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FIELD-REVERSED PLASMA RINGS

Charles W. Hartman

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Lawrence
Livermore
Laboratory

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A COLLECTIVE ACCELERATOR USING FIELD-REVERSED PLASMA RINGS

Charles W. Hartman
Lawrence Livermore National Laboratory, University of California
Livermore, CA 94550

1. INTRODUCTION

Recent experiments^{1,2} have demonstrated the production and stable trapping of field-reversed plasma rings (compact Torus configurations³). The rings are formed using a magnetized, coaxial, plasma gun and are stably trapped in a shaped flux conserver located at the gun muzzle as shown in Fig. 1.

Following initial transients, typical parameters of the ring are:

$U_{\text{magnetic}} \leq 10 \text{ kJ}$, $R = 20 \text{ cm}$, $E = 5\text{-}10 \text{ kG}$, total number of ions

$N = 10^{20}$ ($n = 2 \times 10^{15} \text{ cm}^{-3}$), a temperature $T = 5\text{-}20 \text{ eV}$, and

$B = 2 \text{ neT} / (B^2 / 2\mu_0) \gtrsim 0.01$. Since the B is small, the rings are nearly force-free. Decay times of the rings range up to $200 \mu\text{s}$ which is adequately long for the applications considered here. The rings are observed to be highly resilient to large distortions ($\delta B/B \approx 1$) both during formation and decay.

This note discusses the possibility of magnetically accelerating the plasma rings. At low-to-moderate ring kinetic energy, application to heating, fueling, and efficient current drive of conventional fusion reactors appears possible. At high ring kinetic energy, applications to inertial-confinement fusion through pellet heating and to transuranic element synthesis appear possible. The rings may be considered to be a self-linking flux bundle having net helicity. From an accelerator point of view, the rings represent collective particle entities held together by magnetic forces and may be viewed as macroparticles or micropellets having large magnetic moment per unit mass. Because of the relatively long lifetime and resiliency of the rings, it

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Fig. 1. Ring formation configuration.

appears possible to accelerate to multimegajoule kinetic energy over reasonable distances and to focus the rings to centimeter-size dimensions.

Although this note is mainly concerned with Spheromak-like compact toruses, alternative possibilities are briefly considered, using relativistic electron rings.⁴

11. RING ACCELERATION

Consider first a basic ring configuration as shown in Fig. 2. In the following, the pressure will be neglected and large aspect ratio R/a will be assumed and extended to $R/a \approx 1$.

The outward hoop stress of the ring can be supported by an equilibrium field,⁵

$$B_{eq} = \frac{aB_p}{2R} \left[\ln \frac{8R}{a} - \frac{5}{4} \right] \approx \frac{aB_p}{2R}$$

where $B_p = \mu_0 I_{toroidal}/2\pi a$. Alternatively the hoop stress can be supported by eddy currents in a nearby conducting shell as shown in Fig. 1.

The magnetic energy of the ring,

$$U_m = U_{poloidal} + U_{toroidal} \approx 2U_{poloidal}$$

is given by,

$$U_m \approx \mathcal{L} I_t^2 = \frac{\psi_p^2}{\mathcal{L}}$$

with $\mathcal{L} \approx \mu_0 R$. The hoop force for equilibrium is,

$$F_{eq} = \frac{dU_m}{dR} = -\frac{U_m}{R}$$

where ψ_p is held constant. If, instead of a uniform B_{eq} , an axial gradient is introduced, an axial force $F_z = -\mu \nabla B_{eq}$ appears and the ring is accelerated.

Fig. 2. Basic ring configuration.

Because of the resiliency of the rings, assume that sufficient ∇B_{eq} can be introduced to create $F_z = \kappa F_{eq}$, $\kappa \leq 1$. Holding R constant (as in a uniformly shaped traveling magnetic wave) then,

$$M\ddot{Z} = \kappa \frac{U_m}{R}$$

or,

$$\ddot{Z} = \kappa \Omega^2 R$$

where M is the ring mass, Z its position, and

$$\Omega^2 = \frac{U_m}{R^2 M} = v_{\text{Alfvén}}^2 / 2R^2 .$$

The ring velocity is,

$$V_r = \kappa \Omega t \Omega R$$

and the kinetic energy is

$$U_k = \kappa U_m \frac{Z}{R} .$$

Let $G = U_k / U_m$ be the energy gain for acceleration over a length L . Some quantities of interest are,

