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COMPUTATION AND GRAPHICS
IN MATHEMATICAL RESEARCH

Progress Report
for Period June 1, 1990 - May 31, 1991

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MASTER

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TABLE OF CONTENT

Cover Sheet.	i
Table of Content	ii
Description of Computation Facility	1
Research Activities	8
Reserach of Joel Spruck.	8
Research of David Hoffman.	13
Personnel Wholly of Partially Supported by this Grant.	17
Research Visitors.	17
Appendix:.	18
Curriculum Vitae of David Chopp	19
Geometry, Computation and Graphics Series List.	21

Computing Facility Description

Center for Geometry, Analysis, Numerics, and Graphics

The *Center for Geometry, Analysis, Numerics, and Graphics (G.A.N.G.)* maintains a computation and visual analysis facility in the University's Lederle Graduate Research Center. This facility serves principal investigators David Hoffman and Joel Spruck, along with collaborators, visiting scientists and graduate students (about 20 people in total).

Computation

The GANG facility centers around a Silicon Graphics 4D/240GTX graphics computer. This machine has 4 20MIP processors, 32MB of memory, 1.2GB of disk storage and a 120MIPS graphics processor. The close coupling of high-speed processing with high-speed graphics has had a great effect on the Center's focus in differential geometry: researchers are now able to compute, and simultaneously visualize in near real-time, problems which required many minutes of time on earlier computers. Animation sequences are now produced as the problems are being calculated. In Winter 1990, the Center will enhance its real-time 3D graphics capability with the addition of a Silicon Graphics 4D/25TG networked to the 4D/240GTX with the *Fiber Distributed Data Interface (FDDI)*. The 4D/240GTX will also be upgraded to the VGX high-performance graphics subsystem. Two Sun 3/60 color workstations are used for still-frame color graphics and additional windows on the server machines. Two Macintosh II color workstations are used by the principal investigators for *Mathematica* and access to the SGI and Sun servers.

The Center's Sun 4/260 serves as the core development machine, and as the main file server for the Center's client (dataless) workstations. This system has 3.0GB of disk storage, 32MB of memory, and a high-resolution monochrome console. Two Sun SPARCstation-1s provide high-performance monochrome access points to users. The Center also uses two NeXT workstations to run the Thinking Machines, Inc. Connection Machine Simulator for experimentation with data parallel algorithms, for video software development, and for inexpensive on-line storage of the Center's picture library using removable optical disk.

The Center's computer systems are connected to other research laboratories and computing facilities (such as the Engineering Computer Services, Math Research Computing Facility, and University Computing Services) via the campus backbone Ethernet. The University plans to upgrade this backbone to a high speed fiber trunk during 1991. The campus backbone has a T1 speed (1.4Mb/s) link to the NSFNet, and slow-speed external connections to the CSNet and BITNet.

Video Animation

During 1990, the Center has re-engineered its approach to the production of scientific video animations. The new video animation system greatly improves video quality, meets archiving needs and makes the system easier to use and accessible from remote locations. A Lyon Lamb Real Time Converter (RTC) replaces the older RS-170 encoder, and generates very high-quality NTSC video (along with Super-VHS and RS-170 RGB) directly from high-resolution computer graphic video. The RTC has three "live" high-resolution inputs and is computer-controllable. Using additional high-resolution video switching and distribution, all of the Center's graphics computers can be connected to the RTC. A Sony LVR5000 Laser Disk Recorder was added to the Center's existing Sony BVU-950 3/4" tape deck to record output from the RTC. Up to 50 minutes of video can be recorded on each LVR disk (write-once), either as still frames or sequences. Media life is expected to exceed 30 years, making the laser disk a far better archiving media than tape. During 1990, software is being added to put these three units - the RTC, laser disk, and tape deck - entirely under computer control (the sole exception being the loading of media). With the addition of broadcast capability in Winter 1990, the new system will broadcast to (and accept video from) remote locations and through the NSFNet, will be controllable from other sites.

