}) \quad (39)$$

$$\bar{J}_{n,m} \equiv \int_0^{\theta_0} d\theta \left[\sin^2 \left(\frac{n\pi}{2} \right) \cos(s^{(m)} \theta) \cos(n\omega_b \hat{t}) + \cos^2 \left(\frac{n\pi}{2} \right) \sin(s^{(m)} \theta) \sin(n\omega_b \hat{t}) \right] \quad (40)$$

$$I_n \equiv \int_0^{\theta_0} d\theta \cos(n\omega_b \hat{t}) \quad (41)$$

$$K_{n,p,m} \equiv \int_0^{\theta_0} d\theta \left[1 - \frac{\Lambda}{h(\theta)} \right]^{-1/2} \cos[s^{(p)} \theta - (n+s^{(m)}) \omega_b \hat{t}] \quad (42)$$

$$\bar{K}_{n,p,m} \equiv \int_0^{\pi} d\theta \cos[s^{(p)} \theta - (n+s^{(m)}) \omega_b \hat{t}] \quad (43)$$

In Eqs. (26) and (27), the prime means the derivative with respect to the minor radius, d/dr . Derivative terms come from the Taylor expansion of ϕ and $A_{\parallel}$ with the assumptions that each harmonic varies only weakly over a typical orbit radial excursion and that the radial wavelength is large compared to the ion banana width, $k_{rB} \rho_{bi} \ll 1$. Thus the scalar potential and the parallel component of the vector potential can be expanded in Taylor series and terms are retained up to the second derivative term. $A_{pm}^{Q\phi}$ represents the term which comes from the quasineutrality condition (Q) and which multiplies the scalar potential ϕ . Other terms can be interpreted in the same way. Also note that the width of the radial excursion of circulating particles is smaller than that of trapped particles by the factor $\epsilon^{1/2}$. So matrices A_{pm} and B_{pm} have contributions only from trapped particles.

We retain contributions which come from the trapped electrons in the A, B, and C matrices and from the nonadiabatic circulating electrons, which were all neglected in Ref. 13.

Equations (26) and (27) are converted into a matrix equation by expanding the spatial dependence of the various terms in these equations in a set of

basis functions ψ_k , called cubic B-spline finite elements.^{13,15,18}

$$\phi_m(x) = \sum_{k=1}^{N+2} \alpha_{mk} \psi_k(x) \quad (44)$$

$$A_{lm}(x) = \sum_{k=1}^{N+2} \gamma_{mk} \psi_k(x) \quad (45)$$

where $x \equiv r/a$ and a is the minor radius of the plasma boundary.

Substituting Eqs. (44) and (45) into Eqs. (26) and (27) and then operating with $\int_0^1 dx \psi_k$, on these equations, we get the following matrix equation:

$$M \begin{bmatrix} \alpha_{mk} \\ \gamma_{mk} \end{bmatrix} = \begin{pmatrix} M_{pmkk'}^{Q\phi} & M_{pmkk'}^{QA} \\ M_{pmkk'}^{A\phi} & M_{pmkk'}^{AA} \end{pmatrix} \begin{bmatrix} \alpha_{mk} \\ \gamma_{mk} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad (46)$$

where

$$M_{pmkk'}^{Q\phi} = \int_0^1 dx \psi_{k'}(x) \left[A_{pm}^{Q\phi}(x) \psi_k''(x) + B_{pm}^{Q\phi}(x) \psi_k'(x) + C_{pm}^{Q\phi}(x) \psi_k(x) \right],$$

$$M_{pmkk'}^{A\phi} = \int_0^1 dx \psi_{k'}(x) \left[A_{pm}^{A\phi}(x) \psi_k''(x) + B_{pm}^{A\phi}(x) \psi_k'(x) + C_{pm}^{A\phi}(x) \psi_k(x) \right],$$

$$M_{pmkk'}^{QA} = \int_0^1 dx \psi_{k'}(x) \left[A_{pm}^{QA}(x) \psi_k''(x) + B_{pm}^{QA}(x) \psi_k'(x) + C_{pm}^{QA}(x) \psi_k(x) \right],$$

and

$$M_{pmkk'}^{AA} = \int_0^1 dx \psi_{k'}(x) \left[A_{pm}^{AA}(x) \psi_k''(x) + B_{pm}^{AA}(x) \psi_k'(x) + C_{pm}^{AA}(x) \psi_k(x) \right].$$

Equation (46) is the form to be implemented in the computer code, which basically tries to find roots of $\det(M) = 0$ to obtain eigenvalues and eigenvectors. Having found the eigenvectors, we can evaluate the radial structures of the perturbed scalar potential and parallel vector potential through Eqs. (44) and (45) and then we have the complete two-dimensional structure of the perturbation with the help of Eq. (8).

