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## ADSORPTION ON MIXTURES OF ION EXCHANGERS

Roberto Triolo and M. H. Lietzke<br>Chemistry Division

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Prepared by the
OAK RIDGE NATIONAL LABORATORY
Oak Ridge, Tennessee 37830
ope rated by
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## iii

## CONTENTS

Page
Abstract ..... 1
Introduction ..... 1
Ion Exchange as a Two-Phase Equilibrium ..... 3
Mixed Ion Exchangers ..... 5
Distribution Coefficients in Mixed Lon Exchangers in the Linear Isotherm Region (low loading) ..... 7
Distribution Coefficients in Mixed Ion Exchangers as a Function of Supporting Electrclyte Concentration at Constant $M_{M}$ ..... 己
Distribution Coefficients for Mixed Exchangers as a Function of $\mathbb{B}_{M}$ at Constant Supporting Electrolyte Concentration ..... 14
Acknowledgements ..... i6
Appendix 1 ..... 35
Appendix 2 ..... 37

## ADSORPTION ON YIXTURES OF IUN EXCHALGERS

Koberto Triolv and M. H. Lietzke


#### Abstract

A theoretical study has been made of adsorption on mixtures of ion exchangers. The effect of variables such as the concentration of the ion being adsorbed. the concentration of the supporting electrolyte, loading, the values of the capacities and equilibrium constants for the various exchange processes, and the fraction of each adsorber in the mixture on the observed distribution coefficient has been investigated. A computer progran has been written to facili tate the calculation of jistribation coefficients for the adsorption of an ior on a given mixture of ion exchangers under a specified set of conditions.


## INTRODUCTION

In the past few years the study of the adsorption oi ions on naturally occurring materials, such as clays and other mineralo, has received increasing attention because of the initiation of applied programs in areas surh as enhanced oil recovery and nuclear waste isolation. In enfanced oil recovery the comples interactions which secus between the micellar floods and the geologic fromations containitig the oll are to a large extent determined by the ion exchange characteristics both ${ }^{\text {f }}$ the formations and of the surfartants used in the process. These interactions must be investigated over a wide range of salinities and of alkali metal to alkaljne earth ratios. Work in this araa also involves an effort to identif: ions which might be used as tracers for underground water flow both under the conditicns

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of salinity and alkali metal to alkaline earth ratios encountered in the geologic formations and at the interfaces with the micellar floods. To be a good watir tracer an in should exhibit negligible adsorption under these conditions.

In nuclear waste isolation an fmportant criterion in selection of geological formations as disposal sites for wastes from the nuclear power industry is the degree to which the radioactive substances can be retarded by the storage environment. The retention of these wastes is a function of the idsorption characteristics of the particular minerals invoived and of the interaction with ground waters of widely varying compositions which may pass through the formation. Thus work in this area involves the measurement of distribution coefficients for the fission product, actinide, and other ions involved over the range of solution composition which mignt be encountered in various ground waters, including the highly saline water which could possihly issue from a breach in a salt mine sed for waste storage.

The natural formations inyolved in these applications are complex mixtures. They may contain several substances which can function as adsorders or $i o n$ exchangers. In some cases the minerals may contain very small amounts of impurities which have high adsorption or distribution coefficients for the various (ionic) species encountered. To help appreciate the implications of the behavior of observed distribution coefficients when an ion is adsorbed on a mixed ion exchanger we have written a computer piogram for calculating the overall distribution coefficient for an ion adsorbed on a mixed exchanger as a function of concentration of the ton being adsorbed, concentration of the supporting
electrolyte, loading, the values of the capacities and equilibrium constants for the various exchange processes, and the fraction of each adsorber in the mixture. In developing the program ideal exchange has been assumed in all cases. By ideal exchange we mean that the value of K/:, as defined below, remains constant over che entire range of the computation; that there is no ion-complexing in solution; and that there is no co-icn invasion of the adsorber. Before describing the computer program, however, we will give a description of the propertier of mi:ed ideal ion exchangers.

ION EXCHANGE AS A TWO-PHASE EQUILIBRTLM

Ion exchange is a two-phas librium. The equilibrium for the exchange of ar: ion $M^{n+}$ with an ion $A^{p^{+}}$may be represented by

$$
\begin{equation*}
\mathrm{pM}^{\mathrm{n}+}+\mathrm{n} \bar{A}^{\mathrm{p}^{+}} \rightleftarrows \mathrm{p}^{\bar{M}^{n+}}+\mathrm{nA} \mathrm{~A}^{+} \tag{1}
\end{equation*}
$$

with the corresponding equilibrium constant giver by

$$
\begin{equation*}
K=\frac{\left(\bar{m}_{M}\right)^{p}\left({ }^{n} A\right)^{n}\left(\bar{\gamma}_{M}\right)^{p}\left({ }_{\gamma}\right)^{n}}{\left(m_{M}\right)^{p}\left(\bar{m}_{A}\right)^{n}\left(\gamma_{M}\right)^{p}\left(\bar{\gamma}_{A}\right)^{n}} \tag{2}
\end{equation*}
$$

In these equations the bars refer to concentrations in the adsorber, no bar refers to concentrations in the solution phase, the $\gamma$ 's are the activity coefficients of the respective species, and $I \underline{m}$ represents concentration in moles/liter of solvent or kg of adsorber The capacity $\underline{C}$ of the adsorber in mole:s/kg is given by $C=\frac{\bar{m}_{A}}{A}+\bar{n}_{M}$. Hence $\bar{m}_{A}=\left(C-n \bar{m}_{M}\right) / P$. If we designate the stoichiometifc activity coefficient quotfent by $\Gamma$ and use the foregoing expression for $\bar{m}_{A}$ in terms
of t.ie capacity of the exchanger, equation (2) becomes

$$
\begin{equation*}
\frac{K}{\Gamma}=\frac{\left(\bar{m}_{M}\right)^{p}\left(p m_{A}\right)^{n}}{\left(m_{M}\right)^{p}\left(c-n \bar{m}_{M}\right)^{n}} \tag{3}
\end{equation*}
$$

The stoichiometric activity ruefficient crotient ! is a measurable ratio of the appropriate powers of the mean ionic activily coefficients i\# of the electroi, ies invoived in the exchange reactions. However, in the present discussion the quantity $\pi / \Gamma$ is assumed constant for adsorption of an ion on a given excnanger.