Software Development

Because the Center hosts a greater number of visitors and collaborators each year, additional effort has been directed toward documenting the Center's software. MESH, the adaptive mesh generator, has been completely re-worked for interactive use on the 4D/240GTX. Work began in December 1989 on the DISP array graphics package which runs on all the Center's hardware platforms. A version of DISP will be released to collaborating researchers in early 1990, with supporting documents, installation scripts and examples. Work will begin in 1990 on documenting the DIS (rasterfile display), VEW (polygon file display), SOLSURF (surface generation) and VRUN (VPL script execution) software packages. Further work is being done to document the several data formats used with these packages so that data can be more easily exchanged with visitors and collaborators who wish to use their own software tools in combination with these packages. The Center anticipates making several of these packages generally available via anonymous FTP on the INTERNet in Winter 1990¹.

¹Center software packages are made available on an "as is" basis with no warrantee expressed or implied. All software is copyrighted by the Center for Geometry, Analysis, Numerics, and Graphics. It is likely that software will be available in both binary and source forms.

Center for Geometry, Analysis, Numerics,
and Graphics
University of Massachusetts, Amherst
Computation Facility Diagram
November 1990

Equipment List FY90
Center for Geometry, Analysis, Numerics, and Graphics
Lederle GRC 1535

Computer Systems	location
Silicon Graphics 4D/240GTXB graphics workstation 4 processors; 3 380MB ESDI disks; 32MB memory; 1280x1024x24db 19" color monitor; mouse; keyboard; NTSC video genlock	Lederle 1535 machine rm
Sun Microsystems 4/260 development workstation 1 processor; 2 300MB ESDI disks; 32MB memory; 1536x1152 19" mono monitor; mouse; keyboard Aviv 1.2GB SMD disk subsystem (attached) Exabyte 8MM helical scan tape subsystem (attached)	Lederle 1535 machine rm
Ridge 3200 computing workstation 1 processor; 16MB memory; 2 300MB ESDI disks; 16MB memory; 1152x900 16" mono monitor; mouse; keyboard	Lederle 1535 machine rm
(2) Sun Microsystems 3/60 color workstations 1 processor; no disk; 6MB memory 1152x968x8 16" color monitor; mouse; keyboard	Lederle 1535 lab
(1) Sun Microsystems 3/50 monochrome workstation 1 processor; 1 71MB SCSI disk; 4MB memory 1152x968x8 19" mono monitor; mouse; keyboard	Lederle 1535 lab
(2) Sun Microsystems 3/50 monochrome workstations 1 processor; no disk; 4MB memory 1152x968x8 19" mono monitor; mouse; keyboard	Lederle 1524, Lederle 1526
(2) NeXT Computer 1 processor; 1 340MB disk; 8MB memory 1024x890 17" mono monitor; mouse; keyboard	Lederle 1535 lab
(2) Apple Macintosh II color personal computers	Lederle 1536, Lederle 1542

1 processor; 1 40MB disk; 4MB memory
640x480x8 12" color monitor; mouse; keyboard

(2) Apple Macintosh SE personal computers

Lederle 1534, Lederle 1544

1 processor; 1 40MB disk; 1MB memory
500x350 8" mono monitor; mouse; keyboard

Printers

Apple Laserwriter Laser Printer

Lederle 1535 lab

Networking Equipment

Kinetics Fast Path AppleTalk Ethernet Gateway

Lederle 1535 machine rm

Digital Equipment DELNI Ethernet Multiport Transceiver

Lederle 1535 machine rm

Digital Equipment DEMPR Ethernet Multiport Transceiver

Lederle 1535 machine rm

Graphics Displays

Raster Technologies Model One/380 Graphic Display

Lederle 1535 lab

1280x1024x32 19" Sony color monitor; genlock option
(attached to Ridge 3200)

Raster Technologies Model One/10 Display Terminal

Lederle 1535 machine rm

640x480x4 12" color monitor

(attached to Raster Tech Model One/380)

Star Technologies Graphicon 1700 Graphic Display

Lederle 1535 lab

1280x1024x32 19" Hitachi color monitor

(to be attached to Sun 4/260)

