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Anomalies, Unitarity and Renormalization * **

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In these remarks, I wish to indicate how renormalizability of a gauge theory with γ_5 couplings to fermions can require additional dynamical constraints to those already well-known in connection with the triangle anomaly. Here, I can only outline how I differ from previous work [for more details, see Refs. 1,2].

Without γ_5 couplings, we know a spontaneously broken gauge theory--Higgs model with massive vectors--is renormalizable by using 't Hooft's gauge.^{3,4}

With γ_5 couplings, there is the additional complication of the anomaly problem. Here, I have a strong conjecture that hopefully can be elevated to the status of a theorem.

Recall that for the lowest-order triangular diagram with two external vectors (couplings γ_μ, γ_ν ; outgoing momenta p_1, p_2) and one external axial-vector (coupling $\gamma_5 \gamma_\lambda$; incoming momentum $q = p_1 + p_2$) the vertex satisfying

$$p_{1\mu} \Gamma_{\mu\nu\lambda} = p_{2\nu} \Gamma_{\mu\nu\lambda} = 0 \quad (1)$$

has an anomalous axial Ward identity⁵

$$q_\lambda \Gamma_{\mu\nu\lambda} = \frac{1}{4\pi} \epsilon_{\mu\nu} (p_1 p_2) \quad (2)$$

which has been successfully related to the $\pi^0 \rightarrow 2\gamma$ decay rate.

Soon after 't Hooft's work, it was pointed out by several people⁶ that this anomaly must be canceled in flavor dynamics to preserve renormalizability. In the general non-abelian case, let the flavor

matrices T^a, T^b, T^c be associated with the vertices $\gamma_\mu, \gamma_\nu, \gamma_5 \gamma_\lambda$ respectively and let the T^a contain the sign (± 1) corresponding to the fermion helicity. Then including the crossed diagrams and summing over flavors gives, at lowest order,

$$q_\lambda \Gamma_{\mu\nu\lambda}^{abc} \sim \text{Tr}(\{T^a, T^b\}_+ T^c) \frac{1}{4\pi^2} \epsilon_{\mu\nu} (p_1 p_2) \quad (3)$$

and hence we require that

$$\text{Tr}(\{T^a, T^b\}_+ T^c) = 0 \quad (4)$$

In an $SU(2) \otimes U(1)$ theory with only left-handed doublets and right-handed singlets, Eq.(4) reduces to $\sum_L Q_L = 0$, which is satisfied separately for each quark-lepton generation: $g_1 = (u, d, e), g_2 = (c, s, \mu), g_3 = (t, b, \tau)$ since $3(\frac{2}{3} - \frac{1}{3}) - 1 = 0$.

What was shown in Ref. 6 is that if Eq.(4) is not satisfied then there is a disaster: there remains the question of the proof of renormalizability when Eq.(4) is satisfied.

Concerning such a proof, with the usual anomaly cancellation, there is the two-step prescription⁷

1. Dimensionally regularize meson loops
2. Handle the closed fermion loops one by one, performing the Dirac trace in four dimensions then regularizing.

My point is that during step (1), unless the triangular vertex has been defined for generic dimension, n , there is a danger of violating unitarity. This is especially so when the triangle is involved in an overlapping divergence,

as in Fig. 1. In such a case, the $(4-n)^{-1}$ in the overlapping divergence may give a finite contribution with the $(4-n)$ term from the triangle. To throw away such a $(4-n)/(4-n)$ piece of the Feynman amplitude can lead to violation of unitarity, in particular of the cutting rules.⁸

With this motivation, we examine the expansion of the triangle in $(4-n)$. In general, the fermion masses $m_\alpha, m_\beta, m_\gamma$ (corresponding to the flavor labelling $T_{\alpha\beta}^a, T_{\beta\gamma}^b, T_{\gamma\alpha}^c$) may be different for the three sides of the triangle. After straightforward algebra, one finds

$$q_\lambda \Gamma_{\mu\nu\lambda}^{abc} = \frac{1}{4\pi^2} \epsilon_{\mu\nu} (p_1 p_2) \times \sum_{\text{flavors}} T_{\alpha\beta}^a T_{\beta\gamma}^b T_{\gamma\alpha}^c \left[1 + (4-n) f(m_\alpha, m_\beta, m_\gamma, p_1^2, p_2^2, q^2) + \dots \right] \quad (5)$$

after ensuring that the vector identities are satisfied. The right-hand-side of Eq.(5) should be zero for all energies p_1^2, p_2^2, q^2 and the necessary and sufficient condition for this is mass degeneracy within each quark-lepton generation. This is sufficient to kill all terms including higher powers $(4-n)^P$; my conjecture is that it is actually necessary for renormalizability.

Thus, with such mass degeneracy the proof of renormalizability seems complete but without this constraint the proof is at best incomplete and may not even hold.

The conventional wisdom appears to be based largely on the intuition that the ultra-violet structure must be unaffected by the Higgs mechanism,

as is the case without γ_5 couplings.³ But such a general argument needs rigorous verification.

To nail down my result, it appears necessary to complete a 6th order calculation to find whether or not the new anomaly somehow cancels. In partial calculations done so far, no sign of such cancellation is seen, but such work is still incomplete. It is technically difficult.

Also, one should show independence of the dimensional regularization method. There exists a method with higher derivatives preserving gauge invariance⁹--this could show the $(4-n)$ expansion is not crucial in obtaining the result.

References

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8. It is necessary but not sufficient for a prescription to give unique, finite answers satisfying all Ward identities. There is also unitarity.
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Figure Caption

Fig. 1 General Feynman diagram containing triangle in overlapping divergence.

Fig. 1