Also note that as $\beta \rightarrow 0$, we can obtain results in the electrostatic limit, which are the main subjects of Ref. 13: The term $(-1/4\pi) (c/v_e) (T_e/n_e e^2) \delta_{pm}$ in A_{pm}^{AA} of Eq. (28) can be rewritten as

$$- 2 \frac{c}{v_e} \frac{T_e}{e^2 B_0^2} \frac{\sum_j (n_j/n_e) \tau_j}{\beta} \delta_{pm} \quad (47)$$

Therefore as $\beta \rightarrow 0$, A_{mm}^{AA} becomes larger and larger, causing M_{mmkk}^{AA} to be larger and larger and rendering Y_{mk} negligible. Hence, Eq. (46) is effectively reduced to $M_{pmkk}^{O\phi} \cdot \alpha_{mk} = 0$, which is the matrix equation solved in reference 13.

III. NUMERICAL RESULTS

In this section, we present results obtained when the finite- β effects on a trapped-in instability are taken into account. In getting numerical results, we use the following equilibrium profiles for $n=n_e=n_i$, density, T_j , temperature, and q , safety factor:

$$n = n(0) \exp\left[-\left(\frac{x}{x_n}\right)^{\alpha_n}\right] \quad (48)$$

$$T_e = T_e(0) \exp\left[-\left(\frac{x}{x_{Te}}\right)^{\alpha_{Te}}\right] \quad (49)$$

$$r_i = T_i(0) \exp\left[-\left(\frac{x}{x_{Ti}}\right)^{\alpha_{Ti}}\right] \quad (50)$$

$$q(x) = q(0) + [q(a) - q(0)] x^2 \quad (51)$$

where $x \equiv r/a$ and a is the minor radius of the plasma boundary. Values of parameters are listed in Table 1. They correspond fairly accurately to those of Gladd and Ross,⁸ except for the density and the temperature.¹³

The temperature scaling of the mode frequency was investigated. During the analysis, the ratio of the electron temperature to the ion temperature was fixed at a constant, $T_i/T_e = 2$. The results are displayed in Fig. 1 and their numerical values are listed in Table 2. For $\beta = 3 \times 10^{-4}$, the real and imaginary parts of the eigenfrequency agree almost exactly with those in the electrostatic limit ($\beta_{av} \equiv$ radially averaged beta $= 0$) within an error of about one percent. As shown in Fig. 1, however, they deviate from those in the initial case as T_e , T_i , and β_{av} increase: For β_{av} less than 4.2×10^{-4} , or T_e less than 2.8 keV, the growth rate increases as β_{av} increases, while after β_{av} passes that point and continues to increase, the growth rate goes down, so that increasing T_j and β has a stabilizing effect on a trapped-ion mode. What causes such a change in the growth rate is not obvious here, but the interaction of drift modes with shear-Alfvén modes as β_{av} increases may be one cause for that variation of the growth rate, along with the temperature scaling of v_j and other quantities. The real part of the mode frequency increases fairly linearly up to $T_e = 3.1$ keV, which is a very different behavior from that of the growth rate. For T_e above 3.7 keV, the curves for the real and imaginary parts of the frequency both seem to turn around: the real frequency decreases and the growth rate increases again as T_j and β_{av} increase. We should mention that such behavior may not be reliable because we

see sudden variation, or spikes, on the eigenfunctions of the real and imaginary parts of Φ and $A_{\parallel}$. They may come from the fact that the number of cubic B-spline finite elements is not large enough to resolve the spatial variation in the perturbation, or from the fact that our basic assumptions that $k_{\perp}\rho_j < 1$ and $k_{\perp}\rho_{bj} < 1$ may be breaking down because ρ_j and ρ_{bj} are increasing with temperature.