Video Equipment

Sony BVU-950 3/4" Editing Video Tape Recorder

Lederle 1535 lab

Sony BVR-50 Remote Control Unit

Lederle 1535 lab

Sony PVM-2030 19" NTSC color monitor

Lederle 1535 lab

Sony PVM-1271Q 12" NTSC color monitor

Lederle 1535 lab

(3) NAD MR 13A NTSC Broadcast monitors

Lederle 1535 lab

Lyon-Lamb RTC Scan Converter

Lederle 1535 lab

Lyon-Lamb VAS-Delta Editing Controller

Lederle 1535 lab

Misubishi HS-359UR 1/2" VHS Tape Recorder
Sony LVR5000 / LVS5000 Laser Videodisc Recorder
Tektronix 1730 Waveform Monitor
(2) Catel Broadcast Modulators
Catel Broadcast Demodulator
Grass Valley Group Ten-XL switch
Grass Valley Group 8500 Distribution Amplifiers

Lederle 1535 lab
Lederle 1535 lab
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Lederle 1535 lab
Lederle 1535 lab

RESEARCH ACTIVITIES FROM PRIOR DOE SUPPORT

Research of Joel Spruck

Joel Spruck has worked on a variety of pure and applied problems in Differential Geometry, Calculus of Variations and Mathematical Physics. With a group of collaborators at the University of Massachusetts (Eydeland, Lifshits and Turkington) he has made substantial progress in modelling and developing algorithms to compute equilibria and slow evolution of plasmas. With L. Caffarelli, he has developed new ideas for the study of geometric variational problems and with L.C Evans, he has made substantial progress in developing a new p.d.e approach to study motion by mean curvature.

Spruck, in collaboration with Caffarelli and Gidas completed his work on the classification of isolated singularities of the conformally invariant scalar field equation:

$$\Delta u + u^{\frac{n+2}{n-2}} = 0$$

$$u \geq 0.$$

In doing so, they developed a powerful new variant of the Alexandrov reflection technique that is measure theoretic and localizable. This work appeared in

Asymptotic Symmetry and the Local Behavior of Semilinear Elliptic Equations with Critical Sobolev Growth, C.P.A.M 42 (1989) 271-297.

Spruck with Caffarelli continued the above mentioned work to study variational problems with critical Sobolev growth. In particular, they proved that the Dirichlet problem

$$\Delta u + \lambda u^{\frac{n+2}{n-2}} = 0 \quad \text{on } \Omega$$

$$u = \phi \quad \text{on } \partial\Omega$$

for nonnegative ϕ , $\phi \not\equiv 0$, is well-posed. This result is of particular interest because it clarifies the reasons for blow up. This work recently appeared in

Variational Problems with Critical Sobolev Growth and Positive Dirichlet Data, Indiana University Math. J. 39 (1990) 1-18.

Spruck, in collaboration with Eydeland and Turkington completed a first paper on variational problems with many constraints that models semilinear problems

$$-\Delta u = \Lambda(u)$$

where the profile function $\Lambda(u)$ is determined implicitly from a minimization problem subject to a family of nonlinear constraints. The motivation for this problem comes from the classic paper of Kruskal and Kulsrud, which gives the natural formulation of the magnetohydrodynamic (MHD) equilibrium problem governing configurations of a plasma in a toroidal geometry. This formulation can be succinctly stated as a variational principle: an equilibrium configuration minimizes its total (potential) energy subject to two families of constraints, which determine the flux and mass within the magnetic surfaces in the plasma. These constraints are derived from the families of conserved quantities associated with ideal MHD evolution.

In the present paper we study the case of an incompressible plasma (whose magnetic field has no toroidal component) in a cylindrical geometry. In this case, there is only one constraint family and the constraints are convex. We are able to formulate a natural finite constraint approximation to the original variational problem and develop a globally convergent algorithm for its solution. This finite constraint problem gives exact solutions to the physical problem and is only an approximation in the sense that the full set of constraints are not used.

Our constrained variational principle and algorithms furnish us with a foundation for many investigations in plasma physics, especially those concerned with the design of magnetic confinement devices and the analysis of their performance (see below).

The paper will appear as

Multiconstrained Variational Problems of Semilinear Eigenvalue Type: New Formulation and Algorithms, to appear in *Mathematics of Computation*.

