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MASTER

COMPUTATIONS OF THE SPHERICAL $E \rightarrow I$ CONVERTOR

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ABSTRACT

One proposed approach to the overlap/deposition problem for multi-channel electron beams is to place a thin hollow sphere in the overlap region and allow the incoming electrons to form a virtual cathode inside this sphere. The resulting electric field causes ions from the inner surface of the sphere to flow inward radially towards a target. The efficiency of this process is studied analytically and with a 1-D spherical, two-species electrostatic particle code. The maximum possible (power conversion) efficiency is concluded to be about 10 percent. Two-dimensional effects are briefly considered, and angular momentum in particular is found to be a possibly serious limitation.

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I. THE BASIC SCHEME

There are a number of questions concerning direct use of the electrons in a REB multi-channel scheme. These concern getting the electrons to a small (mm-sized) target in the overlap region, and the efficiency of deposition in the target if the electrons do get there.^{1,2,3} One proposed way to avoid these problems is to convert the e-beams to i-beams using a hollow sphere of radius r_1 (a few cm) as indicated in Fig. 1.⁴ Ideally, most electrons will make several passes through the sphere at r_1 , being reflected on the inside by a virtual cathode VK and on the outside by the magnetic field of the combined channel currents. An electron's ultimate fate is either 1) absorption in the shell at r_1 , 2) drifting away outside of r_1 , or 3) loss to the target at r_2 . A plasma will form on the inside of the shell at r_1 , and space-charge-limited ions will flow inward towards the target.

This type of fusion scheme has a long history. It is basically the same as suggested by Goldstein and Guillory,⁵ except that here the shell is placed in the overlap region of the REB-channel system instead of inside a single diode. Similar ideas have been advanced by Verdeyen⁶ and earlier by Hirsch,⁷ although at lower current levels. Computer calculations of the 1-D spherical inertial confinement device were performed by Barnes.⁸ The work reported in the present paper differs from this last reference in (1) injecting electrons, and allowing space-charge-limited ions, (2) including relativistic (but not magnetic) effects, and (3) considering much higher current

Figure One: Schematic of $e \rightarrow i$ Converter

densities. Nevertheless, several features observed in the present work were also seen by Barnes, as will be pointed out later.

The main question concerning the scheme is the efficiency with which electron energy can be converted to ion energy incident on the target. The calculations will be time-independent in the sense that only equilibria, or "quasi-equilibria" (where the system oscillates about a defined average state), will be computed. Thus we shall not consider obvious questions such as electron pre-heat, plasma formation or motion, or pulse shaping or other methods to obtain ion bunching. Hence the "efficiency" will be determined simply by $I_i \bar{E}_i / I_e E_e$, where I_i is ion current to the target, $\bar{E}_i$ is average ion energy to the target, and I_e and E_e are the current and energy of the incident electrons.

The computations to be discussed are one-dimensional in the spherical coordinate r . This was done despite the fact that there are a number of obviously important 2-D (and 3-D) effects in any real system, such as magnetic fields, angular momentum, asymmetry in the incoming e-beam, return current path, non-uniformity in the ion-emitting plasma, etc. The first reason for adopting a 1-D model for this problem is purely practical. For the current levels of interest, the appropriate length scales which must be resolved are of order $C/\omega_p \ll r_1$. This makes even 1-D code calculations relatively expensive. Attempts were made to model the problem in 2-D, nevertheless, using a previously developed spherical diode code which included magnetic effects,⁹ but all such efforts thus far have failed

to produce useful solutions. A second complicating feature of the present problem is that unlike an ordinary spherical diode, which has been studied analytically,^{10,11} the inner sphere at r_2 is floating in potential so that we have vanishing electric field boundary conditions at both r_1 and r_2 . Physically this means that the total electron and ion fluxes to r_2 must be equal, and that the net charge in the system $r < r_1$ must be zero. This is an easy condition to enforce in 1-D, but is much more difficult in 2-D. A third reason for using a 1-D model is that all the 2-D effects seem to lower the conversion efficiency, and there is historical precedent for first studying the "optimum" case. A further discussion of 2-D effects will be given in Sec. IV below. Finally, an essential feature of the problem, and an interesting problem in itself, is the formation and oscillation of the virtual cathode (VK in Fig. 1). The basic physics involved here involves only the radial direction.