It is convenient to speak of the distribution coefficient $D_{M}$ of the adsorbed icn as defined hy $D_{M}=\bar{m}_{M} / m_{M}$. Introducing this into equation (3) and reairanging gives

$$
\begin{equation*}
\left(D_{M}\right)^{p}=\left(\frac{k}{\Gamma}\right) \frac{\left(C-n \bar{m}_{M}\right)^{n}}{\left(\mathrm{pm}_{A}\right)^{n}} \tag{4A}
\end{equation*}
$$

or

$$
\begin{equation*}
\left(D_{M}\right)=\left(\frac{K}{\Gamma}\right)^{1, p} \frac{\left(C-n \bar{m}_{M}\right)^{n / p}}{\left(p m_{A}\right)^{n / p}} \tag{4B}
\end{equation*}
$$

The term $K / \Gamma$ is a measure of the relative selectivity of an exchanger for one ion ove: another. In the following discussion we will focus our attention on the effect of a posisibly varying and usually much greater concentration of the ion $A^{p+}$ rn the distribution coefficient $D_{M}$ for the adsorption of an ion $M^{m+}$.
where the sum is taken over all the individual exchangers in the mixture.

Combining equations (4) and (9) gives the following general expression for the overall distribution coefficient for the adsorption of an ion $\mathrm{M}^{\mathrm{n}+}$ en a mixed exchanger:

$$
\begin{equation*}
D_{\text {observed }}=\frac{1}{\left(F_{A}\right)^{n / p}} \sum_{i} F_{i}\left(\frac{K}{\Gamma}\right)_{i}^{1 / p}\left(C_{i}-n \bar{m}_{i}\right)^{n / P} \tag{10}
\end{equation*}
$$

In this equation $\bar{m}_{i}$ represents the concentration of $\underline{M}$ in the $i^{\text {th }}$ exchanger. For simplicity, in the subsequent development mixed exchangers containing only two components will be considered.

To study the adsorption of an ion in solution on a mixed ion exchanger over a wile range of conditions a computer program has been written. In this program, equation (3) is solved numerically by successive approximations for each exchanger separately to give the value of $\tilde{m}_{M}$ corresponding to fixed values of $(K / \Gamma), M_{M}, r_{A}, \underline{C}, \underline{n}$, and $p$. The vaiues of $\bar{m}_{M}$ obtained in each case are then used to calculate the respective distribution coefficients for the adsorption of the ion in question. Taking into account the fraction of each solid component in the mixed adsorber the value of $D_{o b s}$ is the: calculated using equation (9). Further details concerning the mathematical methods used in the computer program are givelt in Appendix 1. This program has been used ts carry out the calculations and produce all the figures in this report.

## MIXED ION EXCHANGERS

Since we are interested in the properties of rixed exchangers we need an expression for the overall distribution coefficient for the adsorption of an ion on the exchanger in terms of the fractions of each adsorber in the excinanger. To derive such an expression, consider a mixed exchanger containing $g_{1} \mathrm{~kg}$ of exchanger 1 and $g_{2} \mathrm{~kg}$ of exchanger 2. Let $\bar{m}_{1}$ be the amount of an adsorbed ion in exchanger 1 and $\bar{m}_{2}$ the amount of the same ion in exchanger 2, in each case per kg of adsorber. Then the total amount of the ion adsorbed per kg of adsorber is given by

$$
\begin{equation*}
\text { Total adsorbed } / \mathrm{kg}=\frac{\bar{m}_{1} g_{1}}{g_{1}+g_{2}}+\frac{\bar{m}_{2} s_{2}}{g_{1}+g_{2}} . \tag{5}
\end{equation*}
$$

However, $\frac{g_{1}}{g_{1}+g_{2}}=F_{1}$, the weight fraction of exchanger 1 , and $\frac{g_{2}}{g_{1}+g_{2}}=F_{2}$, tie weight fraction of exchanger 2. Hence, equation (5) may be written

$$
\begin{equation*}
\text { Total adsorbed } / \mathrm{kg}=\bar{m}_{1} \mathrm{~F}_{1}+\bar{m}_{2} \mathrm{~F}_{2}, \tag{6}
\end{equation*}
$$

where $F_{1}+F_{2}=1$. Dividing through by $m$, the concentration of the ion in solution gives

$$
\begin{equation*}
\frac{\text { Total adsorbed } / \mathrm{kg}}{\mathrm{~m}}=\frac{\bar{m}_{1} \mathrm{~F}_{1}}{\mathrm{~m}}+\frac{\bar{m}_{2} F_{2}}{\mathrm{~m}} \tag{?}
\end{equation*}
$$

Hence,

$$
\begin{equation*}
D_{\text {observed }}=F_{1} D_{1}+F_{2} D_{2} ; \tag{8}
\end{equation*}
$$

or in general

$$
\begin{equation*}
D_{\text {observed }}=\sum_{i} F_{i} D_{i}, \tag{9}
\end{equation*}
$$