Continuing this work, Spruck (with Eydeland, Lifshits and Turkington) showed that the algorithm of the previous paper extends to cover the case of a general axisymmetric compressible plasma. In the usual cylindrical coordinates the toroidal region $D = \{x = (r, \phi, z) \in \mathbb{R}^3 : (r, z) \in \Omega, 0 \leq \phi < 2\pi\}$ is determined by its cross-section Ω , a domain in the half-plane $\mathbb{R}_+^2 = \{(r, z) : r > 0\}$. In D the plasma-vacuum system is governed by the ideal magnetohydrodynamic equilibrium equations

$$(0.1) \quad \mathbf{J} \times \mathbf{B} = \nabla p, \quad \nabla \times \mathbf{B} = \mathbf{J}, \quad \nabla \cdot \mathbf{B} = 0.$$

The plasma lies within the free-boundary surface S on which the natural interface conditions hold—namely, $p = 0$, $\mathbf{n} \cdot \mathbf{B} = 0$ on S and $\mathbf{n} \times \mathbf{B}$ is continuous across S , where $\mathbf{n}$ is the unit normal on S . On the fixed boundary ∂D , which is taken to be a perfectly conducting shell, the normal component of $\mathbf{B}$ vanishes.

Under the assumption of axisymmetry with respect to the toroidal angle ϕ , the fields in (1.1) admit the representation

$$(0.2) \quad \mathbf{B} = \nabla \Psi \times \nabla \phi + f \nabla \phi$$

$$(0.3) \quad \mathbf{J} = \nabla f \times \nabla \phi + (L\Psi) \nabla \phi$$

in terms of the scalar unknowns $\Psi = \Psi(r, z)$, $f = f(r, z)$ and the linear elliptic partial differential operator

$$(0.4) \quad L := -r \frac{\partial}{\partial r} \left(\frac{1}{r} \frac{\partial}{\partial r} \right) - \frac{\partial^2}{\partial z^2}.$$

The basis for our computational method is a variational principle which makes use of the functionals

$$(0.5) \quad E(\psi, f, g) = \frac{1}{2} \int_{\Omega} [r^{-1} |\nabla \psi|^2 + r^{-1} f^2 + r g^2] dr dz$$

$$(0.6) \quad F_0(f) = \int_{\Omega} r^{-1} f dr dz$$

$$(0.7) \quad F_{\sigma}(\psi, f) = \int_{\Omega} r^{-1} f(\psi + \bar{\psi} - \sigma)_+ dr dz \quad (\sigma \geq \sigma_0)$$

$$(0.8) \quad G_{\sigma}(\psi, g) = \int_{\Omega} r g^{2/\gamma} (\psi + \bar{\psi} - \sigma)_+ dr dz \quad (\sigma \geq \sigma_0),$$

where the notation $s_+ = \max(s, 0)$ is used. Here the pressure p is replaced by the unknown

$$(0.9) \quad g := \left(\frac{2p}{\gamma - 1} \right)^{\frac{1}{2}},$$

as this substitution is convenient for technical reasons.

The basic variational problem governing the equilibrium configurations we study can now be stated as

$$(P_{\infty}) \begin{cases} E(\psi, f, g) \rightarrow \min \text{ subject to} \\ F_0(f) = F_0^*, F_{\sigma}(\psi, f) = F_{\sigma}^*, G_{\sigma}(\psi, g) = G_{\sigma}^* \quad (\sigma \geq \sigma_0), \end{cases}$$

where $F_0^*, F_{\sigma}^*, G_{\sigma}^*$ denote given constraint values. This minimization problem is novel in the sense that it imposes two (continuously) *infinite* families of constraints (as σ runs through the magnetic surfaces within the plasma). Consequently, the calculation of the variational equation (a Lagrange multiplier rule) satisfied by a minimizer

(ψ, f, g) is not straightforward, since certain regularity properties are required to justify such a calculation. In order to arrive at a variational problem that is tractable to analysis and computation we therefore formulate a natural discretization of the constraint families in (P_∞) . The resulting multiconstrained variational problem, which we call (P_n) is solved by a globally convergent iterative algorithm. An important feature of our formulation is that *solutions to (P_n) give exact solutions* to the equilibrium equations (1.1) (and not approximations!) Also, many of the ideas introduced have application to other optimization problems with nonconvex constraints.

This work will appear as

Multiconstrained Variational Problems in Magnetohydrodynamics I: Equilibrium, preprint.