$$\frac{L}{R} = \frac{G}{\kappa}$$

$$\Omega \tau_{\text{acc}} = \frac{1}{\kappa} (2G)^{1/2} ,$$

$$V_r = (2G)^{1/2} \Omega R = G^{1/2} v_A ,$$

and the power $P_r = F_z V_r$ which must be supplied to the ring during acceleration is,

$$P_r = \Omega U_m (2G)^{1/2} \left(\frac{Z}{L} \right)^{1/2} \text{ watts} .$$

Roughly,

$$V_A = 2.2 \times 10^8 \frac{B_{kg}}{\sqrt{m_i} n \text{ cm}^{-3}} \text{ cm}/\mu\text{s}$$

or for the ring, $U_m = (B^2/2\mu_0) 2\pi R \pi R^2$,

$$V_A = \left(\frac{1.2 \times 10^{19} U_m}{Nm_i} \right)^{1/2} \text{ cm}/\mu\text{s}$$

where U_m is in joules, Nm_i is the total number of nuclei in the ring, m_i is the ion mass in amu. Finally, the kinetic energy/nucleon is

$$\epsilon_n = 6 \times 10^{18} \frac{GU_m}{Nm_i} \text{ eV} .$$

In some cases it appears that the power requirements may limit the maximum ring energy for a given accelerator length. From the previous expressions the power supplied to the ring P_r is,

$$P_r = \left[\frac{1.2 \times 10^{19}}{Nm_i} \right]^{1/2} \frac{U_r^{3/2}}{\kappa L} .$$

Using this expression L vs U_r is plotted in Fig. 3 for $N = 1.2 \times 10^{19}$ ions and $\kappa = 1$. (Note, L depends only on the ring parameters N , m_i and not on U_m or the ring radius R .) From Fig. 3 it can be seen that with a lot of power (10^{12} W) ring kinetic energies up to 10^8 joules possibly can be attained with

Fig. 3. Acceleration length, L , acceleration time, τ_{acc} , and kinetic energy per nucleon, E_n , vs the ring kinetic energy U_k for $P_{max} = 10^{12} W$, $\kappa = 1$, $U_m = 10^4$, $R = 10$ cm, and $N = 1.2 \times 10^{19}$ nuclei.

"reasonable" accelerator lengths. Also shown in Fig. 3 are τ_{acc} and the kinetic energy per nucleon E_n . For light rings, $m_i = 1$, E_n of 1-10 MeV appears possible, while for heavier ions, $m_i \approx 100$, $E_n \approx 10$ -100 KeV. In the spirit of collective acceleration, a small percentage of heavy ions in a low mass ring could achieve multi MeV/nucleon.

III. RING STABILITY

In addition to providing the necessary conditions for acceleration, macroscopic stability must be maintained. The most serious instability of Spheromak-like rings discussed thus far is the tilting instability. A ring situated in a uniform equilibrium field or a cylindrical flux conserver tends to tilt as shown in Fig. 4 and open the closed flux surfaces in the field case or assume a racetrack shape in a cylindrical flux conserver. Tilting in an equilibrium field quickly annihilates the ring in a time τ which is roughly,

$$\tau \approx \frac{R}{a} v_{Alfvén} \cdot$$

while the "racetrack" configuration appears to be grossly stable with 100 μs or so lifetimes in keeping with the aforementioned resiliency.

Figure 5 shows four alternative possibilities for avoiding tilt motion during acceleration. Figure 5(a) relies on spinning the ring at near Alfvén speed to stabilize tilting.⁶ The upright, symmetric ring configuration would be accelerated by inductive coupling to a traveling magnetic wave as indicated. This possibility has certain advantages associated with not having to have conducting walls near the ring. Focusing requires the rotational speed to be controlled. The remaining possibilities of Fig. 5(b) - 5(d), involve stabilization of the ring by $v_{toroidal}$ acceleration (a coaxial rail gun) [Fig. 5(b)] or by acceleration orthogonal to the ring axis.⁷ Each remaining

a) In $B_{\text{equilibrium}}$

b) In cylindrical flux conserver

Fig. 4. Tilt instability.

Fig. 5. Alternative acceleration methods.

possibility is adaptable to rail-type acceleration as shown in Fig. 6, provided good contact can be maintained with the electrodes.