## DISTRIBETION COEFFICIENTS IN MIXED ION EXCHANGERS

 IN THE LINEAR ISOTHERM REGION (LOW LOADING)In considering the adsorption at low loading of an ion on mixed exchanger containing two adsorbels it is convenient to write equation (10) as
$D_{o b s}=\frac{1}{\left(p m_{A}\right)^{n / p}}\left[F_{1}\left(\frac{k}{\Gamma}\right)_{1}^{1 / p} c_{1}^{n / p}\left(i-\frac{n m_{1}}{C_{1}}\right)^{n / p}+F_{2}\left(\frac{K}{\Gamma}\right)_{2}^{1 / p} c_{2}^{n / p}\left(1-\frac{n m_{2}}{C_{2}}\right)^{n / p}\right]$.

In this equation the terms $n \frac{\bar{m}_{i}}{C_{i}}$ represent the loadings of $\underline{M}$ on the $i^{\text {th }}$ erchenger. If we designate the terms $n \frac{1}{C_{i}}$ by $L_{i}$ and the terms $F_{i}\left(\frac{K}{i}\right)_{i}^{1 / p} C_{i}^{n / p}$ by $x_{i}$, equation (11) becomes

$$
\begin{equation*}
D_{o b s}=\frac{i}{\left(p_{A}\right)^{n / p}}\left[\alpha_{1}\left(i-L_{1}\right)^{n / p}+\alpha_{2}\left(1-L_{2}\right)^{n / p}\right] \tag{12}
\end{equation*}
$$

If now $L_{1}$ and $L_{2} \ll 1$ and the $i_{i}$ by assumption are constant at fixed vaıues of $F_{1}$ and $F_{2}$, then

$$
\ln D_{o b s}=-\frac{n}{p} \ln m_{A}-\frac{n}{p} \ln p+\ln \left(\alpha_{1}+\alpha_{2}\right)
$$

and

$$
\begin{equation*}
\frac{d \ln D_{o b s}}{d \ln m_{i}}=-\frac{n}{p} \tag{13}
\end{equation*}
$$

Thus, at negligible loading pluts of $\log U_{\text {obs }} v s l_{\text {ing }} m_{A}$ are parallel and have slopes of $-n / P$, irrespective of the values of $F_{1}$ and $F_{2}$. This
is illustrated for $1-1,2-1$, and $3-1$ excinange by consideration of the extreme right hard portions of the curves in Figures 2, 4. and 1:, which give $\log D_{o b s}$ vs $\log m_{A}$ at constan: $M_{M}$. All the curves are parallel as the relative proportions of the two excnangers vary from pure adsorber 2 to pure adsorber 1 ; since in this case $p=$ ? the curves have limiting slopes at low loading (the linear isothern region) of $-1,-2$, and -3 , respectively, for $1-1,2-1$, and $3-1$ exchange.

DISTRIBUTION COEFFICIENTS IN MIXED ION EXCHANGERS AS A FLNCTION OF SUPPORTING ELECTROLYTE CONCENTRATION AT CONSTANT $m_{M}$

In deriving expressions for the limiting values of $D_{\text {obs }}$ for mixed exchangers as a function of $m_{A}$, it is convenient to carry out the co:nputations at constant $M_{M}$. However, we should point out that this does not correspond to a single sequence of experiments where perhaps the initial concentration $m_{M}$, but not the equilibrium value, is controlled. Modeling this more realistic case could be achieved by constructing a family of curves of the type to be described for a range of values of $M_{M}$ which cover the experimental conditions.

For convenjence, we shall derive the expressions for the distribution coefficients in mixed exchangers with the value of $p$ in equation (3) taken as unity. Hence the expressions will aoply to 1-1, 2-1, and 3-1 exchange. For completeness, however, an expression will be given for the general case where $p$ may iffer from unity.

With $p$ taken as unity, rearrangement of equation (3) gives for each solid

$$
\begin{equation*}
\left(\frac{k}{\Gamma}\right) \frac{\left(m_{M}\right)}{\left(m_{A}\right)^{n}}\left(C-n \bar{m}_{M}\right)^{n}-\left(\bar{m}_{M}\right)=0 \tag{14}
\end{equation*}
$$

We row consider separately the cases where $n=1 ; n=2$; ard $n=3$. (a) With $n=1$ further rearrangement of (14) gives

Then,

Now, if we let $m_{A} \rightarrow n$ at constant $m_{M}, D_{o b s} \cdot \frac{C}{M_{M}}$, the maximum value of the distribution coefficient for ideal l-l exchange at constant $M_{M}$ When $D_{\text {obs }}$ reaches this value the adsorber is fully loaded. In view of equation (9), for $1-1$ exchange on a mixed adsorber coracaining $\underset{i}{ }$ components

$$
\begin{equation*}
\mathrm{g}_{\mathrm{obs}} \cdot \stackrel{\because F_{i}}{C_{i}} \frac{C_{M}}{m_{M}} \tag{17}
\end{equation*}
$$

as $m_{A} \rightarrow 0$ at constant $M_{M}$. Since the terms on the right hand side of equation (17) are constant,

$$
\begin{equation*}
\left[\frac{\mathrm{d} \ln \mathrm{D}_{\text {obs }}}{\mathrm{d} \ln m_{A}}\right]_{M_{M}}=0 \tag{18}
\end{equation*}
$$

under these conditions. Thus plots of $\log D_{o b s} \because \log m_{A}$ will approach the ordinate axis at very low $\mathrm{m}_{\mathrm{A}}$ with zero slone. The numerical value of $D_{\text {obs }}$ in all cases will be given by equation (17).