Continuing this work, Spruck (with A. Lifshits, Eydeland and Turkington), has applied their previous work to study the slow evolution of a plasma that describes flux conserving and adiabatically compressed tokamaks. The method allows us to easily study magnetic island formation and annihilation. This work will appear as

Multiconstrained Variational Problems in Magnetohydrodynamics II: Slow Evolution, preprint.

In collaboration with L.C. Evans, Spruck completed several papers on "Level set motion by mean curvature". In this p.d.e. approach, given an initial surface Γ_0 , select some continuous function $g: \mathbb{R}^n \rightarrow \mathbb{R}$ so that

$$(0.10) \quad \Gamma_0 = \{x \in \mathbb{R}^n | g(x) = 0\}.$$

Consider then the parabolic PDE

$$(0.11) \quad u_t = \left(\delta_{ij} - \frac{u_{x_i} u_{x_j}}{|Du|^2} \right) u_{x_i x_j} \quad \text{in } \mathbb{R}^n \times [0, \infty)$$

$$(0.12) \quad u = g \quad \text{on } \mathbb{R}^n \times \{t = 0\},$$

for the unknown $u = u(x, t)$ ($x \in \mathbb{R}^n$, $t \geq 0$). Now the PDE (1.11) says that each *level set of u evolves according to its mean curvature*, at least in regions where u is smooth and its spacial gradient Du does not vanish. Consequently, focusing our attention on the set $\{u = 0\}$, it seems reasonable to *define*

$$(0.13) \quad \Gamma_t \equiv \{x \in \mathbb{R}^n | u(x, t) = 0\}$$

for each time $t > 0$ as our generalized mean curvature flow.

Using an appropriate notion of weak solution inspired by the so-called "viscosity solution" approach, we are able to prove the existence and uniqueness of a global in time solution to our mean curvature evolution pde and to show that Γ_t is *independent* of the choice of g . This approach allows the definition of mean curvature flow even after the onset of singularities. Many surprising and interesting properties of the motion follow from a strong comparison principle for solutions. This work will appear in

Motion of Level Sets by Mean Curvature I, to appear in J. Diff. Geometry.

A second paper of Spruck and Evans gives a new and surprising proof of classical short time existence (as well as some other properties) by using the distance function to the motion. The motion is now the zero set of the distance function.

This work will appear in

Motion of Level Sets by Mean Curvature II, to appear in Trans. A.M.S.

Recently, Spruck and Evans have completed a third paper studying their generalized mean curvature flow whose purpose is to effect a partial reconciliation of the level set point of view with Brakke's geometric measure theory approach. The point of this paper is to develop tools to analyze the structure of the singularities of the generalized mean curvature flow. Along the way, several interesting results are proved.

For example, it is shown that $H^{n-1}(\Gamma_t^*) < \infty$ for each time $t > 0$ provided Γ_0 is compact, H^{n-1} -rectifiable with $H^{n-1}(\Gamma_0) < \infty$. Here $\Gamma_t^* = \partial\Gamma_t$, the topological boundary of Γ_t .

Another result of interest is that if some part of our generalized mean curvature evolution can be written as a graph, then that portion is in fact a smooth hypersurface. This is a first regularity assertion for our generalized flow. As a corollary, we show that an initial surface consisting of the boundary of an arbitrary convex set becomes smooth and convex.

This work will appear in

Motion of Level Sets by Mean Curvature III, preprint.

Research of David Hoffman

In this section we will describe scientific publications and progress made in the last year .

Complete embedded minimal surfaces of finite total curvature

Hoffman has continued research concerning embedded minimal surfaces. This has been done together with W. Meeks, H. Karcher, M. Wohlgemuth, H. Rosenberg and others. The fundamental paper *Embedded minimal surfaces of finite topology*, *Annals of Mathematics* 131 (1990), 1-34 established the existence of an infinite sequence of embedded minimal surfaces of genus $k \geq 3$ with three topological ends, generalizing the genus-one example of Costa and giving a proof of embeddedness for all the examples at once. It is proved in that paper that these surfaces are the most symmetric examples of genus k , with three ends. A symmetry extension theorem is presented that gives circumstances under which symmetries of the Gauss mapping of a minimal surface can extend to the surface in $\mathbb{R}^3$. Included at the beginning of this paper is a synopsis of known results about embedded minimal surfaces of finite total curvature and finite topology, as well as an estimate (in terms of the total winding at infinity) of the order of self intersection a minimal surface can have.