II. ANALYTIC MODEL

The potential $\phi(r)$ in the expected solution is sketched in Fig. 2. The electrons enter the conducting sphere at r_1 with some radial velocity v_0 , and may form a virtual cathode at VK, where $\phi = \phi_{\min}$. Some electrons turn around at VK and go back to r_1 , while some continue on to hit the target at r_2 . Space-charge-limited ions flow inward from r_1 to r_2 , and the ion current I_i must equal the electron current I_e to the target. Except for a reversal of species, this is just the "type two" steady state of Barnes.⁸

Figure Two: Schematic of Potential vs. Radius, for Case Where a Virtual Cathode VK Forms

If the injected e-beam is cold (no spread in v_0), then in the static solution $\phi_{\min}$ must equal the injected energy eV , where V corresponds to the voltage of the diode producing the beam (neglecting losses of energy in the channel and the sphere). Such a system is intrinsically unstable,^{12,13} because an infinitesimal perturbation in $\phi_{\min}$ results in a finite change in the current I_e (either all or nothing). A system into which a truly cold beam is injected will avoid this problem by not forming a steady state; as shown previously,¹⁴ such a system will oscillate forever, with $\phi_{\min}$ sometimes above, sometimes below, V . This will allow (on the average) the proper $I_e = I_i$ to be drawn by the target.

Any real system, however, will have some spread in v_0 , and according to previous work,⁸ although a static solution is still unstable in the sense of possessing oscillations, these oscillations may be controlled to some extent by increasing the initial velocity spread. In choosing a model for the $e-i$ convertor, we must recall that the injected electrons will be experiencing complicated magnetic and 2-D electric fields, foil scattering and energy loss, and will possess considerable angular momentum. In terms of a 1-D model, in fact, the e-beam will be very "hot", so for the incoming radial velocity distribution we will simply say

$$f_{in}(r_1, v_0) = \begin{cases} f_0 & 0 < v_0 < v_d \\ 0 & v_0 > v_d \end{cases} \quad (1)$$

where f_0 is a constant, and $v_d = c(1 - 1/(1 + eV/mc^2)^2)^{1/2}$ is the maximum possible radial velocity.

The total injected electron current is found to be

$$I_{e_{in}} = NI_{e_{exp}} = 2\pi er_1^2 f_0 c^2 (1 - 1/\gamma_d^2) \quad (2)$$

where $\gamma_d = (1 - v_d^2/c^2)^{-1/2}$, $I_{e_{exp}}$ is the actual current

flowing down the channels, and N is the average number of passes through the foil at r_1 . The electron current hitting the target, and hence the ion current, is then

$$I_i = 2\pi er_1^2 f_0 c^2 \left(\frac{1}{(1 - \eta_{min})^2} - \frac{1}{\gamma_d^2} \right) \quad (3)$$

where $\eta_{min} = e\phi_{min}/mc^2$, m being the electron mass.

Therefore,

$$I_i = I_{e_{in}} \left(\frac{\gamma_d^2}{(1 - \eta_{min})^2} - 1 \right) \frac{1}{\gamma_d^2 - 1} \quad (4)$$

and since γ_d is known, it remains to find η_{min} . (Note the obvious result that if $\phi_{min} = -V$, $I_i = 0$.)

We follow the time-honored procedure of finding the electron and ion densities in terms of ϕ , or $\eta = e\phi/mc^2$, and obtain the following boundary-value problem:

$$\begin{aligned}
\frac{d}{dr} r^2 \frac{d\eta}{dr} = \frac{I_{e_{in}}}{8.5 \text{ kA}} & \left[2 \int_{1-\eta}^{\gamma_m} \frac{d_0}{\gamma_o^3 (1 - 1/(\gamma_o + \eta)^2)^{1/2}} \right. \\
& + \int_{\gamma_m}^{\gamma_d} \frac{d\gamma_o}{\gamma_o^3 (1 - 1/(\gamma_o + \eta)^2)^{1/2}} \\
& \left. - \frac{1 - \frac{\gamma_m^2}{2}}{\gamma_d^{1/2}} \frac{\gamma_d^2}{\gamma_m^2 (-\eta)} \right] \quad (5)
\end{aligned}$$

$$\eta'(r_1) = \eta'(r_2) = \eta(r_1) = 0 \quad (6)$$

where prime means d/dr and $\gamma_m \equiv 1 - \eta_{\min}$ is the smallest value of γ which can reach the target. Here $\gamma_o = (1 - v_o^2/c^2)^{-1/2}$. We note that in this model, there is no well-defined virtual cathode, since electrons are turning around at all radii. The expected solution for $\eta(r)$ is still basically like that in Fig. 2, except that $\eta(r)$ will decrease all the way in to the target (r_2). Equation (5) is a second order differential equation for $\eta(r)$, and the extra boundary condition (6) can be used to find $\eta_{\min} = 1 - \gamma_m$, at least in principle. In practice, Eq. (5) does not seem to have an analytic first integral, so we resort to making a rough approximation. Ignoring spherical effects by assuming $(d/dr)(r^2 d\eta/dr) = r^2 d^2\eta/dr^2$ for some r^* in (r_2, r_1) , we get $\frac{1}{2}(d\eta/dr)^2 = r^{*-2} \int \rho(\eta) d\eta$ where ρ is the right side of Eq. (5). Using Eqs. (6) leads to

