Nonlinear Drift Tearing Modes

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ABSTRACT

It is shown analytically and with a numerical code that the nonlinear growth and saturation of magnetic islands in tokamaks is not affected by diamagnetic corrections. Diamagnetic effects change the linear growth and introduce mode rotation, which persists in the nonlinear regime. The analysis is used to construct a model for the effect of feedback stabilization schemes in the nonlinear regime.

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I. INTRODUCTION

In a recent study of Mirnov oscillations in Ormak, Carreras et al., have found good agreement between elementary nonlinear tearing mode theory predictions and the experimentally measured values of the variations of poloidal magnetic field.¹ Similarly, it was reported in Ref. 2 that elementary theory gave a good description of the behavior of magnetic islands in the ST tokamak.

By elementary theory we mean a magnetohydrodynamic description of the plasma with resistivity as the only nonideal effect. We here report on the inclusion of other nonideal effects such as the hall current and gyroviscosity. The results are, that the nonlinear behavior of tearing modes cited in the above works^{1,2} is unaffected by the inclusion of these diamagnetic effects. Most notably the nonlinear growth and saturation of the magnetic islands are unchanged when the diamagnetic effects are included.

The organization of this paper is as follows. First we give a simple derivation of the diamagnetic equations. Then the linear theory of these equations is reviewed. In Section IV we expand the nonlinear solution in $1/S$ (S being the ratio of the resistive time to the poloidal Alfvén time). We then show how the solution of the elementary equations is suitably modified to obtain a solution to the diamagnetic equations. Numerical verification of the results of Section IV is presented next. Further application of these results are made to feedback stabilization of rotating islands, and finally a general discussion is presented in Section VI.

II. BASIC EQUATIONS

The starting point for our investigation is an analysis of the two fluid equations

$$\rho \frac{dv}{dt} = -\nabla P + \mathbf{J} \times \mathbf{B} - \nabla \cdot \Pi^m \quad (1)$$

$$\mathbf{E} = -\nabla \times \mathbf{B} + \frac{\mathbf{J} \times \mathbf{B}}{nq} - \frac{\nabla P_e}{nq} + \eta \mathbf{J}, \quad (2)$$

where Eq. (1) is the ion equation of motion and Eq. (2) is the electron equation of motion ignoring electron inertia. The magnetic or gyroviscous part of the stress tensor Π^m is given by Braginskii.³ We expand these equations in ϵ using the usual low β tokamak ordering $\beta \sim \epsilon^2$ and $q \sim 1$. Here β is the ratio of fluid energy to magnetic energy, q is the safety factor and ϵ is the inverse aspect ratio a/R . This ordering allows us to treat the tokamak as a cylinder in lowest order and use helical symmetry (i.e., all quantities are functions only of r , τ , and t , where $\tau = m\theta + n\phi$, m and n being the poloidal and toroidal mode numbers respectively). We then have from $\nabla \cdot \mathbf{B} = 0$ that the magnetic field can be expressed as

$$\vec{B} = \nabla \psi \times \hat{z} + B_z \vec{h}, \text{ with } \vec{h} = \frac{-n}{m} \left(\frac{r}{R} \right) \hat{\theta} + \hat{\tau}.$$

The equations are now completed by using

$$\nabla \times \vec{B} = \vec{J} \quad \text{and} \quad \nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}.$$

From the z component of Faraday's law we find that $\nabla \cdot \mathbf{v} = 0$ to lowest order and that $v_z \approx 0$ from the equation of motion.

We therefore have $\mathbf{v} = \nabla A \times \hat{\mathbf{z}}$.

The resulting equations for λ , ψ , and N (the ion number density) are

$$\frac{d\psi^2}{dt} = \hat{\mathbf{z}} \cdot \nabla \psi \times \nabla (J_z) S^2 + \Pi^* \quad (3)$$

$$\frac{d\psi}{dt} = \omega_{*T} \frac{\nabla \psi \times \hat{\mathbf{z}} \cdot \nabla N}{N} - \eta J_z + E \quad (4)$$

$$\frac{dN}{dt} = -\omega_{\nabla \cdot \mathbf{v}} \hat{\mathbf{z}} \cdot \nabla \psi \times \nabla (J_z) \quad (5)$$