Figures 2 to 7 show what the eigenfunctions for Φ and $A_{\parallel}$ look like. These are results in the case $T_e = 2.0$ keV, $T_i = 1.0$ keV, and $\beta_{av} = 3 \times 10^{-4}$. The electrostatic potential Φ , has the same features as in the electrostatic limit,¹⁸ $\beta_{av} = 0$: the perturbation in the electrostatic potential is almost localized between two rational surfaces and balloons to the outside of the tokamak. Note that there are two rational surfaces at $x \approx 0.7$ and 0.95 , calculated from our model equilibrium profile of the safety factor with the toroidal mode number 3 and poloidal mode number 5 and 6.

As shown in Figs. 5 to 7, the perturbation in the parallel component of the vector potential has both similar and different features from that in the electrostatic potential: the perturbation in $A_{\parallel}$ is also localized between rational surfaces while it seems to balloon to the inside of the tokamak. It is quite flutelike around the rational surface at $x \approx 0.95$.

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APPENDIX

In this appendix, a detailed calculation of the perturbed distribution function and the quasineutrality condition and the parallel Ampere's law which lead to Eqs. (26) and (27) will be presented. Many of the results in Ref. 13 will be used in doing such calculations.

Let us consider the perturbed distribution function $\tilde{g}_j$ first:

$$\tilde{g}_j = \sum_m \exp(-im\theta) \int_{-\infty}^t dt' \left(\frac{-ie_j F_{mj}}{T_j} \right) (\omega - \omega_{*Tj}) \left(\phi_m - \frac{v_{||} A_{||}}{c} \right) \exp\{i[\lambda(\zeta' - \zeta) - m(\theta' - \theta) + (\omega + iv_j)(t' - t)]\} \quad (A-1)$$

We note that $v_{||} = \sigma v_j \{Y[1 - \Lambda/h(\theta)]\}^{1/2}$ is dependent on time through $\theta = \theta(t)$, where $v_j = (2T_j/m_j)^{1/2}$, $Y = E/T_j$, $\Lambda = \mu B_0/E$, $h(\theta) = 1 + \epsilon \cos\theta$, and σ_j represents the sign of the parallel velocity along the equilibrium magnetic field line. We divide the integration Eq. (A-1) into two parts, one of which is related to the scalar potential and basically is the same as in Ref. 13 and the other of which is related to the parallel vector potential and is the new term to which we need to pay attention:

$$\begin{aligned} \tilde{g}_j = & \sum_m \exp(-im\theta) \int_{-\infty}^t dt' \left(\frac{-ie_j F_{mj}}{T_j} \right) (\omega - \omega_{*Tj}) \\ & \times \phi_m(r') \exp\{i[\lambda(\zeta' - \zeta) - m(\theta' - \theta) + (\omega + iv_j)(t' - t)]\} \\ & + \sum_m \exp(-im\theta) \int_{-\infty}^t dt' \left(\frac{-ie_j F_{mj}}{T_j} \right) (\omega - \omega_{*Tj}) \\ & \times \left(\frac{-v_{||} A_{||m}(r')}{c} \right) \exp\{i[\lambda(\zeta' - \zeta) - m(\theta' - \theta) + (\omega + iv_j)(t' - t)]\} \quad (A-2) \end{aligned}$$

According to Ref. 13, the first part of (A-2) is just

$$\begin{aligned} & \left(\frac{e_j F_{Mj}}{T_j} \right) \sum_m \exp[-im\theta] \sum_n a_n \left\{ \phi_m G(n) + \phi'_m \sum_{n_1} \hat{r}(n_1) \exp[in_1 \omega_b t] [G(n+n_1) - G(n)] \right. \\ & \quad \left. + \frac{1}{2} \phi''_m \sum_{n_1, n_2} \hat{r}(n_1) \hat{r}(n_2) \exp[i(n_1+n_2) \omega_b t] [G(n+n_1+n_2) - 2G(n+n_1) + G(n)] \right\}, \end{aligned} \quad (A-3)$$

where the derivative terms come from the Taylor expansion of ϕ_m with the assumptions that each harmonic varies weakly over a typical orbit excursion and that $k_y \rho_{bi} < 1$:

$$\phi_m(r) = \phi_m(r) + (r'-r) \phi'_m(r) + \frac{1}{2} (r'-r)^2 \phi''_m(r) \quad . \quad (A-4)$$

Also

$$r'-r = \sum_m r^{(n)} \exp[in \omega_b t] \quad (A-5)$$

for the radial excursion of the trapped particles and the corresponding terms for the circulating particles are ignored since the radial excursion of the circulating particles is smaller than that of the trapped particles by the factor of $\epsilon^{1/2} \ll 1$. So the second and the third terms in Eq. (A-3) contain contributions only from the trapped particles whereas the first term in Eq. (A-3) has contributions from both the circulating and the trapped particles.