$$\begin{aligned}
2 \int_1^{\gamma_m} \frac{d\gamma}{\gamma^3} (\gamma^2 - 1)^{1/2} + \int_{\gamma_m}^{\gamma_d} \frac{d\gamma}{\gamma^3} \left((\gamma^2 - 1)^{1/2} \right. \\
\left. - \left[(\gamma + \eta_{\min})^2 - 1 \right]^{1/2} \right) \\
= \frac{(\eta_{\min})^{1/2} \left(\frac{1}{\gamma_m^2} - \frac{1}{\gamma_d^2} \right)}{(2m/m_i)^{1/2}} \quad (7)
\end{aligned}$$

As a crude approximation, if $\gamma_d \approx \gamma_m \approx 1$,

$$\cos^{-1} \frac{1}{\gamma_m} - \frac{1}{2} \sin(2 \cos^{-1} \frac{1}{\gamma_m}) \approx \sqrt{\frac{m_i}{2m}} \left(\frac{1}{\gamma_m^2} - \frac{1}{\gamma_d^2} \right) \sqrt{\eta_{\min}}$$

or roughly,

$$(\gamma_m - 1) \left(\frac{1}{\gamma_m^4} - \frac{2}{\gamma_d^2 \gamma_m^2} + \frac{1}{\gamma_d^4} \right) \approx \left(\frac{\pi}{2} \right)^2 \frac{2m}{m_i}$$

or

$$\gamma_m = \gamma_d - \frac{\pi}{2} \sqrt{\frac{m}{2m_i}} \gamma_d^{5/2}$$

so that we find from Eq. (4)

$$I_i = I_{e_{in}} \pi \sqrt{\frac{m}{2m_i}} \frac{1}{\sqrt{\gamma_d}} \quad (8)$$

According to this result, the conversion from electron to ion current will be rather inefficient (a few %), unless $I_{e_{in}} = N I_{e_{exp}}$

can be increased substantially by forcing the average number of passes N to be at least ~ 10 .

III. Computer Simulation in 1-D

A one-dimensional particle simulation code was written for spherical geometry. The code includes relativistic electrons, non-relativistic ions, and solves $\nabla \cdot \vec{E} = \rho/\epsilon_0$ including any net charge accumulated on the target. In addition to the electron beam injected at $r = r_1$, the code emits space charge limited electrons from $r = r_2$ (if necessary), and space charge limited ions (or electrons, from $r = r_1$). A type of Gauss law emission was used in all cases. Electron and ion currents to both boundaries are computed, as are particle energies, velocity distributions, etc.

The first cases studied (as tests of the code) were those in which only the injected electron beam was included. Here a well-defined VK forms at about $r = r_1 - c/\omega_b$, where ω_b = beam plasma frequency.¹⁴ Cases were run at injected currents of .3, 3, and 30 MA with cold beams at 2MeV; $r_1 = 4.8$ cm. No electrons reach the target at these currents. Besides testing the code, these runs provided information on the required zoning, which of necessity becomes very fine at the higher injected currents, because of the need to adequately resolve c/ω_b , which is about 1mm at 30MA. This zoning requirement at currents of interest in turn made it necessary to use more particles ($\sim 10^4$), and made the runs relatively expensive for a 1-D simulation.

The early runs including ions were done at an artificial mass ratio, $m_i/m = 100$, to more efficiently study the oscillations which were present in all runs, and the dependence of the solutions on time step, particle number, cell size, and emission method. All remaining runs (discussed below) were done with $m_i = \text{proton mass}$. The code confirmed the $I_i \propto m_i^{-1/2}$ scaling of Eq. (8).