where

$$\Pi^* = -\omega_{*i} \hat{\mathbf{z}} \cdot \nabla \times [(\mathbf{z} \times \vec{\nabla} N) \cdot \vec{\nabla} \mathbf{v}]$$

$$J_z = -\nabla^2 \psi - 2$$

$$\omega_{*i} = \frac{T_i}{qB_z a^2} \tau_R, \quad \omega_{*T} = \frac{T_i + T_e}{qB_z a^2} \tau_R$$

$$\omega_{\nabla \cdot \mathbf{v}} = S \left(\frac{1}{\tau_H \Omega_i} \right), \quad S = \frac{\tau_R}{\tau_H}$$

$$\tau_R = \frac{a^2}{\eta_{rs}}, \quad \tau_H = \frac{\sqrt{\rho R}}{B_z} \left(\frac{m}{n} \right)$$

$$\Omega_i = \frac{qB_z}{m_i}.$$

The units of time, flux, resistivity and number density in Eq. (3) - (5) have been scaled by $\tau_R, (n/m) (B_z a^2/R), \eta_{rs}$ (resistivity at the singular surface, where $q = m/n$), and N_{rs} respectively. Equation (3) follows easily from Eq. (1) by applying $-\hat{z} \cdot \nabla x$, keeping $\rho = \text{constant}$ and noting that $(\vec{B} \cdot \nabla) J_z \gg (\vec{J} \cdot \nabla) B_z$. Equation (4) follows from Faraday's law by dotting with the vector $\vec{h}$ and using $\vec{J} \times \vec{B} \approx \nabla_\perp P$ i.e., ignoring terms of order ω/Ω_i . We have also assumed that the parallel thermal conductivity is infinite. The term on the right hand side of Eq. (5) allows for electron flow parallel to the field and is found keeping the lowest order nonzero term in $\nabla \cdot v$ arising from the z component of Faraday's law. In the present model n, T_i and T_e are not advanced. These equations are discussed in more detail in Refs. 4,5.

III. SOLUTION OF THE DIAMAGNETIC EQUATIONS IN THE LINEAR REGIME

The modification of the dispersion relation for the tearing mode caused by the introduction of nonideal drift terms was accomplished some time ago.⁶ The modified mode is called a drift tearing mode. If the time dependence is taken as $e^{-i\omega t}$ we have^{6,7}

$$(\omega - \bar{\omega}_{*T})^3 \omega (\omega - \bar{\omega}_{*i}) = i\gamma_T^5$$

where

$$\bar{\omega}_{*T} = -\frac{\omega_{*T} N'_m}{r}$$

$$\bar{\omega}_{*i} = -\frac{\omega_{*i} N'_m}{r}$$

and

$$\gamma_T = 0.55 \eta_{rs}^{3/5} (S)^{2/5} \left(\frac{m}{r} \psi''_0 \right)^{2/5} (\Delta')^{4/5}.$$

Here γ_T is the growth rate of the pure tearing mode and Δ' is the discontinuity in the logarithmic derivative of the exterior solution. The main effect of the drift terms is to cause the mode to rotate with a frequency of the order of $\bar{\omega}_*$ and to decrease the growth rate as ω_* approaches γ_T . Recently Biskamp⁸ has done numerical work that indicates that γ may again increase as $\omega_* \gg \gamma_T$.

IV. SOLUTION IN THE NONLINEAR REGIME

Here we assume that the width of the island has greatly exceeded the tearing layer width and that the gradient scale lengths are now of the order of the island width as was done by Rutherford in his description of the elementary nonlinear tearing mode.⁹ We now show how Rutherford's solution must be modified when the drift terms are included.