Consider a mixed exchanger consisting of two adsorbers each with the same ion exchange rayacity $\left(C_{1}=C_{2}=C\right)$. According to equation (17) the limiting value of $D_{o b s}$ at constant $M_{M}$ will be given by $D_{\text {obs }}=$ $C / m_{M}$ for all relative proportions of the two exchangers. In Figure 1 is shown a plot of $\log D_{o b s}$ us $10 g m_{A}$ for $1-1$ exchange of an ion on a mixed adsorber with the following characteristics: $C_{1}=C_{i}=1.0 ;(\mathrm{K} / \Gamma){ }_{1}=$ 1.0; $(K / \Gamma)_{2}=10^{4}$. With $m_{M}=10^{-4}$ moles/liter the limiting value of $D_{o b s}$ is given by $1 / 1 \times 10^{-4}=10^{4}$ for all reiative proportions of the two exchangers. As shown in the figure, all the curves converge to a single line which approaches the ordinate axis with zero slope. The limiting value $G: D_{\text {obs }}$ is $10^{4}$.

If the values of $c_{i}$ for the two adsorbers are different, then, as the relative proportions of the two adsorbers vary, the limi ing values of $D_{\text {ob }}$ at low $m_{A}$ will also vary, but in all cises will be given by equation (17). This is illustrated for $1-1$ exchange by the left hadd portions of the curves in Figure 2. In this case an ion at $\mathrm{m}_{\mathrm{M}}=10^{-5}$ is adsorbed on a mixed exchanger with $C_{1}=0.1, C_{2}=1.0,(\mathrm{~K} / \Gamma)_{1}=1.0$, and $(K / \Gamma)_{2}=10^{3}$. Application of equation (17) to a mixture containing $50 \%$ of each of these components gives a limiting value for $D_{o b s}$ of 55,000 , the value shown on the plot for this mixture.
(b) Putting $n=2$ into equation (14) rosults in a quairatic expression that may be solved using the quadratic formula. The expression for $D_{\text {obs }}$ becomes

$$
\begin{equation*}
D_{o b s}=\frac{4\left(\frac{K}{\Gamma}\right) m_{M}^{C}+m_{A}^{2}-\sqrt{8\left(\frac{K}{\Gamma}\right) m_{M} C_{A}^{2}+m_{A}^{4}}}{8\left(\frac{K}{\Gamma}\right) m_{M}^{2}} \tag{19}
\end{equation*}
$$

Although the quadratic equation has two roots it can readily be shown by substituting numbers into the equation that adding the radical term in the $n$ yerator of equation (i9) leads to meaningless values of $D_{\text {obs }}$. If we let $m_{A} \rightarrow 0$ at constant $m_{M}, D_{o b s} \rightarrow \frac{1}{2} \frac{C}{m_{M}}$, and equation (18) again describes the variation of $\log D_{\text {obs }}$ with $\log m_{A}$ at very low values of $m_{A}$. For a mixed exchanger containing $\underline{i}$ components the limiting expression for $D_{o b s}$ at very low values of $m_{A}$ in the case of 2-1 exchange is

$$
\begin{equation*}
D_{\text {obs }}=\frac{1}{2} \quad \sum_{i} F_{i} \frac{C_{i}}{M_{M}} \tag{20}
\end{equation*}
$$

at constant $M_{M}$.
If we consider a mixed exchanger consisting of two components each with the same ion exchange capacity $\left(C_{1}=C_{2}=C\right)$ then the limiting value of $D_{o b s}$ at corstant $M_{M}$ for $2-1$ exchange wili be given by $D_{\text {obs }}=$ $\frac{1}{2} \frac{C}{m_{M}}$ for ali relative proportions of thr two exchangers. This is illustrated by the left hand portion of the $\log _{\text {obs }}$ vs $\log \mathrm{m}_{\mathrm{A}}$ plot in Figure 4. In this case an ion at a concentration of $10^{-4}$ moles/ $\ell$ is adsorbed on a mixed exchanger in which both adsorbers have the same capacity $\left(C_{1}=C_{\hat{2}}=1\right)$ bit different $K / \Gamma$ values: $(K / \Gamma)_{1}=1.0,(K / \Gamma)_{2}=$ 100. Under these conditions the limiting value of $D_{o b s}$ is given by $D_{o b s}=\frac{l}{2} 1.0 \times 10^{4}=5000$ for all relative proportions of the two exchangers. If however, the ion exchange capacities of the two adsorbers are different, then as the relative proportions of the two adsorbers vary, the limiting values of $D_{\text {obs }}$ at low $m_{A}$ are all different and are given by equation (20). This is illustrated by considering the left, hand portions of the curves shown in Figure 8. Here an ion at
concentration $m_{M}=10^{-5}$ is adsorbed on a mixed exchanger in which the adsorbers 1 and 2 have capacities $C_{1}=0.1$ and $C_{2}=1.0$, while $(K /:)_{1}=$ SU and $(K / \Gamma)_{<}=10^{4}$. In tilis case the limiting values of $D_{\text {obs }}$ vary from 5000 for adsorption on pure component 1 to 50,000 for adsorption on pure component 2.
(c) Putting $n=3$ into equation (14) gives a cubic equation which is diffirult to solve explicitly for the three roots corresponding to possible values of $\overline{\bar{m}}_{M}$. However, by substituting reasonable values of $(K / r), C$, and $m_{M}$ int the cublc expression and letting $m_{A} \rightarrow 0$ it can be shown that under these conditions there is only one real root. In solving for this root numerically the same value for $\bar{m}_{M}$ is obtained regardless of whether the starting estimates are higher or lower than the converged value. Lising this value of $\bar{m}_{M}$, the limiting value of $D_{o b s} \rightarrow$ $\frac{1}{3} \frac{C}{m_{M}}$ in all cases as $m_{A} \rightarrow 0$. Hence for 3-1 exchange of a trivalent ion on a mixed adsorber ai very low values of $m_{A}$

$$
\begin{equation*}
n_{\text {obs }}=\frac{1}{3} \quad \sum_{i} \quad F_{i} \quad \frac{C_{i}}{m_{M}} \tag{21}
\end{equation*}
$$