Table I gives a summary of the main simulations. Runs 1-8 had $r_1 = 3 \text{ cm}$, $r_2 = 1 \text{ cm}$, and were for parameters possibly relevant to EBFA.⁴ Runs 9-11 had $r_1 = 4.4 \text{ cm}$, $r_2 = 1.1 \text{ cm}$, and were intended to be relevant to the upcoming Hydra experiments of P. Miller.¹⁵ The injected 8.8 MA in the latter comes from a large assumed source radius of 1.5 cm, with .25 MA per channel: $I_{e_{in}} \doteq .25 (4\pi r_1^2 / \pi (1.5\text{cm})^2)$. In the table, E_e is the injected e-beam energy, T_e describes the beam as cold (no spread in injected radial velocities) or hot (velocities selected randomly from $(0, v_d)$). The last four columns in the table describe the results: I_i and $\bar{E}_i$ are the ion current and average energy to the target at r_2 , VK is the position of the virtual cathode (if one forms), and P. E. is the power efficiency, defined as $\bar{E}_i I_i / E_e I_{e_{in}}$ with E_e replaced by $\bar{E}_e$ in cases where a hot beam is injected.

In run #1, 30 MA of electrons, all at 2MeV, is converted to about 2.6 MA of 2.6 MeV protons. The VK forms close to r_1 because the current is so large. In runs 2-5, lower energy beams (each cold) were added to the 30MA, to simulate the

Table I. Summary of 1-D Simulations

Run	r_1 (cm)	r_2 (cm)	E_e (MeV)	$I_{e_{in}}$ (MA)	T_{in}	I_i (MA)	E_i (MeV)	vK (cm)	P.E.
1	3	1	2	30	cold	2.55	2.64	2.84	11%
2	3	1	$\begin{pmatrix} 2 \\ .08 \end{pmatrix}$	$\begin{matrix} 30 \\ 15 \end{matrix}$	$\begin{matrix} \text{cold} \\ \text{cold} \end{matrix}$	2.7	2.7	2.85	12%
3	3	1	$\begin{pmatrix} 2 \\ .31 \\ .12 \end{pmatrix}$	$\begin{matrix} 30 \\ 15 \\ 15 \end{matrix}$	$\begin{matrix} \text{cold} \\ \text{cold} \\ \text{cold} \end{matrix}$	3.0	2.6	2.84	12%
4	3	1	$\begin{pmatrix} 2 \\ .31 \\ .12 \\ .04 \\ .01 \end{pmatrix}$	$\begin{matrix} 30 \\ 10 \\ 10 \\ 10 \\ 10 \end{matrix}$	$\begin{matrix} \text{cold} \\ \text{cold} \\ \text{cold} \\ \text{cold} \\ \text{cold} \end{matrix}$	2.8	2.6	2.84	11%
5	3	1	$\begin{pmatrix} 2 \\ 0-2 \end{pmatrix}$	$\begin{matrix} 30 \\ 40 \end{matrix}$	$\begin{matrix} \text{cold} \\ \text{hot} \end{matrix}$	2.8	2.6	2.84	11%
6	3	1	$\begin{pmatrix} 2 \\ 0-2 \end{pmatrix}$	$\begin{matrix} 10 \\ 20 \end{matrix}$	$\begin{matrix} \text{cold} \\ \text{hot} \end{matrix}$	1.0	2.3	2.7	10%
7	3	1	$\begin{pmatrix} 2 \\ 0-2 \end{pmatrix}$	$\begin{matrix} 6 \\ 24 \end{matrix}$	$\begin{matrix} \text{cold} \\ \text{hot} \end{matrix}$	.7	2.5	---	10%
8	3	1	0-2, $\bar{E}_e = .2$	30	hot	.6	1.9	---	19%
9	4.4	1.1	1	8.8	cold	.3	1.9	4.2	17%
10	4.4	1.1	0-1, $\bar{E}_e = .18$	8.6	hot	.2	1.1	---	14%
11	4.4	1.1	0-1, $\bar{E}_e = .18$	35	hot	.9	1.5	---	21%

situation where the electrons make several passes through the foil but with much reduced energy each pass. It is seen from the table that adding these extra beams had no substantial effect on the conversion to ions; the cold 30MA beam completely dominates.

In runs 6-8, we go from the limit of perfectly cold to extremely hot, keeping the total $I_{e, in}$ constant. In fact run 8 is the supposedly realistic case which we adopted for our analytic model in Sec. II, wherein we assumed the radial velocities were uniformly distributed over the interval $(0, v_d)$. In runs 1, 6, 7, 8 the trend is for the virtual cathode to move in towards the target and vanish, after which the $|\phi_{min}| \approx V$, where ϕ_{min} occurs at the target. There is also a trend of decreasing ion current; the value predicted by Eq. (8) is .7 MA, in surprisingly good agreement with the code (run 8) considering the rough approximations made in deriving Eq. (8).