Formally we introduce the parameter $\delta = 1/s^2$ and take

$$\frac{\partial}{\partial t} \sim 1$$

$$A \sim A_0 + \delta A_1 + \dots$$

and similarly for ψ and N . The equation of motion [Eq. (3)] then implies that $\nabla^2 \psi = f(\psi) + o(\delta)$. Introducing the notation

$$\langle f \rangle = \oint f \frac{d\ell}{|\nabla\psi|} / \oint \frac{d\ell}{|\nabla\psi|}$$

$$[\vec{f}] = \oint \nabla\psi \times \hat{z} \cdot \vec{f} \frac{d\ell}{|\nabla\psi|}$$

we find

$$\langle \dot{\psi} \rangle = -\langle \eta \rangle J + E \quad (6)$$

$$J = -\nabla^2 \psi - 2 = J(\psi) \quad (7)$$

$$[\dot{\vec{v}}] + [\vec{v} \cdot \nabla \vec{v}] = \omega_{*i} [\hat{z} \times \nabla \eta \cdot \nabla \vec{v}] \quad (8)$$

$$\dot{N} = -\vec{v} \cdot \nabla N \quad (9)$$

$$\vec{v} \cdot \nabla \psi = -\eta J + E - \dot{\psi} - \omega_{*T} \hat{z} \times \nabla \ln(N) \cdot \nabla \psi \quad (10)$$

Where we have used $\omega_{\nabla \cdot \nabla} \ll s^2$ in arriving at Eq. (9) and Eq. (8) comes from integrating Eq. (3) once to find

$$\frac{\partial \vec{v}}{\partial t} + \vec{v} \cdot \nabla \vec{v} = \nabla \psi \nabla^2 \psi \left(\frac{1}{s^2} \right) + \omega_{*i} \hat{z} \times \nabla N \cdot \nabla \vec{v} + \nabla \lambda \quad (11)$$

We take $\psi(0)$ and $N(0)$ as given, where $\psi(0)$ denotes $\psi(t=0)$. Now we note that Eqs. (6) and (7) are independent of drift effects and so may be integrated analytically following Refs. 9 or 2 or

integrated numerically in any of the standard¹⁰ or non-standard ways.¹¹ There is however some arbitrariness in the obtained solution for $\psi(o)$ since the vector $\omega \partial \psi(o) / \partial \theta$ is a null vector for the $\langle \rangle$ operator. $[\partial \psi(o) / \partial \theta]$ is also a null vector for the operator $\nabla^2 + (\partial J / \partial \psi)$ used to find $\dot{\psi}$ in the standard approach of differentiating the constraint equation with respect to time.¹⁰ We therefore denote the non-rotating solution as $\psi_n(o)$. Hence

$$\dot{\psi}(o) = \dot{\psi}_n(o) + \omega(o) \frac{\partial \psi(o)}{\partial \theta} \quad (12)$$

From Eq. (10) we have

$$\vec{v}(o) \cdot \nabla \psi(o) \approx \eta J(o) + E - \dot{\psi}_n(o) - \omega(o) \frac{\partial \psi}{\partial \theta} - \omega_{*T} \hat{z} \times \nabla \ell n N(o) \cdot \nabla \psi(o) \quad (13)$$

and therefore

$$A(o) = \int_0^{\chi(o)} [-\eta J(o) - \dot{\psi}_n(o) - \omega_{*T} \hat{z} \times \nabla \ell n N(o) \cdot \nabla \psi(o)] J d\chi + \frac{r^2 \omega(o)}{r} + g(o) \quad (14)$$

Here, χ is an angle-like coordinate around each ψ line,

$$J = \frac{1}{\nabla \psi \times \nabla \chi \cdot \hat{z}}$$

is the Jacobian for these variables, and g is a function of ψ and t only. We therefore take $\omega(o)$, and $g(o)$ as given initial conditions. This merely amounts to knowing $\vec{v}(o)$ consistent with

$(\partial/\partial t) \sim 1$. Now we can solve these equations as an initial value problem. First we calculate

$$L(0) = \hat{z} \cdot \oint \vec{r} \times \vec{v}(0) d\tau \quad (15)$$

the angular momentum at $t = 0$.