Figures 11 and 12 illustrate plots of $\log D_{o b s}$ us $\log m_{A}$ for 3-1 exchange on a mixed adsorber under conditions where the capacities of the two exchangers are the same and where the capacities are different. In all cases the limiting values of $D_{o b s}$ at very low values of $m_{A}$ are given by equation (21).
(d) $n-p$ Exchange on a Mixed Adsorber. In the general case of $n$ - $p$ exchange on a mixed adsorber $i t$ is easy to show that as $m_{A} \rightarrow 0$ at constant $T_{M}$ the limiting value of $D_{o b s}$ becomes

$$
\begin{equation*}
\mathrm{D}_{\mathrm{cob}} \rightarrow \frac{1}{\mathrm{n}}=\mathrm{F}_{\mathrm{i}} \frac{C_{M}}{m_{M}} \tag{22}
\end{equation*}
$$

We have discussed the behavior of the overall distriburion coefficient for the adsorptior of an ion at fixed concentration in solution on a mixed ion exchanger, both at low loading, which occurs ar high values of $m_{A}$, and at high loading, which occurs at low values of $m_{A}$. In the intermediate range :he behavior of the overall distribution coefficient is strongly influenced by the relative values of $K / \Gamma$ for the various components in the mixed exchanger. This is illustrated in Fi,ures 4 through 7 fct adsorption of a divalent ion at $m_{M}=1 \times 10^{-4}$ on a mixed adsorber in which both solid components have a capacity of 1. With $\left(K / F_{2}\right)_{2}$ equal to 100 and $(K /:)_{1}$ equal to 1 there is a slight suggestion of a change of slope in the plot for the mixture containing $5 \%$ of component 2. As the value of $(\mathrm{K} / \mathrm{I})_{2}$ increases from 100 to 100,000 with $(K / I)_{1}$ fixed at 1 , this effect becomes much more pronounced because the second adsorber becomes lnaded much more quickly than does the first. This behavior observed in plots of $\log D_{o b s} \cup S \log m_{A}$ is indicative of a mixed adsorber. However, in many cases, especially when the (K/r) values for the components in a mixed exchanger are not too dissimilar, the effect may hardly be noticeable, as in Figure 4. This is also the case for the $\log D_{\text {obs }}$ iog $\log m_{A}$ plots shown for $1-1$ exchange in figures 2 and 3. As will be showt later, a far better method for differentiaring a mixture ${ }^{[ }$rom a "pure" acsorber is to determine log $D_{o b s}$ yif loading of the adsorber at constant $m_{A}$.

The presence in 3 rixed exchanger of $\in v \in n$ à very small amount of an adsorber having a high value of $K /:$ relative to the value of $K /$ : for the adsorber present as the major component may have a dramaric effect on the value of $D_{o d s}$ for the adsorption of an ion even at trace loading. This is illustrated in Figure 7 for the adsorption of a divalent ion at $m_{M}=1 \times 10^{-4}$ on a mixed exchanger in which both adsorbers have the same capacity $C=1$ but different $K / \Gamma$ values: $(K / \Gamma)_{2}=100,000,(K / \Gamma)_{1}=1$. Note that the presence of only $0.1 \%$ of component 2 in the mixture increases the value of $D_{\text {obs }}$ by a factor of 40 , at a supporting electrolyte corcentration of 4.5 m , over the value observed for adsorption on pure componeat 1. Thus, supposediy pure samples of a clay from different sources may exhibit different values of $D_{o b s}$ for the adsorption of a particular ion due to the presence in the samples of differing small amounts of impurities with high distribution coofficients.

In Figures 9 and 10 are shown $\log V_{\text {obs }} \because 2 \log$ a plots for 2-1 exchange on a mixed adsorber at two different values of $M_{M}$. The values of $m_{A}(.02$ to 4.5 m$)$ in these plots span a range comonly encountered in experimental situations. Here again the strong effect on $D_{o b s}$ of small amounts of a component with a large value of $K / i$ in a mixture is illustrated.

## DISTRIBUTION COEFFICIENTS FOR MIXED EXCHANGERS AS A FUNCTION OF mm AT CONSTANT SUPPORTING ELECTROLYTE CONCENTRATION

He pointed out in the previous section that distribution cuefficients are not experimentally measured at constant values of $m_{M}$. However, it is relatively easy under most conditions to measure distribution coefficienss as a function of $m_{M}$ at constant values of $m_{A}$, the supporting
electrolyte concentzation.
In an ion $f$ xchange equilibrium, as the concentration in solution of the $i o n$ adsorbed $m_{M} \rightarrow 0$ at constant $m_{A}$, the concentration of the ion in the adsorber $\bar{m}_{M}$ also approaches zero. Hence, the term $\left(C-n \tilde{m}_{M}\right)^{n}$ in equation (l) approaches the ion exchange capacity $\underline{C}$ and

$$
\begin{equation*}
\mathrm{D}_{\mathrm{M}} \rightarrow\left(\frac{\mathrm{~K}}{\mathrm{~F}}\right)^{1 / \mathrm{p}}\left(\frac{\mathrm{C}}{\mathrm{pm}}\right)^{\mathrm{n} / \mathrm{p}} \tag{23}
\end{equation*}
$$