Finally, plasma rings formed with relativistic electrons (gyroradius = R) have been formed. Such rings are observed to be stable to tilting and other motions, the lifetime ($\tau \approx 1$ ms) is adequately long for acceleration, and the rings have been transported by moving mirrors. It is unlikely, however, that electron rings will prove to be as resilient as the rings discussed here, thereby limiting κ and increasing the accelerator length. The possibilities of electron rings have not been evaluated yet; although a related study of ion rings in this context has been made.⁸

IV. FOCUSING

In applications such as pellet bombardment it is necessary to focus the ring. What is imagined here is that the ring is accelerated, emerging with radius R_0 and energy $U_{ke} = GU_{m0}$ with $G \gg 1$. The ring then enters a converging conducting funnel (see Fig. 7) which is shaped so as to exert a focusing force

$$F_r = \lambda \frac{U_m}{R} .$$

At the exit of the funnel the ring is either focused to the desired radius or emerges with a radially inward velocity to focus at some point beyond the exit. The final focal-spot size depends on the interplay of factors such as ring radius during acceleration, plasma species, ohmic heating, radiation loss, and limiting stable β . Estimates of these factors and possible focused states are discussed later in this section.

Two ring configurations during focusing can be examined: an upright ring (symmetry and velocity axes aligned), and a horizontal ring (axes orthogonal) as shown in Fig. 4(b). In both cases simple flux-conservation arguments

a) Coaxial rail

b) Racetrack ring, $B_{eq} = 0$

c) Frisby ring

Fig. 6. Rail-type acceleration.

Fig. 7. Focusing with a convergent funnel.

suggest that in the frame reference of the ring the compression is self-similar, i.e., $L \propto P$. Since focusing (compression) takes place rapidly, $pV^\gamma \approx \text{constant}$ while $U_m \propto 1/R$ gives $\beta \propto pV/U_m \propto 1/R$ for $\gamma = 5/3$. Thus, one factor which may limit slow compression is the maximum stable β of the configuration, $\beta_{\max} \approx 0.2$.

For the upright ring with spin stabilization, it appears necessary to control the rotational speed by "rifling" because if $MRV = \text{const}$, $U_\theta \sim 1/R^2$. Spin up and spin down presumably would be accomplished by rifling the gun and the accelerator--no more smooth bores! A horizontal or slowly tilting ring compresses similarly to the upright ring so that no spin appears necessary for focusing.

To estimate funnel focusing, as before, assume the radial force exerted on the ring is a fraction δ of the equilibrium force, i.e.,

$$F_r = -\frac{\delta U_m}{R} = -\frac{\delta \psi_p^2}{\mu_0 R}$$

where plasma pressure is neglected. Letting $y = R/R_0$ and $x = \Omega_0 t$,

$\Omega_0 = V_{A0}/R_0$, the ring trajectory $y(x)$ is given by,

$$\sqrt{2\delta} x = [y(1-y)]^{1/2} + \frac{\pi}{2} - \tan^{-1} \left(\frac{y}{1-y} \right)^{1/2}$$

where

$$\frac{dy}{dx} = -\sqrt{2\delta} \left(\frac{1-y}{y} \right)^{1/2}.$$

The trajectory curve is plotted in Fig. 8. Roughly, from Fig. 8 it can be seen that the time to focus is approximately

$$\Omega_0 \tau \approx \frac{\pi}{2} \frac{1}{\sqrt{2\delta}}$$

Fig. 8. Funnel shape, $F_r = F_{eq}$. Here $y = R/R_0$, $x = \Omega_0 t$, and $\delta = 1$.

or about one Alfvén transit for $\delta = 1$. The distance to focus, $V_r = G^{1/2} V_A$, is $L_f = V_r \tau = R/\delta$ where $\delta < 1$ can be adjusted to increase L_f . (Here it is assumed $V_r = \text{constant}$, $MV_r^2/2 \gg U_m$).