Next we advance ψ , N , J

$$\psi(\Delta t) = \psi(0) + \left[\dot{\psi}_n(0) + \omega(0) \frac{\partial \psi(0)}{\partial \theta} \right] \Delta t \quad (16)$$

$$N(\Delta t) = N(0) - \vec{v}(0) \cdot \nabla N(0) \Delta t$$

$$J(\Delta t) = -\nabla^2 \psi(\Delta t) - 2$$

after which a new coordinate χ can be selected. Equations (6) and (7) can now be integrated to find $\psi_n(\Delta t)$ and $A(\Delta t)$ is found from Eq. (10)

$$A(\Delta t) = \int_0^{\chi(\Delta t)} [-\eta J(\Delta t) - \dot{\psi}_n(\Delta t) - \omega_{*T} \hat{z} \times \nabla \ell n N(\Delta t) \cdot \nabla \psi(\Delta t)] J d\chi \\ + \frac{r^2 \omega(\Delta t)}{2} + g(\Delta t) \quad (17)$$

or

$$\vec{v}(\Delta t) = \vec{v}_1 + r\omega(\Delta t)\hat{\theta} + g'(\Delta t)\nabla\psi \times \hat{z} \quad .$$

Here $' = (\partial/\partial\psi)$ and

$$\vec{v}_1 = \nabla \int_0^{\chi} (-\eta J + -\dot{\psi}_n - \omega_{*T} \hat{z} \times \nabla N \cdot \nabla \psi) J d\chi \times \hat{z} \quad . \quad (18)$$

An expression for $g'(\Delta t)$ is found from Eq. (8)

$$\begin{aligned} g'(\Delta t) & \oint \frac{|\nabla\psi|}{|\nabla\chi|} \partial\chi + \omega(\Delta t) [\hat{r}\hat{\theta}] + [v_1(\Delta t)] \\ & = g'(0) \oint \frac{|\nabla\psi|}{|\nabla\chi|} \partial\chi + \omega(0) [\hat{r}\hat{\theta}] + [v_1(0)] \\ & - [v(0) \cdot \nabla v(0)] \Delta t + \omega_{*i} [z \times \nabla \eta(0) \cdot \nabla v(0)] \nabla t . \end{aligned} \quad (19)$$

We now use the solubility condition (the conservation of angular momentum) that comes from Eq. (11) to solve for $\omega(\Delta t)$

$$L(\Delta t) = L(0)$$

$$\frac{\pi}{2} \omega(\Delta t) + \hat{z} \cdot \int \vec{r} \times \vec{v}_1(\Delta t) d\tau + \hat{z} \cdot \int \vec{r} \times g'(\Delta t) \nabla\psi \times \hat{z} d\tau = L(0) . \quad (20)$$

Now all quantities have been stepped and the procedure can be repeated.

A word on the solubility condition. This condition is merely a global constraint that follows from the equation of motion Eq. (3) but that does not follow from Eq. (8). Satisfying the solubility condition insures us that we will be able to continue the expansion process and solve for ψ_1 , N_1 , etc., and removes the arbitrariness of the solution by determining $\omega(t)$. The other obvious global constraint, the conservation of vorticity, does follow from Eq. (8) and therefore is automatically satisfied.

As a first example of the application of the above stepping procedure we show how in the case of $\omega_* = 0$, the solutions of the

above equations, as described in Ref. 2 and 9, can be used to construct the solution in the presence of an initially rotating fluid. Taking $\omega(0) = \omega_0$ and $g(0) = 0$ we have from Eqs. (14), (15), and (16)

$$\psi(\Delta t) = \psi(0) + \left[\dot{\psi}_n(0) + \omega_0 \frac{\partial \psi}{\partial \theta} \right] \Delta t \quad (21)$$

$$L(0) = \frac{\pi \omega_0}{2}$$

$$A(0) = \int_0^{\chi} [-nJ(0) - \dot{\psi}_n(0)] J d\chi + \frac{r^2 \omega_0}{2}$$

or

$$A(0) = \dot{A}_n(0) + \frac{r^2 \omega_0}{2} .$$

Then Eqs. (17) and (19) imply that

$$A(\Delta t) = A(0) + \left[\dot{A}_n(0) + \omega_0 \frac{\partial A_n}{\partial \theta} \right] \Delta t + \frac{r^2 \omega(\Delta t)}{2} . \quad (22)$$