Also

$$G(n) \equiv [\omega - \omega_{Dj}^{(0)} - (n + H\sigma_{||} S) \omega_{b,t} + i\nu_j]^{-1}, \quad (A-6)$$

where

$$H = \begin{cases} 0 & \text{for trapped particles} \\ 1 & \text{for circulating particles} \end{cases}$$

and

$$S = lq(r) - m.$$

Here,

$$a_n = \begin{cases} \exp[-i(S\theta - n\omega_b \hat{t} + w)] \left(\frac{\epsilon^{1/2}}{\pi L_b}\right) J_{nm} & \text{for trapped particles,} \\ \exp\{-i[S\theta - (n + \sigma_j S) \omega_c \hat{t} + w]\} \left(\frac{\epsilon^{1/2}}{\pi L_c}\right) & \text{for circulating particles} \\ \times \int_0^{t_d\theta} \left(1 - \frac{\Lambda}{h(\theta)}\right)^{-1/2} \cos[S\theta' - (n + \sigma_j S) \omega_c \hat{t}] & \end{cases} \quad (A-7)$$

Let us consider the second part of Eq. (A-2): substituting $v_{||} = \sigma_j v_j$ $\{Y(1 - \Lambda/h(\theta))\}^{1/2}$.

$$\sum_m \exp[-im\theta] \int_{-\infty}^{t_d\theta'} \left(\frac{-ie_j F_{mj}}{T_j}\right) (\omega - \omega_{*Tj}) \{-\sigma_j v_j [Y(1 - \frac{\Lambda}{h(\theta')})]^{1/2} \frac{\Lambda_{jm}}{c}\} \\ \times \exp\{i[l(\zeta' - \zeta) - m(\theta' - \theta) + (\omega + iv_j)(t' - t)]\} \quad (A-8)$$

Following a similar procedure to that for the first part of Eq. (A-2) and performing the σ_j summation, which should be done in Eqs. (23) and (24) by using results in Ref. 13 and noting that $\Lambda_{jm}(r')$ is also expanded in a Taylor series, Eq. (A-8) can be rewritten as follows:

$$\begin{aligned}
& \left(\frac{e_j F_j}{T_j} \right) \left(\frac{-v_j}{c} \right) \sum_m \exp[-im\theta] (\omega - \omega_{*Tj}) \sum_n b_n \\
& \times \{ G(n) A_{jm} + A_{jm}^* \sum_{n_1 \text{ odd}} \hat{r}(n_1) \exp(in_1 \omega_b \hat{t}) [G(n + n_1) - G(n)] \\
& + \frac{1}{2} A_{jm}^* \sum_{n_1, n_2 \text{ odd}} \hat{r}(n_1) \hat{r}(n_2) \exp[i(n_1 + n_2) \omega_b \hat{t}] [G(n + n_1 + n_2) - 2G(n + n_1) + G(n)] \},
\end{aligned}
\tag{A-9}$$

where

$$\begin{aligned}
b_n &= \int_0^{\tau_b} dt' \sigma_{||} \left(1 - \frac{\Lambda}{h(\theta')} \right)^{1/2} \exp \{ i [S(\theta' - \theta) + w' - w - (n + H \sigma_{||} S) \omega_b \hat{t} (t' - t)] \} \\
&= \left(\frac{4}{\tau_b} \right) \exp[-i(S\theta + w - n\omega_b \hat{t})] \\
&\quad \times \int_0^{\tau_b/4} dt' \left(1 - \frac{\Lambda}{h(\theta')} \right)^{1/2} \left[\cos^2 \left(\frac{n\pi}{2} \right) \sin(S\theta') \sin(n\omega_b \hat{t}' - w') \right. \\
&\quad \left. + \sin^2 \left(\frac{n\pi}{2} \right) \cos(S\theta') \cos(n\omega_b \hat{t}' - w') \right] \quad \text{for trapped particles}
\end{aligned}
\tag{A-10}$$

$$\begin{aligned}
&= \left(\frac{2}{\tau} \right) \exp[-i(S\theta + w - n\omega_b \hat{t})] \\
&\quad \times \int_0^{\tau/2} dt' \sigma_{||} \left(1 - \frac{\Lambda}{h(\theta')} \right)^{1/2} \cos[S\theta' - (n + \sigma_{||} S) \omega_b \hat{t}'] \quad \text{for circulating particles.}
\end{aligned}$$