Once there are examples, it is natural to ask about the structure of the "space of all examples". One of the first results we were able to prove is that the sequence of surfaces in the previous paragraph has a well defined limit surface. What that surface is depends on how they are normalized. First of all, we will assume that the limiting normals are always vertical. If the surfaces are normalized so that the maximum absolute value of the Gauss curvature is equal to 1, and a point where this value is achieved is put at the origin, then any limit surface must be Scherk's doubly-periodic surface (with the ends orthogonal). If, on the other hand, the normalization is done so that the points of maximum curvature lie on a regular polygon of diameter equal to one then the limit is a catenoid together with a horizontal plane meeting it at the waist. The regular polygon converges to the waist-circle and the convergence is smooth away from this circle. This work will appear shortly as *Limits of minimal surfaces and Scherk's fifth surface* in the *Archive for Rational Mechanics and Analysis*.

We were also able to show that each of these examples sits in a 1-parameter family

of examples. In analyzing the possible degeneracy of these families, we were able to prove the following compactness result. Let $S_{k,r}$ be the space of complete embedded minimal surfaces of finite total curvature, with genus at most k , and with at most r ends, all assumed to be vertical. Normalize the space so that all surfaces have maximum absolute value equal to one. Further normalize $S_{k,r}$ so that a point where this maximum occurs is located at the origin in $\mathbb{R}^3$. Then $S_{k,r}$ is compact. The genus and the number of ends can drop. These results are contained in a preprint *One-parameter families of embedded minimal surfaces*. in this area.

Singly-periodic minimal surfaces

Following an earlier paper in which we prove the existence of families of singly-periodic embedded minimal surfaces with more than one end and an infinite symmetry group, Callahan, Meeks and I published *The structure of singly-periodic minimal surfaces*, *Inventiones Math.* 99, (1990) 455–481. Here, we proved that the properties of these new examples were to a large extent forced by the condition of having an infinite number of ends and large symmetry group. We also proved that the examples constructed in our earlier paper could be deformed through properly embedded minimal surfaces invariant under a *screw motion*. In this paper the notion of the *limit tangent plane* for a properly embedded minimal surface was introduced and it was proved to be unique. We also showed that the quotient of such a surface by its screw-motion (or translation) isometries had finite total curvature if and only if it had finite topology, a result that has already been applied elsewhere.

A survey of these new results up to and including this paper appeared in *New examples of singly-periodic minimal surfaces and their qualitative behavior*, *Progress in Geometry: Contemporary Mathematics* 101, 1989, 97–106. Since then, Wohlgemuth and I have discovered a way to insert additional tunnels in the examples constructed with Callahan and Meeks. The working title of this paper in progress is *New examples of singly periodic minimal surfaces*. Karcher and I have found new singly-periodic surfaces with Scherk-type ends, based on triply-periodic constructions of Fischer and Koch. These examples are the first known embedded minimal surfaces whose Weierstrass data is given on a rhombic torus. We have also found helicoidal deformations of these surfaces. A picture of one helicoidal deformation appears on the cover of the October 1990 issue of the *American Math. Monthly*, an issue devoted to geometry.

This research is still active. The part described here will be written up as *One*

parameter families of singly-periodic minimal surfaces derived from triply-periodic examples. Karcher, Rosenberg and I have studied questions related to the classical Riemann example. We show that an embedded minimal annulus bounded by a pair of parallel lines, which also lies in a slab must be a fundamental domain of a Riemann example. We do not need to assume finite total curvature because we can apply the results of *The structure of singly-periodic minimal surfaces*, cited above. This is part of the paper *Embedded minimal annuli bounded by a pair of straight lines in parallel planes*, which is completed and about to be submitted for publication.