For a mixed adsorber the corresponding limiting value of $D_{o b s}$ as $m_{M} \rightarrow$ 0 at constant $m_{A}$ is thus giver by

$$
\begin{equation*}
D_{c b s}=-\bar{r}_{i} D_{i}=\frac{1}{\left(p m_{A}\right)^{n / p}} \sum_{i} F_{i}\left(\frac{K}{\Gamma}\right)_{i}^{1 / p} C_{i}^{n / p} \tag{i}
\end{equation*}
$$

Figure 13 shows plots of $\log D_{o b s}$ ve $\log M_{M}$ for $1-1$ exchange on a mixed adsorber with $C_{1}=C_{2}=1,(K / \Gamma)_{1}=1$, and $(K / \Gamma)_{2}=100$. With $\mathrm{m}_{\mathrm{A}}$ equal to 0.1 the limiting value of $\mathrm{D}_{\mathrm{obs}}$ as $\mathrm{m}_{\mathrm{M}}+0$ for a mixture containing 0.5 weight fraction of each comfonent is 505 as computed using equation (24). This is the value shown on the plot. As the value of $M_{M}$ increases the values of $D_{\text {olbs }}$ begin to decrease and eventual!y go down with a slope of minus one. Note that only the curves for the mixtures with the three lowest weight fractions of the second component show slight inflexinas indicative of a mixed adsorber. A much more dramatic way to differentiate between a "pure" adsorber and a mixed adsorber is to plot $\log D_{\text {obs }}$ vs loading of the adsorber at fixed values of $\mathrm{m}_{\mathrm{A}}$. Figure 14 shows the loading plots for $1-1$ adsorption on a mixed exchanger under the same conditions as those for the $\log \mathrm{D}_{\mathrm{obs}}$ us $\log _{\mathrm{m}} \mathrm{m}$ plots
show in Figure 13. Of course, the limiting values of $D_{\text {ot. }}$ (corresponding to zero loading) are the same in both Figures 13 and 14 and the values of $D_{\text {obs }}$ eventually all approach a limiting value at full loading of both adsorbers. However, in the intermediate regions the overall "observed" values of $D$ for the mixed adsorbers much more clearly reflect the composite nature of the adsorber. Similar plots for 2-1 and 3-1 exchange on mixed exchangers are show in Figures 15-18.

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Figure 1


Figure 2


Figure 3


Figure 4


Figure 5


Figure 6


Figure 7


Figure 8


Figure 9
$\log D$ vs. Log $A$ for $2-1$ exchange


Figure 10

Log D iss Log A for 3 l exchange


Figure 11

Log $D$ vs. Log A for $3-1$ exchange


Figure 12


Figure 13

## Log $D$ vs. Loading for $1-1$ exchange



Figure 14


Vigure 15



Figure 17


Yigure 18

## APPENDIX 1

## Description of the Mathematics in the Computer Program

The calculations performed by the computer program involve the solution of equation (3) for the value of $\bar{M}$ for each adsorber in the mixture with a range of fixed values of the other quantities in the equation. The values of $\underline{D}$, the distribution coefficient, and $\underline{L}$, the loading, are computed for each adsorber under specified conditions. These values are then combined to give the overall values of $\underline{\mathrm{D}}$ and $\underline{I}$ for the mix.d adsorber.

With $\underline{n}$ and $p$ both equal to one ( $1-i$ exchange) equation (3) can be solved directly for $\bar{m}_{M}$. However, higher order equations must be solved when $n$ and/or $p$ are greater than unity. Thus different methods wcuid have to be used in the latter cases, not only to solve for che roots of the equations, but also to choose the appropriate root for the compatation of $\underline{D}$ and $\underline{L}$. In order to make the computer program as compact and efficient as possibie, the same method is used for all three cases. This results in a slight loss of efficiency for the case where $\underline{n}$ and $p$ are equal to one. This is more than offset, however, ty the gain in efficiency for the cases where $n$ or $p$ are greater than unity.

In the computer program equation (3) is solved numerically for the value of $\bar{m}_{M}$ for each adsorber using the Newton-Raphson method. In implementing this method we rearrange equation (3) and define

$$
\begin{equation*}
f\left(\bar{m}_{M}\right)=\frac{K}{\Gamma} \frac{m_{M}^{p}}{\left(p m_{\Lambda}^{n}\right)} \quad\left(c-n \bar{m}_{M}\right)^{n}-\bar{m}_{M}^{p} \tag{25}
\end{equation*}
$$

Suppose $\bar{m}_{M(0)}$ is an initial estimate of the value of the desired root of equation (25). Then an improved estimate of the root is given ty

$$
\begin{equation*}
\bar{m}_{M(1)}=\bar{m}_{M(0)}+\Delta \bar{m}_{M} \tag{26}
\end{equation*}
$$

where

$$
\begin{equation*}
\Delta \bar{m}_{M}=\frac{f^{\left(\bar{m}_{M(0)}\right)}}{f^{\prime}\left(\overline{\bar{m}}_{M(0)}\right)} \tag{27}
\end{equation*}
$$

In this equation $f^{\prime}\left(\bar{m}_{M(0)}\right)$ represents the derivative of $f\left(\bar{m}_{M}\right)$ with respect to $\bar{m}_{M}$ evaluated for the initial estimate of the root $\bar{m}_{M(0)}$. The derivative of equation (25) with respect to $\overrightarrow{\mathrm{m}}_{\mathrm{M}}$ is given by

$$
\begin{equation*}
f^{\prime}\left(\bar{m}_{M}\right)=-\frac{K}{\Gamma} \frac{\left(\bar{m}_{M}\right)^{p}}{\left(p \bar{m}_{A}\right)^{n}}\left(n^{2}\right)\left(C-n \bar{m}_{M}\right)^{n-1}-p \bar{m}_{M}^{p-1} \tag{28}
\end{equation*}
$$

The value of $\bar{m}_{M(1)}$ is then used similarly to obtain a better estimate of the value of the ruct, ard the process is continued until the difference between so sessive calculated values of $\bar{m}_{M}$ becomes arbitra~ily small.