Hence, combining Eqs. (A-3) and (A-9) together,

$$\begin{aligned}
\tilde{g}_j &= \sum_m \exp(-im\theta) \left(\frac{e_j F_m j}{T_j} \right) \\
&\times \sum_n \left\{ \left(a_n \phi_m - \frac{v_j A_{jm} b_n}{c} \right) G(n) \right. \\
&+ \left(a_n \phi'_m - \frac{v_j A'_{jm} b_n}{c} \right) \sum_{\substack{n_1 \\ \text{odd}}} \hat{r}(n_1) \exp(in_1 \omega_B \hat{t}) [G(n+n_1) - G(n)] \\
&+ \frac{1}{2} \left[a_n \phi_m - \frac{v_j b_n A''_{jm}}{c} \right] \sum_{\substack{n_1, n_2 \\ \text{odd}}} \hat{r}(n_1) \hat{r}(n_2) \exp[i(n_1+n_2) \omega_B \hat{t}] [G(n+n_1+n_2) \\
&\quad \left. - 2G(n+n_1) + G(n)] \right\} \quad . \quad (A-11)
\end{aligned}$$

Substituting Eq. (A-11) into Eqs. (23) and (25), respectively, replacing $\int d^2v$ with $(\pi/2) v_j^{3/2} \sum_{\sigma_j} \int dY Y^{1/2} \int_0^{h(\theta)} d\Lambda [1 - \Lambda/h(\theta)]^{-1/2}$ and performing the σ_j summation by using the transformation properties of w and $\hat{t}$ when $\sigma_j \rightarrow -\sigma_j$ at constant θ for both trapped and circulating particles as in Ref. 13, we get the final results, Eqs. (26) and (27), respectively.

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Table 1

R_0	80 cm
A	23 cm
B_0	2.8 T
n_0	$0.5 \times 10^{13} \text{ cm}^{-3}$
x_n	1.0
x_{Te}	1.0
x_{Ti}	1.0
α_n	3.5
α_{Te}	3.5
α_{Ti}	3.5
$T_e(0)$	variable
$T_i(0)$	variable

Numerical values of parameters used to determine equilibrium profiles.

$\tau = T_e(r)/T_i(r) = 2$ is fixed throughout the numerical analysis.

Table 2

Numerical values of eigenfrequencies with temperature

T_e (keV)	δ_{av} (%)	$\omega_r (\times 10^4 \text{ sec}^{-1})$	$\gamma = \omega_i (\times 10^4 \text{ sec}^{-1})$
2.0	0	-1.560	2.040
2.0	0.0298	-1.542	2.018
2.2	0.0328	-1.72	2.210
2.4	0.0357	-1.906	2.342
2.6	0.0387	-2.087	2.412
2.8	0.0417	-2.260	2.420
3.0	0.0447	-2.454	2.369
3.2	0.0477	-2.651	2.267
3.4	0.0506	-2.865	2.131
3.6	0.0536	-3.096	1.987
3.8	0.0566	-3.306	1.838

Note that the ratio of the electron temperature to the ion temperature is fixed: $\tau = T_e(r)/T_i(r) = 2$.

FIGURE CAPTIONS

- FIG. 1. Variation of mode frequencies with electron temperatures and β_{av} .
 $\tau = T_e(r)/T_i(r) = 2$.
- FIG. 2. Real part of the perturbed electrostatic potential (arbitrary units).
- FIG. 3. Imaginary part of the perturbed electrostatic potential.
- FIG. 4. Magnitude of the perturbed electrostatic potential for $\ell = 3$, $T_e = 2.0$ keV, $\omega = (-1.542 + 2.018i) \times 10^4 \text{ sec}^{-1}$.
- FIG. 5. Real part of the parallel component of the perturbed vector potential.
- FIG. 6. Imaginary part of the parallel component of the perturbed vector potential.
- FIG. 7. Magnitude of the parallel component of the perturbed vector potential for $\ell = 3$, $T_e = 2.0$ keV, $\omega = (-1.542 + 2.018i) \times 10^4 \text{ sec}^{-1}$.

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Fig. 1

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Fig. 2

Fig. 3

Fig. 4

Fig. 5

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Fig. 6

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Fig. 7

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