If, instead of the above, the ring emerges from a funnel section with radius R_0 and inward velocity $y'_1 = R_1/V_{A1} < 0$, it will reach a minimum radius $y_{\min}$ and then expand indefinitely. For this case $y(x, y_1)$ is given by

$$\sqrt{2} cx = y'_1 \left[\left[\frac{y(cy - 1)}{c - 1} \right]^{1/2} - 1 \right] + \frac{1}{c^{1/2}} \ln \frac{[(cy - 1)^{1/2} + (cy)^{1/2}]}{[(c - 1)^{1/2} + c^{1/2}]}$$

where

$$c = 1 + \frac{y_1'^2}{2}$$

and

$$y' = y'_1 \left[1 + 2 \frac{(y - 1)}{y(c - 1)} \right]^{1/2}.$$

The minimum radius is when $y' = 0$, or

$$y_{\min} = \frac{1}{c} = \frac{1}{1 + \frac{y_1'^2}{2}}.$$

The time to reach $y_{\min}$ is

$$\tau = \frac{-\dot{R}_1}{\sqrt{2} V_{A1} \Omega_1}$$

and for a ring having velocity $V_r = G^{1/2} V_{A0}$ the distance traversed in reaching $y_{\min}$ is

$$\frac{L_{\text{focus}}}{R_1} = - \left(\frac{G}{2} \right)^{1/2} \frac{\dot{R}_1}{V_{A1}}.$$

The results of the focusing calculations are summarized in Fig. 9.

In the preceding discussion of acceleration and focusing, the ring was considered to be treated "gently," i.e., forces of order $F \sim F_{eq}$ were applied. Arguments can be advanced that on the timescale of focusing, at least, both the β limit and the force magnitude $\delta = F/F_{eq}$ can be large. In effect, for much larger forces and β , the electrons probably remain tied to field lines and because of high overall density charge neutrality requires that the ions follow them.

Considering focusing, the angular change in the Alfvén transit frequency

$$\theta = \int_0^{\tau_{\text{focus}}} \Omega \, dt$$

is of order $\theta \approx 1/\sqrt{\delta}$ to focus in a funnel. To focus with $\delta \approx 1$, $\theta \approx 1$ so that a single Alfvén wave transit occurs. As the β limit is exceeded, in the frame of the ring, shear inhibited interchange is expected to cause the plasma to be transported across the ring field so that focusing might be lost. Transport takes place at no greater than the local ion sound speed, and probably ~ 0.1 less so that net transport is probably small during focusing. At $\beta = 1$, $V_{\text{Alfvén}} \approx V_{\text{sound}}$ and transport may become important however no obvious limit to focusing exists except in the total kinetic energy of the ring. Similarly, no upper limit to δ is obvious other than the very high electrostatic charge-separation limit.

Focusing may provide access to extreme plasma and magnetic field parameters. Consider, for example, a ring with

Fig. 9. Summary of focus calculations.

$$U_{m0} = 10^4 \text{ j} \quad (B_0 \approx 10 \text{ kg})$$

$$U_k = 10^7 \text{ j} \quad (G = 10^3)$$

$$R_0 = 10 \text{ cm}$$

$$I_0 = 0.3 \times 10^6 \text{ A}$$

$$n_0 = 10^{15} \text{ cm}^{-3}$$

$$T_0 = 10 \text{ eV}$$

which enters a funnel exerting a focus force with $\delta \approx 1$. Suppose further that during compression β is limited to $\beta_{\max} \approx 0.2$. Since

$$U_m = U_{m0} \frac{R_0}{R}$$

$$I = I_0 \frac{R_0}{R}$$

($\psi_p = \text{const}$) maximum compression occurs when the ring stagnates, i.e.,

$$U_m = U_k$$

since β is low, giving

$$\frac{R_0}{R} = \frac{U_k}{U_{m0}} = G$$

or,

$$R = 10^{-2} \text{ cm} .$$

Since $B \sim 1/R^2$, the initial $B_0 = 10$ kg would rise to,

$$B = 10^5 \text{ kg} = 100 \text{ mg}$$

In the absence of losses the plasma would compress adiabatically, $p v^\gamma = \text{const}$,
i.e.,

$$v \sim \frac{1}{R^3}$$

$$p \sim \frac{1}{R^5} .$$

However,

$$B \sim \frac{1}{R^2}$$

so,

$$\beta = \frac{2p}{B^2} \sim \frac{1}{R}$$

and β would exceed β_{max} . Let plasma losses hold β down so that

$$n \sim \frac{1}{R^2}$$

and,

$$T \sim \frac{1}{R^2} .$$

The final plasma parameters would be,

$$n = 10^{21} \text{ cm}^{-3}$$

$$T = 10^7 \text{ eV} !$$

Clearly a number of factors may limit focusing to a smaller value of R (some rotational mass motion, motion of the funnel walls, field soak-in, etc.). Possibly, the final focusing section should be coupled to a high field plasma focus or z-pinch.