General theorems about properly embedded and immersed minimal surfaces

For complete minimal surfaces, finite total curvature implies finite topology . That these conditions are not equivalent may be seen in the classical example of the helicoid, which is simply connected but has infinite total curvature. The helicoid is the only known example of a complete embedded minimal surface of finite topology with infinite total curvature. In *The asymptotic behavior of properly embedded minimal surfaces of finite topology*, Journal of the AMS 2 No. 4 (October, 1989), 667–681, Meeks and I studied this situation and were able to prove that a properly embedded minimal surface can have, at most, two annular ends of infinite total curvature. This implies that such a surface with finite topology has at most two ends that are not asymptotic to a plane or a catenoid.

In *The strong halfspace theorem for minimal surfaces*, Inventiones Math. 101, (1990) 373–377, Meeks and I proved that a properly immersed minimal surface cannot lie in a halfspace of $\mathbb{R}^3$ and as a consequence, two properly immersed minimal surfaces must necessarily intersect, unless they are parallel planes. This result has proved to be extremely useful and is applied in many different contexts in the study of embedded minimal surfaces.

In *Some basic facts, old and new, about triply periodic embedded minimal surfaces*, which will appear in the *les Éditions de Physique: Proceedings of the Geometry and Interfaces Conference (Aussois, France 9/90)*, I prove that there is a forced loss of symmetry in any deformation of the classical triply-periodic Schwarz P-surface. This result was motivated by a computation of Maggs and Leibler.

Other publications

With P. Concus and R. Finn, I edited *Computer Graphics and Geometric Analysis: Proceedings of the Conference on Differential Geometry, Calculus of Variations*

and *Computer Graphics*, which has just been published in the Math. Sci. Res. Inst. Conference Proceedings Series (Springer Verlag). It also contains the abstract entitled *Periodic area minimizing surfaces in microstructural science* (with E. Thomas, D. Anderson, D. Martin, and J. Hoffman). Among other things, this abstract describes some of the contents of *Graphical simulation of 2-D TEM data from surface information* (with D. Anderson, E. Thomas, J. Hoffman), which has been completed, and is in the process of being submitted.

Meeks and I were invited to write an article for the special geometry issue of the *American Mathematical Monthly*. This article, *Minimal surfaces bases on the catenoid*, has just appeared (Volume 97, No. 8 702-730).

Joel Spruck, Harold Rosenberg and I have begun to investigate the general boundary value problem for constant Gauss curvature. Some results in the annular case have been written up as the manuscript *Boundary value problems for surfaces of constant Gauss curvature*.

Personnel Wholly or Partially Supported by this Grant

Ph.D. students

Bo Guan - Motion of Interfaces and curvature (working with Spruck)

Guoguang Sun - Motion of interfaces by curvature functions (working with Spruck)

Edward Thayer - Scientific programming involving problems in differential geometry, assisting with graphics development for Ph.D and minimal surface and beginning his Ph.D work.

Wayne Rossman - Pursuing a Ph.D involving problems in differential geometry in euclidean and hyperbolic space.

Scientific and Graphics Programmers

James T. Hoffman - Graphics programming for all scientific projects, development of graphics interface for visual analysis of numerical simulations, development of algorithms for the analysis of data, graphics software development.

Eric Boix - Graphics and scientific programming, assisted Jim Hoffman and worked intensively with David Hoffman on research projects.

James Riordan - System programming and scientific programming.

Postdoctoral Research Associate

David Chopp or someone equally qualified who will be working with David Hoffman on geometric analysis and computation.

Research Visitors

Yisong Yang (New Mexico)

Carolyn Gordon (Dartmouth University)

Harold Ziegler (Ruhr-University, Bochum)

Alan Weinstein (Berkeley and Curant Institute)

Brian White (Stanford)

Takashi Kako (Saitama University, Japan)

Jon Pitts (Texas A&M University)

Richard Schoen (Stanford and Courant Institute)

Vladimir Oliker (Emory University)

Jurgen Jost (Harvard University)

Ivan Sterling (Technische Universitat, Berlin)

Appendix

1. Curriculum Vitae of David Chopp
2. Geometry, Computation and Graphics Preprint List

APPENDICES

David L. Chopp

Home Address

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Berkeley, CA 94720
E-mail: chopp@math.berkeley.edu

Education

University of California, Berkeley

Ph.D. in Mathematics, degree expected May 1991
Thesis: Computation of Minimal Surfaces via Mean-Curvature Flow
GPA 3.98 out of possible 4.0