Next we discuss the Hydra runs 9-11. The cold-beam run 9 is similar to the EBFA run 1, having slightly higher PE presumably because of the lower v_d . Run 10 just injects the 8.8 MA as a hot beam, assuming each electron makes only one pass into the system ($N = 1$), whereas run 11 assumes $N = 4$, considered by some to be the optimum in ordinary reflex diodes.¹⁶ Neither 10 nor 11 formed a VK. The predictions of Eq. (8) for these cases are .26 and 1.05 MA, respectively, and we see from the table that the agreement between the code and the analytic model is good enough to warrant taking Eq. (8) with some seriousness. The PE values in runs 8, 10, and 11, however, are not to be taken

too seriously, because in dividing by E_e , the average radial energy, they overestimate the efficiency by neglecting the non-radial kinetic energy. Using $E_e = 1$ MeV instead, the PE values in 10 and 11 become 3% and 4%. On the other hand, in run 11 one should remember that $N = 4$ so that the "realistic" PE here is 15%. As a semi-empirical result, we may write approximately

$$PE^* = \frac{E_i I_i}{2 V I_e \exp} \approx 1.3 N \sqrt{\frac{m}{2 m_i \gamma_d}} \approx .067 \frac{N}{\sqrt{\gamma_d}} \quad (9)$$

for the hot beam cases. To pursue the efficiency question further here would be to stretch the applicability of the 1-D model beyond reasonable limits, but it is interesting that most of the PE values we have obtained are about 10 to 15%; to quote Ref. 16 on the behavior of 1-D reflex diodes, "... in our simulations, the ion current efficiency was about 50%. Nevertheless, when remembering that we used $m_i/m_e = 25$, the result for a real m_i/m_e ratio would be correspondingly lower, in agreement with the maximum theoretical estimate of 10 - 15%."

IV. Possible 2-D Effects

The two or three dimensional problem, including magnetic fields, angular momentum, etc., seems to be quite intractable analytically. And, for reasons which should now be apparent, a straightforward 2-D code is not feasible either, although this does not preclude the possibility of making some simplifying assumptions concerning the electron dynamics, and/or using

non-uniform grid spacing to help with the severe resolution requirements. For present purposes, we mention what may be the most serious 2- or 3-D effects, all of which tend to make our 1-D results err on the optimistic side.

The first problem is that there will be serious spherical asymmetries in the electron behavior, and consequently in the ion trajectories. The incoming e-beams will intercept the sphere at r_1 with a built-in asymmetry, and the magnetic fields near r_1 will have a complicated dependence on all three coordinates. Inside r_1 there will be non-radial electric fields which will prevent the ions from focusing at the center. Despite resolution and boundary condition difficulties, a 2-D (r, θ) particle code was attempted, based on a previously developed spherical diode code.⁰ Although no acceptable steady solutions were ever found with this code, the work clearly showed the asymmetries in fields and ion trajectories.

The second problem, as pointed out by P. Miller,⁴ is the loss of electrons outside r_1 , due to the VB (and possibly other) drifts. The preliminary work with the 2-D code suggested this is not as severe a problem as might be expected, although no definitive statement can yet be made. Of course, this effect has not been included in the 1-D model of the present paper, but it would show up in the equations as a decrease in the parameter N .

Finally, the problem of angular momentum may be the most serious multi-dimensional effect. The hot electrons from the channels will have large non-radial velocity components, which

will probably keep them from reaching the target. If the fields are independent of the spherical coordinates ϕ and θ , then $P_\phi = \pi r^2 v_\phi$ and $P_\theta = \pi r^2 v_\theta \sin^2 \theta$ will be constants of the motion. If, e.g., an electron has $v_0 = 10^6 = 1 \times 10^{10}$ cm/sec (a conservative value) at $r_1 = 4$ cm, then the minimum possible radius for this electron is $1 \frac{1}{3}$ cm, and this does not include the reduction in r as the electron passes through the potential well:

$$r_{\min} = \frac{r_1 v_{\theta 1}}{v_{t0}} = 3.61 r_1 \quad (10)$$

where subscript "1" denotes values at r_1 , while "t" means target. Unless one invokes the magic words "turbulent electric fields" or "anomalous diffusion," it is difficult to see why Eq. (10) does not make the whole e - i convertor scheme of dubious viability. Even if non-radial E - fields are present which allow enough electrons to reach the target, these will defocus the ions.

V. Conclusions

In this paper we have used analytic and numerical methods to study the e - i convertor. The 1-D results indicate that interestingly large (~MA) ion currents may reach the target, but the power efficiency of the process is only ~10%. Folding in deleterious 2-D effects, especially angular momentum, the scheme may be expected to have serious difficulties.

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