Finally, by the symmetry of the non-rotating solution, terms 2 and 3 in Eq. (20) vanish yielding

$$\omega(\Delta t) = \omega_0 .$$

This is, of course, the expected result since Eqs. (3) - (5) remain unchanged when they are written in a frame moving with the angular velocity ω_0 , and Eqs. (21) and (22) are just the first

terms in the expansion of the exact solution

$$\psi(r, \theta, t) = \psi_n(r, \theta - \omega t, t)$$

$$A(r, \theta, t) = A_n(r, \theta - \omega t, t) + \frac{r^2 \omega_0}{2}.$$

It is now easy to see how we may take a solution $\psi(t)$ for the $\omega_* = 0$ case and generate a solution $\psi_*(t)$ for the $\omega_* \neq 0$ case. We can accomplish this because Eqs. (6) and (7) are independent of ω_* . The solution $\psi_*(t)$ is therefore the same as $\psi(t)$ except that it is rotated at some frequency ω , determined by the ω_* terms. (We note that this rotation is a rotation of islands, i.e., mode rotation, and not a rotation of the fluid as a whole, since the angular momentum remains constant.) The fact that $\psi_*(t)$ and $\psi(t)$ are the same except for a rotation means that the rate at which the island grows and the saturation width of the island will be independent of ω_* terms. The flow pattern, $A_*(t)$, is however greatly affected by the ω_* terms and is not a simple rotation of $A(t)$.

We choose a solution of the $\omega_* = 0$ equations with a definite symmetry about $\theta = \theta_0$ i.e., of the form

$$\psi(\bar{r}, t) = \sum_M \psi_M(r, t) \cos(m\theta - m\theta_0)$$

where θ_0 is independent of r and m . There are a number of reasons why these are preferred solutions. First, these are the solutions that have been quoted in the literature and used to make

comparison with experiments. Secondly, this is the form the solution would have if the island were to grow from an eigenmode in the $\omega_* = 0$ case. (The nonlinear coupling does not destroy this symmetry.) And this seems to be approximately the form that is observed in the numerical runs to develop out of an eigenmode in the $\omega_* \neq 0$ case (at least for the lowest harmonics). This is reasonable since outside the island the mode is ideal with no radial phase shift and in the island "constant ψ " is appropriate for these modes. Nonlinear theory^{2,9} indicates that only the principle and the $m = 0$ harmonics are necessary to obtain good agreement with the numerical experiments, other harmonics being negligible. Thus the symmetry assumption is probably irrelevant for the higher harmonics.

V. NUMERICAL INTEGRATION OF THE DRIFT TEARING EQUATIONS

The conclusions of the last section have been verified by numerically integrating Eqs. (3), (4), and (5). The solution is accomplished by finite differencing the equations on an r, θ grid, using a predictor corrector time step, and an alternating direction implicit scheme is employed to stabilize the diffusion term in Eq. (4).

The results of this numerical integration are shown in Figs. 1 and 2. Figure 1 shows the time history of two nonlinear runs, one with $\omega_* = 0$ and the other with $\omega_* \sim \gamma_T$. We see that both runs saturate at the same width and have the same $\dot{W}$ near the end of the run. The growth rates do not agree early in time because the tearing layer width (δ_T) for this case is $\approx .05$ and the above theory does not hold

until $W \gg \delta_T$. Agreement with the above analysis over a larger range in W is provided by Fig. 2. Here, the tearing layer width is about the same as in Fig. 1 but the saturation width is $W \sim 0.4$. Diamagnetic effects were initiated at $W \approx 0.1$.

The variation of the mode frequency ω with the width of the island is not great for the cases we have run with $S > 10^5$ and $W \gg \delta_T$, being typically 10 to 20 percent with a period of the order of $1/\omega_*$ for these cases. In very resistive cases, with $S < 10^4$ we have observed large changes in ω , probably related to the fact that the island width was not much greater than the tearing width δ_T . In these cases a nonlinear decrease in the frequency is observed, with ω even reversing sign. This frequency decrease is associated with a flattening in the density profile in the vicinity of the island. In Fig. 3 is shown the mode frequency as a function of time for a typical high resistivity case.

Results similar to those described in the last two sections have been obtained by D. Biskamp,¹² who also introduces the angular momentum to determine the rotation. He assumes that density is a constant on a flux surface, i.e., $N = N(\psi)$, while our derivation is independent of this assumption. Our work^{13, 14} also differs from his in that we keep all harmonics, not just the principle and $m = 0$, and that we use the complete form of the viscosity tensor.