1. AN ACCELERATOR CONFIGURATION

Possibly the simplest form of accelerator is the "super gun" or coaxial rail configuration shown in Figs. 5(b) and 10. An isolated ring is formed by the gun and accelerated by the B_θ field in the ring wake. Taking the acceleration force $F = U_m/R$, ($\kappa = 1$, $R = (a + b)/2$), the force exerted by the B_θ field is

$$F = \frac{d}{dz} \frac{\mathcal{L} I^2}{2}$$

where $\mathcal{L} = (\mu_0/2\pi) z \ln b/a$ or,

$$F = \frac{\mu_0}{2\pi} \ln \frac{b}{a} \frac{I^2}{2}$$

$$= \frac{U_m}{R}$$

and since $U_m \approx \mu_0 R I_r^2$,

$$I = \frac{4\pi}{\ln \frac{b}{a}} I_r$$

The voltage between the gun center conductor and the outer conductor at the ring is supplied by sequentially firing capacitor banks 1, 2, etc. is

$$V = \frac{d}{dt} (\mathcal{L} I) = I \frac{\mu_0}{2\pi} v_r \ln \frac{b}{a}$$

Fig. 10. "Super Gun" accelerator.

where I is held constant or, since $V_r = G^{1/2} V_A$,

$$V = G^{1/2} I \frac{\mu_0}{2\pi} V_A \ln \frac{b}{a} .$$

at the end of acceleration, $G = L/R$.

Consider a gun somewhat like the BETA II gun producing a ring with

$$R \approx 15 \text{ cm} \approx \frac{a+b}{2}$$

$$a = 10 \text{ cm}$$

$$b = 20 \text{ cm}$$

$$I_r = 0.3 \text{ MA}$$

$$m_i N = 1.2 \times 10^{20} \quad (n_{D^+} \approx 2 \times 10^{15} \text{ cm}^{-3})$$

$$V_A = 3 \times 10^7 \text{ cm/s}$$

$$U_m \approx 2 \times 10^4 \text{ J} .$$

To accelerate, $I \approx 3I_r \approx 1 \text{ MA}$ and,

$$V \approx 60 G^{1/2} \text{ kV} .$$

Then the following table can be constructed,

$G = 10$	$V = 180 \text{ kV}$	$L = 150 \text{ cm}$	$U_k = 0.2 \text{ mJ}$
100	600	15,00	2 MJ
1000	1800	15,000	20 MJ

Similar arguments can be constructed for the traveling wave version. If no effort is made to recover the magnetic energy in the coaxial rail after acceleration, the maximum efficiency is 50%.

VI. APPLICATION TO MAGNETIC FUSION

Several aspects of plasma rings may have application to magnetic fusion. First, the ring itself is a magnetic bottle. The possibility of converting the kinetic energy of the ring into compressional heating of the plasma has been discussed earlier in conjunction with focusing. A very high field ($B \sim 1$ mg) moving ring reactor with no first wall based on the above possibility has been investigated.⁹

Other possibilities exist in which the ring is injected into a fusion device and acts as a carrier of flux, fuel, or energetic particles depending on conditions. A schematic illustration of one possible cross-field injection scheme is shown in Fig. 11. An accelerated ring passes through a circular flux conserver region in a racetrack configuration [see Fig. 4(b)]. The channel cross-section changes to rectangular shape as shown and the ring emerges moving across the confinement field of the fusion device (e.g., Tokamak or Tandem Mirror central cell). The confinement field also serves to maintain the ring equilibrium⁷ and as the ring drifts towards the center of the device, tilting takes place annihilating the ring poloidal field and depositing flux¹⁰ and particles.

It is entirely possible that by varying the ring size, speed, and mass it can penetrate completely to the center of the fusion device and that tilting can be made to occur there. If so, this approach may provide a solution to penetration which appears to require excessively energetic neutral beams for heating, or very high particle velocities¹¹ for fueling. As discussed

Fig. 11. Cross-field ring injection.

earlier, ion kinetic energy in the range 0.1-1 MeV appears possible at say, 1 MJ/ring for heating on the 1-10 MW level. For fueling (say tritium at the center of a Tokamak to minimize inventory) rings with 10-100 coulombs of ions appear possible for 100-1000 A injectors.

Another application, transporting energetic electrons into a fusion device, has been discussed earlier.¹²

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