University of Washington

Bachelor of Science, June 1985
Summa cum laude with distinction in Mathematics,
Major: Mathematics and Applied Mathematics in Numerical Analysis
GPA 3.98 out of possible 4.0

Prizes

IBM Graduate Fellow, 1989, 1990
Distinguished Teaching Award, 1988
Phi Beta Kappa, 1985
Undergraduate Merit Scholar, 1982, 1983
Dean's list member 1982, 1983, 1984, 1985

Research Area Numerical Analysis, Scientific Computing, and Fluid Dynamics

Work Experience

Research Associate, Physics Division, Lawrence Berkeley Laboratory, 1989-present
Teaching Assistant, Mathematics Dept., UC Berkeley, 1985-1989
Freshman Calculus 1985-1987
Graduate course in Numerical Analysis 1987-1989

Computer Experience

Fluent in C, C++, Fortran, and Pascal programming languages.
Worked on large computers including Vax, Cray X/MP, Stardent, Sun, CDC, and Prime.

References

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Thesis Summary for David Chopp

The simplest form of the minimal surface problem in $\mathbf{R}^3$ may be stated as follows: Let $\gamma(t)$ be a simple closed curve in $\mathbf{R}^3$. Let $\mathcal{U}$ be the collection of all 2-dimensional compact C^2 manifolds which have boundary $\gamma(t)$. The question is, does there exist a $U \in \mathcal{U}$ such that $\text{Area}(U) = \min_{V \in \mathcal{U}} \text{Area}(V)$. The problem becomes more difficult when the boundary is allowed to be the union of any finite number of closed curves, not necessarily disjoint. In addition, the collection $\mathcal{U}$ may be expanded to include surfaces which have dimension 2 everywhere except on a set of measure zero.

In the broadest sense, neither existence nor uniqueness is guaranteed for such a "minimal surface" U . However, Euler developed a condition under which a surface U is at least a local minimal surface, meaning that U has least area among all surfaces a small perturbation away from U . That condition is that for every point $x \in U$ where U is a C^2 manifold in a neighborhood containing x , the mean-curvature $\kappa(x)$ of U at x is zero.

In a paper by Osher and Sethian, an algorithm is introduced which evolves a surface with speed dependent on the local mean-curvature. The surface at time t must be representable as the level set $\{x : \phi(x, t) = 0\}$ for some function ϕ . The motion of any point x on a given surface V is in the direction normal to the surface at the point x with speed $F(\kappa)$, where F is a function dependent only on the local curvature κ . In that paper, the equations of motion are reduced to a "pseudo-Hamilton-Jacobi" system of differential equations suitable for numeric approximation.

My thesis incorporates this algorithm in a numerical method to solve the minimal surface problem. To begin, F is chosen to be $F(\kappa) = \kappa$ so that a given initial surface will flow towards a final surface with mean-curvature zero. The key to the minimal surface algorithm is to have the evolving surface remain attached to the fixed boundary $\gamma(t)$. To do so requires interpolatory boundary conditions at specified grid points in a neighborhood of γ . A new local reinitialization procedure both ensures that the surface stays at the boundary without instability at minimal cost.

Thus far, the algorithm has proven successful for all curves γ which admit an isomorphism of the disk with γ as its boundary. Also, it has been successful for problems which include two or more curves $\gamma_1, \dots, \gamma_n$ with initial surface an isomorphism of a sphere S minus open connected subsets of S which have $\gamma_1, \dots, \gamma_n$ as their boundary. The convergence of the method is linear with the size of the mesh.

A key distinguishing factor of this new algorithm from other minimal surface algorithms is that it is capable of letting the initial surface change topology naturally in the course of its evolution. This property is significant because it allows for finding solutions with different topology than the initial surface, thus removing the need to know the general topology of the sought minimal surface beforehand.

In the future I plan to study the algorithm's effectiveness when the initial surface is not itself a manifold (i.e. surfaces which are locally manifolds except on a set of measure zero). I also intend to adapt this method to other domains than $\mathbf{R}^2$ and $\mathbf{R}^3$. So, for example, a possible new application will be to compute the shortest path between two points on a given 2-dimensional manifold.

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