VI. APPLICATION TO FEEDBACK STABILIZATION

We now propose simple model equations that may be used to study a variety of feedback schemes to stabilize the drift-tearing

mode. In Refs. 1 and 2, the mode equations

$$\tilde{\psi} = \psi_I \cos(m\theta)$$

$$\dot{\psi}_I = \frac{\Delta'(w)}{w} \psi_I$$

were found to give very good results in describing the nonlinear evolution of the tearing mode when $\omega_* = 0$. Here, $\tilde{\psi}$ is the perturbed flux at the island center, and Δ' is the nonlinear analog of the jump in the logarithmic derivative (see Ref. 2), and w is the width of the island given approximately by

$$w = 4(\psi_I/\psi_0'')^{1/2}.$$

From the results of Section IV, i.e., that Eqs. (6) and (7) are independent of ω_*

$$\tilde{\psi} = \psi_I \cos[m\theta + \omega(t)t] \quad (23)$$

$$\dot{\psi}_I = \frac{\Delta'(w)}{w} \psi_I \quad (24)$$

should give a correct description when $\omega_* \neq 0$. Equations (23) and (24) can be written as two coupled first order differential equations

$$\dot{\psi}_C = \frac{\Delta'_C}{W} \psi_C - \omega \psi_S \quad (25)$$

$$\dot{\psi}_S = \frac{\Delta'_S}{W} \psi_S + \omega \psi_C \quad (26)$$

where

$$\psi_C = \psi_I \cos(\omega t)$$

$$\psi_S = -\psi_I \sin(\omega t)$$

and

$$\omega = 4(\psi_I/\psi_0'')^{1/2}.$$

Feedback stabilization can be achieved by means of helical coils mounted inside the vacuum vessel. The effect of the coils is to modify the boundary conditions for $\tilde{\psi}$ at the wall, which can be described in terms of a modulation of the value of Δ' . We now generalize Eqs. (25) and (26) to apply to the situation where the boundary conditions determining Δ' are not the same for the $\cos(m\theta)$ and $\sin(m\theta)$ components of the mode.

$$\dot{\psi}_C = \frac{\Delta'_C}{W} \psi_C - \omega \psi_S \quad (27)$$

$$\dot{\psi}_S = \frac{\Delta'_S}{W} \psi_S + \omega \psi_C \quad (28)$$

where Δ'_S and Δ'_C are to be found for each harmonic as in Ref. 2.

The validity of this representation has been checked in detail using the numerical code. In particular, we have verified that a modulation of the boundary conditions at the chamber wall through $\psi_w = -f\psi_C \cos m\theta$ leaves the value of $\Delta'_S/w \equiv (\dot{\psi}_S - \omega\psi_C/\psi_S)$ unchanged, and modulates the value of $\Delta'_C/w \equiv (\dot{\psi}_C + \omega\psi_S/\psi_C)$ in a manner consistent with the expected linear dependence of Δ'_C on f .¹⁴ Equations (27) and (28) have provided a useful guide in a search for effective feedback stabilization schemes, giving results which are in reasonable agreement with the results obtained using the full nonlinear code. An exact model has not been constructed due to the difficulty in modeling the dependence of the frequency ω on island width for all values of ω_{*T} and profile parameters. However, numerical runs with no feedback indicate that $\omega \approx \text{constant}$ is a good approximation with values of $S > 10^5$, appropriate for present day tokamaks. This information then allows in conjunction with Eqs. (27) and (28) a reliable model for the effect of feedback stabilization schemes. In particular, we have examined the possibility of a stabilization scheme not requiring phase information; i.e., with Δ'_C modulated at a frequency ω_f , to be chosen to ensure stabilization. Combining Eqs. (27) and (28), however, gives a second order differential equation which unfortunately is not of the Mathieu type for which there are values of ω_f which can produce stable behavior. In Fig. 4 are shown the results of such an attempted stabilization. The results from the model

agree well with those obtained using the full nonlinear equations. Although the island growth is reversed when the phase of the feedback opposes island growth, no dynamic stabilization is achieved.