It can be shown by hand calculation that in the cases with in or $p$ > 1 the desired root is the smallest positive root. Hence an initial estimate of $\bar{m}_{M}$ is taken as $10^{-10}$. Convergence upward to the smallest positive root $\bar{m}_{M}$ is very rapid.

## APPENDIX 2 <br> Description if the Computer Program

The computer program, writter in Fortran, allows the calculation of the total distribution cocfficient and total l-ading for the adsorption of an ion in solution onto an exchanger containing up to ten adsorberis, each with its own valle of $K / \Sigma$ and capacity. (Of course all dimensions in the program may be changed to suit the needs of the individual user.) The definitions of the variables which comprise the input and output of the program are shown in Table 1 , while Table 2 gives the field and format information for transferring the variables to the input cards.

The program, as listed below, contains a statement CALL DPLOT. We have not included a ploting routine since such routines are often uritten to take advantage cf the pecularities of a specific computing system. If the user wishes to plot his output, as we have done in this report, he may supply his $3 w n$ plot package. Since all necessary outpui variables are stored in subscripted arrays, adapting any plotiing routine to the program should be fairly simp?e.

In order that a prospective user may check ort the p.ogram at his cwn installation we have included a sample problem. Values of the input variables for this problem are shown in Table 3. The problem illustrates 2-1 exchange on an adsorbcr consisting of three exchangers. Output from the program corresponding to the input variables in Table 3 is shown in Table 4. Input cards generating this output are included as the last eight COMMENT cards at the head of the program. In order to use these last

COMEENT cards as i.uput for the sample ?roblem it is only necessary to remove the $\underline{C}$ from Column 1 and the asterfsk fic $\cap$ Column 79 on each card.

Thble 1
definitions of the variables which comprise the I/O OF THE MAIN PROGRAM*

INPUT

| NKGC | Number of different adsorbers in the exchanger ( $\leq 10$ ) Number of different values of $K G$ (defind below), or lumber of different values of $C$ (defined below). |
| :---: | :---: |
| NA | Number of different values of A (defined below). |
| NM | Number of different values of M (defined below). |
| NIT | Maximum number of iterations allowed for convergence. |
| EN | Valence of the exchangeable ion. |
| P | Valence of the adsorbed ion. |
| EPS | Convergence parameter. |
| C | Exchange capacity of different adsorbers in the exchanger (moles/kg). |
| KG | $\mathrm{K} / \mathrm{\Gamma}$ for different adsorbers in the exchanger. |
| A | Concentration of supporting electrolyte (moles/l). |
| M | Equilibrium concentration of exchanged ion (moles/l). |
| FR | Weight fraction of each adsorber in the exchanger. |
|  | OUTPUT |
| DTOT | Distribution Coefficient |
| LTOT | Loading fraction of the exchanger |

[^0]TABLE 2

INPUT DATA FORMAT FOR MAIN PROGRAM

|  | Field |  |  | Variable |
| :--- | :---: | :---: | :---: | :---: |
| Card No. 1 | $1-5$ |  |  | Format |
|  | $6-10$ | NA |  | I5 |
|  | $11-15$ | NKGC | I5 |  |
|  | $16-20$ | NIT | I5 |  |
|  | $21-30$ | EN | I5 |  |
|  | $31-40$ | EPS | E10.0 |  |
|  | $41-50$ | $P$ | E10.0 |  |
|  |  |  | E10.0 |  |

Card No. 2


Notz: If NA > 8 then enough cards to accomodate all values of $A$ will follow.

Card No. 3

| $1-10$ | $M_{1}$ | E10.0 |
| :---: | :---: | :---: |
| $\vdots$ | $\vdots$ | $\vdots$ |
| $71-80$ | $M_{8}$ | E10.0 |

Note: If $N M>8$ enough cards to accomodate all values of $M$ will follow.

Card No. 4
$1-10$
$\vdots$
$71-80$
$\mathrm{KG}_{1}$
$\vdots$
$\mathrm{KC}_{2}$
E10. 0
$\vdots$
E10. 0

Note: If NKGC $>8$ enough cards to accomodate all values of KC will follow.

Card No. 5
$1-10$
$\vdots$
$71-80$
$\mathrm{C}_{1}$
$\vdots$
$\mathrm{C}_{8}$
E10. 0
$\vdots$
E10.
O
Note: If NKGC $>8$ enough cards to accomodate all values of $C$ will follow.

Card No. 6


Note: If NKGC > 8 enough cards to accomodate all values of FR will follow.

41
TABLE 3

## input variables for the sample problem

| Variable | Vaiue | Variable | Value | Variable | Value |
| :---: | :---: | :---: | :---: | :---: | :---: |
| NA | 5 | NM | 2 | NKGC | 3 |
| NIT | 100 | EN | 2.0 | EPS | $1.0 \cdot 10^{-7}$ |
| P | 1.0 | A(1) | 0.1 | A (2) | 0.5 |
| A (3) | 1.0 | A(4) | $2 . \mathrm{C}$ | A(5) | 4.0 |
| M(1) | $1.0 \cdot 10^{-4}$ | M (2) | $1.0 \cdot 10^{-2}$ | KG(1) | 1.0 |
| KG(2) | $1.0 \cdot 10^{3}$ | KG(3) | $1.0 \cdot 10^{5}$ | C(1) | 1.0 |
| C (2) | 1.0 | $\mathrm{C}(3)$ | 1.0 | FR(1) | 0.95 |
| FR(2) | 0.04 | FR(3) | 0.01 |  |  |

TABLE 4

## OUTPUT FROM SAMPLE PROBLEM

```
2-1 PICHA&GE OE A HITTCRE JF A ASSORERS
```



```
K/G(1)= 1. C\1)=1.00C00 PRACTIOM=0.950
K/E\ Z)=100C. C(2)=1.00D 03 PaACTIO# = 0.0#0
R/G(1)=100%00. C{ 3)=1.000 00 FasctiJE=0.010
\begin{tabular}{|c|c|c|c|c|}
\hline \(\cdots\) & A & D & & 1 \\
\hline 1.0000-04 & \(1.0000-01\) & 3.0030 & 02 & \(6.0050-02\) \\
\hline 1.000c-04 & \(5.0 C C D-C 1\) & 1.1730 & 02 & 2.3070-02 \\
\hline 1.0005-04 & 1.000000 & 7.0130 & 01 & 1.403n-02 \\
\hline 1.000 I-04 & 2.OCCD CO & 4.1430 & 01 & 8.2870-03 \\
\hline 1.0000-04 & 4.00c0 00 & 2. 3510 & 01 & -.7020-03 \\
\hline 1. \(\operatorname{coct-02}\) & 1.0cad-01 & 2.6200 & 01 & 5.2410-01 \\
\hline 1.0'500-02 & 5.0000-01 & 5.5750 & 00 & 1.1150-01 \\
\hline 1.000t-02 & 1.0con 00 & 3.0030 & 00 & 6.005D-02 \\
\hline 1.0000-02 & 2.000000 & 1.9970 & 00 & 3.9940-02 \\
\hline 1.000r-02 & 4.OCOD CO & 1.357D & 00 & 2.7140-02 \\
\hline
\end{tabular}
```




```
    THPLICIT EENLEO (A-M, S-2)
    DEALEA H,EG,LOAD,GTOT
    MIMEESION A(30), 目(35), KG(10),C(10), FP(10), CN(10), FI(10, DPR(10),
```




```
    PORAAT(415,3E19.0)
        IP(BA - ED. 0) CALL EEIT
        READ (5,2) (A(I),I=1, ПA)
        REAO (5,2) (n)(I),I=1, ह昗)
        REND (5,2) (KG(1),I=1,DESZ)
        BEAD (5,2) (C(I), i=1, WEGC)
        200 NEAD (5,2) (PN (I),T=1.EIEE:)
    2 FONHAT(8E1O.多
    IP (PE (1) - EQ J.) 6O IO 100
    00 500 LA=1,目若
    MA=A(LM)
    DO SOO MEF 1. #n
    EN=M (% #)
    DO 210 I=1, %rrc
210CN(I)=1.0D-15
    DO450 ED=1, IIT
    00 000 I=1,#EGC
```



```
    EM1=2%-1.000
    PW1=P-1.0D0
```



```
    Am (T)=-FH(I)/D PH (I)
000 An(I)=An(I) *-CN (I)
    DO & 10 I= 1, FEEC
    CHT(1) =DABS(AF(I) CH(1)-1.000)
    IP(COIT(I)-EPS) 410,410,420
40 CONPIWOE
    CO TO $60
40 DO & 10 I=1, HKGC
40 Ch(I) = AM(I)
$50 CONTIMEE
    EPITR(6,10)
    10 PORAAT (1H116HDID NOP CONERGE)
40000 4! I= 1, IKGC
470 CN(I) = AT(I)
    BP=1.0/P
    PP= TN/P
    DO 480 I=1, TRGC
    LOAD(I) = &%CH(1) N(I)
```



```
    DT=0.000
    TL=0.000
    DO &90 I=1, [TGC
    DR=DT*D(I) © PR(N)
490 TL=TL*LOAD(I)*PR(I)
```

```
        DTOT(HR,LA) = DT
        LTOP(M,.LA) = TR
    400 COTTIEOE
        MEAD(5,604 (PITLE (RTTT), ITIT=1.20)
    60 pos,ay(20AM
        HEITE(6,TO) (TIFLE(IRIP),RPIT= P, 20)
    7.) POETAT(1R1.20AB)
        #RITE (5,20) #Ñ%
    20 PORTATilEO."
        DO 600 I=1,NEGC
    600 neITE (6,30) I,KG(I),I,C(I), PIS(I)
```



```
        l'PFACTIOE =".0PF6.3.1n
        unITE (6,00)
```



```
        DO 650 BE=1, 明
        00 650 LA=1, 吾
```



```
    SO POPMAT(TEO,IPE{5.3.E1%.3,8:3.3,E11.3)
1000 comtinme
    CALL DPLOT
    CO TO 200
    Z0
        SOBEODTINE DPLOT
        !2P*en
        EMD
10
/GO.FTOSPOOI DO 
    llllllllll
    1.0 - 1.0 -2
    1.0 1.0 *3 1.0 05
    1.0 1.0 1.0
    0.95 0.04 0.01
        2-1 EPCHABGE OEA mITPJRE Of 3 ADSOEBERS
1%
```

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[^0]:    *No description of plotting subroutines is given in the following.