VII. CONCLUSION

We have shown that the diamagnetic effects on magnetic islands are primarily to cause the islands to rotate at the diamagnetic frequency. The nonlinear growth and saturation of the island are not affected. This result has a rather intuitive interpretation. The growth of the island is proportional to the nonlinear $\Delta'(w)$, which describes the magnetic energy available in changing to a configuration with an island of width W , saturation occurring when further growth is no longer energetically advantageous.² This intuitive picture in terms of basic magnetohydrodynamic driving forces is not modified by the inclusion of diamagnetic corrections, the available magnetic energy for growth continues to dominate the nonlinear behavior. Thus, previous comparisons with experimental data using elementary theories which neglect diamagnetic effects are valid.

This result has allowed us to derive model equations that have been successfully applied to the examination of feedback schemes.¹⁴ The essential result is that, while a feedback signal with the proper phase relation can successfully stabilize magnetic island growth, a phase independent or dynamical stabilization scheme does not exist.

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REFERENCES

- ¹B. Carreras, B. V. Waddell, and H. R. Hicks, Oak Ridge National Lab. Rept. ORNL/TM-6403, Oak Ridge, TN (1978).
- ²R. B. White, D. A. Monticello, M. N. Rosenbluth, and B. V. Waddell, Phys. Fluids 20, (1977) 800.
- ³S. I. Bnaginskii, in Reviews of Plasma Physics, edited by M. Leontovich (Consultants Bureau, NY 1965), Vol. I. p. 205.
- ⁴B. V. Waddell, G. Laval, and M. N. Rosenbluth, Oak Ridge National Lab. Rept. ORNL/TM-5968, Oak Ridge, TN (1977).
- ⁵G. Ara, B. Basu, B. Coppi, G. Laval, M. N. Rosenbluth, and B. V. Waddell, Annals of Phys. 112 (1978) 443.
- ⁶P. H. Rutherford and H. P. Furth, Princeton University Plasma Physics Lab. Rept. MATT-872, Princeton, NJ (1971).
- ⁷R. D. Hazeltine, D. Dobrott, and T. S. Wang, Phys. Fluids 18 (1975) 1778.
- ⁸D. Biskamp, Max Planck Institut für Plasmaphysik Rept. IPP6/163, Garching, Germany (1977).
- ⁹P. H. Rutherford, Phys. Fluids 16 (1973) 1903.
- ¹⁰Y. Pao, Phys. Fluids 19 (1976) 1177.
- ¹¹B. V. Waddell, M. N. Rosenbluth, D. A. Monticello, and R. B. White, Trieste College in Theoretical and Computational Plasma Physics IAEA, Vienna (1978) p. 79; and H. Grad, P. N. Hu, and D. C. Stevens, Proc. Nat. Acad. Sci. 72 (1975) 3789.

¹²D. Biskamp, Max Planck Institut für Plasmaphysik Rept. IPP6/174, Garching, Germany (1978).

¹³D. A. Monticello, R. B. White, and M. N. Rosenbluth, Rotating Magnetic Islands, Paper presented at the Annual Controlled Fusion Meeting, Gatlinburg, TN, April 1978.

¹⁴D. A. Monticello, R. B. White, and M. N. Rosenbluth, Seventh International Conference on Plasma Physics and Controlled Nuclear Fusion Research, Innsbruck, Austria, August 1978.

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 Fig. 1. Growth rate of the island, as a function of the width, w , with and without diamagnetic effects. Here, the singular surface is at $r = 0.4$ and the current profile is given by the peaked model with $r_0 = 0.8$. $S = 1.25 \times 10^5$.

Fig. 2. Width of the island, as a function of time with (dotted curve) and without diamagnetic effects. Here, the singular surface is at $r = 0.67$ and the current profile is given by the flat model with $r_0 = 0.5$. $S = 1.25 \times 10^5$ and $\omega_* = 600$. 792039

Fig. 3. Mode frequency ω as a function of time in a highly resistive case, $S = 10^4$. The nonlinear frequency decrease (reversal) is associated with local density profile flattening (reversal). Such strong frequency changes are not observed for $S > 10^5$.

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Fig. 4. Feedback without phase information. Shown is the island width versus time as obtained using the model Eqs. (27), (28) (solid line), and the full nonlinear code (dotted line). Island growth is momentarily reversed when the feedback phase opposes growth, but overall stabilization is not achieved. Here, $S = 10^5$, $r_S = 0.4$, $r_O = 0.8$.